

Deep Learning Neural Net Basics

Outline

- Neural Networks
- *Convolutional* Neural Networks
- Variants
 - Detection
 - Segmentation
 - Siamese Networks
- Visualization of Deep Networks

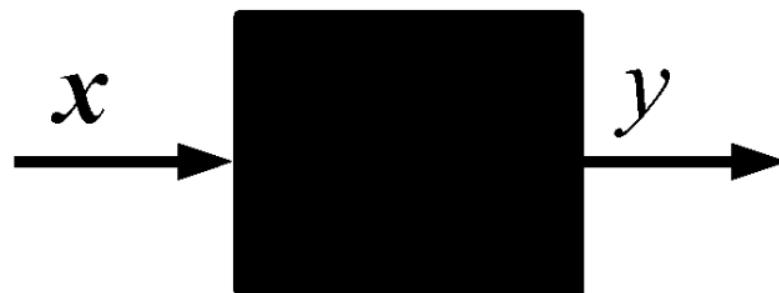
Supervised Learning

$\{(\mathbf{x}^i, y^i), i=1 \dots P\}$ training dataset

\mathbf{x}^i i-th input training example

y^i i-th target label

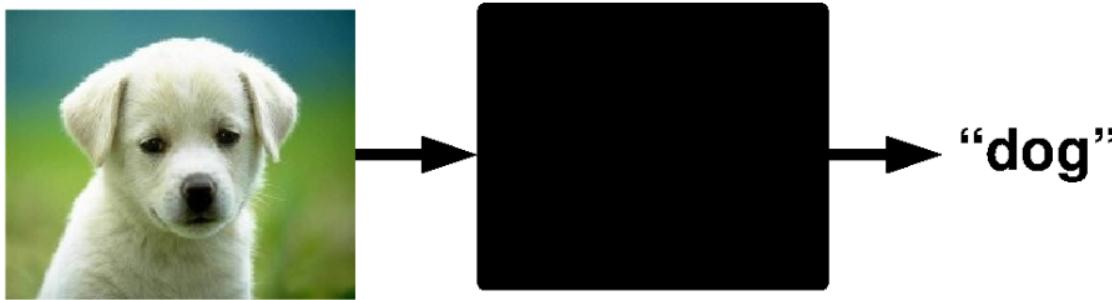
P number of training examples



Goal: predict the target label of unseen inputs.

Supervised Learning: Examples

Classification



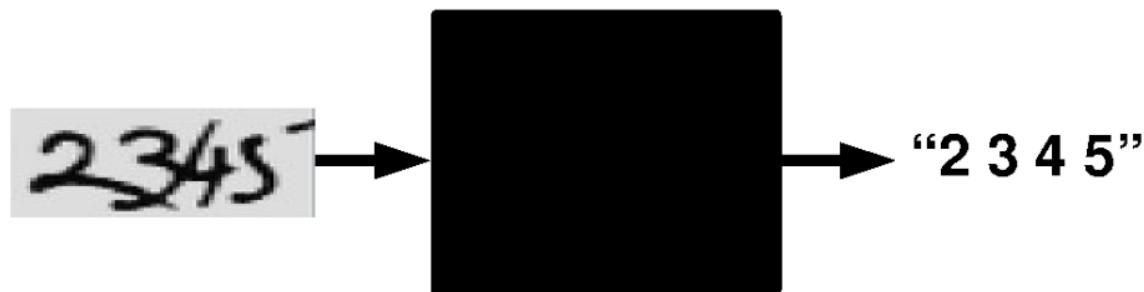
classification

Denoising



regression

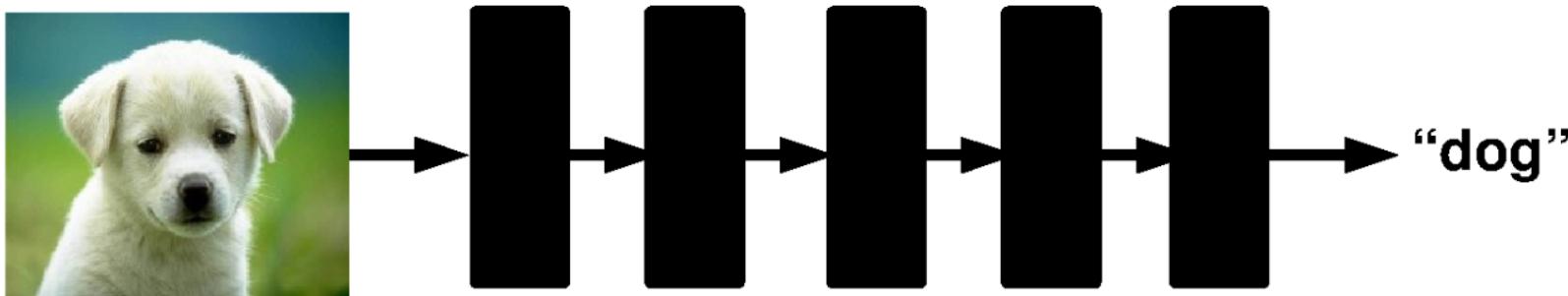
OCR



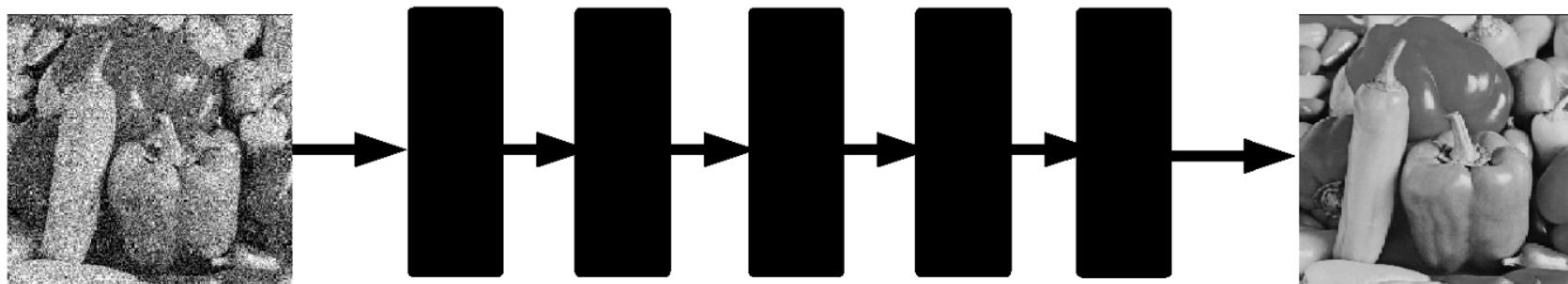
structured
prediction

Supervised Deep Learning

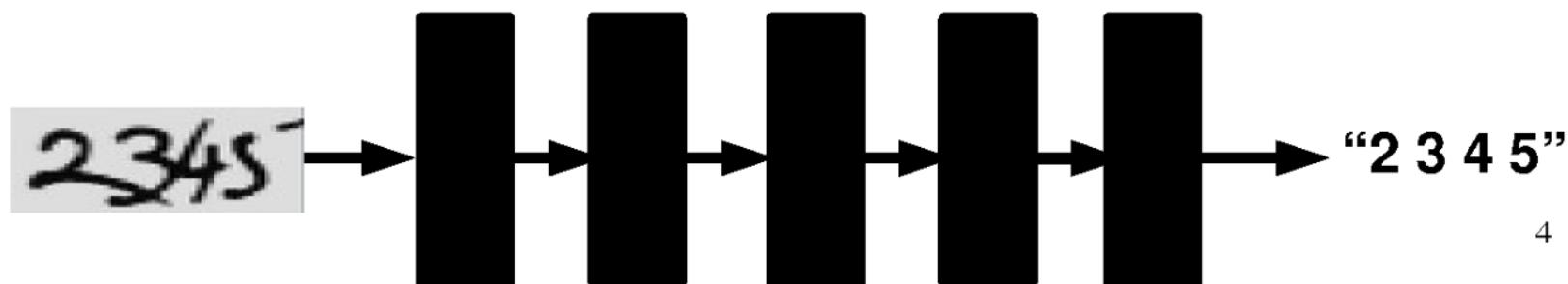
Classification



Denoising



OCR



Project 3: Scene Classification with Deep Nets

Dataset

The dataset to be used in this assignment is the 15-scene dataset, containing natural images in 15 possible scenarios like bedrooms and coasts. It was first introduced by Lazebnik et al, 2006 [1]. The images have a typical size of around 200 by 200 pixels, and serve as a good milestone for many vision tasks. A sample collection of the images can be found below:

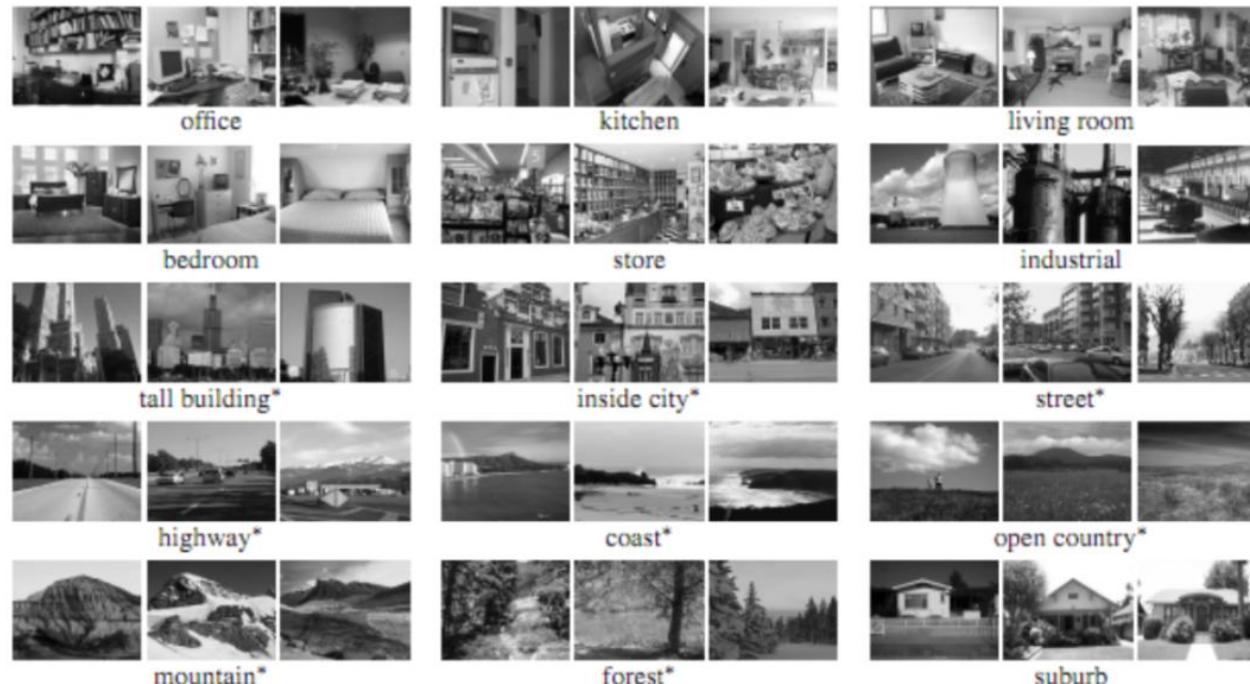


Figure 1: Example scenes from each of the categories of the dataset.

Download the data (link at the top), unzip it and put the `data` folder in the `proj4` directory.

1 Part 1: SimpleNet

Introduction

In this project, scene recognition with deep learning, we are going to train a simple convolutional neural net from scratch. We'll be starting with some modification to the dataloader used in this project to include a few extra pre-processing steps. Subsequently, you will define your own model and optimization function. A trainer class will be provided to you, and you will be able to test out the performance of your model with this complete pipeline of classification problem.

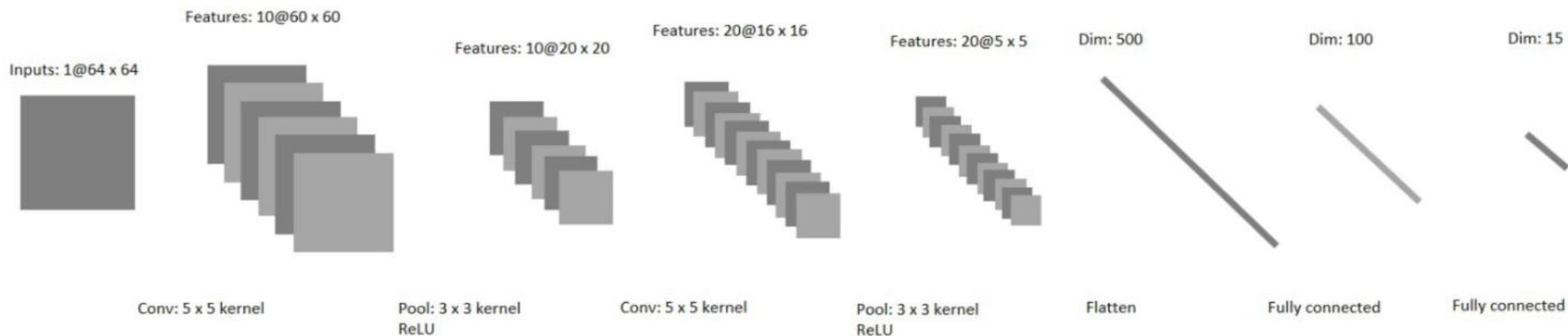


Figure 2: The base SimpleNet architecture for Part 1.

Outline

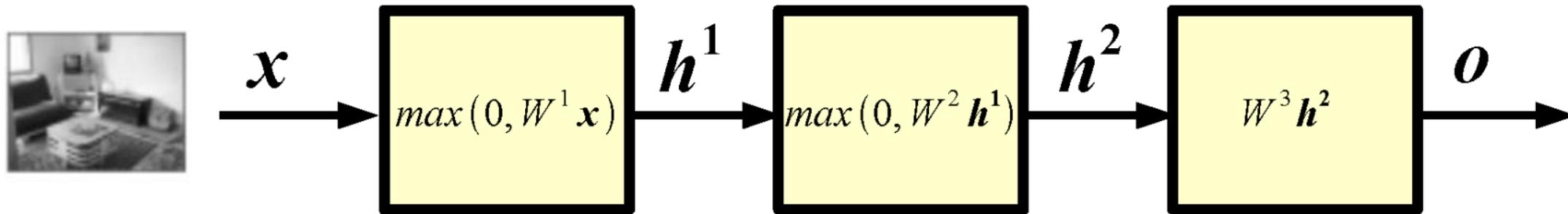
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Neural Networks

Assumptions (for the next few slides):

- The input image is vectorized (disregard the spatial layout of pixels)
- The target label is discrete (classification)

Neural Networks: example



x input

h^1 1-st layer hidden units

h^2 2-nd layer hidden units

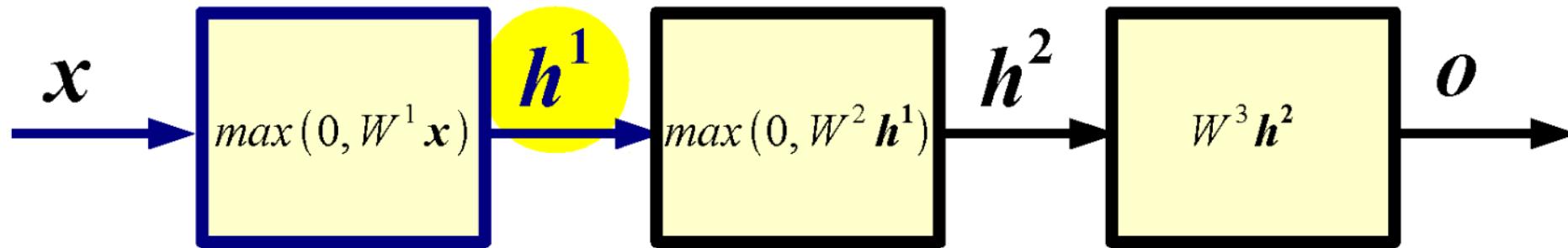
o output

Example of a 2 hidden layer neural network (or 4 layer network,
counting also input and output).

Forward Propagation

Def.: Forward propagation is the process of computing the output of the network given its input.

Forward Propagation



$$x \in R^D \quad W^1 \in R^{N_1 \times D} \quad b^1 \in R^{N_1} \quad h^1 \in R^{N_1}$$

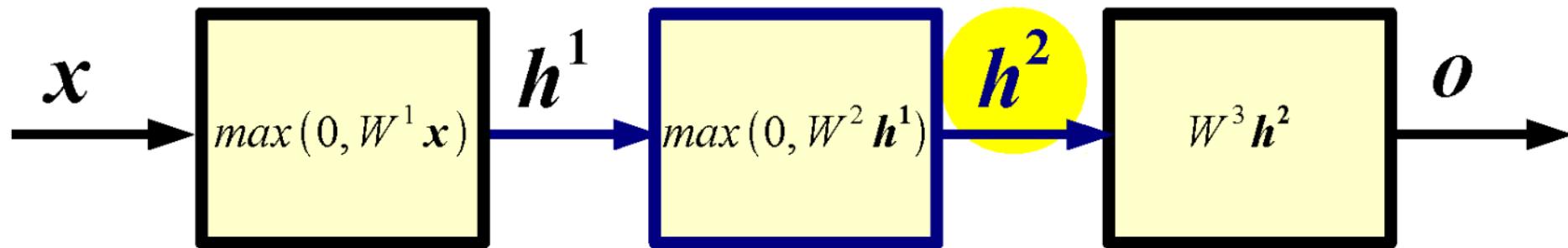
$$h^1 = \max(0, W^1 x + b^1)$$

W^1 1-st layer weight matrix or weights

b^1 1-st layer biases

The non-linearity $u = \max(0, v)$ is called **ReLU** in the DL literature. Each output hidden unit takes as input all the units at the previous layer: each such layer is called “**fully connected**”.

Forward Propagation

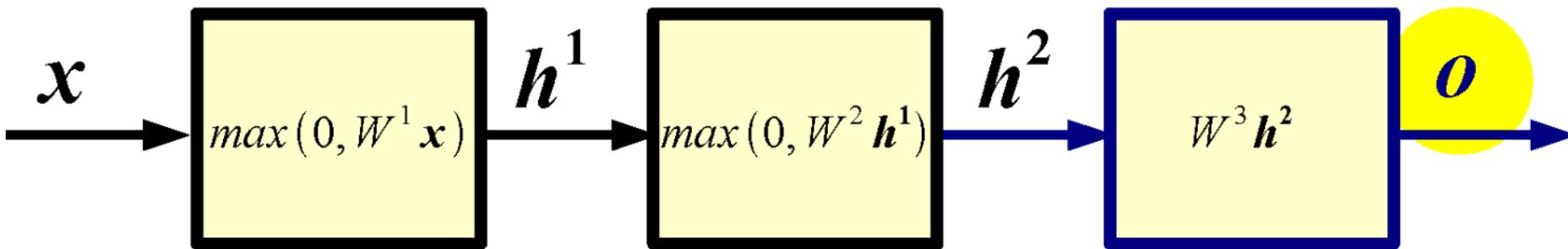


$$h^1 \in R^{N_1} \quad W^2 \in R^{N_2 \times N_1} \quad b^2 \in R^{N_2} \quad h^2 \in R^{N_2}$$

$$h^2 = \max(0, W^2 h^1 + b^2)$$

W^2 2-nd layer weight matrix or weights
 b^2 2-nd layer biases

Forward Propagation

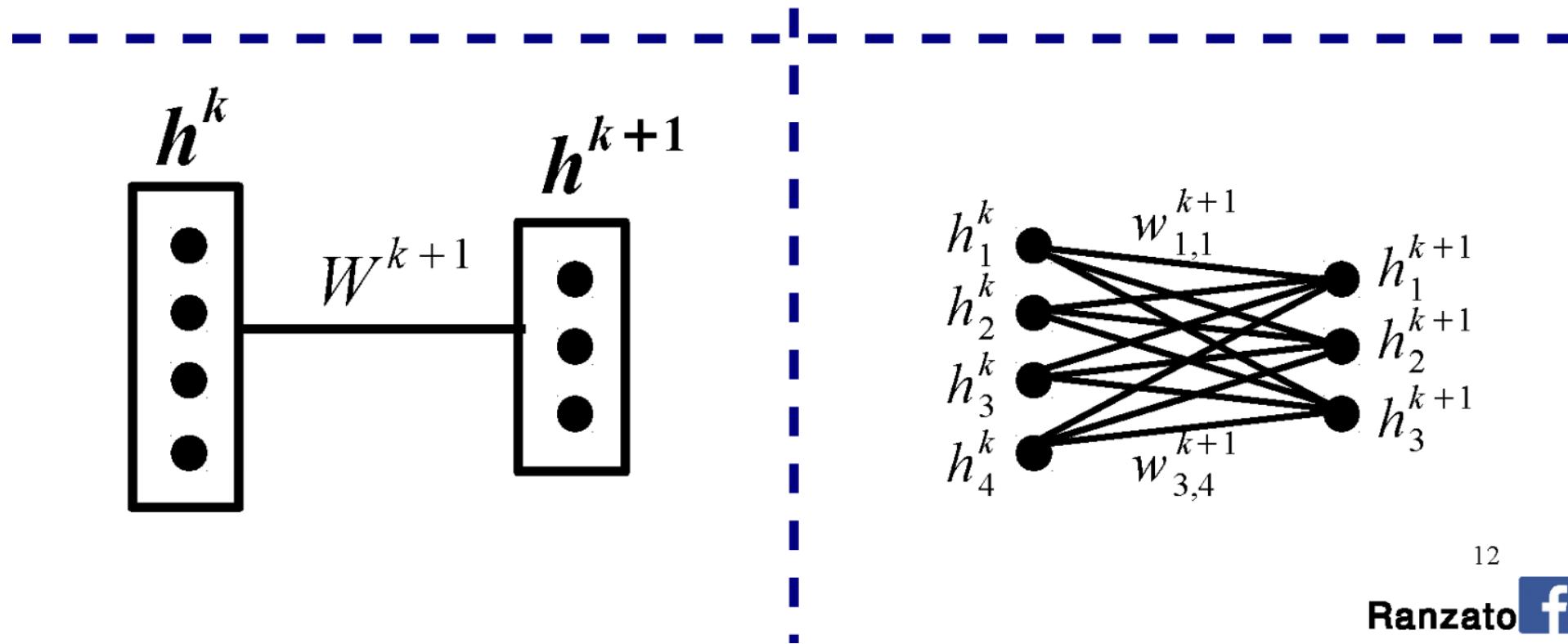
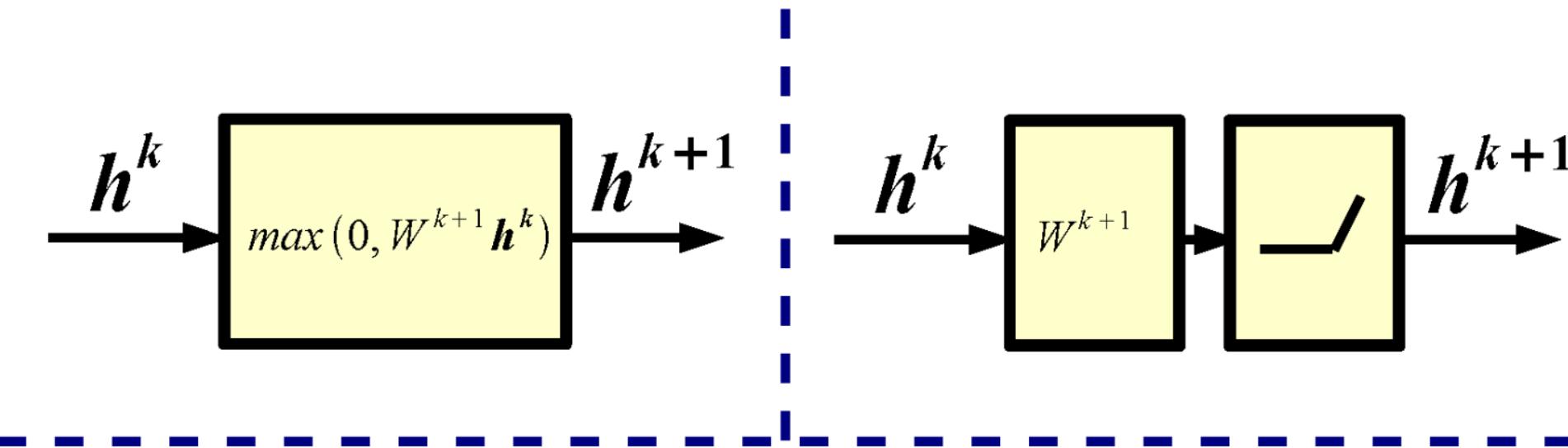


$$h^2 \in R^{N_2} \quad W^3 \in R^{N_3 \times N_2} \quad b^3 \in R^{N_3} \quad o \in R^{N_3}$$

$$o = \max(0, W^3 h^2 + b^3)$$

W^3 3-rd layer weight matrix or weights
 b^3 3-rd layer biases

Alternative Graphical Representation

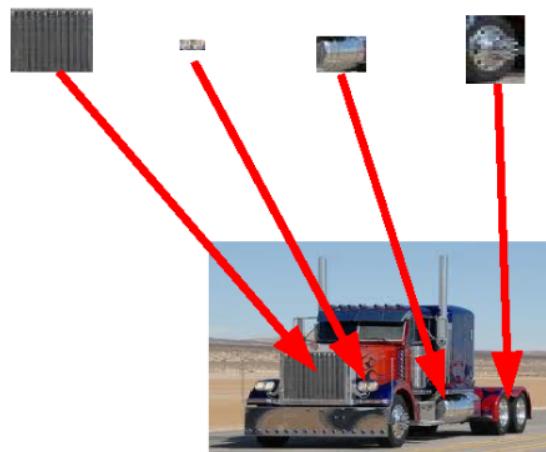


Interpretation

Question: Why do we need many layers?

Answer: When input has hierarchical structure, the use of a hierarchical architecture is potentially more efficient because intermediate computations can be re-used. DL architectures are efficient also because they use **distributed representations** which are shared across classes.

[0 0 1 0 0 0 0 1 0 0 1 1 0 0 1 0 ...] truck feature

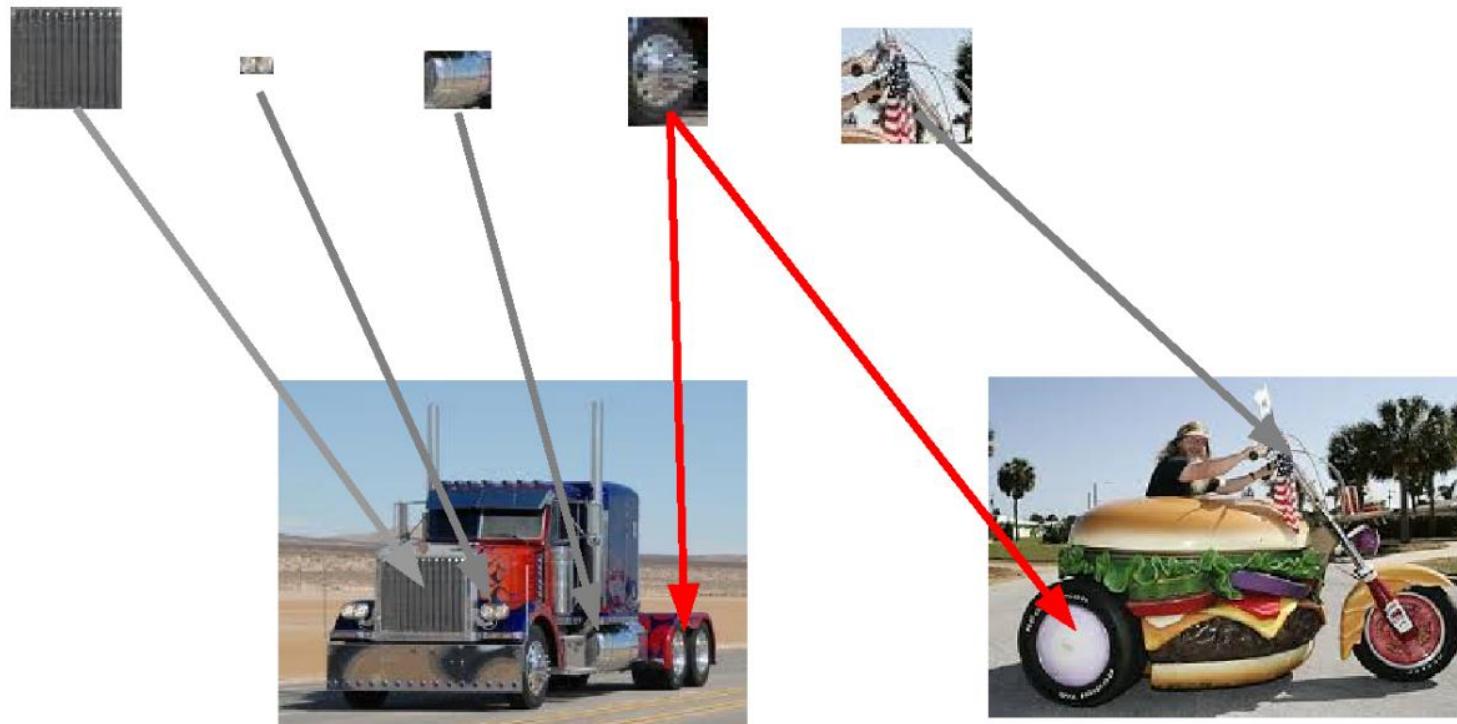


Exponentially more efficient than a
1-of-N representation (a la k-means)

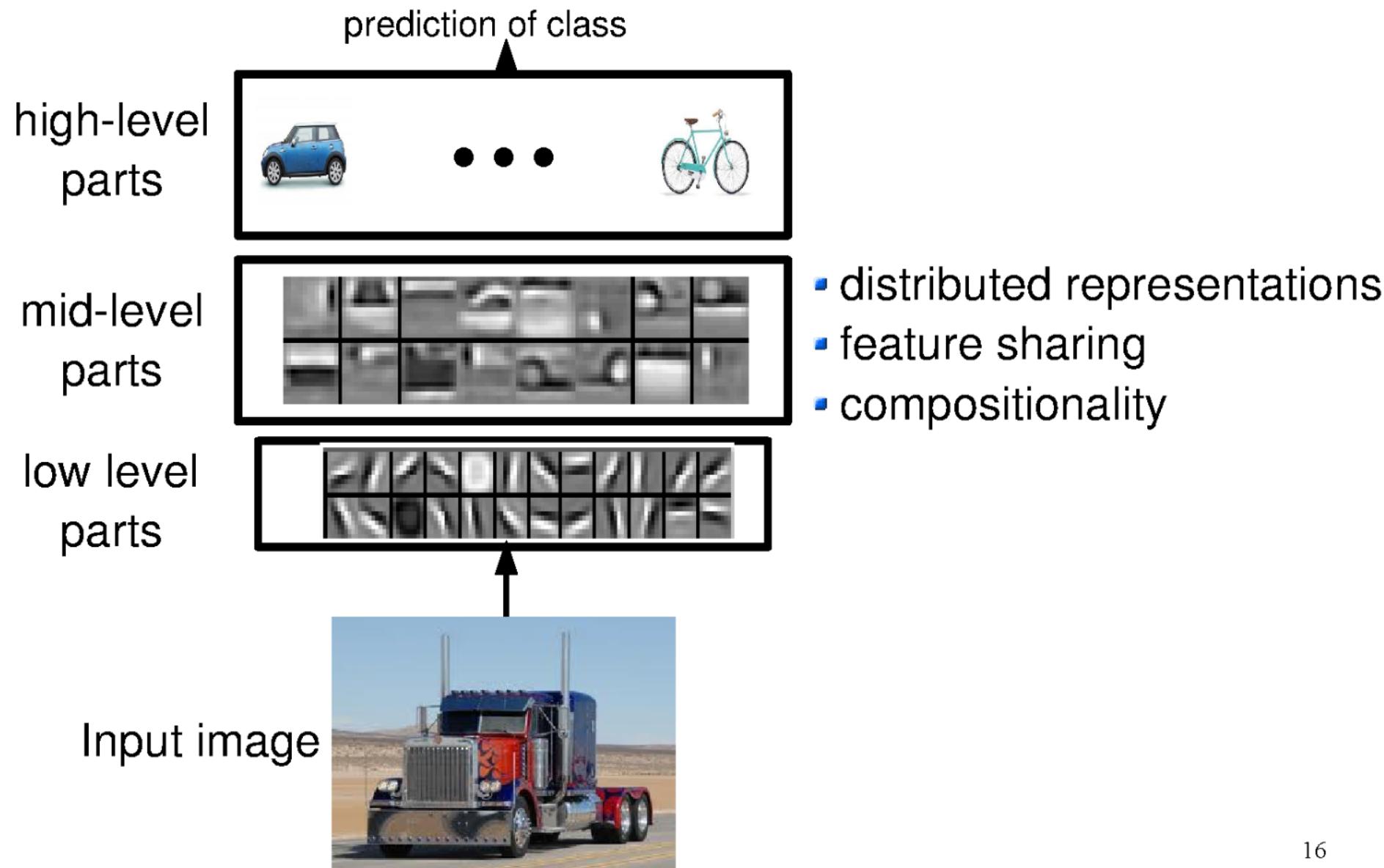
Interpretation

[1 1 0 0 0 1 0 1 0 0 0 0 1 1 0 1...] motorbike

[0 0 1 0 0 0 0 1 0 0 1 1 0 0 1 0...] truck



Interpretation



Interpretation

Question: What does a hidden unit do?

Answer: It can be thought of as a classifier or feature detector.

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Question: How many layers? How many hidden units?

Answer: Cross-validation or hyper-parameter search methods are the answer. In general, the wider and the deeper the network the more complicated the mapping.

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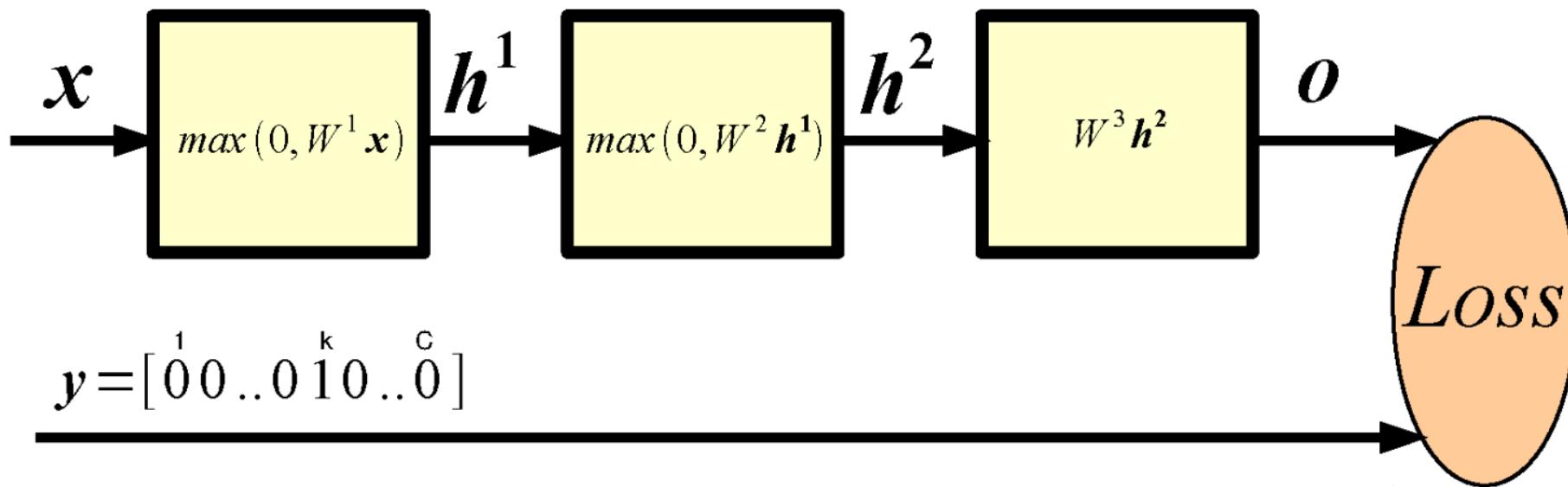
Answer: Cross-validation or hyper-parameter search methods are the answer. In general, the wider and the deeper the network the more complicated the mapping.

Question: How do I set the weight matrices?

Answer: Weight matrices and biases are learned.

First, we need to define a measure of quality of the current mapping.
Then, we need to define a procedure to adjust the parameters.

How Good is a Network?



Probability of class k given input (softmax):

$$p(c_k=1|x) = \frac{e^{o_k}}{\sum_{j=1}^C e^{o_j}}$$

(Per-sample) **Loss**; e.g., negative log-likelihood (good for classification of small number of classes):

$$L(x, y; \theta) = -\sum_j y_j \log p(c_j|x)$$

Training

Learning consists of minimizing the loss (plus some regularization term) w.r.t. parameters over the whole training set.

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} \sum_{n=1}^P L(\mathbf{x}^n, y^n; \boldsymbol{\theta})$$

Training

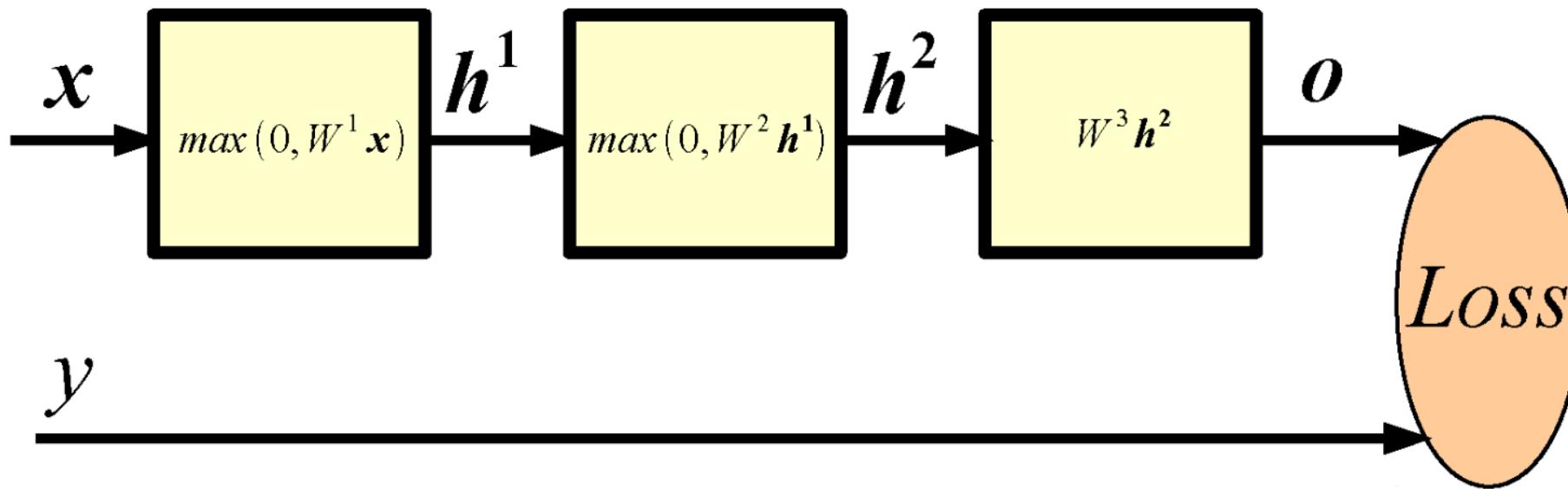
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$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} \sum_{n=1}^P L(\mathbf{x}^n, y^n; \boldsymbol{\theta})$$

Question: How to minimize a complicated function of the parameters?

Answer: Chain rule, a.k.a. **Backpropagation**! That is the procedure to compute gradients of the loss w.r.t. parameters in a multi-layer neural network.

Key Idea: Wiggle To Decrease Loss



Let's say we want to decrease the loss by adjusting $W_{i,j}^1$.

We could consider a very small $\epsilon = 1e-6$ and compute:

$$L(\mathbf{x}, y; \boldsymbol{\theta})$$

$$L(\mathbf{x}, y; \boldsymbol{\theta} \setminus W_{i,j}^1, W_{i,j}^1 + \epsilon)$$

Then, update:

$$W_{i,j}^1 \leftarrow W_{i,j}^1 + \epsilon \operatorname{sgn}(L(\mathbf{x}, y; \boldsymbol{\theta}) - L(\mathbf{x}, y; \boldsymbol{\theta} \setminus W_{i,j}^1, W_{i,j}^1 + \epsilon))$$

Derivative w.r.t. Input of Softmax

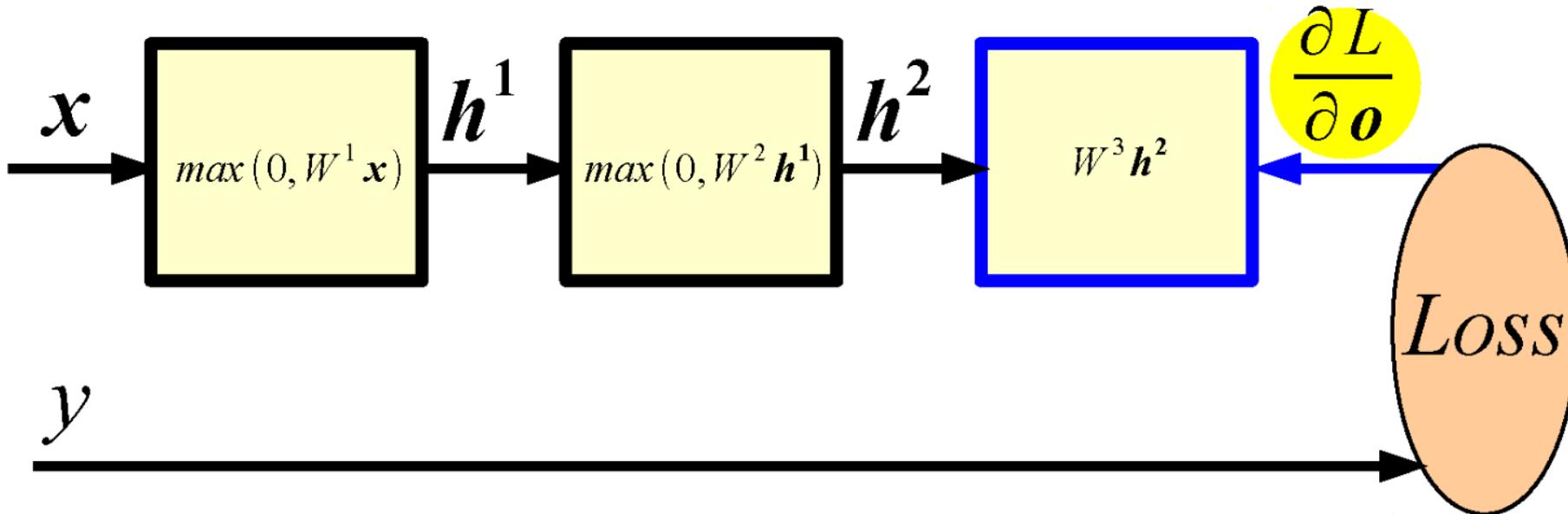
$$p(c_k=1|\mathbf{x}) = \frac{e^{o_k}}{\sum_j e^{o_j}}$$

$$L(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}) = -\sum_j y_j \log p(c_j|\mathbf{x}) \quad \mathbf{y} = [0^1 0..0^1 0..0^c]$$

By substituting the first formula in the second, and taking the derivative w.r.t. \mathbf{o} we get:

$$\frac{\partial L}{\partial o} = p(c|\mathbf{x}) - y$$

Backward Propagation

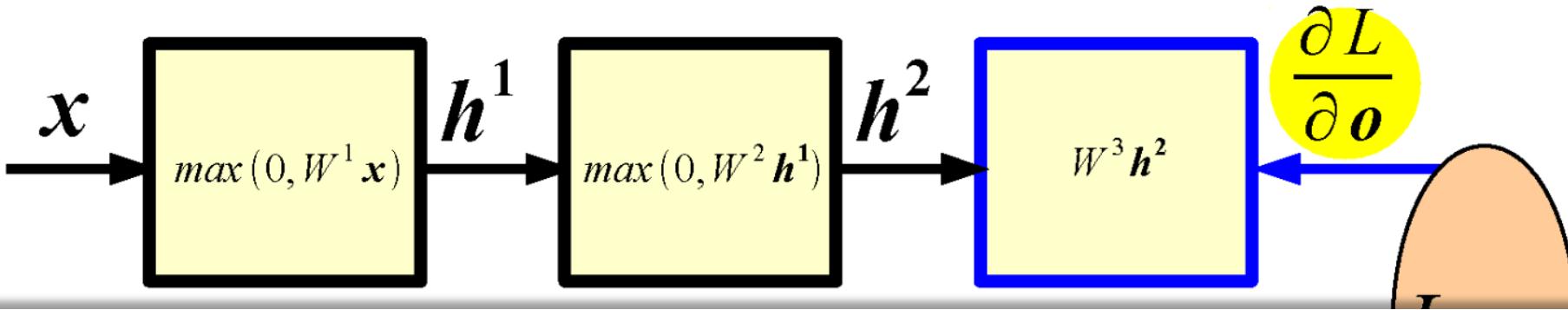


Given $\frac{\partial L}{\partial \mathbf{o}}$ and assuming we can easily compute the Jacobian of each module, we have:

$$\frac{\partial L}{\partial W^3} = \frac{\partial L}{\partial \mathbf{o}} \frac{\partial \mathbf{o}}{\partial W^3}$$

$$\frac{\partial L}{\partial h^2} = \frac{\partial L}{\partial \mathbf{o}} \frac{\partial \mathbf{o}}{\partial h^2}$$

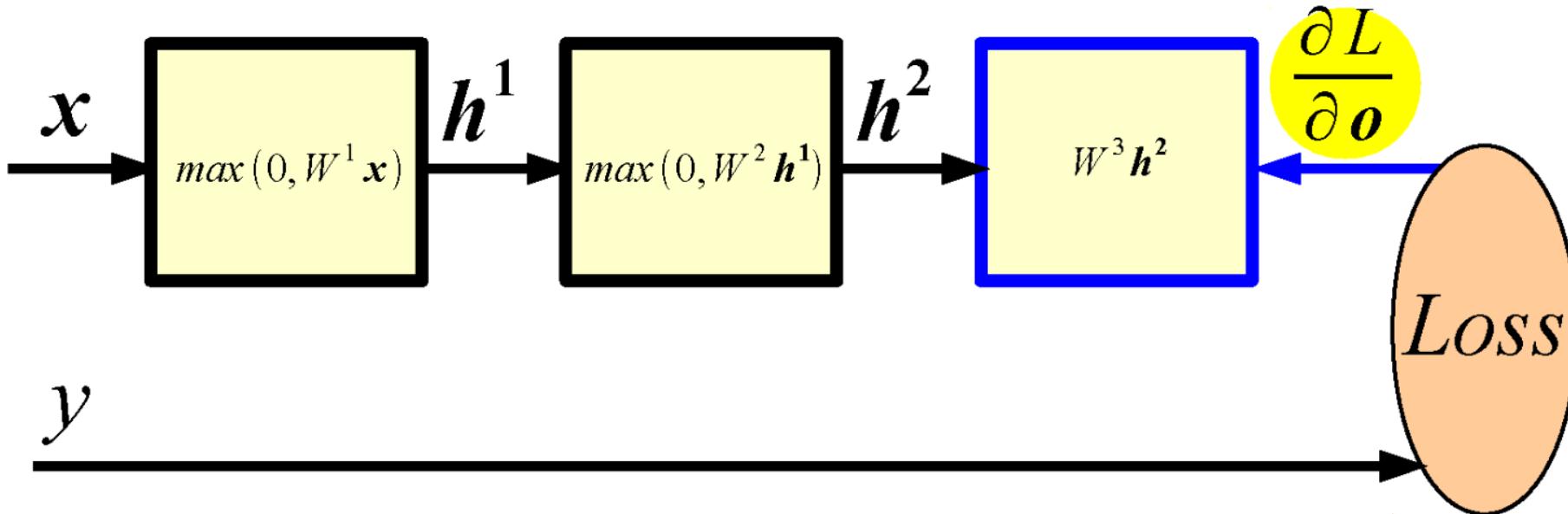
Backward Propagation



Suppose $\mathbf{f} : \mathbf{R}^n \rightarrow \mathbf{R}^m$ is a function such that each of its first-order partial derivatives exist on \mathbf{R}^n . This function takes a point $\mathbf{x} \in \mathbf{R}^n$ as input and produces the vector $\mathbf{f}(\mathbf{x}) \in \mathbf{R}^m$ as output. Then the Jacobian matrix of \mathbf{f} is defined to be an $m \times n$ matrix, denoted by \mathbf{J} , whose (i,j) th entry is $\mathbf{J}_{ij} = \frac{\partial f_i}{\partial x_j}$, or explicitly

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial x_1} & \dots & \frac{\partial \mathbf{f}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla^T f_1 \\ \vdots \\ \nabla^T f_m \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \dots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Backward Propagation

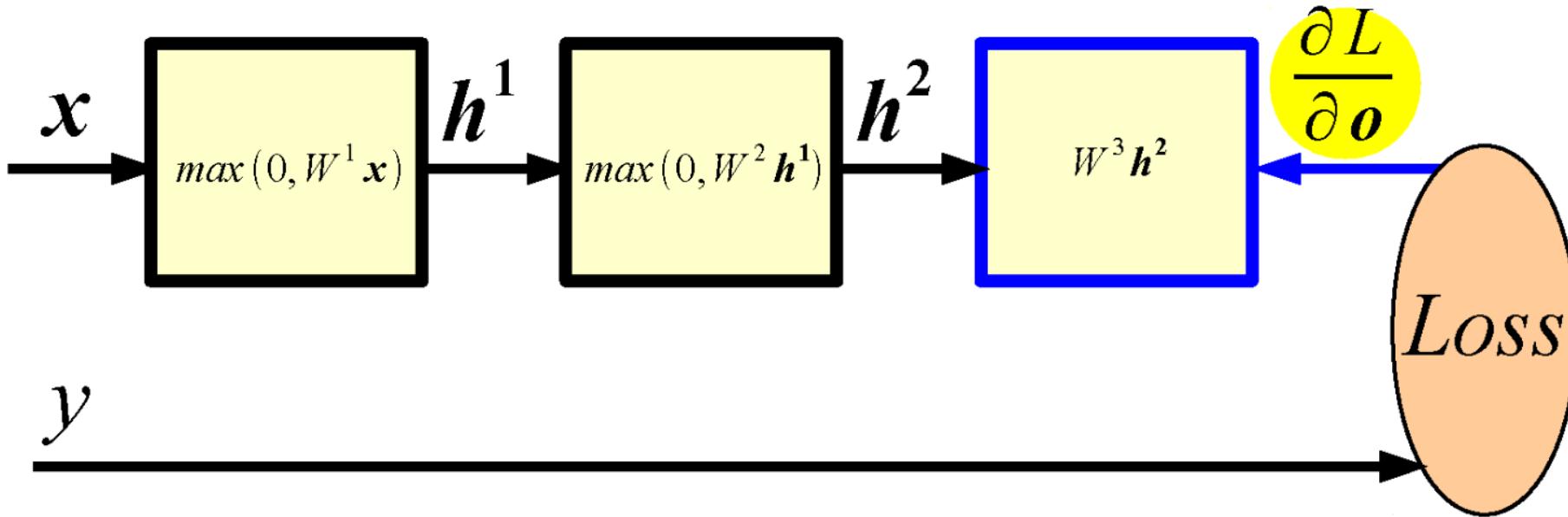


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Backward Propagation



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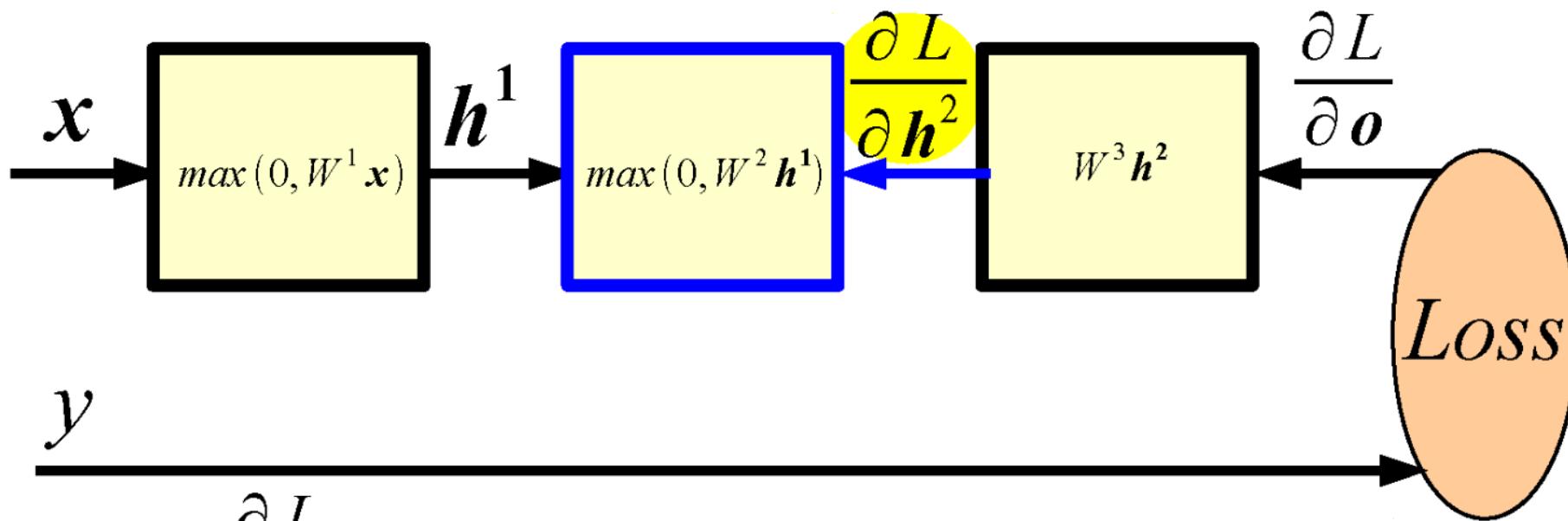
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$$\frac{\partial L}{\partial h^2} = \frac{\partial L}{\partial \mathbf{o}} \frac{\partial \mathbf{o}}{\partial h^2}$$

$$\frac{\partial L}{\partial W^3} = (p(c|x) - y) h^{2T}$$

$$\frac{\partial L}{\partial h^2} = W^{3T} (p(c|x) - y)$$

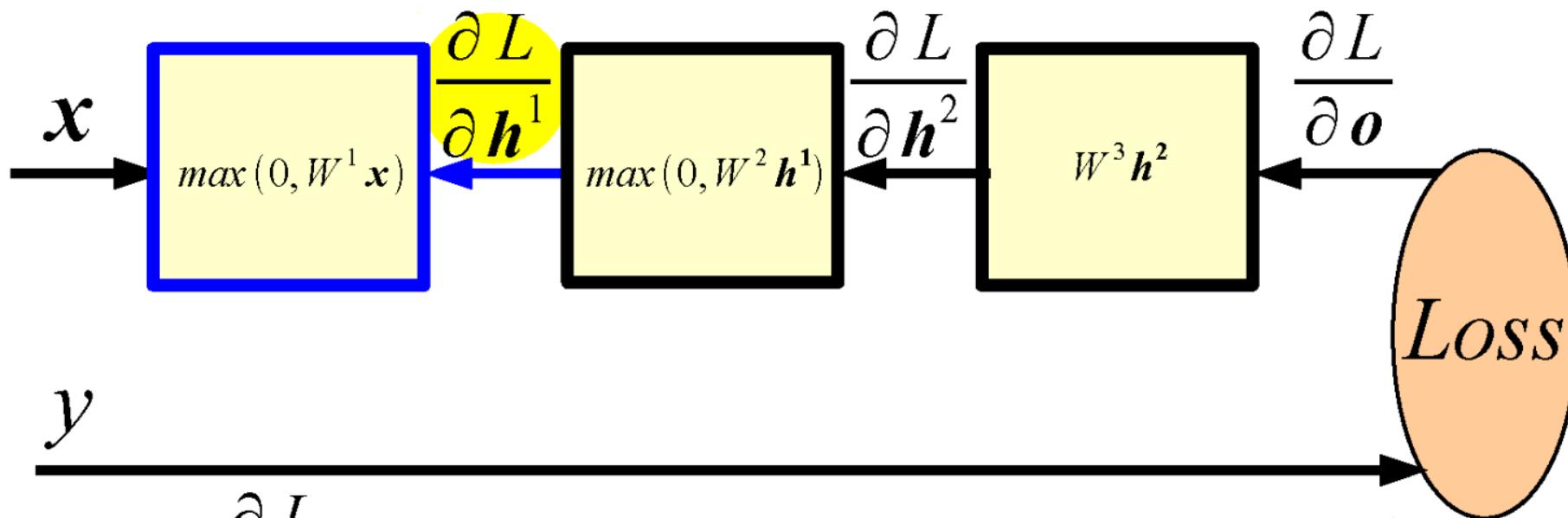
Backward Propagation



Given $\frac{\partial L}{\partial h^2}$ we can compute now:

$$\frac{\partial L}{\partial W^2} = \frac{\partial L}{\partial h^2} \frac{\partial h^2}{\partial W^2} \quad \frac{\partial L}{\partial h^1} = \frac{\partial L}{\partial h^2} \frac{\partial h^2}{\partial h^1}$$

Backward Propagation



Given $\frac{\partial L}{\partial h^1}$ we can compute now:

$$\frac{\partial L}{\partial W^1} = \frac{\partial L}{\partial h^1} \frac{\partial h^1}{\partial W^1}$$

Backward Propagation

Question: Does BPROP work with ReLU layers only?

Answer: Nope, any a.e. differentiable transformation works.

Backward Propagation

Question: Does BPROP work with ReLU layers only?

Answer: Nope, any a.e. differentiable transformation works.

Question: What's the computational cost of BPROP?

Answer: About twice FPROP (need to compute gradients w.r.t. input and parameters at every layer).

Optimization

Stochastic Gradient Descent (on mini-batches):

$$\theta \leftarrow \theta - \eta \frac{\partial L}{\partial \theta}, \eta \in (0, 1)$$

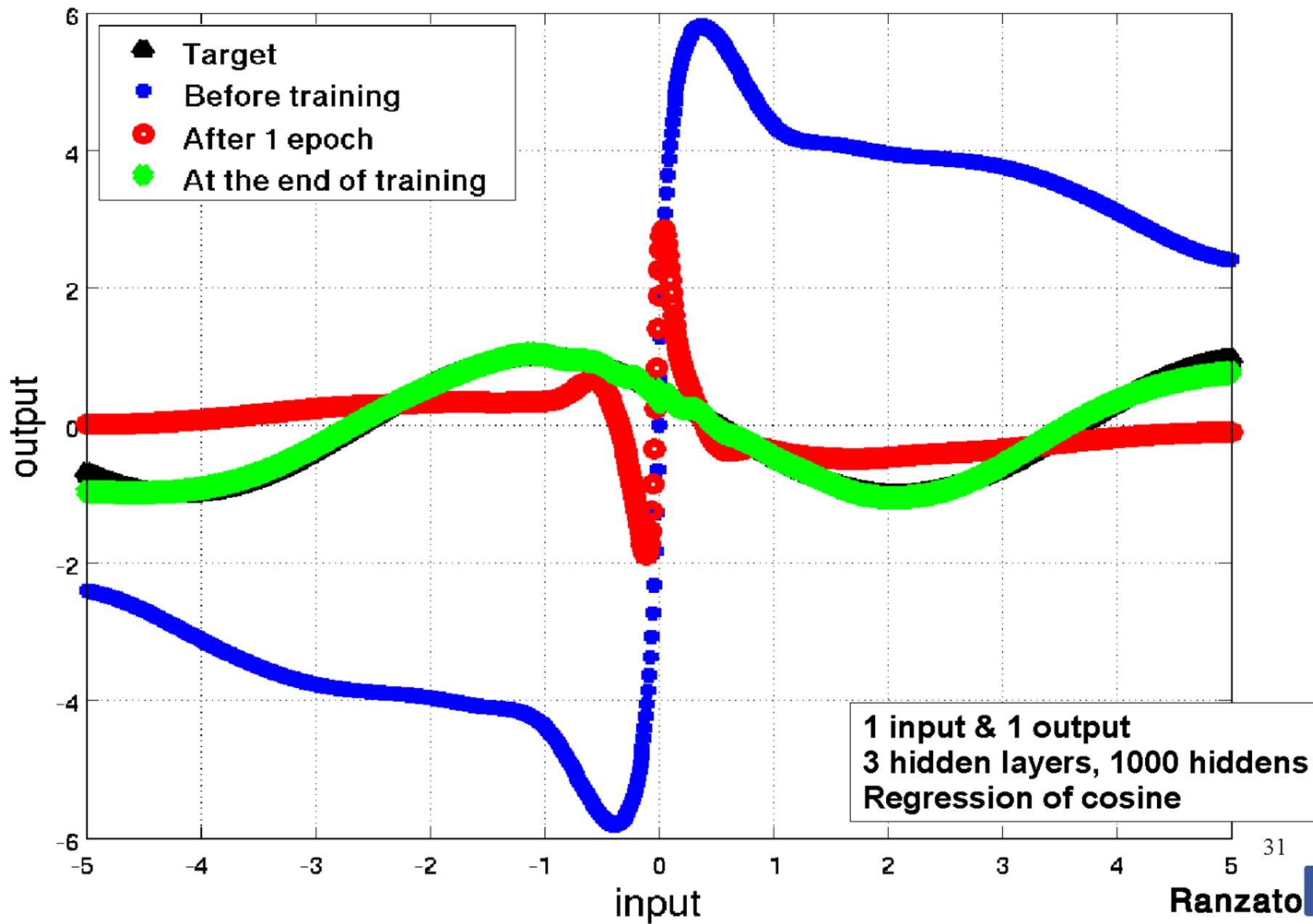
Stochastic Gradient Descent with Momentum:

$$\theta \leftarrow \theta - \eta \Delta$$

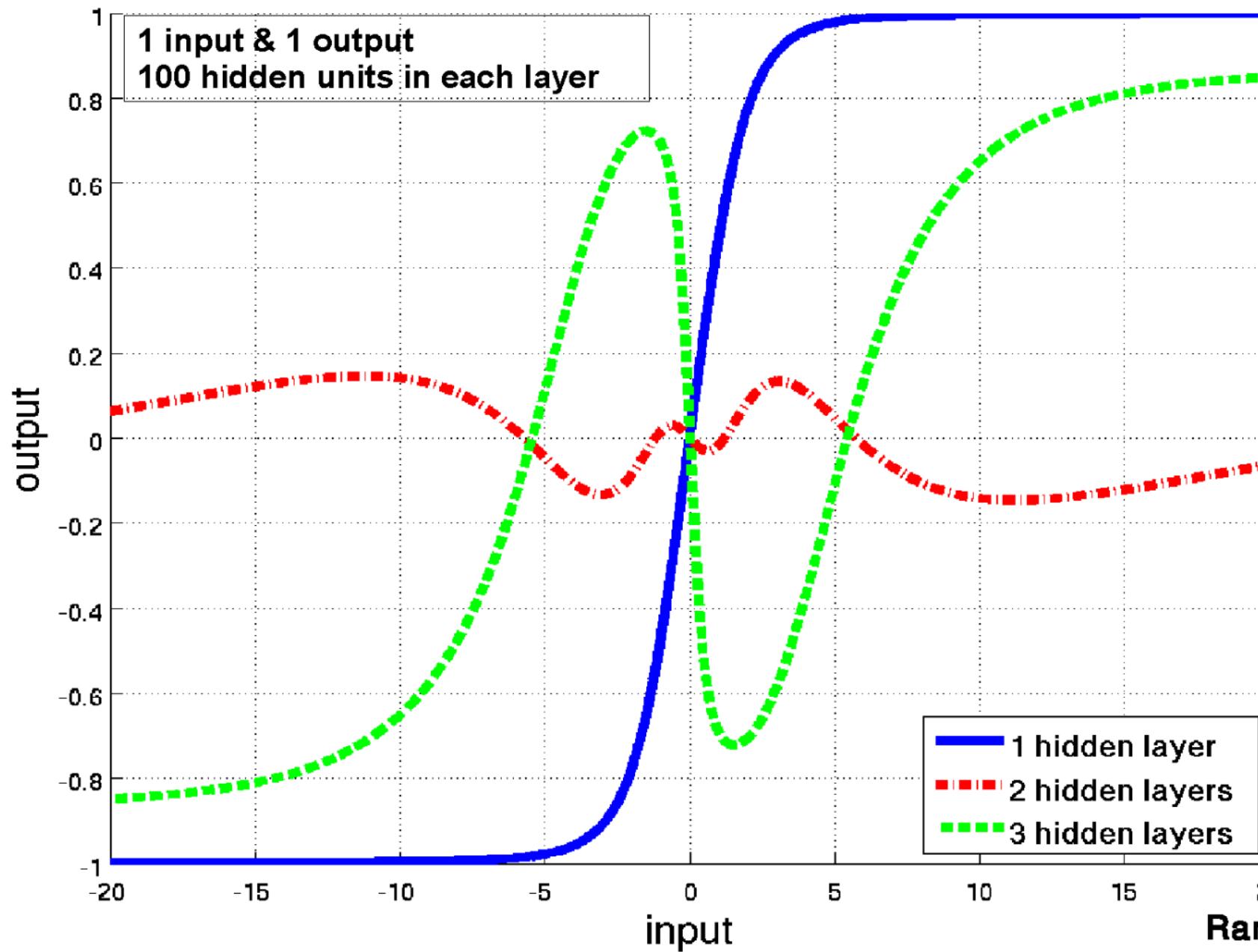
$$\Delta \leftarrow 0.9 \Delta + \frac{\partial L}{\partial \theta}$$

Note: there are many other variants...

Toy Example: Synthetic Data



Toy Example: Synthetic Data



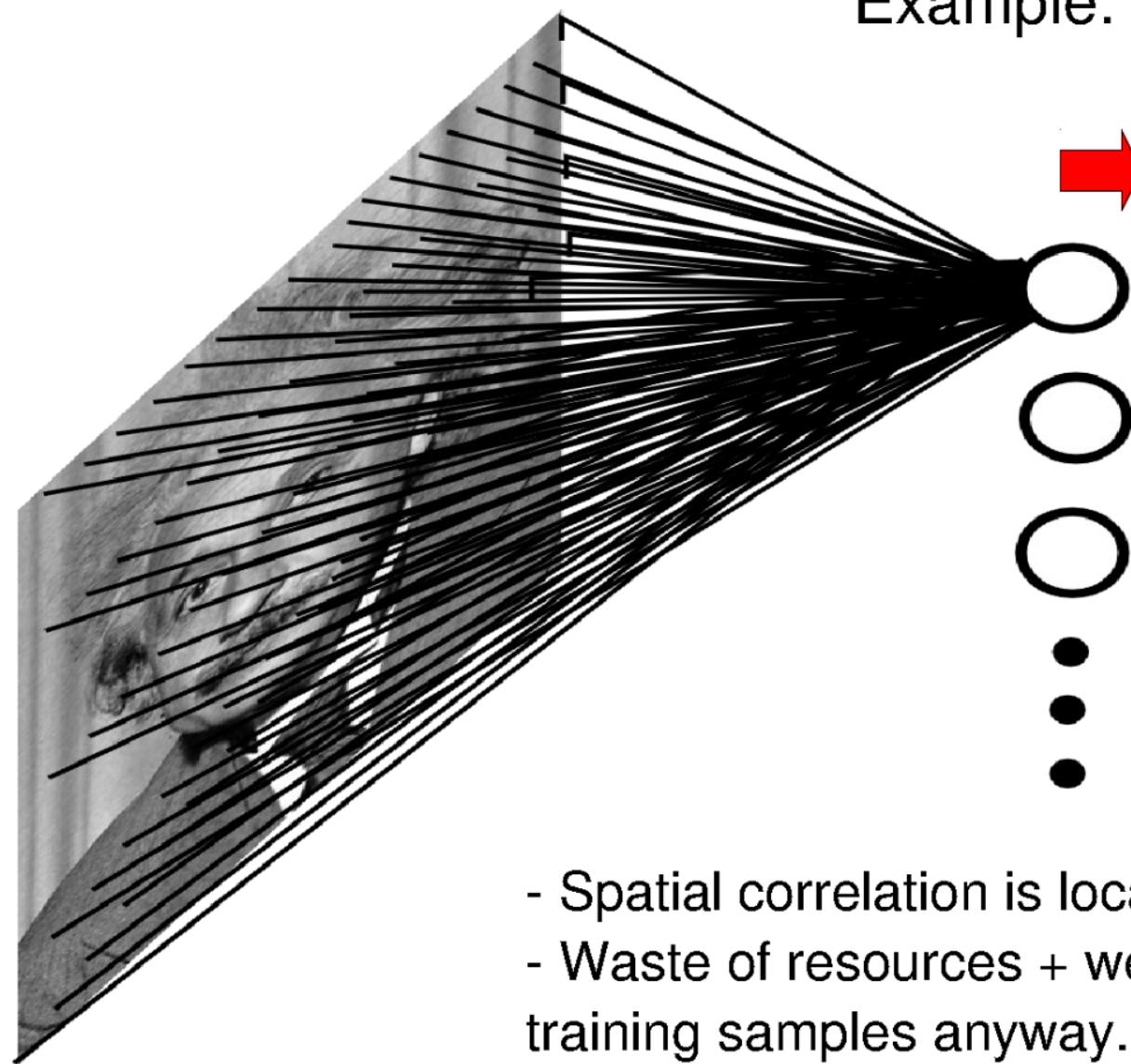
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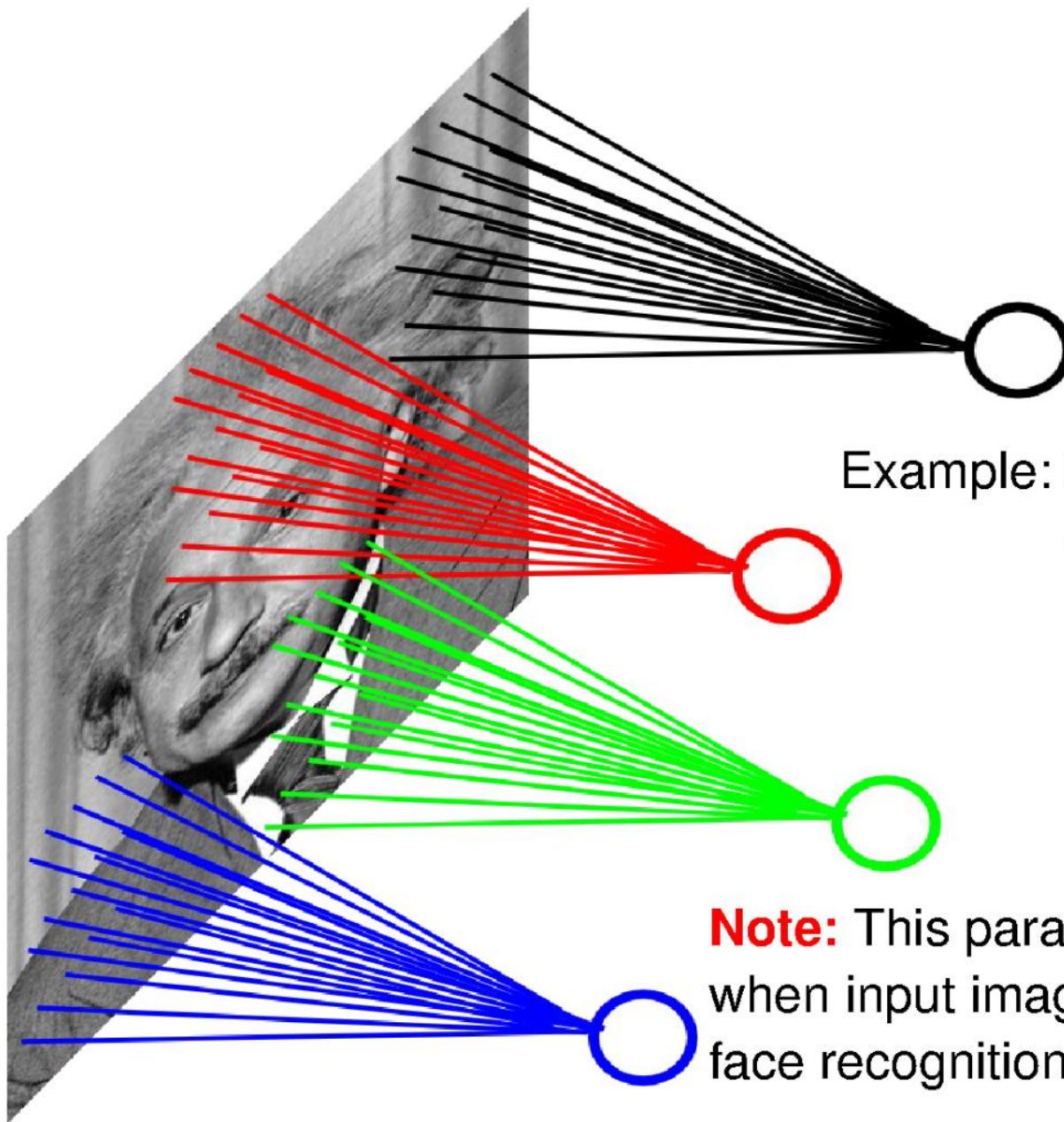
Fully Connected Layer



Example: 200x200 image
40K hidden units
→ **~2B parameters!!!**

- Spatial correlation is local
- Waste of resources + we have not enough training samples anyway..

Locally Connected Layer

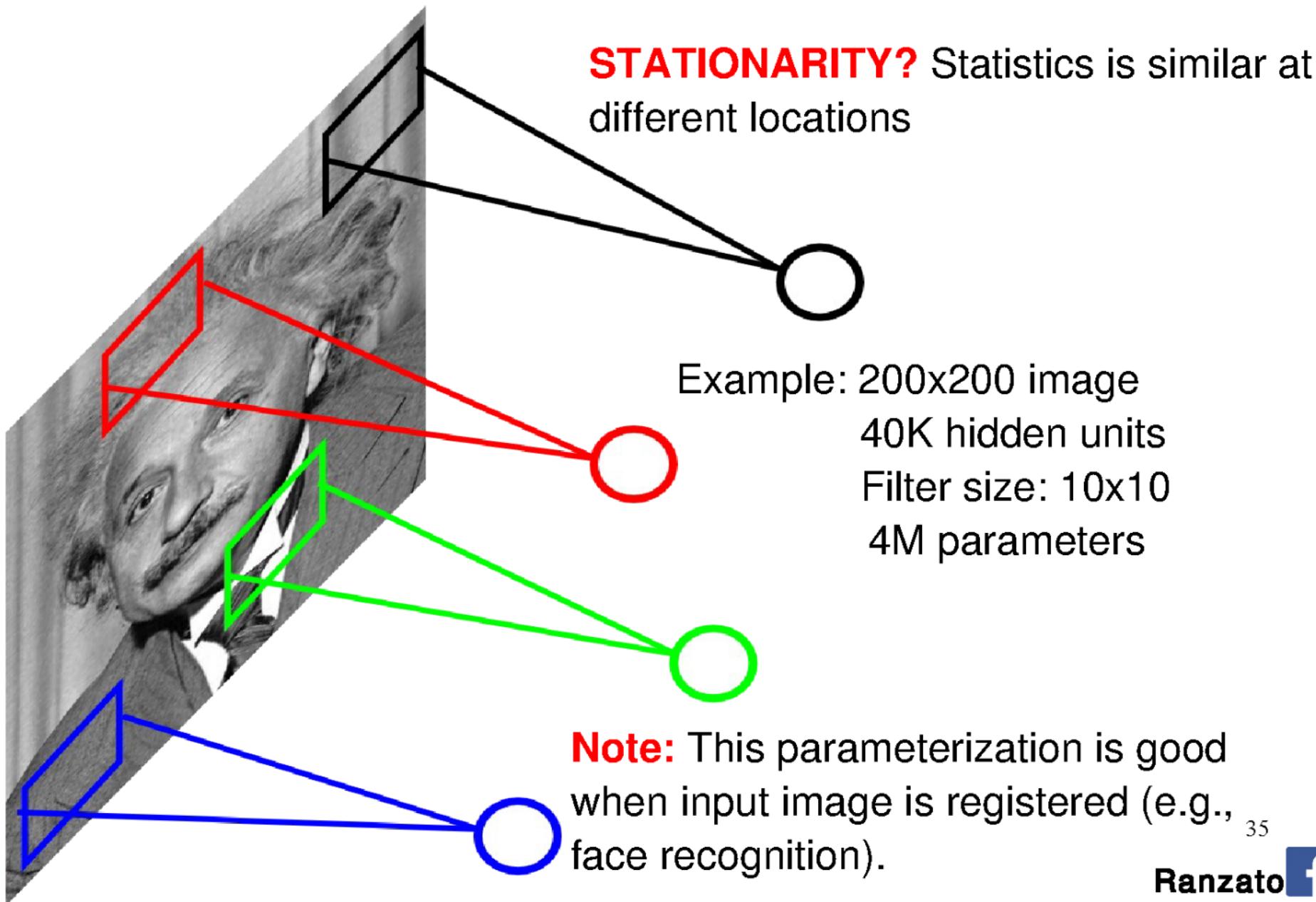


Example: 200x200 image
40K hidden units
Filter size: 10x10
4M parameters

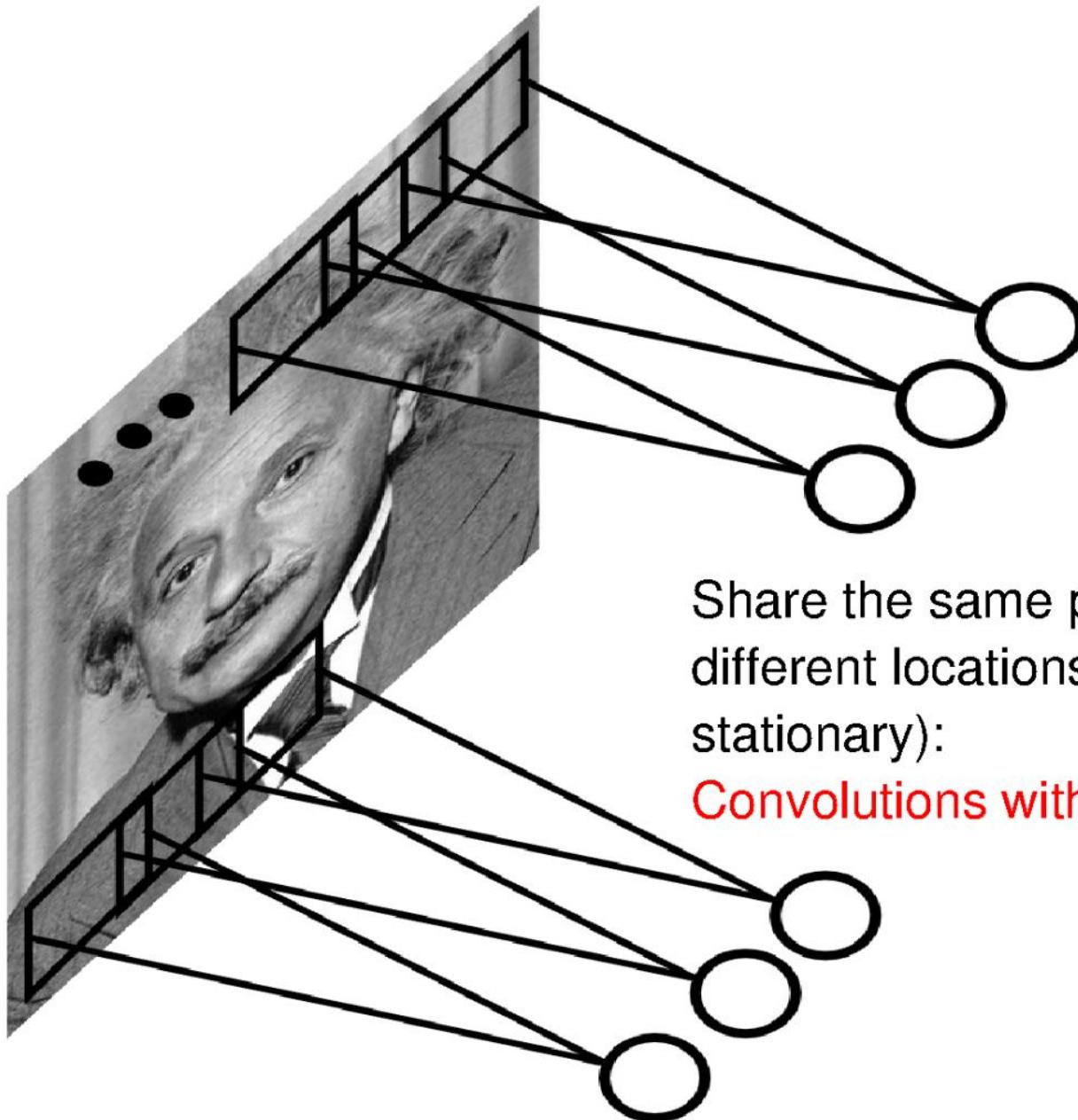
Note: This parameterization is good
when input image is registered (e.g.,
face recognition).

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Locally Connected Layer

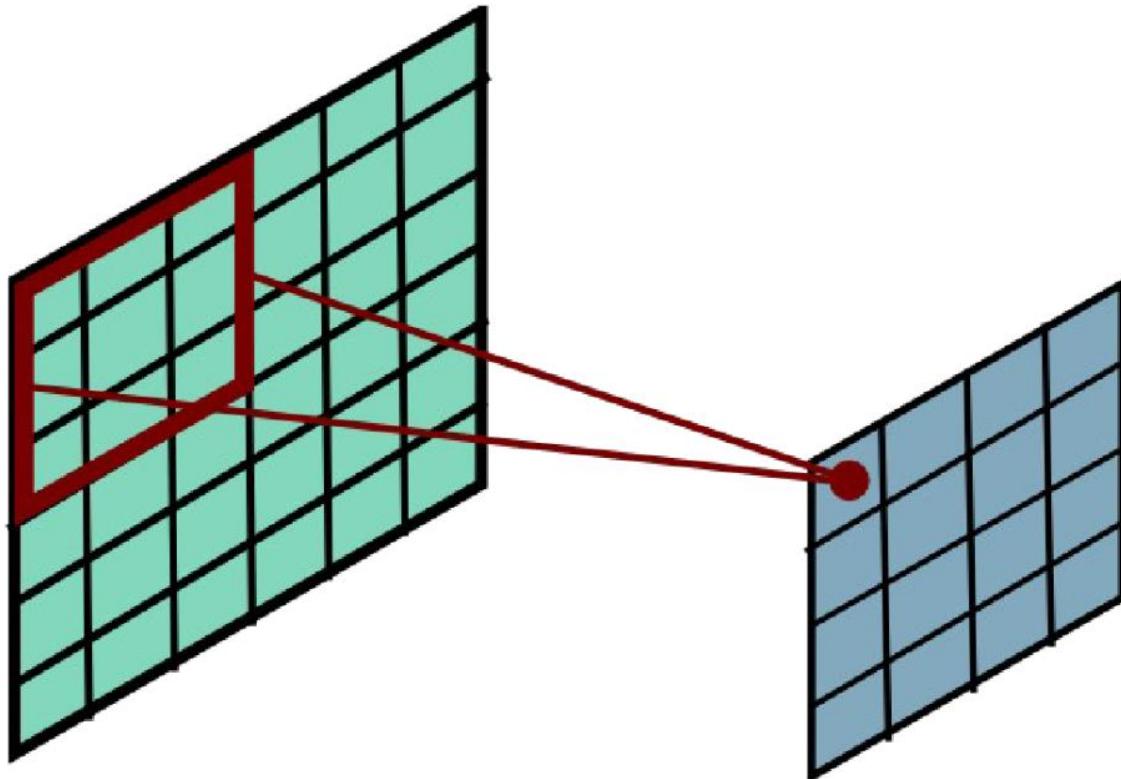


Convolutional Layer

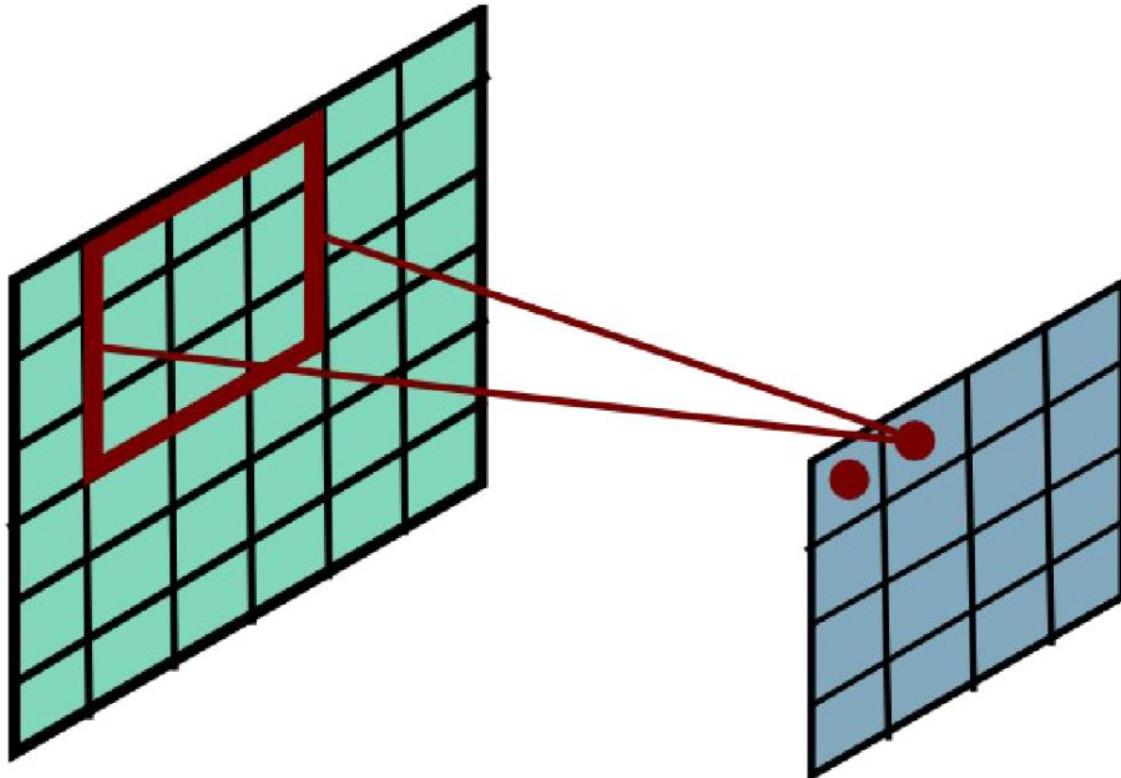


Share the same parameters across
different locations (assuming input is
stationary):
Convolutions with learned kernels

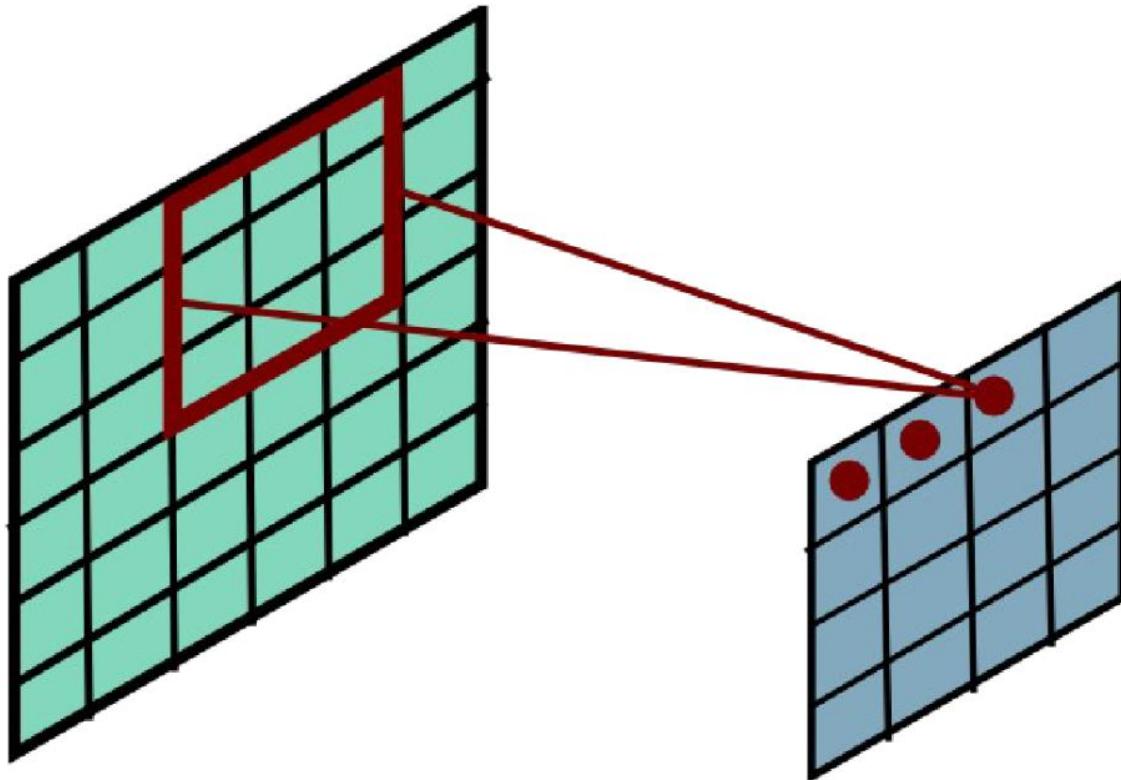
Convolutional Layer



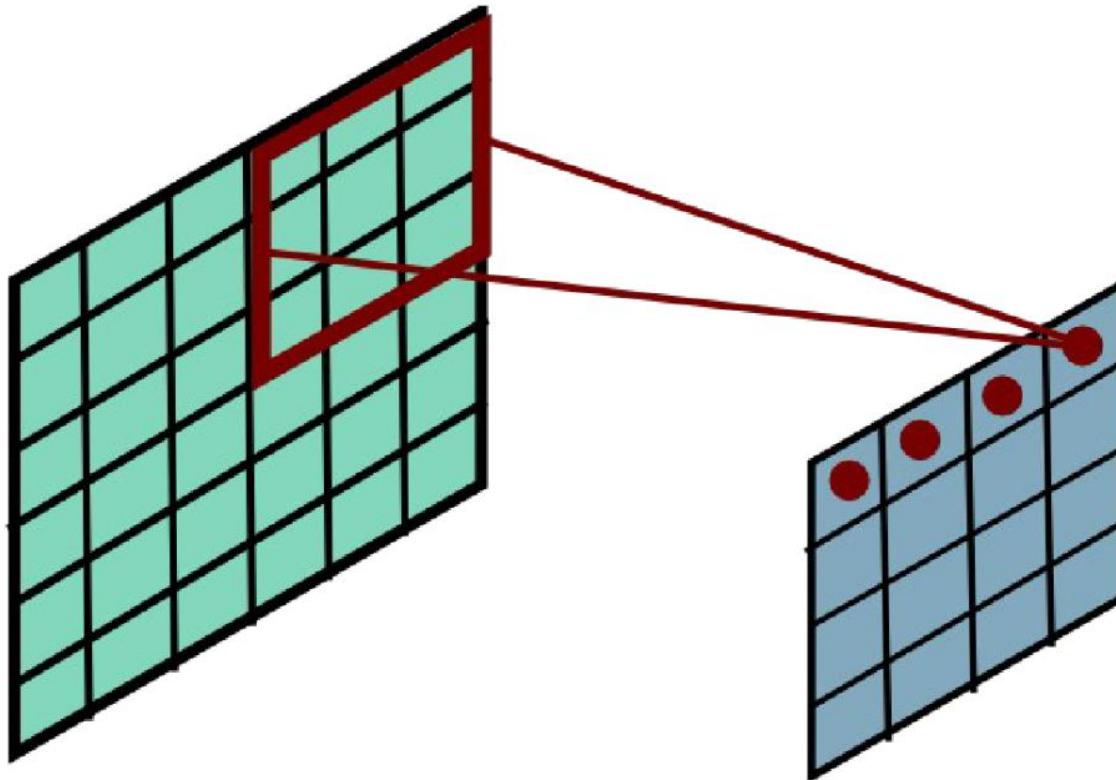
Convolutional Layer



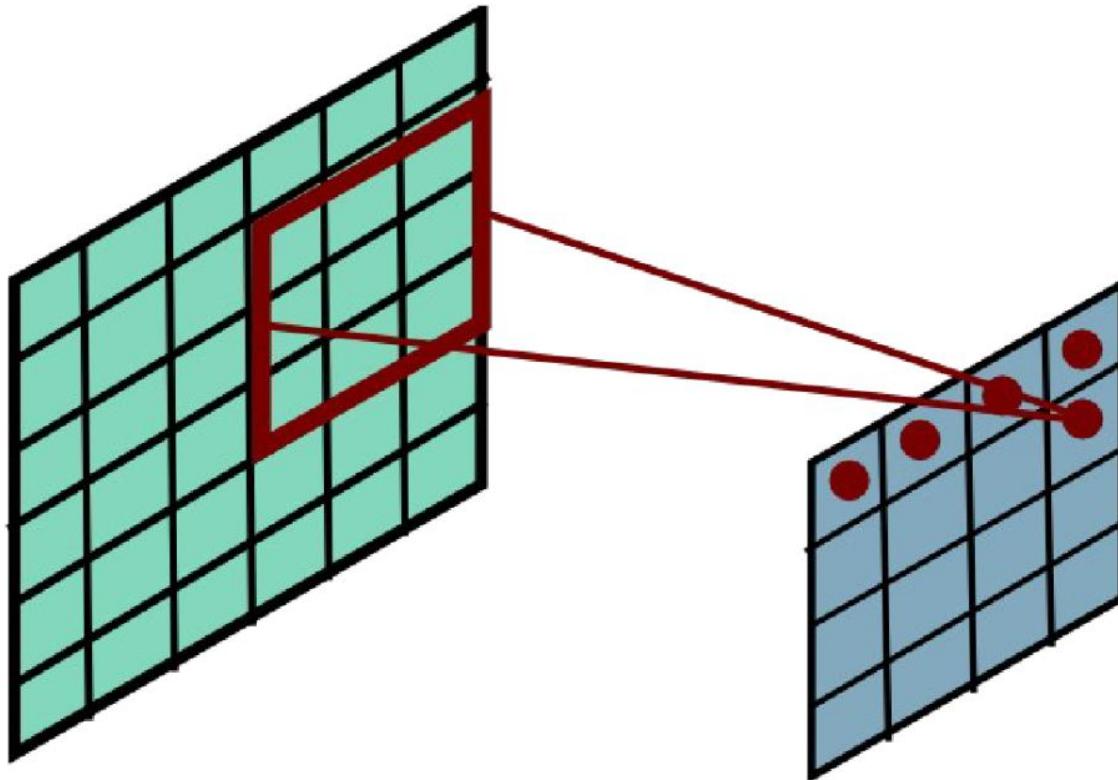
Convolutional Layer



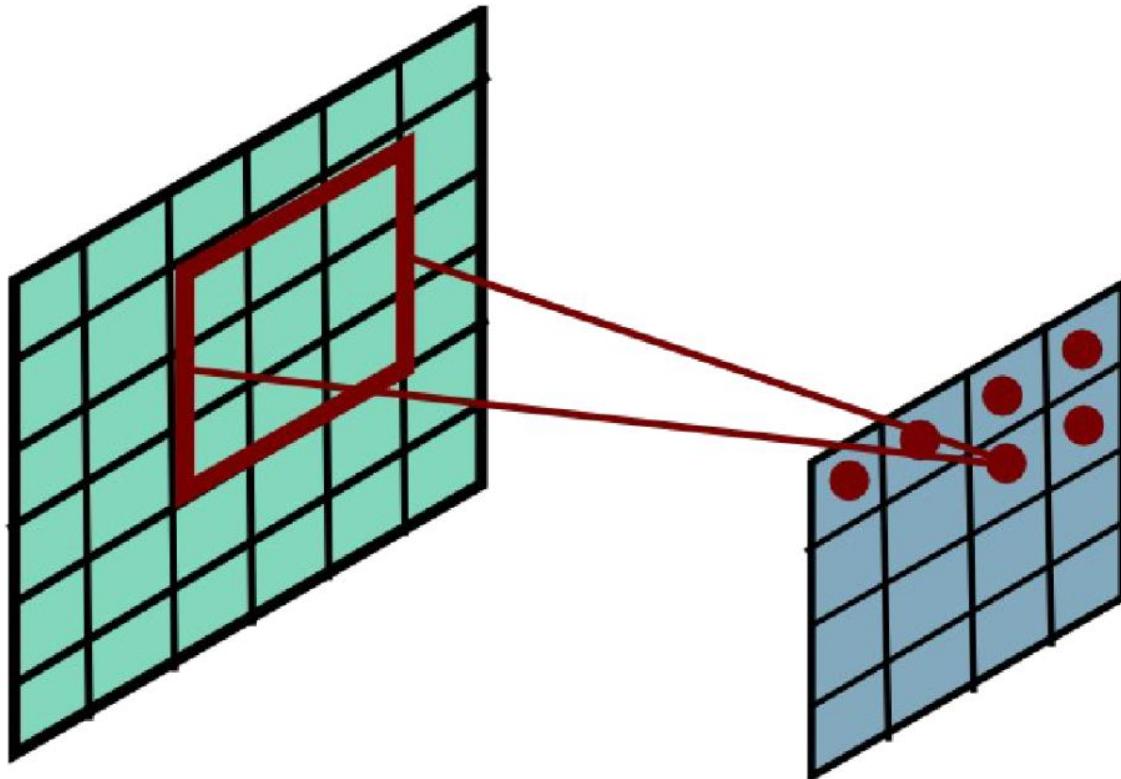
Convolutional Layer



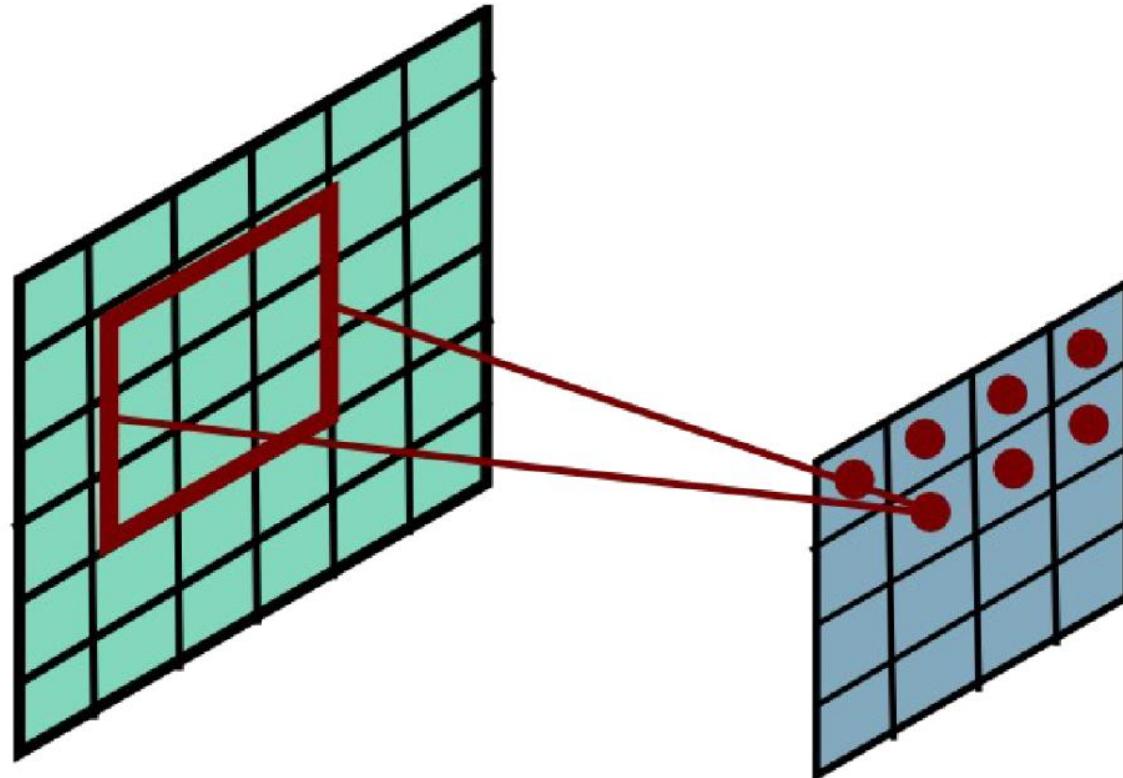
Convolutional Layer



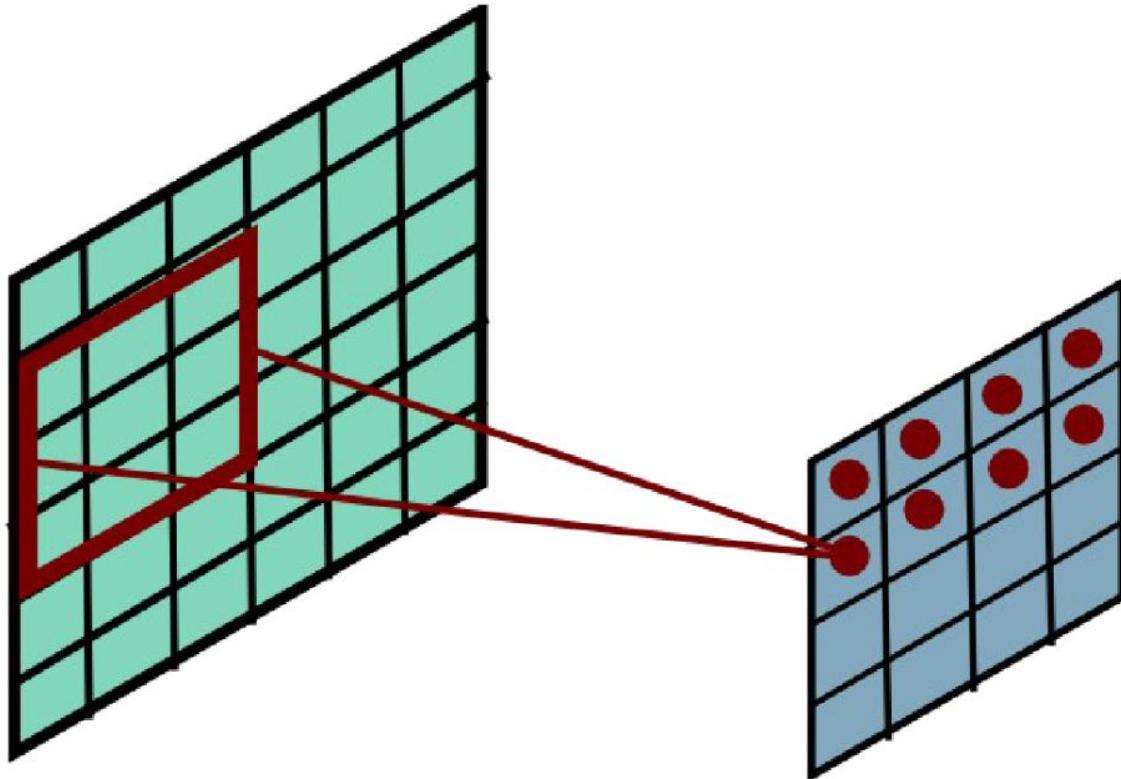
Convolutional Layer



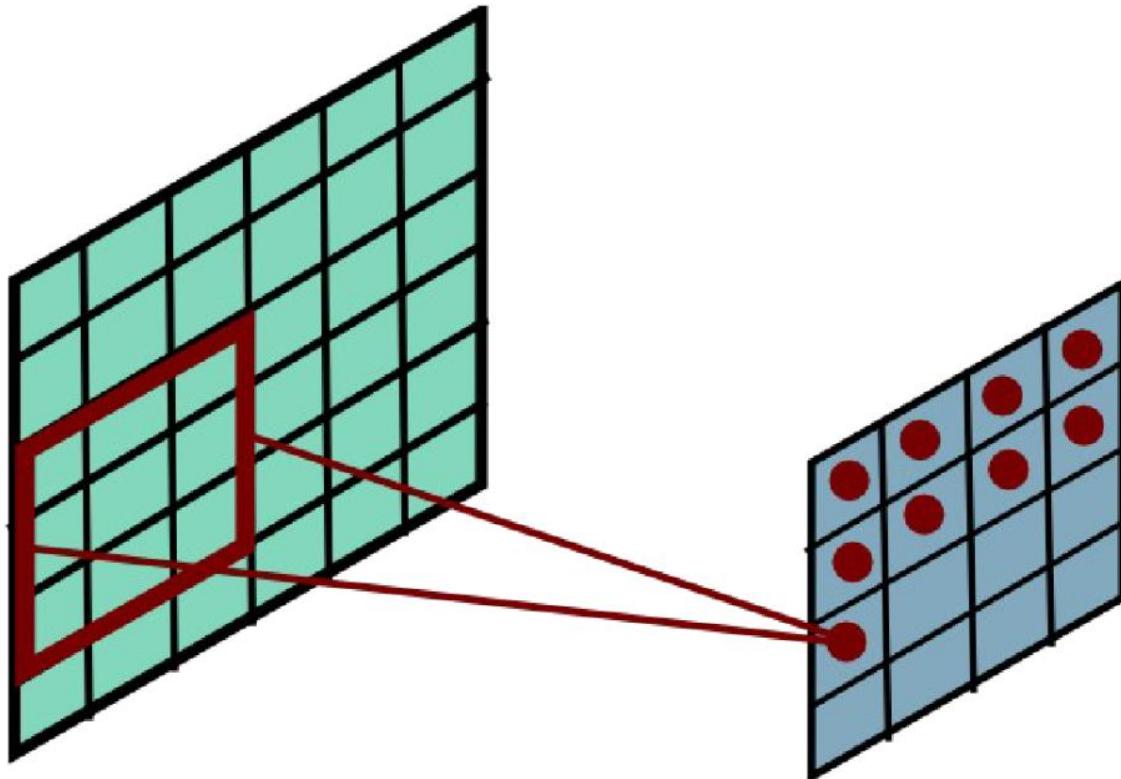
Convolutional Layer



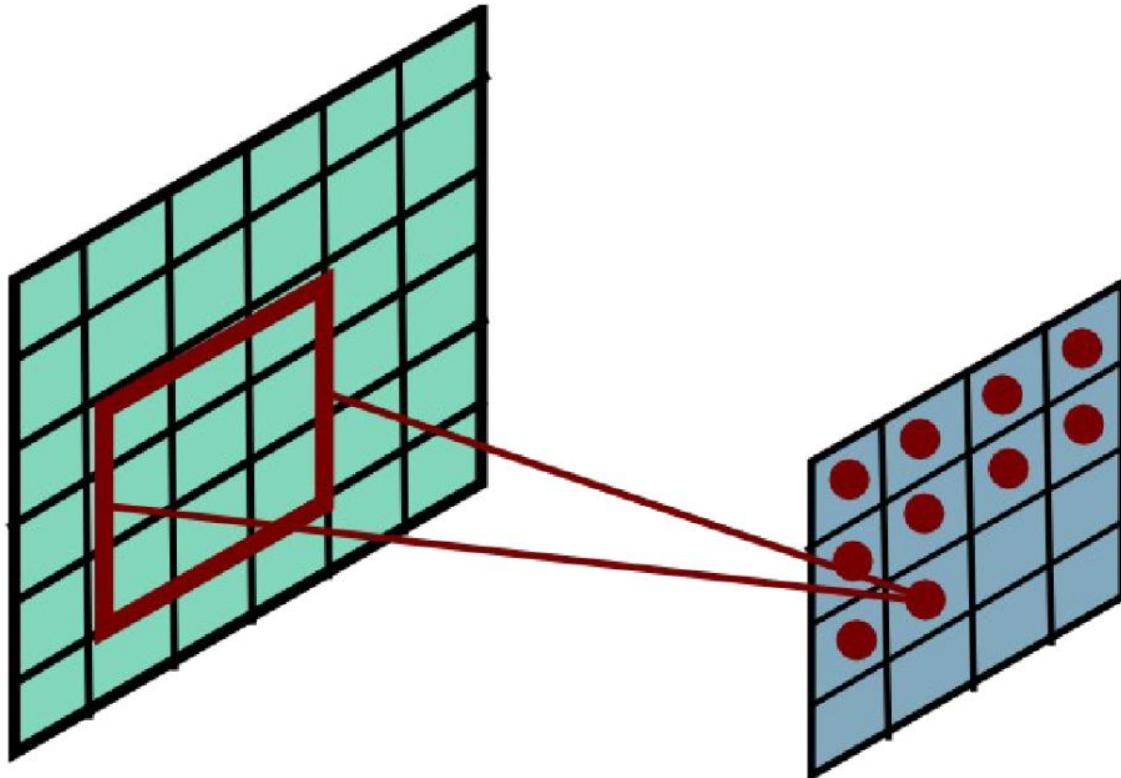
Convolutional Layer



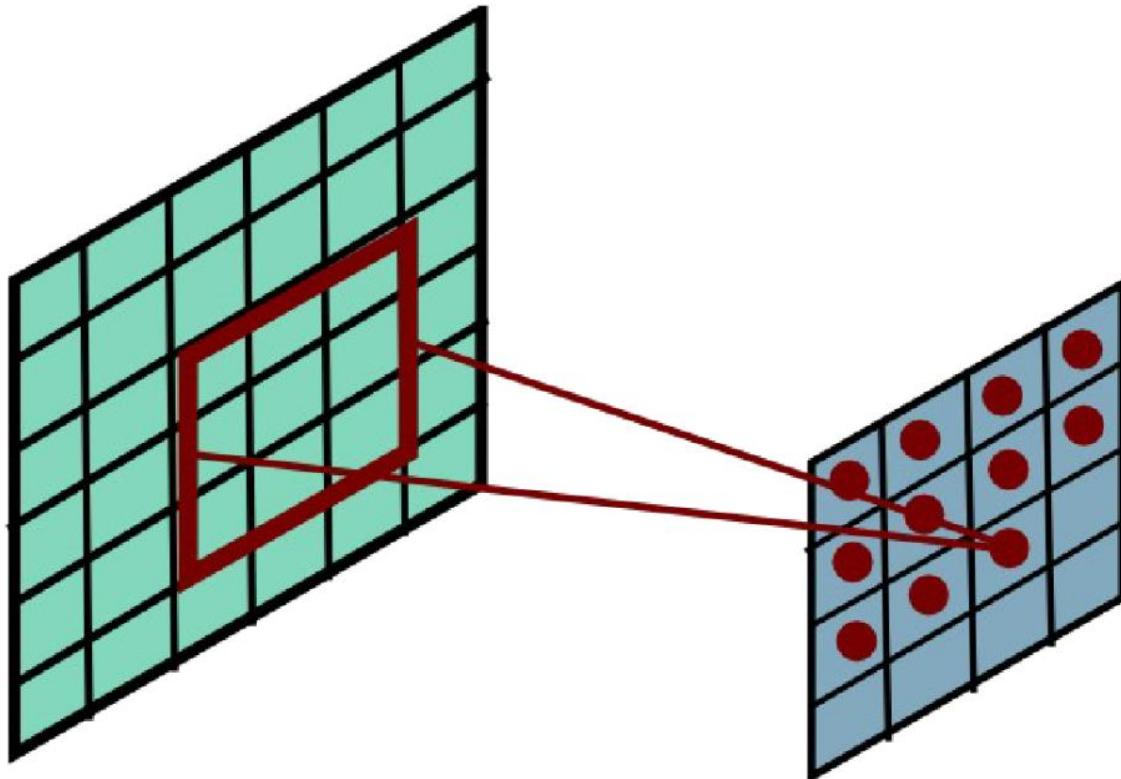
Convolutional Layer



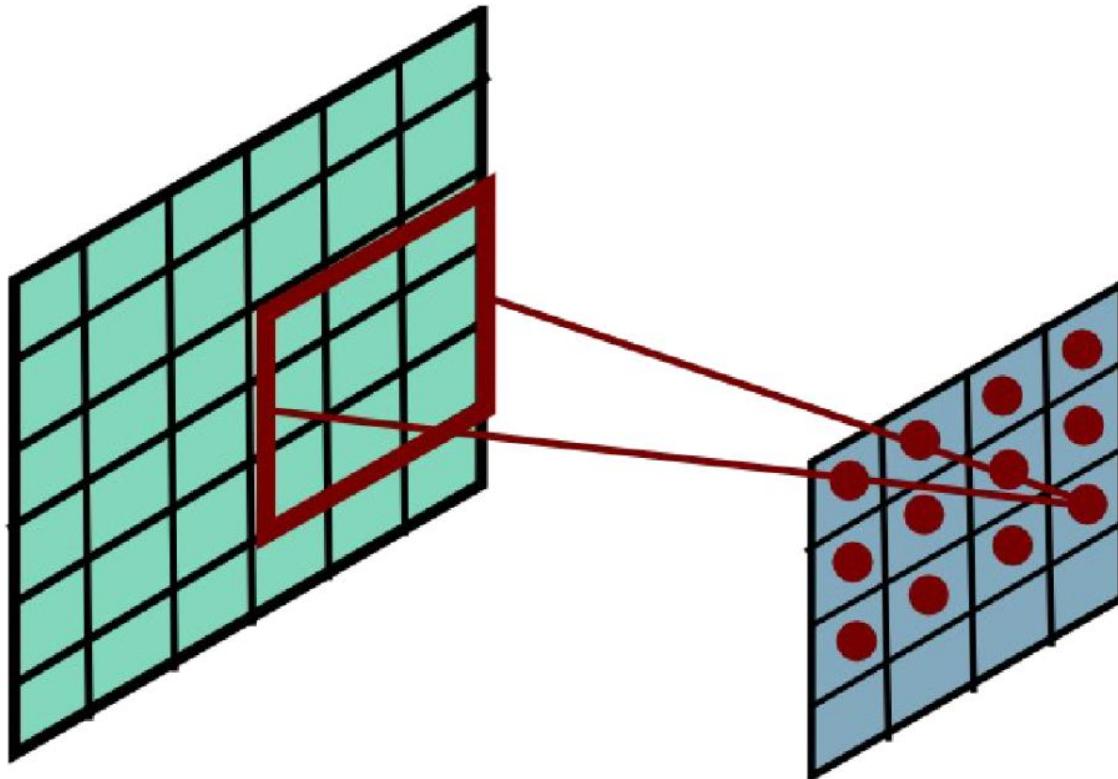
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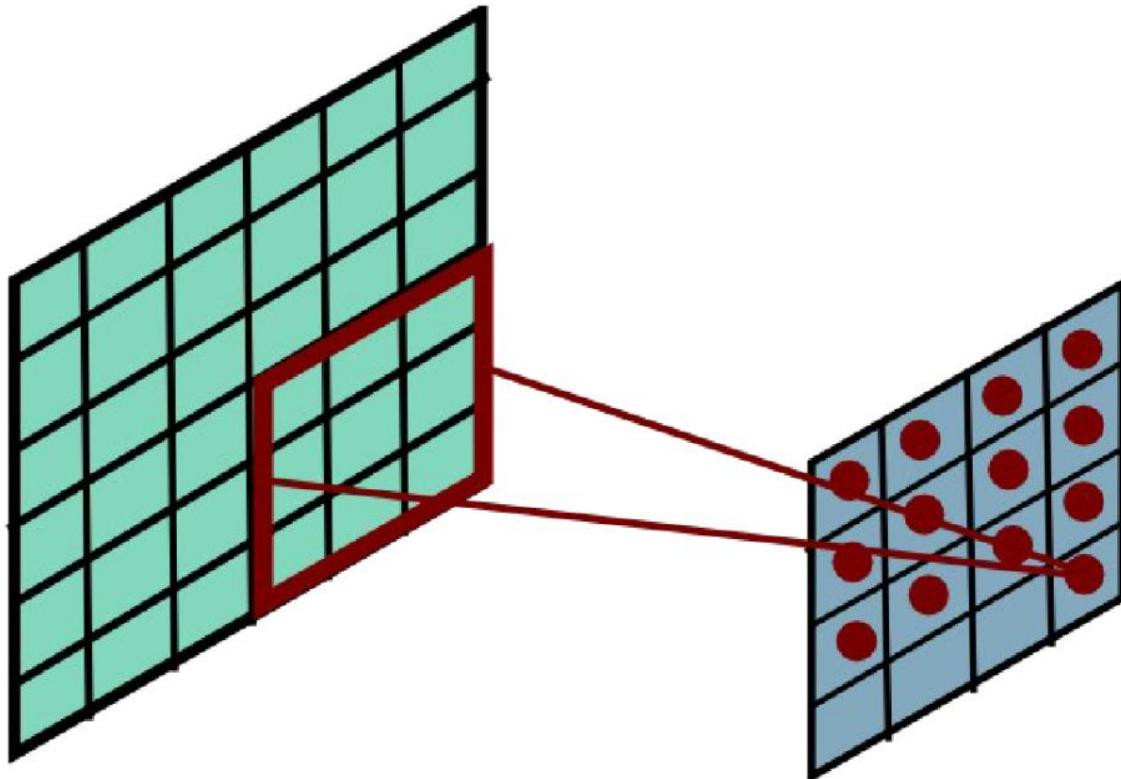
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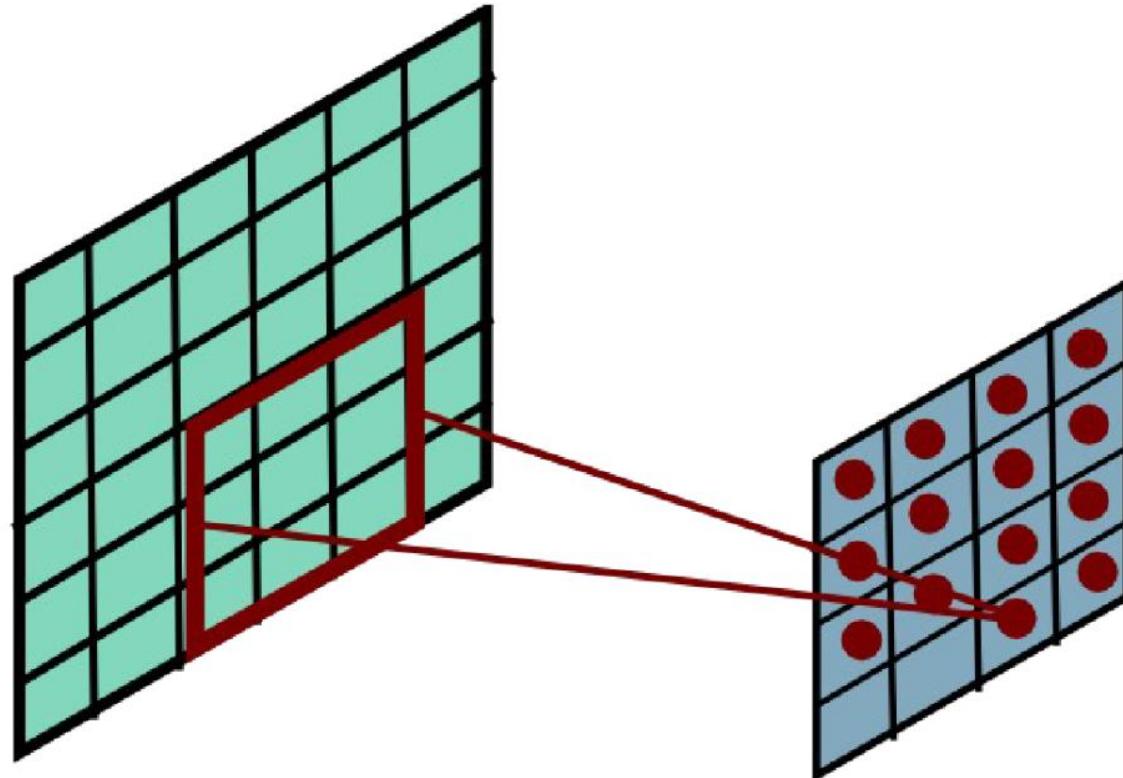
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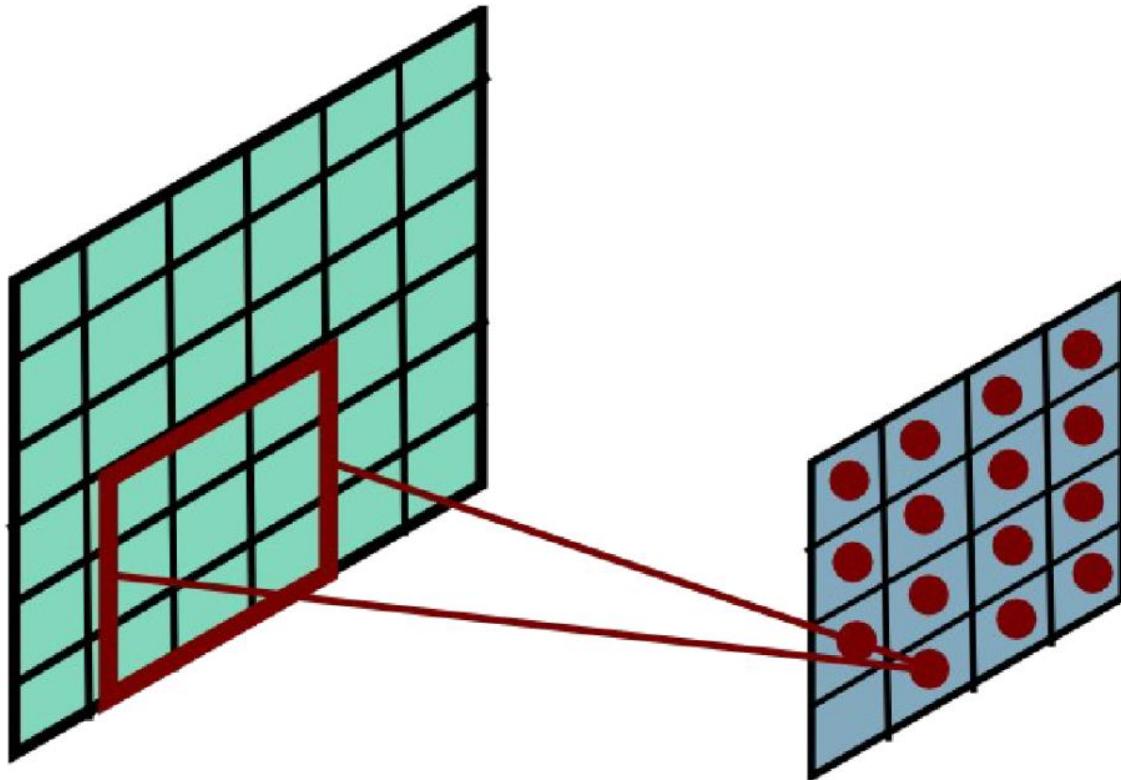
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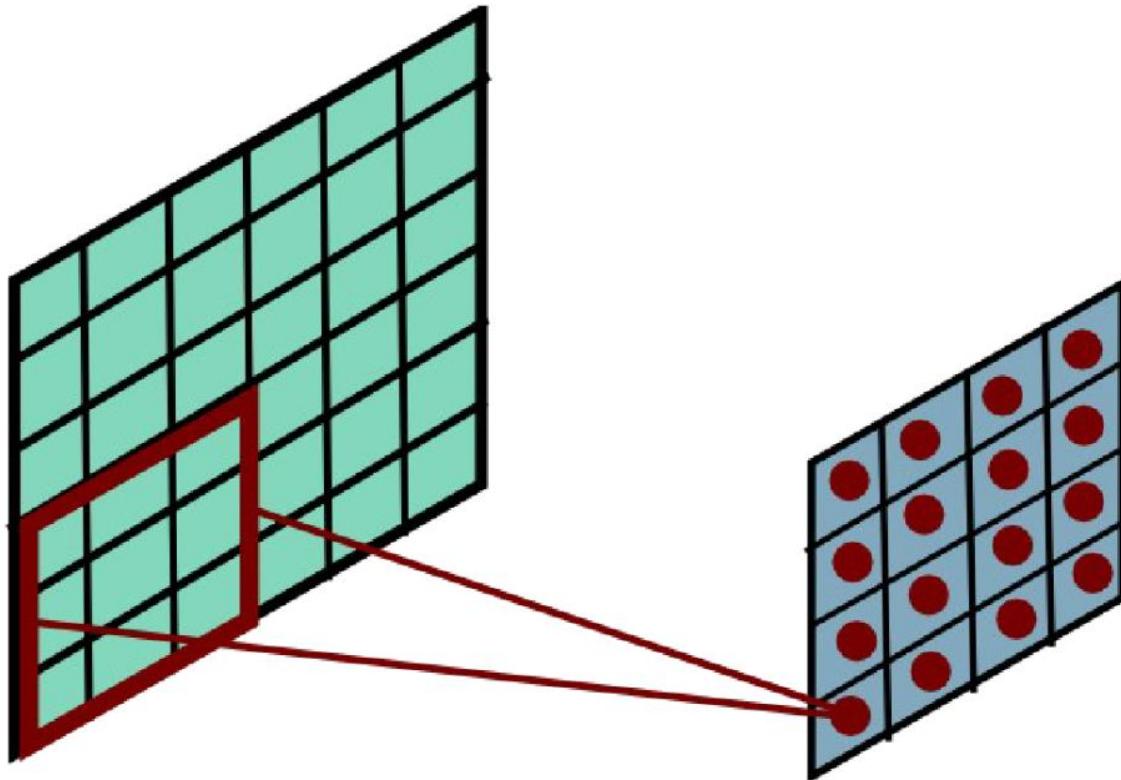
Convolutional Layer



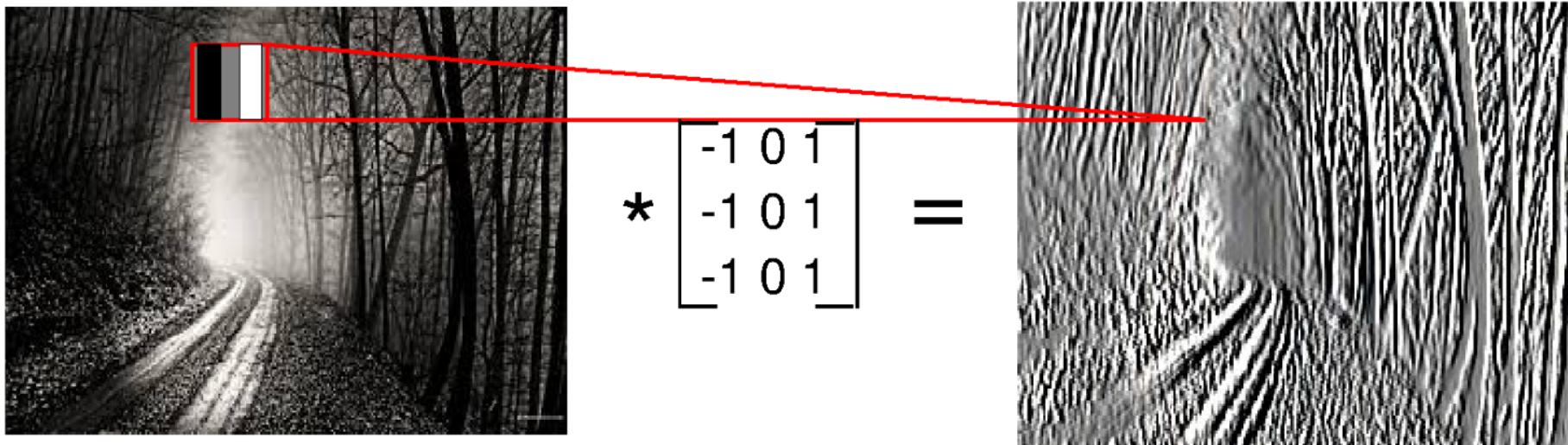
Convolutional Layer



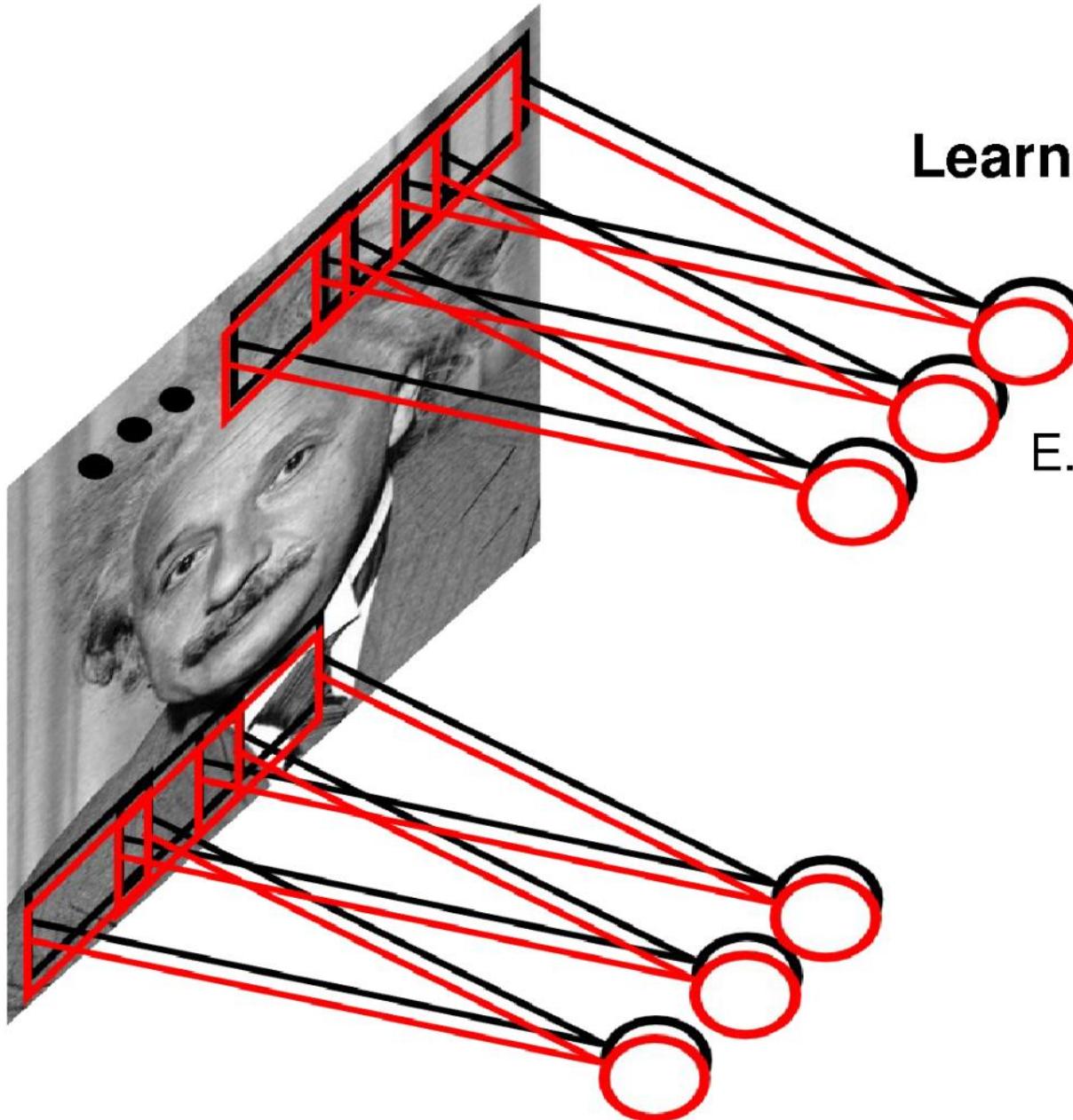
Convolutional Layer



Convolutional Layer



Convolutional Layer



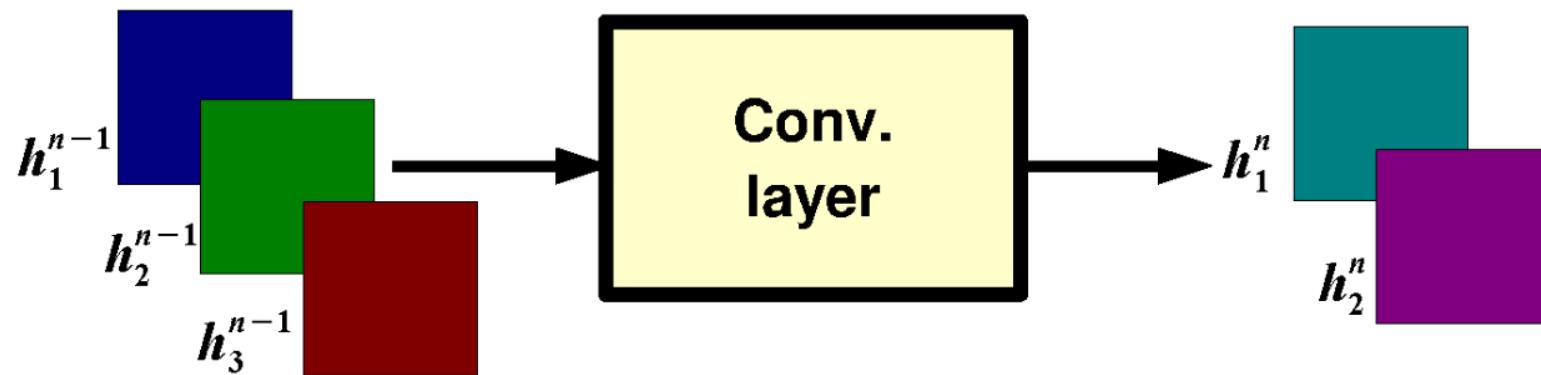
Learn multiple filters.

E.g.: 200x200 image
100 Filters
Filter size: 10x10
10K parameters

Convolutional Layer

$$h_j^n = \max \left(0, \sum_{k=1}^K h_k^{n-1} * w_{kj}^n \right)$$

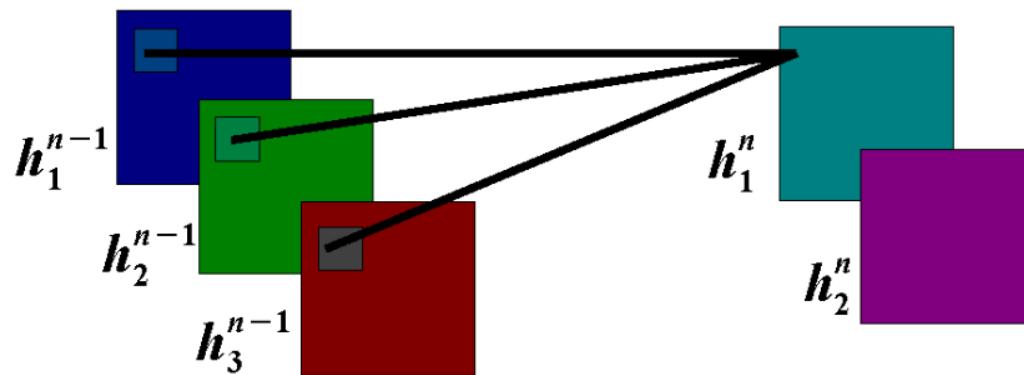
output feature map input feature map kernel



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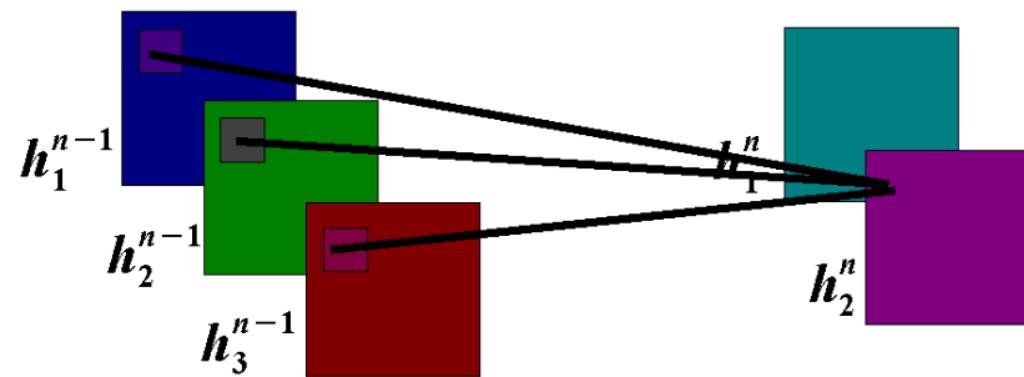
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Convolutional Layer

$$h_j^n = \max \left(0, \sum_{k=1}^K h_k^{n-1} * w_{kj}^n \right)$$

output feature map input feature map kernel



Key Ideas

A standard neural net applied to images:

- scales quadratically with the size of the input
- does not leverage stationarity

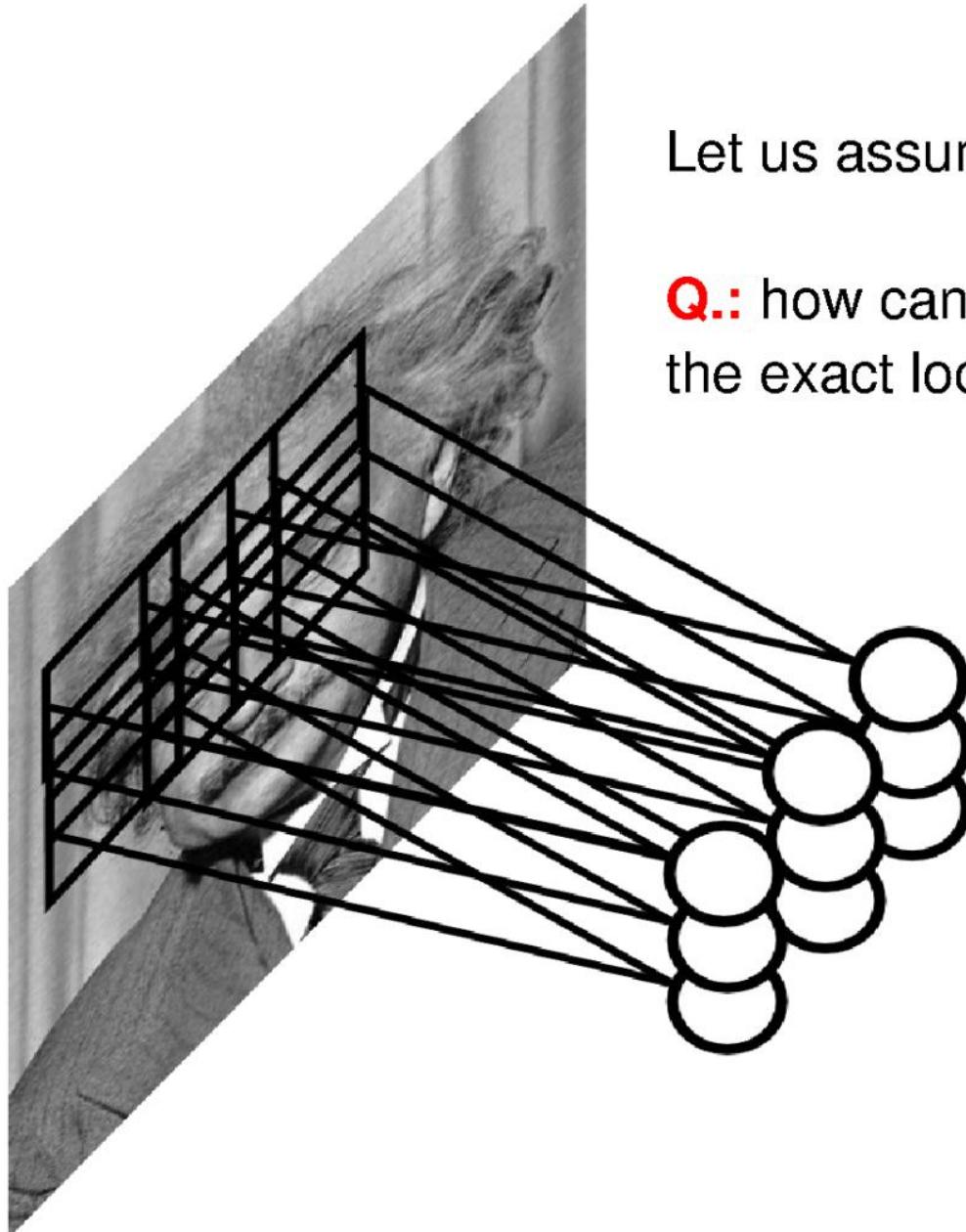
Solution:

- connect each hidden unit to a small patch of the input
- share the weight across space

This is called: **convolutional layer**.

A network with convolutional layers is called **convolutional network**.

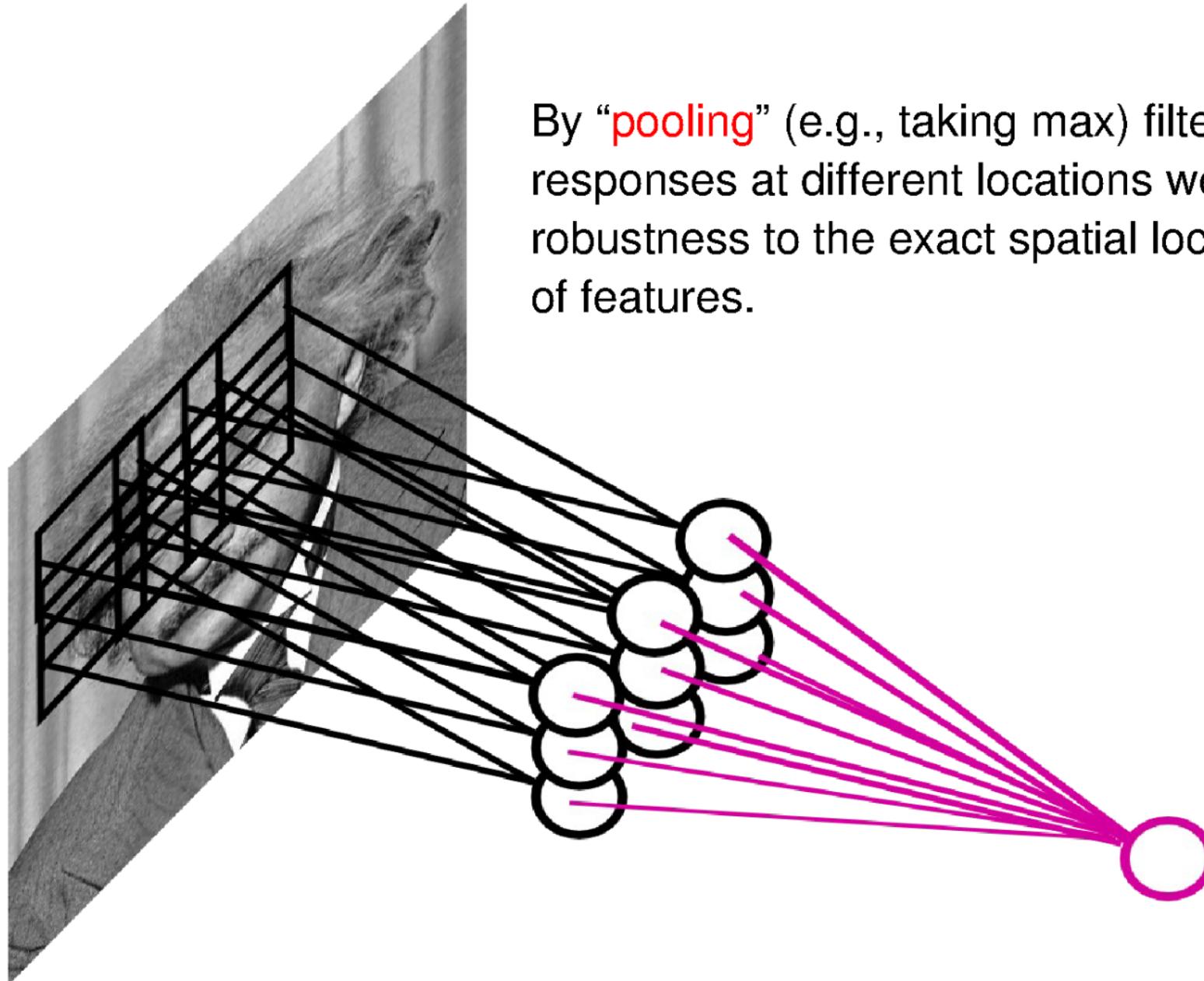
Pooling Layer



Let us assume filter is an “eye” detector.

Q.: how can we make the detection robust to the exact location of the eye?

Pooling Layer



By “**pooling**” (e.g., taking max) filter responses at different locations we gain robustness to the exact spatial location of features.

Pooling Layer: Examples

Max-pooling:

$$h_j^n(x, y) = \max_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

Average-pooling:

$$h_j^n(x, y) = 1/K \sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

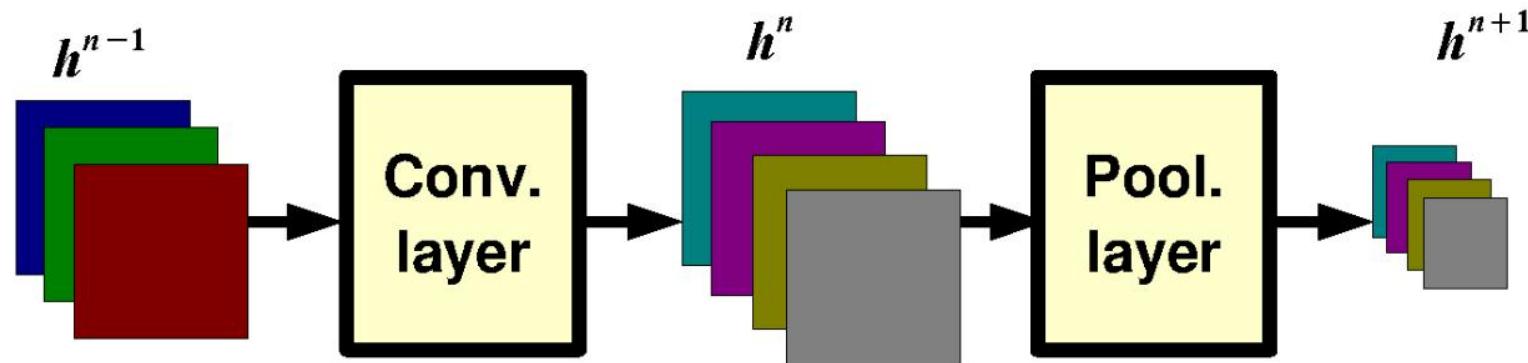
L2-pooling:

$$h_j^n(x, y) = \sqrt{\sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})^2}$$

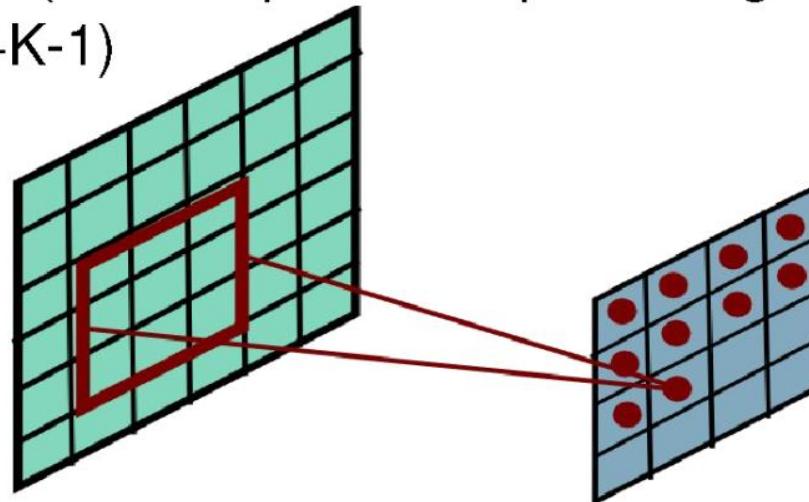
L2-pooling over features:

$$h_j^n(x, y) = \sqrt{\sum_{k \in N(j)} h_k^{n-1}(x, y)^2}$$

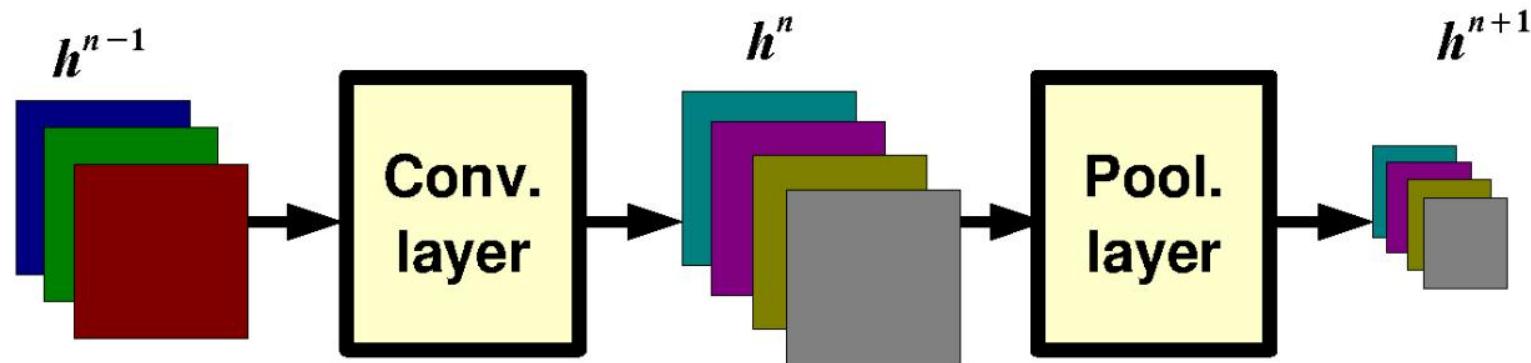
Pooling Layer: Receptive Field Size



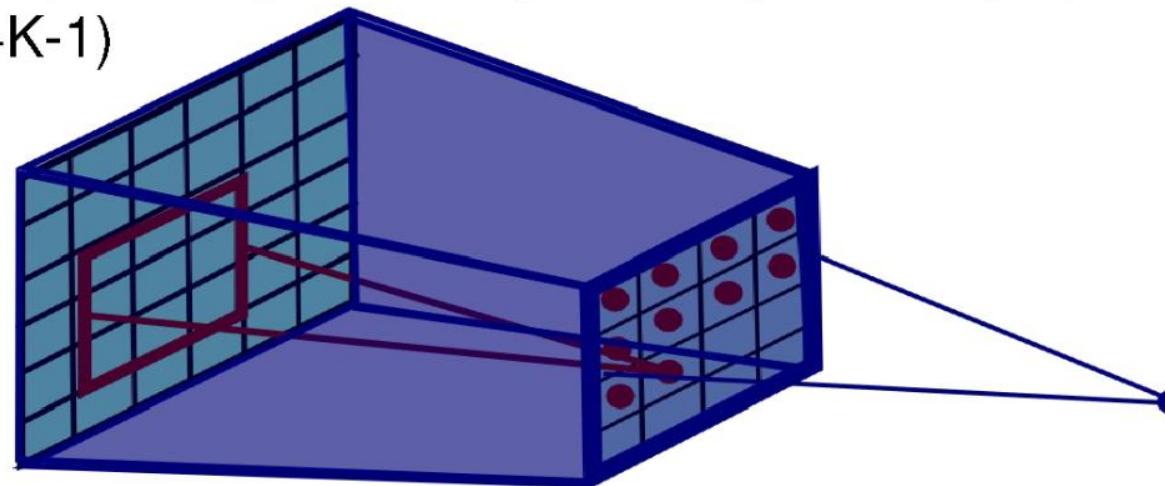
If convolutional filters have size $K \times K$ and stride 1, and pooling layer has pools of size $P \times P$, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size:
 $(P+K-1) \times (P+K-1)$



Pooling Layer: Receptive Field Size

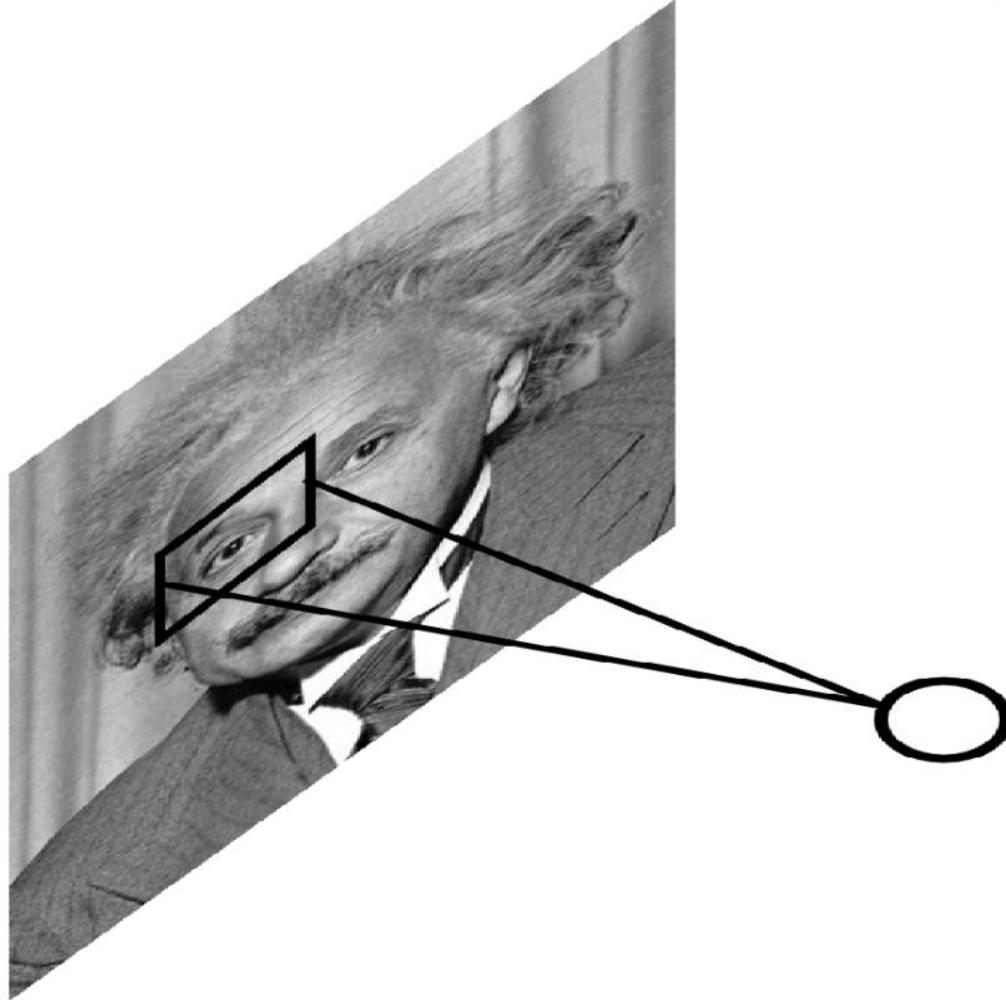


If convolutional filters have size $K \times K$ and stride 1, and pooling layer has pools of size $P \times P$, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size:
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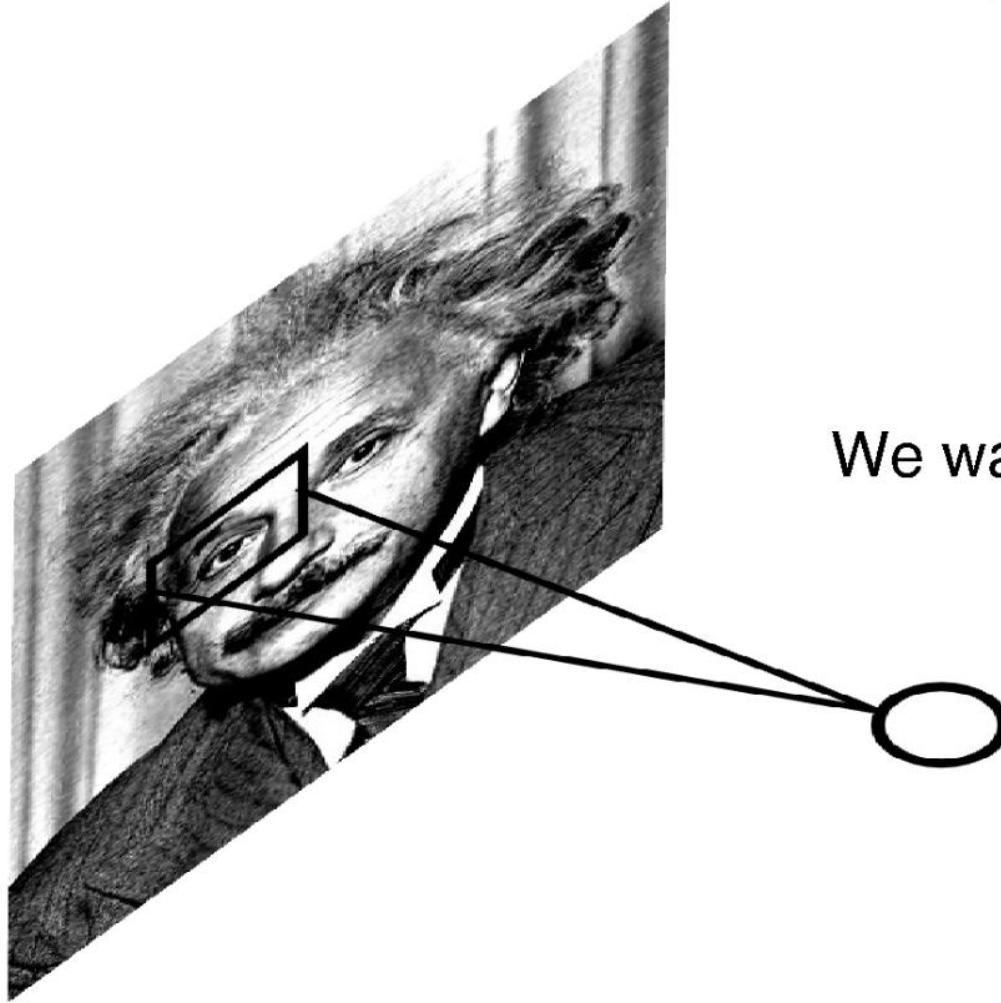
Local Contrast Normalization

$$h^{i+1}(x, y) = \frac{h^i(x, y) - m^i(N(x, y))}{\sigma^i(N(x, y))}$$



Local Contrast Normalization

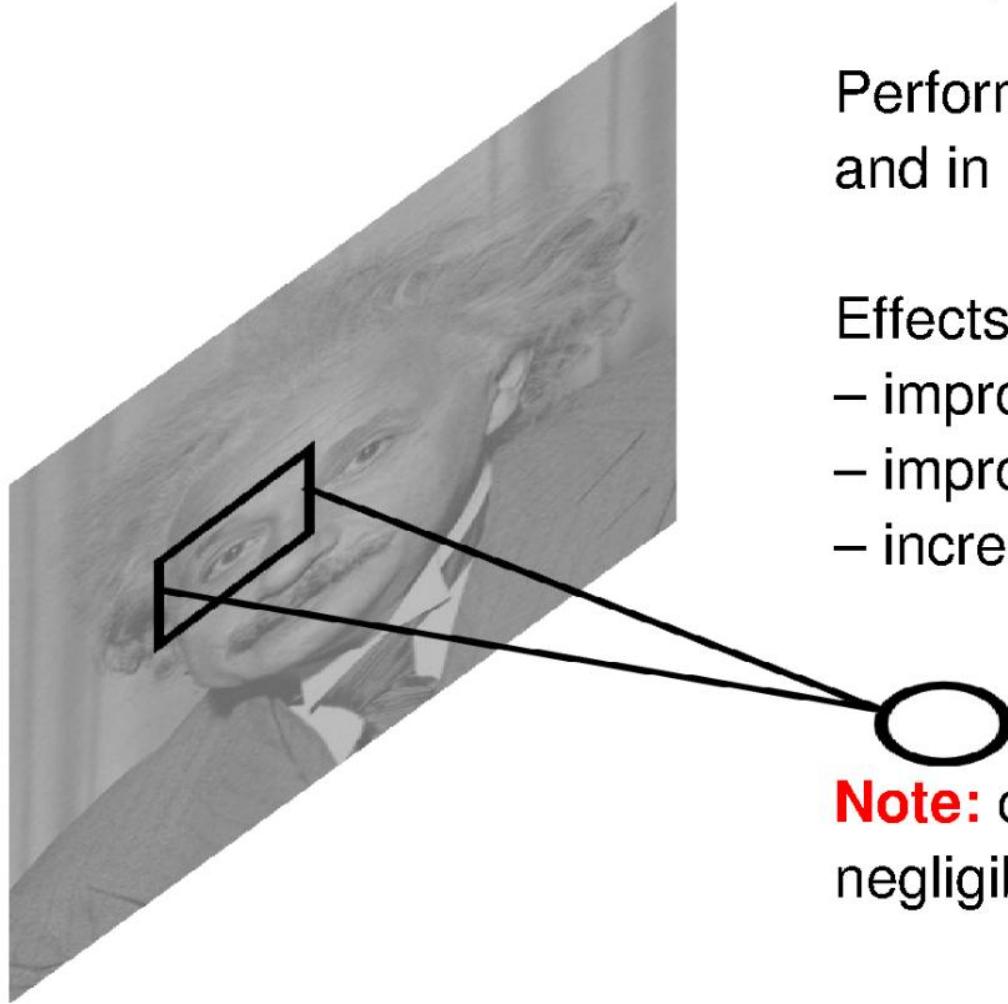
$$h^{i+1}(x, y) = \frac{h^i(x, y) - m^i(N(x, y))}{\sigma^i(N(x, y))}$$



We want the same response.

Local Contrast Normalization

$$h^{i+1}(x, y) = \frac{h^i(x, y) - m^i(N(x, y))}{\sigma^i(N(x, y))}$$



Performed also across features
and in the higher layers..

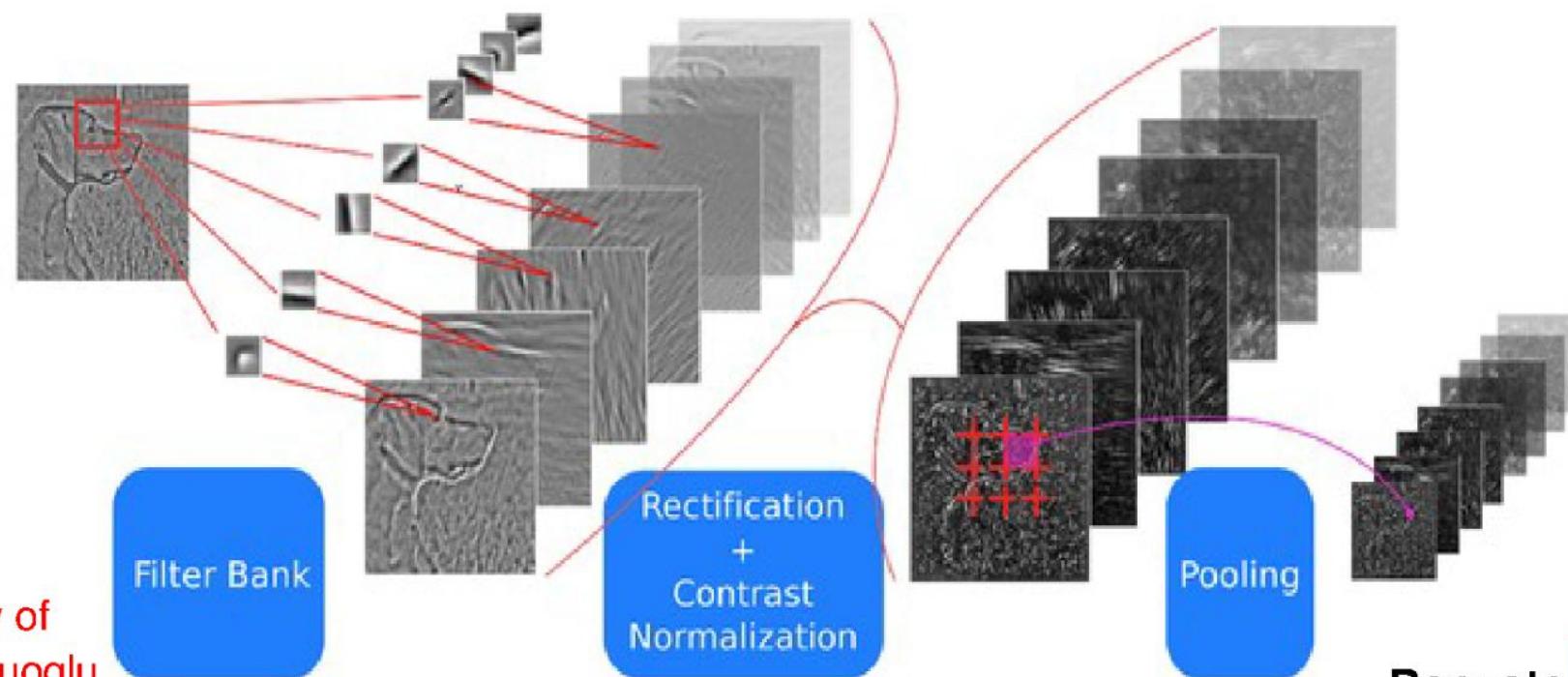
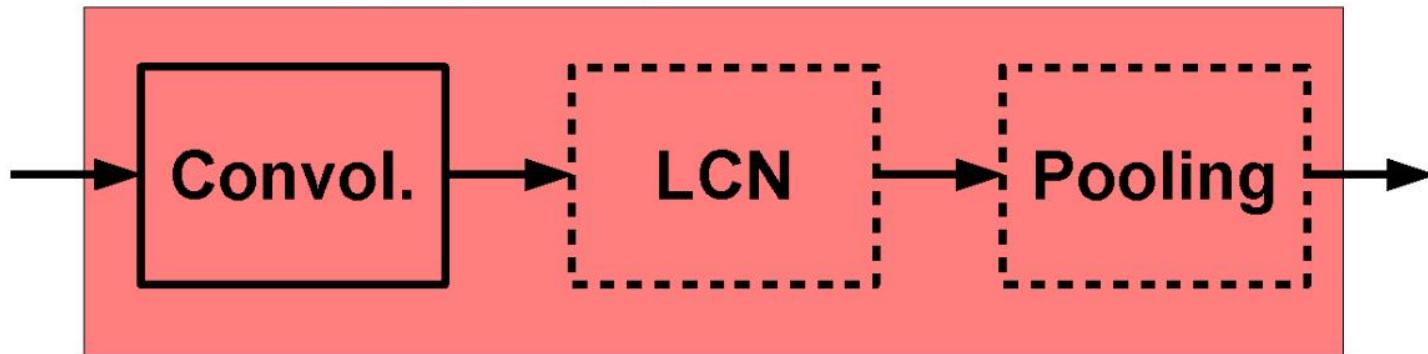
Effects:

- improves invariance
- improves optimization
- increases sparsity

Note: computational cost is negligible w.r.t. conv. layer.

ConvNets: Typical Stage

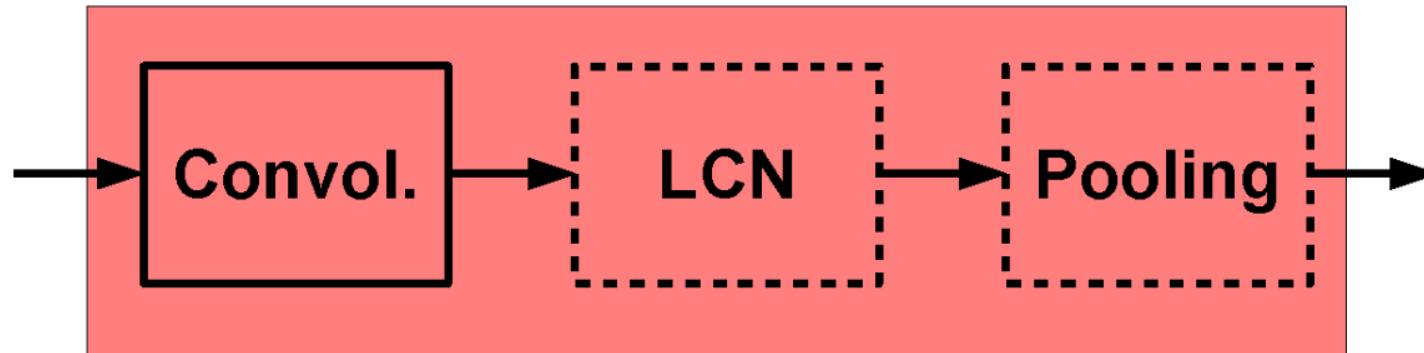
One stage (zoom)



courtesy of
K. Kavukcuoglu

ConvNets: Typical Stage

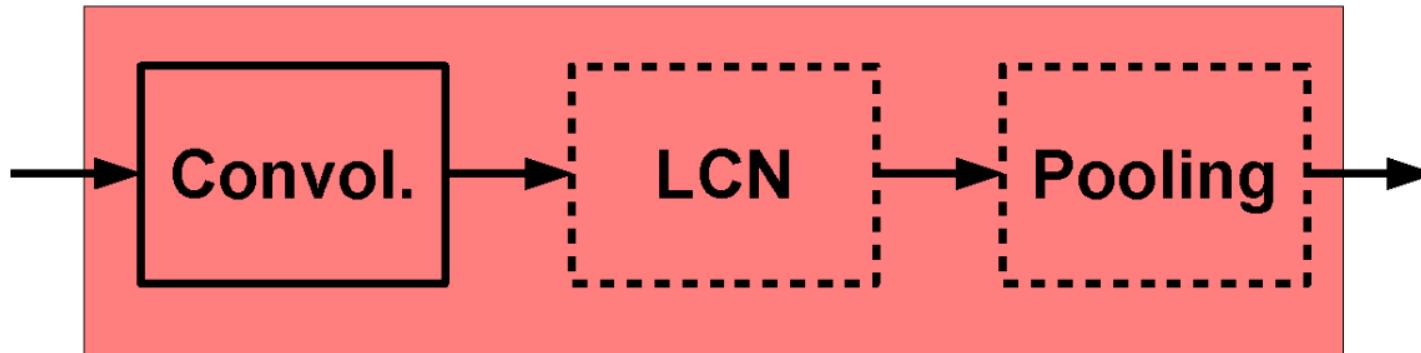
One stage (zoom)



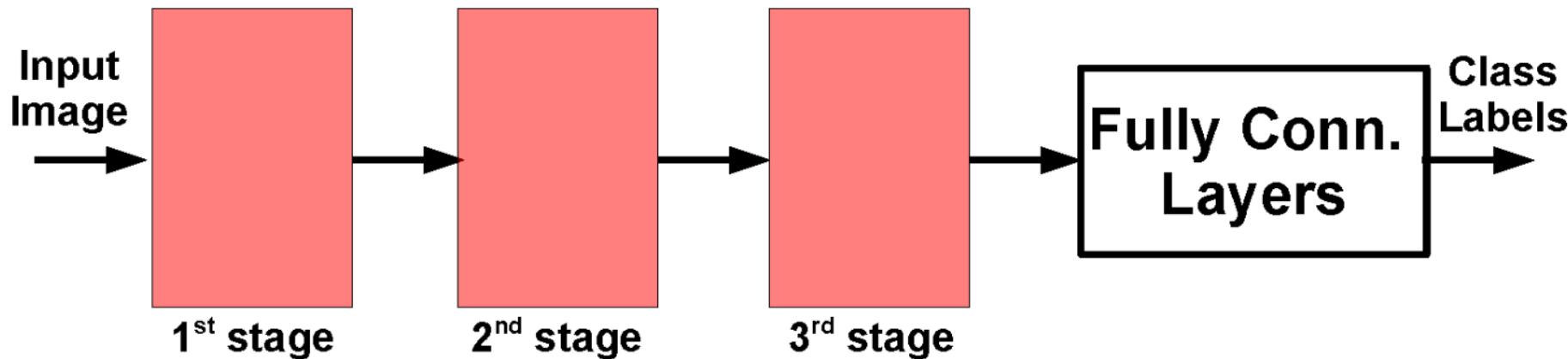
Conceptually similar to: SIFT, HoG, etc.

ConvNets: Typical Architecture

One stage (zoom)

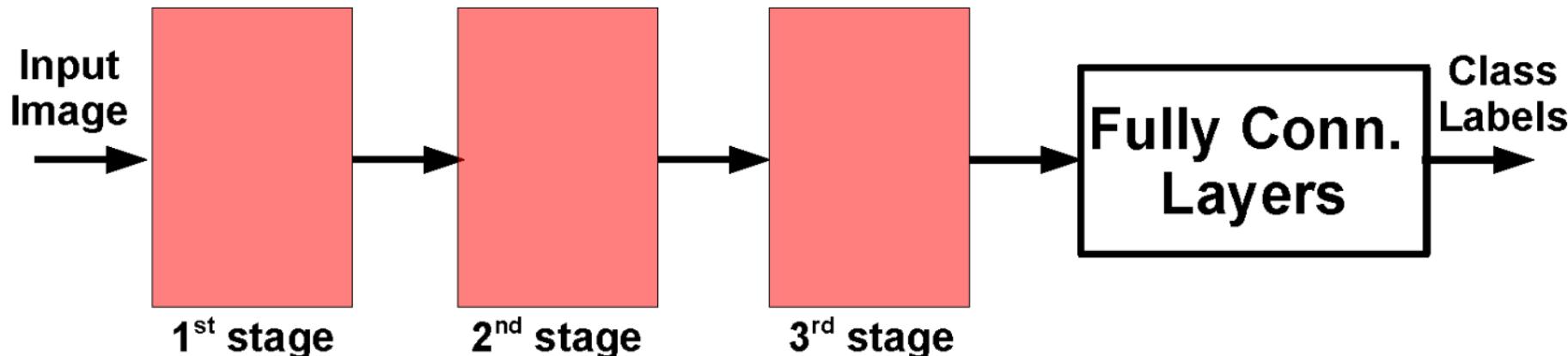


Whole system



ConvNets: Typical Architecture

Whole system



Conceptually similar to:

SIFT → K-Means → Pyramid Pooling → SVM

Lazebnik et al. "...Spatial Pyramid Matching..." CVPR 2006

SIFT → Fisher Vect. → Pooling → SVM

Sanchez et al. "Image classification with F.V.: Theory and practice" IJCV 2012

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration
Neural Network	1
Nearest Neighbor	10
Linear SVM	10
Non-linear SVM	5
Decision Tree or Random Forest	4

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation
Neural Network	1	1
Nearest Neighbor	10	10
Linear SVM	10	9
Non-linear SVM	5	4
Decision Tree or Random Forest	4	4

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation	Speed / memory when training
Neural Network	1	1	1
Nearest Neighbor	10	10	8
Linear SVM	10	9	10
Non-linear SVM	5	4	2
Decision Tree or Random Forest	4	4	4

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation	Speed / memory when training	Speed / memory at test time
Neural Network	1	1	1	6
Nearest Neighbor	10	10	8	4
Linear SVM	10	9	10	10
Non-linear SVM	5	4	2	2
Decision Tree or Random Forest	4	4	4	8

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation	Speed / memory when training	Speed / memory at test time	Accuracy w/ lots of data
Neural Network	1	1	1	6	10
Nearest Neighbor	10	10	8	4	7
Linear SVM	10	9	10	10	5
Non-linear SVM	5	4	2	2	8
Decision Tree or Random Forest	4	4	4	8	7

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Neural Network	1	1	1	6	10
Nearest Neighbor	10	10	8	4	7
Linear SVM	10				
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Decision Tree or Random Forest	4				

Representation design matters
more for all of these