JUST University – CE dept.

CPE 716: Assignment One (Ch 4)
Generating Random Variables for Simulation

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Inverse-Transform Method:

1- Write a code that generates *U*(0,1) using a built in function and transform the first *n* generated numbers into: (*n is an input from the user*)

a) Uniform (-5, 100)

b) Exponential with mean equals 100.

Answer:

Note : python programming language was used to code this problem (numpy & math & random) Libraries used to solve this problem.

1- I used os function to generate random variable U(0,1)
2- The user can insert the number of r.v by using input function which is providing by python as shown in the code file
3- To solve a) part I used two ways of generating r.v :1- mapping rule to transform to uniform (-5,100) which is illustrated in the code as : rv = value*(max-min) + min , Then print the result as shown below.
4- 2- using uniform function u=(x-a)/(b-a) then print out the result
5- For part b) I used an Exponential function to get the inverse function which used to produce r.v X Then print the result as shown in the screenshot below.

☞ **the code file was attached.**

Name	Type	Size	Value
max	int	1	100
min	int	1	-5
n	int	1	5
rv	float64	(5,)	[48.02076027 49.26615568 84.05734658 -1.90189236 7.55621917]
u	float64	1	0.11958303967062311
un	float64	(5,)	[0.05242819 0.05254115 0.05569681 0.04790006 0.04875793]
value	float64	(5,)	[0.50495962 0.51682053 0.84816521 0.02950579 0.11958304]
x	float	1	0.001273596651703414

Screenshot shows the variables and their values in case the user insert N=5

```
______________Generated Random variable ___________

please enter N:5

     u(0,1) :    [0.60496153 0.10225123 0.41752741 0.93986436 0.83919485]
This is uniform function way 1: (x-a)/(b-a)
uniform dist.   [0.05338059 0.04859287 0.0515955  0.05657014 0.05561138]

This is uniform, mapping way 2:
uniform(-5,100): [58.52096114  5.73637893 38.84037776 93.68575782 83.11545923]

Exponential with mean =100 is :
0.009287721381712409
0.001078650135913311
0.0054047314569393165
0.028111526041367917
0.01827561894220559
```

Screenshot of the output results in case N=5

2- Explain how to generate values from a continuous distribution with density function f (t)= 5/(4t^−2)

where 1<t<5 , given u∈U(0, 1) .

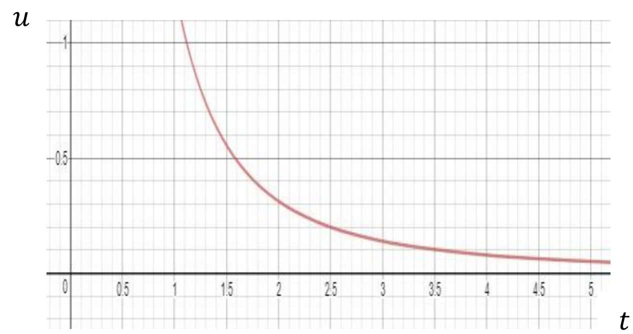

Fig 1.1 illustrate probability density function

Answer :

First :

Calculate CDF from PDF

If x is in the interval (-∞, 1), then

$F(x)=\int_{-\infty}^{x} f(t)dt = 0$

If x is in the interval (1,5)

$$F(x)=\int_1^x f(t)dt \quad = \int_1^x \frac{5}{4t^2}dt$$

$$= \frac{-5}{4x} + \frac{5}{4} \quad = \frac{5}{4}(1-\frac{1}{x})$$

$F(x) = \frac{5}{4}(1-\frac{1}{x})$ ....CDF (1)

Second:

Inverse-Transform Method to generate r.v. X:

1. Generate $u \in U(0,1)$.

2. Return $X = F_X^{-1}(u)$.

$$u = \frac{5}{4}\left(1 - \frac{1}{x}\right) \quad \Rightarrow \quad \frac{4}{5}u = \left(1 - \frac{1}{x}\right)$$

$x = \frac{1}{1-\frac{4}{5}u}$ .... Inverse function (2)

☞ Given $u \in U(0,1)$, setting $x = \frac{1}{1-\frac{4}{5}u}$ produces an instance of X.

e.i : let u=0 then x=1 Or u=1 then x=5.

Notice:

When F(x) must = 1 when x=5

So, F(x) = $\frac{5}{4}\left(1 - \frac{1}{5}\right)$ = 1 ..... proved