## Language and Notation

## March 2024

This is a compendium of things which we would normally explain in maths competitions for teenaged schoolchildren.

- 1. Established mathematical notation will be used.
- 2. We might use a colon or a vertical line as a separator in set notation so  $\{x \mid x \in Z, x > 0\} = \{y : y \in Z, y > 0\}$ .
- 3. Floor and ceiling notation, so for x a real number we let  $\lfloor x \rfloor = \max\{z \mid x \in \mathbb{Z}, z \leq x\}$ . Similarly  $\lceil x \rceil = \min\{z \mid x \in \mathbb{Z}, z \geq x\}$ .
- 4. Fractional part notation. If x is a real number, we define  $\{x\}$  to mean x |x|.
- 5. We write a line over a non-negative integer written in base 10 notation to indicate that it is being viewed as a string of digits rather than a number. Thus the second digit of  $\overline{1729}$  is 7 but 1729 does not have a second digit because it is an integer.
- 6. We allow a phrase such as x is a 3-digit positive integer to mean that if written in Arabic notation as  $x = a_m a_{m-1} \cdots a_1$  with  $a_i$  all digits and  $a_m \neq 0$ , then n = m. We allow 'the sum of the digits of n' to mean: write n in Arabic base 10 notation and then sum the digits.
- 7. We allow informal probability language such as: a point is chosen uniformly at random in the interval [0,1].
- 8. We use  $\binom{n}{r}$  to denote the number of ways of choosing r things from n things.
- 9. The sum over the empty set is 0 and the product over the empty set is 1.
- 10. We use an ellipsis to denote an obvious pattern, either on the line of print of midline (as appropriate) so the set of the first n positive integers can be written  $\{1, 2, ..., n\}$  and their sum is  $1 + 2 + \cdots + n$ .
- 11. For integers l, m, n then  $l^{m^n}$  denotes  $l^{(m^n)}$ .
- 12.  $m^0 = 1$  for all integers m (including 0) if doing combinatorial enumeration. If x is real then  $x^0$  needs to be clarified if x = 0.
- 13. British or American versions of English can be used. Thus "highest common factor" means the same as "greatest common divisor".
- 14. A prefix or subscript may be used to indicate features of a triangle associated with vertices. Thus triangle ABC has three altitudes, and the one dropped from A could be denoted the altitude through A, the A-altitude or the altitude  $h_a$ . Similarly for median lines.
- 15. If the term *natural number* is used, then it will be made clear if 0 is a natural number.
- 16. := means 'is defined to be equal to'.