Lecture 1 - 8^{th} Jan

• Circuit Variables and Elements

• Voltage

↳ charge (q) → electrons or ions

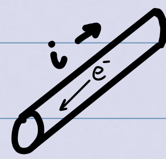
$-$ (electrons) $+$ (ions)

charges create electric fields ⇒ forces ⇒ work (energy).

- $V = \frac{dW}{dq}$ ("amount of work per charge")

  - V = Voltage [V → volts] ↷ $V = J/C$
  - W = Energy [J → joules]
  - q = Charge [C → coulombs].

• Current : Flow of positive charge



$A = C/s$

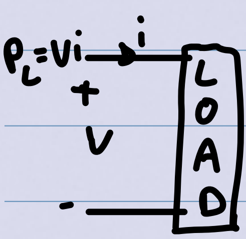
- $i = \frac{dq}{dt}$ ("charge per time") ⇒ [A → amperes]

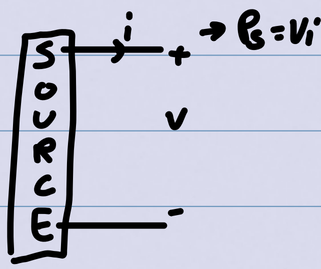
Lecture 2: 10^{th} Jan

Power : $\frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = V \cdot i = p.$

↳ $p = vi$ [W → watts]

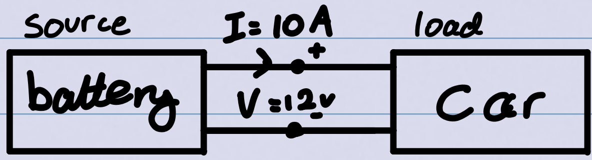
$\Big\}$ power = voltage × current





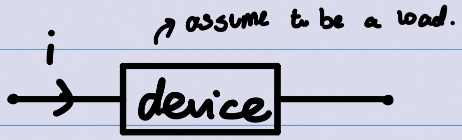
→ $P_L = P_s$.

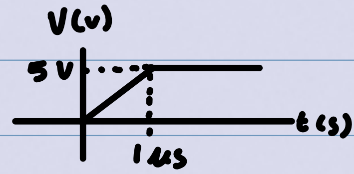
Example:

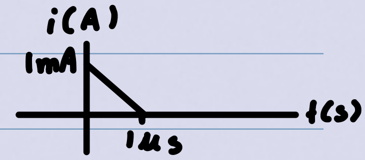


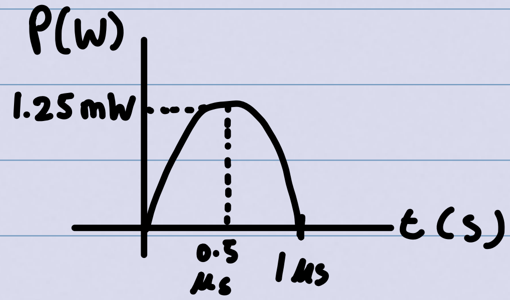
$\therefore P = 12V \times 10A = 120W.$

Example:





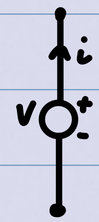




$$P_{max} = V(0.5\mu s) \times i(0.5\mu s)$$
$$\tfrac{5}{2}V \cdot \tfrac{1}{2}mA = 1.25\,mW$$

Ideal Sources:

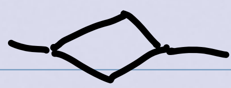
· Independent Source:



Voltage and current sources.

In a voltage source, voltage is defined but current is unknown. The opposite is true for a current source.

Dependent source:



⋮

$R = \frac{V}{i}$ $\quad [\frac{V}{A} = \Omega]$ (resistance)

these elements always absorb power, not produce it!

$G = \frac{1}{R} = \frac{i}{V}$ $\quad [\frac{A}{V} = \Omega^{-1}]$ (conductance)

$$\therefore P = Vi = Ri^2 = \frac{V^2}{R} = GV^2$$

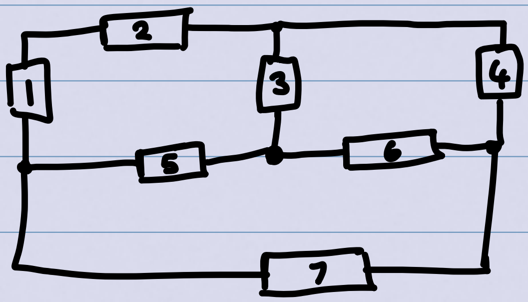
Kirchoff's Laws:

Node → connecting point w/ 3 or more wires

Loop → window panes

Branches → elements

KCL: $\sum_{\text{nodes}} i_{\text{branches}} = 0.$



KVL: $\sum_{\text{loop}} V_{\text{branches}} = 0.$

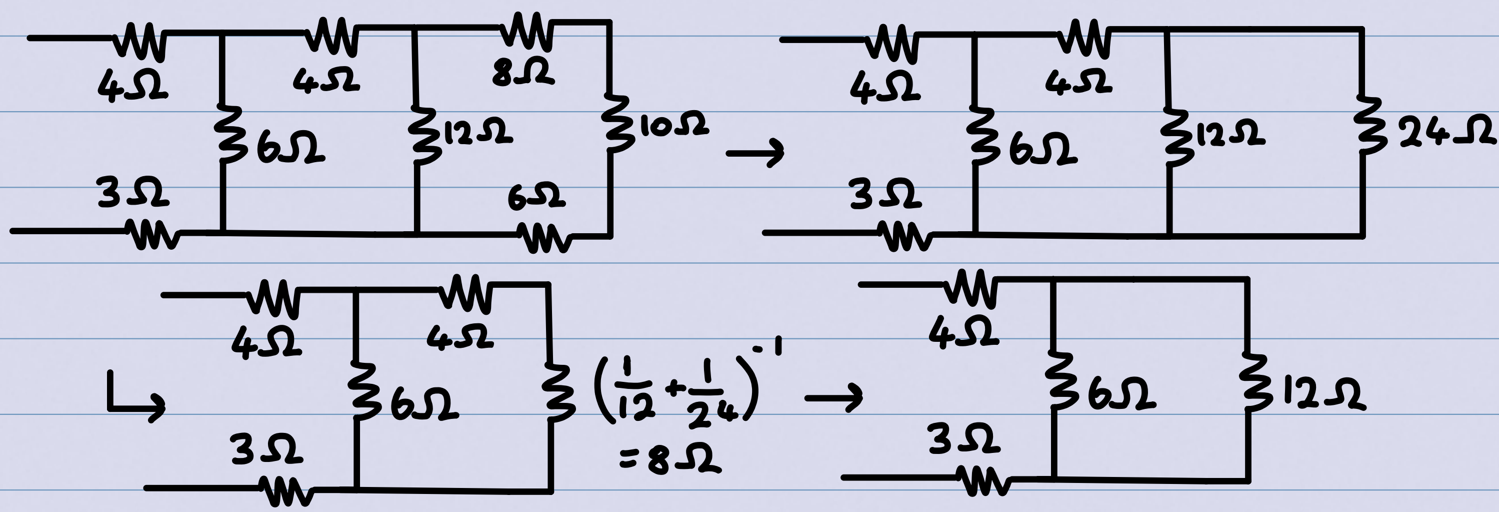
- For N resistors in series, $R_{eq} = R_1 + R_2 + \ldots + R_N = \sum_{n=1}^{N} R_n$

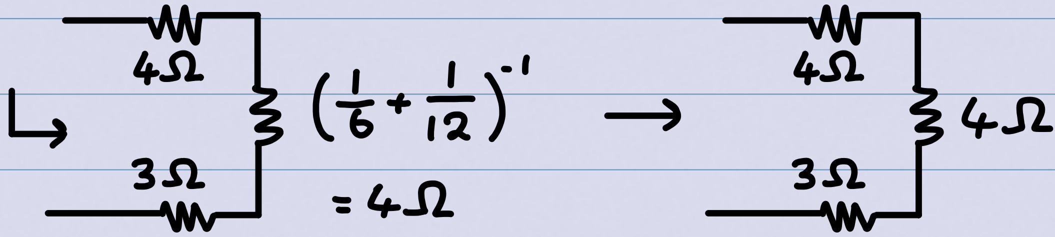
  ↳ Also, $V_n = \dfrac{R_n}{R_1 + R_2 + \ldots + R_n} V$ ↪ the voltage drop across the n^{th} resistor!

- For N resistors in parallel, $R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}\right)^{-1}$

  ↳ If $R_1 = R_2 = \ldots = R_n$, then $R_{eq} = \dfrac{R}{n}$

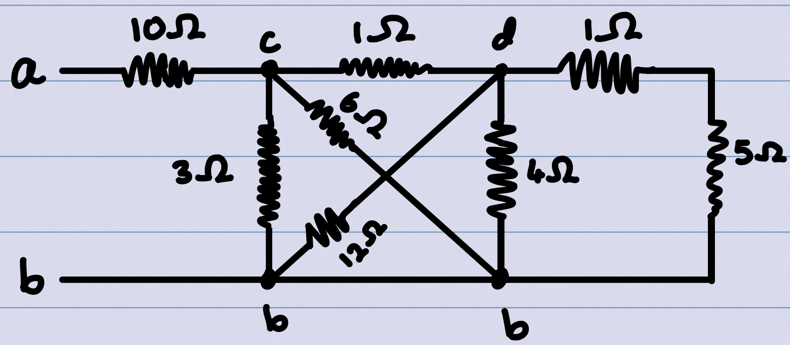
Example: find Req for the following circuit:





$\therefore R_{eq} = 4 + 4 + 3 = 11\,\Omega$

Example: Find R_{ab} in the following circuit:



both nodes b at the bottom are the same, as there is no voltage drop between them!

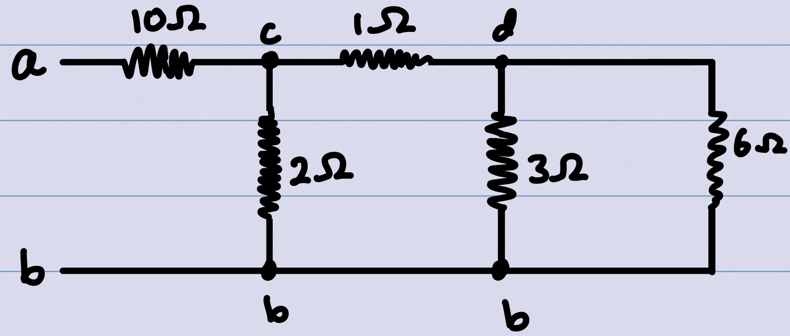
the 3Ω and 6Ω resistors are in parallel, since they are both connected to nodes b and c!

$$\therefore R_{3,6} = \left(\frac{1}{3}+\frac{1}{6}\right)^{-1} = \left(\frac{3}{6}\right)^{-1} = 2\,\Omega.$$

Similarly, the 4Ω and 12Ω resistors are also in parallel, as they are both connected to nodes b and d.

$$\therefore R_{4,12} = \left(\frac{1}{4}+\frac{1}{12}\right)^{-1} = \left(\frac{4}{12}\right)^{-1} = 3\,\Omega.$$

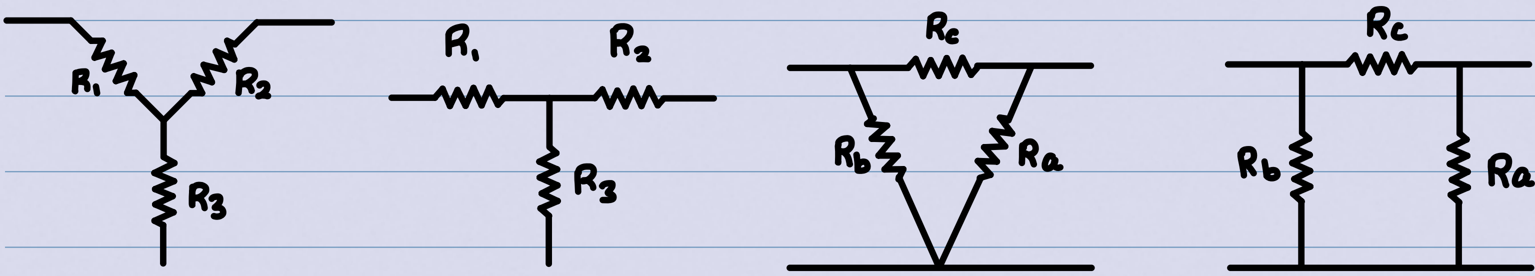
So, the circuit can be re-drawn as follows:



From here, it's a simple circuit:

$$R_{ab} = 10 + \left(\frac{1}{2} + \frac{1}{1 + \left(\frac{1}{3} + \frac{1}{6}\right)^{-1}} \right)^{-1} = 11.2\ \Omega$$

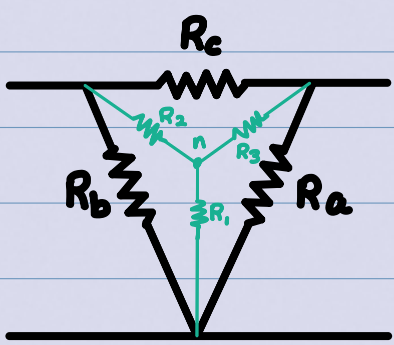
Wye - Delta Transformations



Wye (Y) Tee (T) Delta (Δ) Pi (π)

equivalent ! equivalent !

Generally, we want to convert Δ networks to Y networks. To do so, we superimpose a new central node n in the center, and calculate the new resistances.



We want to calculate R_1, R_2, and R_3 such that the system is the same.

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}, \quad R_2 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_c}{R_a + R_b + R_c}$$

- Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

However, sometimes it might be helpful to transform the other way ($Y \rightarrow \Delta$).

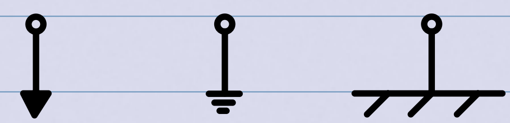
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}, \quad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}, \quad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

- Each resistor in the Δ network is the sum of all possible products of two Y resistors, divided by the opposite Y resistor.

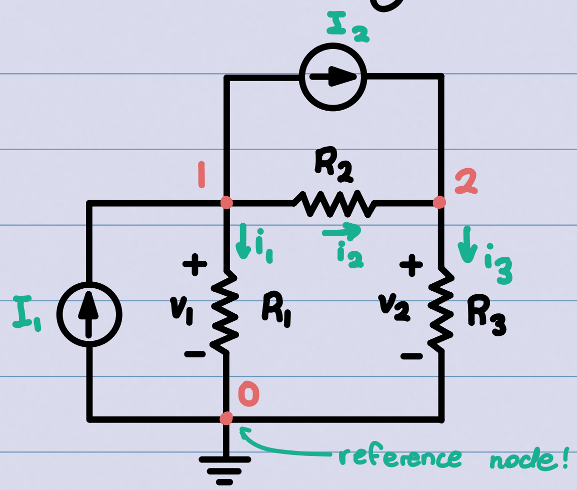
Nodal Analysis without Voltage Sources:

- If the circuit has no voltage sources,
  1) Select a node as the reference node. Assign voltages $V_1, V_2, \ldots, V_{n-1}$ to the remaining $n-1$ nodes.
  2) Apply KCL to each of the $n-1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
  3) Solve the resulting simultaneous equations to obtain the unknown node voltages.

- The reference node is commonly called the ground, since it is assumed to have 0 potential.

↳ It can be indicated by: 

Example: perform nodal analysis on the following circuit.



Performing KCL at node 1: $I_1 = I_2 + i_2 + i_1$

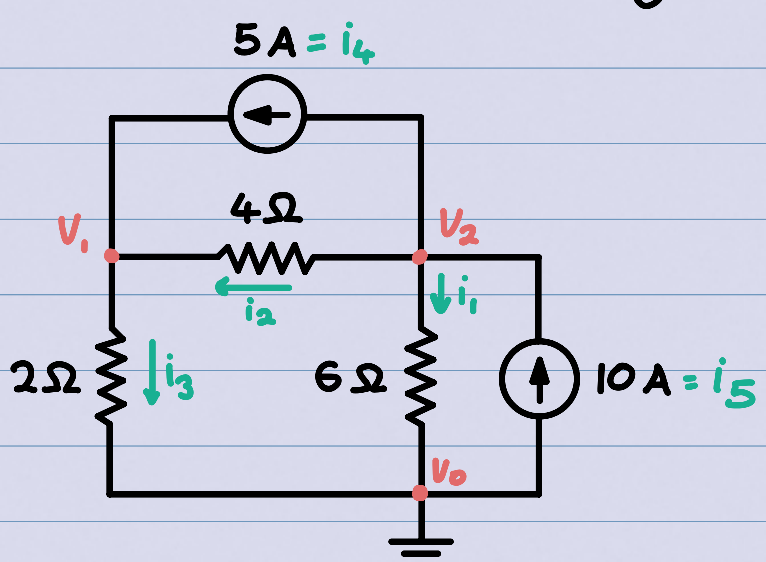
at node 2: $i_2 + I_2 = i_3$

$I_1 = I_2 + i_2 + i_1$: $I_1 = I_2 + \underbrace{\dfrac{V_1 - V_2}{R_2}}_{i_2} + \underbrace{\dfrac{V_1 - V_0}{R_1}}_{i_1}$ → $V_0 = 0$, since it is the reference node!

$i_2 + I_2 = i_3$: $\dfrac{V_1 - V_2}{R_2} + I_2 = \overbrace{\dfrac{V_2 - V_0}{R_3}}^{i_3}$

↳ $I_1 = I_2 + \dfrac{V_1 - V_2}{R_2} + \dfrac{V_1}{R_1}$, $\dfrac{V_1 - V_2}{R_2} + I_2 = \dfrac{V_2}{R_2}$

Example: calculate the node voltages:



KCL at node 1: $i_4 + i_2 = i_3$

node 2: $i_5 = i_1 + i_2 + i_4$

$\hookrightarrow 5 + i_2 = i_3$, $10 = i_1 + i_2 + 5$

$5 + \frac{V_2 - V_1}{4} = \frac{V_1 - 0}{2}$, $5 = \frac{V_2 - 0}{6} + \frac{V_2 - V_1}{4}$

$5 + \frac{V_2 - V_1}{4} = \frac{V_1}{2}$, $5 = \frac{V_2}{6} + \frac{V_2 - V_1}{4}$ → from here, simple system of equations!

$\downarrow$

$V_1 = 10 + \frac{V_2 - V_1}{2} \rightarrow 2V_1 = 20 + V_2 - V_1 \rightarrow V_1 = \frac{20 + V_2}{3}$

$60 = 2V_2 + 3V_2 - 3V_1 \rightarrow 60 = 5V_2 - 20 - V_2$

$\therefore 80 = 4V_2 \rightarrow V_2 = 20, \; V_1 = 40/3$!

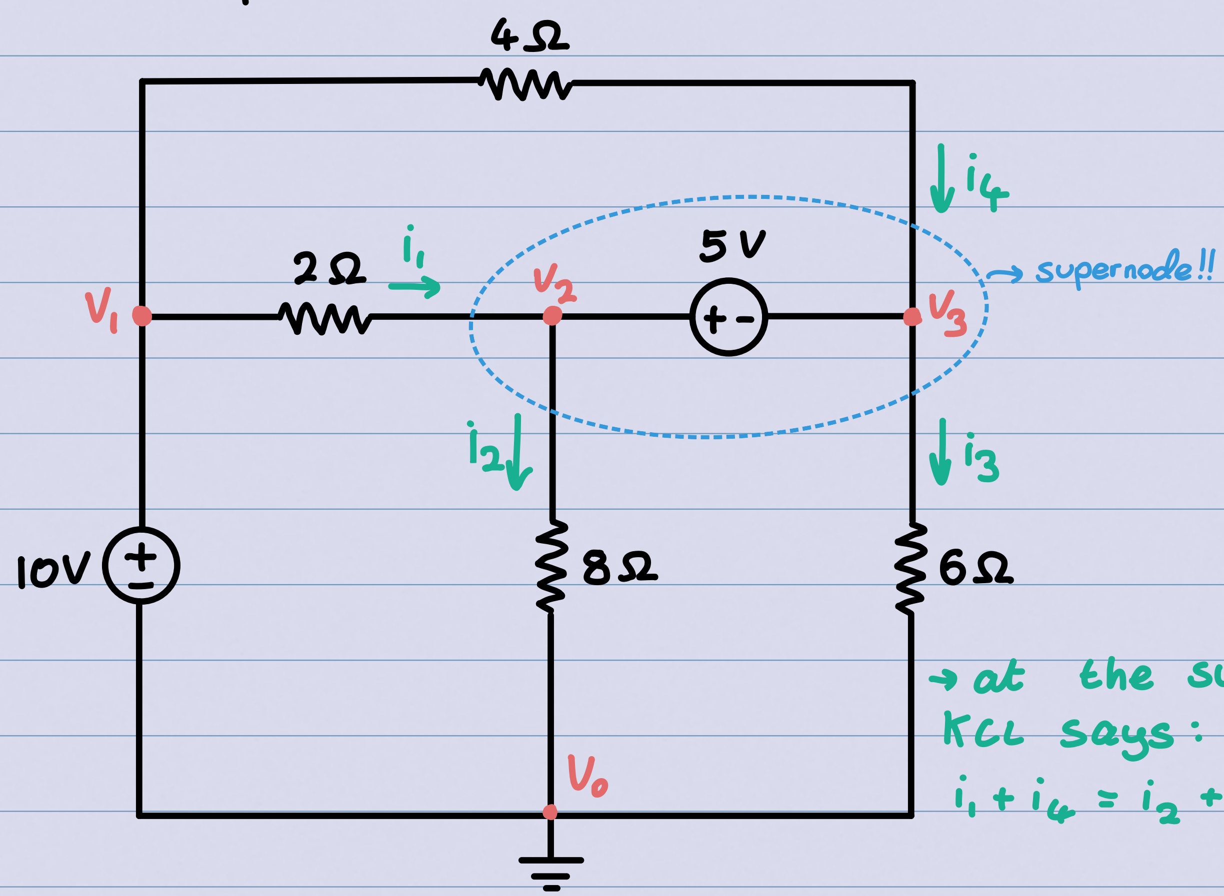
- Current always flows from a higher potential to a lower potential in a resistor.
- The number of nonreference nodes is equal to the number of independent equations we derive.

Nodal Analysis with Voltage Sources:

- Case 1: if a voltage source is connected between the reference node and a nonreference node, simply set the voltage at the nonreference node equal to the voltage of the voltage source.
- Case 2: if a voltage source is connected between two nonreference nodes, these two nodes form a generalized node or a supernode → apply KCL

and KVL to determine node voltages.

Example of possible supernode:

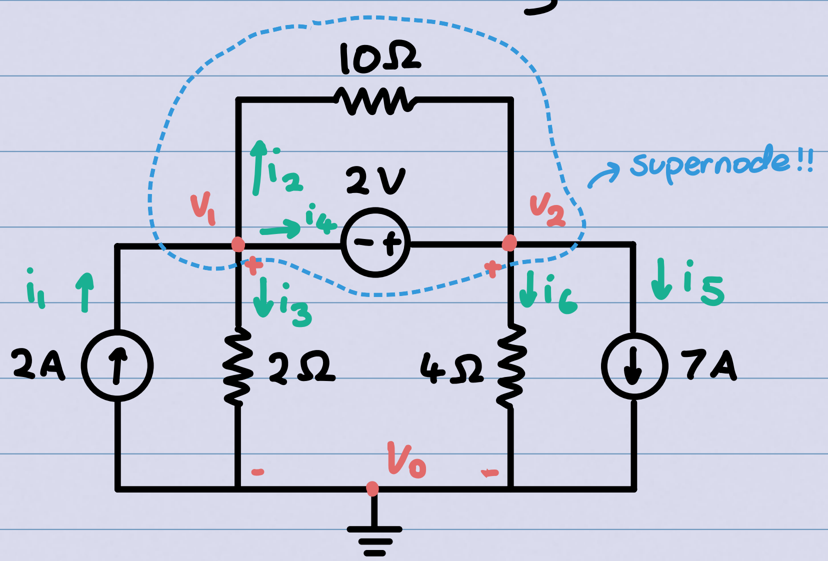


→ at the supernode, KCL says:

$i_1 + i_4 = i_2 + i_3$!

- A supernode may be regarded as a closed surface enclosing the voltage source + the two nodes.

Example: Find the node voltages:



gunuir

KCL on the superode: $i_1 = i_3 + i_6 + i_5 \rightarrow 2 = i_3 + i_6 + 7$

$\hookrightarrow 2 = \dfrac{V_1 - 0}{2} + \dfrac{V_2 - 0}{4} + 7$

$\hookrightarrow 8 = 2V_1 + V_2 + 28 \rightarrow 2V_1 + V_2 = -20$

Then, using KVL on the central loop: $-2i_3 - 2 + 4i_6 = 0$

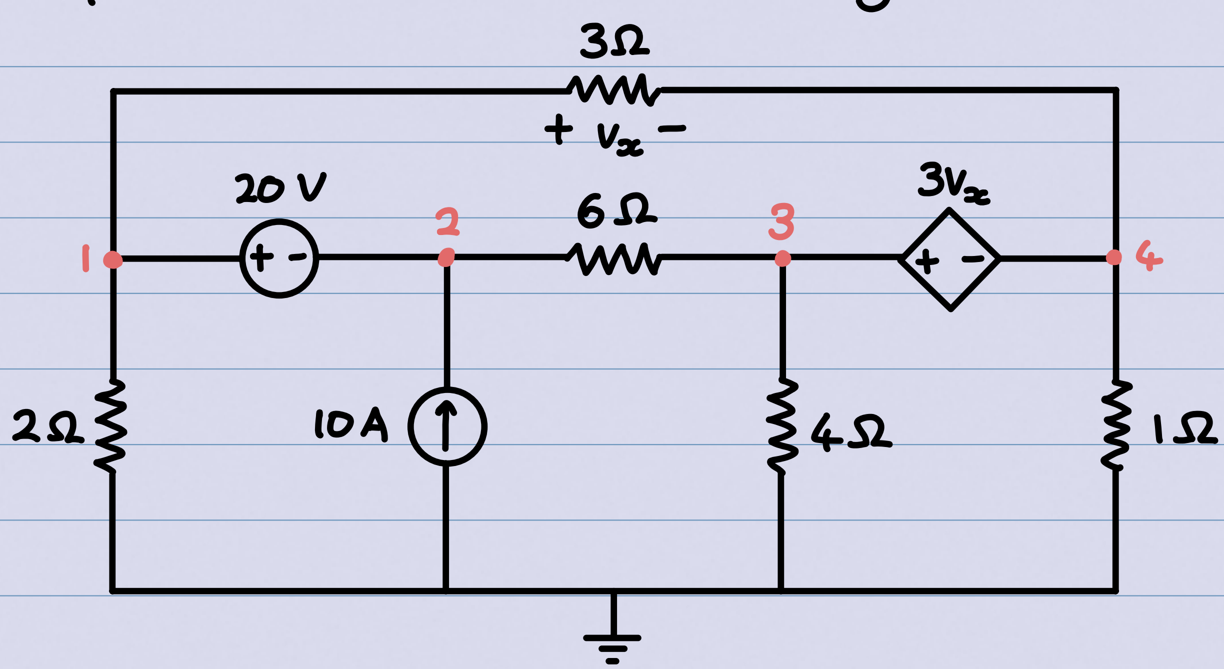
$-2i_3 = -V_1$ and $4i_6 = V_2$.

$\therefore 2V_1 + V_2 = -20, \quad V_2 = 2 + V_1$ simple system of equations from here!

$\hookrightarrow 2V_1 + V_1 + 2 = -20 \rightarrow 3V_1 = -22 \rightarrow V_1 = -\frac{22}{3}$ V.

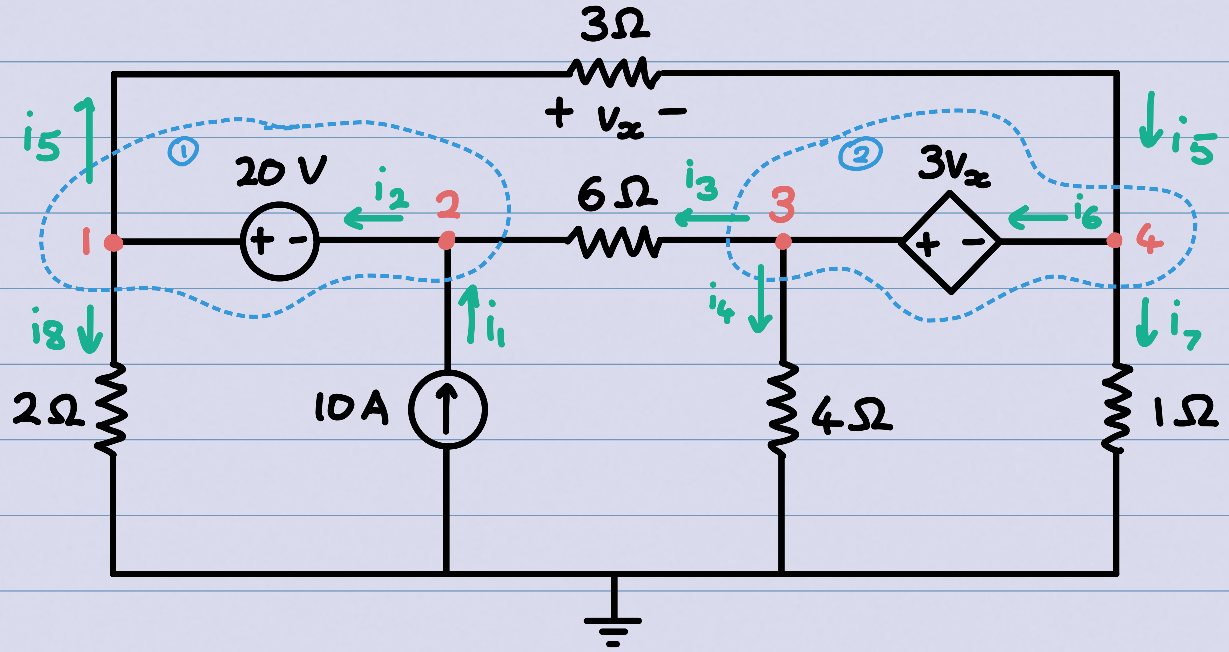
$\therefore$, $V_1 = -\frac{22}{3}$ V and $V_2 = -\frac{16}{3}$ V.

Example: Find the node voltages



Nodes 1/2 and nodes 3/4 are supernodes!

↓



KCL in supernode 1: $i_1 + i_3 = i_6 + i_8 \rightarrow i_3 + 10 = i_5 + i_8$

KCL in supernode 2: $i_5 = i_3 + i_4 + i_7$

$$i_3 + 10 = i_5 + i_8 \rightarrow \frac{V_3 - V_2}{6} + 10 = \frac{V_1 - V_4}{3} + \frac{V_1 - 0}{2}$$

$$\hookrightarrow V_3 - V_2 + 60 = 2V_1 - V_4 + 3V_1$$

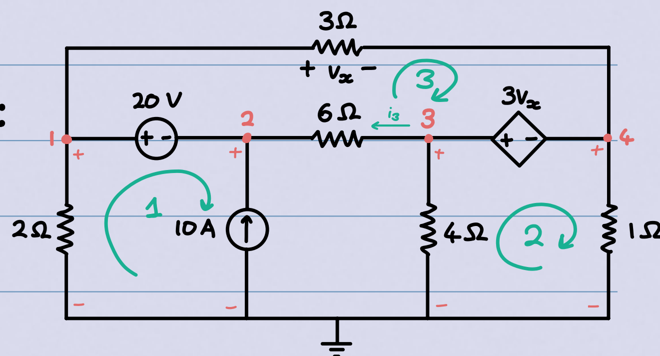
$$\hookrightarrow 5V_1 + V_2 - V_3 - V_4 = 60$$

$$i_5 = i_3 + i_4 + i_7 \rightarrow \frac{V_1 - V_4}{3} = \frac{V_3 - V_2}{6} + \frac{V_3 - 0}{4} + \frac{V_4 - 0}{1}$$

$$\hookrightarrow 4V_1 - 4V_4 = 2V_3 - 2V_2 + 3V_3 + 12V_4$$

$$\hookrightarrow 4V_1 + 2V_2 - 5V_3 - 16V_4 = 0$$

Now, using KVL as shown:



① $-V_1 + 20 + V_2 = 0$

② $-V_3 + 3V_x + V_4 = 0$

③ $V_x - 3V_x + 6i_3 - 20 = 0$

but, $V_1 - V_x = V_4$, so:

$V_x = V_1 - V_4$!

$\therefore$, we have:
$$\left.\begin{aligned} 5V_1 + V_2 - V_3 - V_4 &= 60 \\ 4V_1 + 2V_2 - 5V_3 - 16V_4 &= 0 \\ V_1 &= V_2 + 20 \\ V_3 &= 3V_1 - 2V_4 \\ -2V_1 - 6V_2 + 6V_3 + 2V_4 &= 0 \end{aligned}\right\} \text{5 equations, 4 unknowns!}$$

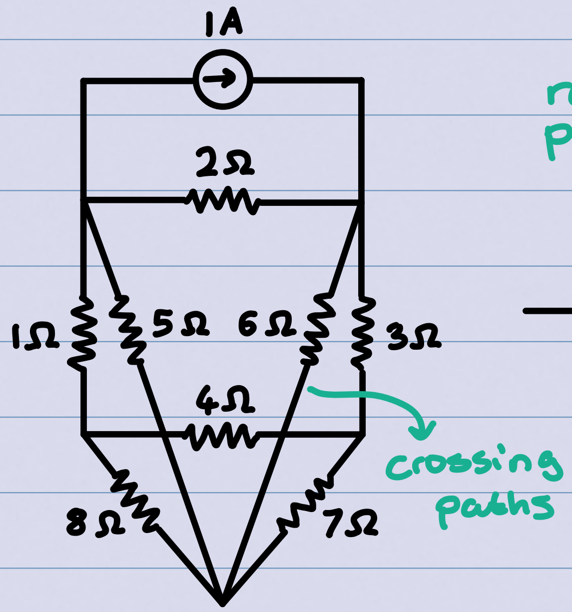
also, $6i_3 = V_3 - V_2$!

$\therefore$, $V_1 = \frac{240}{9}$ V, $V_2 = \frac{20}{3}$ V, $V_3 = \frac{520}{3}$ V, $V_4 = -\frac{140}{3}$ V!

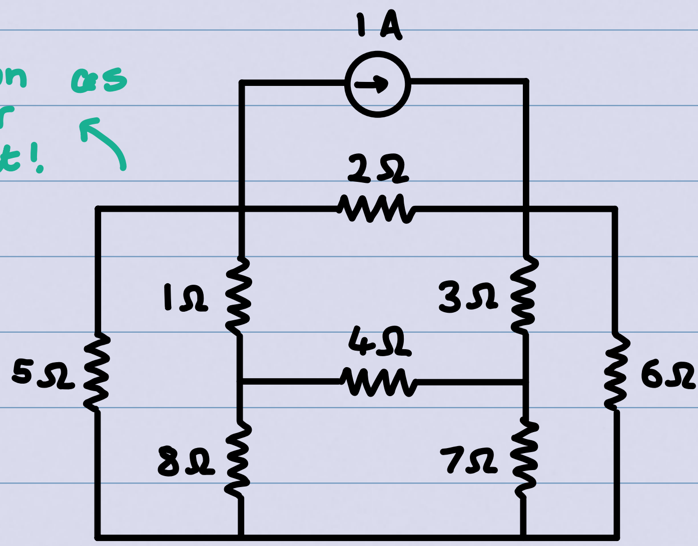
Planar Circuit: can be drawn in a plane with no branches crossing one another.

↳ a circuit may have crossing branches and still be planar if it can be redrawn to avoid crossing branches.

Example: show that this circuit is planar:



redrawn as planar circuit!



• **Mesh:** a loop that does not contain any other loops within it.

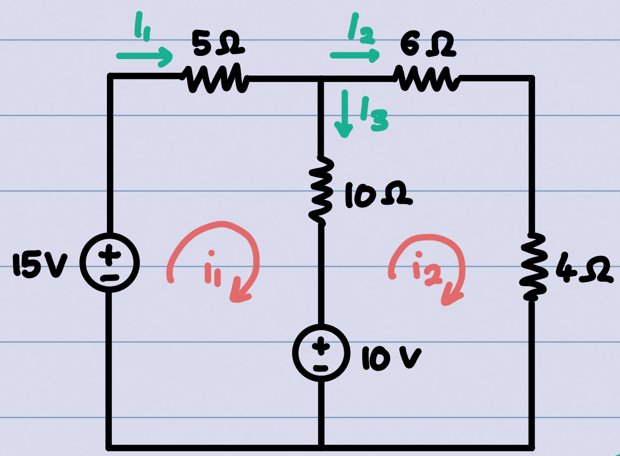
Mesh Analysis: applies KVL to find currents.

↳ only applies to circuits that are planar!

↓

1) Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2) Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3) Solve the resulting n simultaneous equations.

Example: Find the branch currents using mesh analysis.



$i_1 = i_2 + i_3$

↳ $i_3 = i_1 - i_2$.

KVL on mesh 1: $-15 + 5i_1 + 10i_3 + 10 = 0$

$-15 + 5i_1 + 10i_1 - 10i_2 + 10 = 0$

$-5 + 15i_1 - 10i_2 = 0$

↳ $3i_1 - 2i_2 = 1$.

KVL on mesh 2: $-10 - 10i_3 + 6i_2 + 4i_2 = 0$

$-10 - 10i_1 + 10i_2 + 10i_2 = 0$

$20i_2 - 10i_1 = 10$

↳ $i_1 = 2i_2 - 1$.

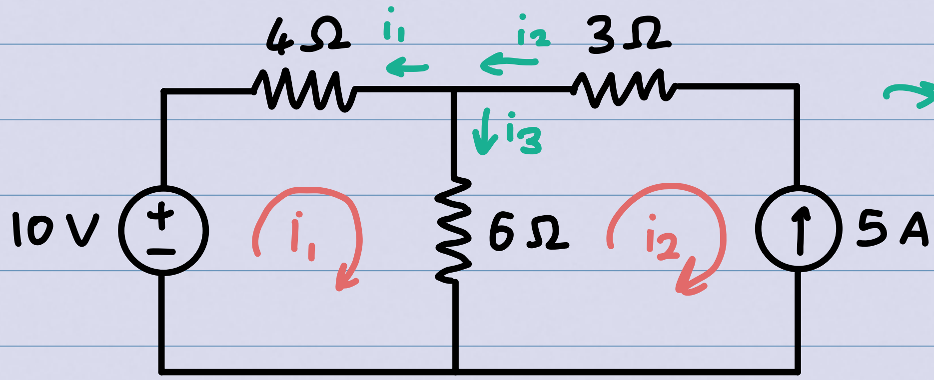
simple system of equations!

$\therefore$, $I_1 = i_1 = 1\,A$, $I_2 = i_2 = 1\,A$, $I_3 = i_1 - i_2 = 0\,A$!

Mesh Analysis with current sources

- **Case 1:** when a current source exists only in one mesh. Set the current of that mesh to the current source, and write mesh equations for the remaining meshs.
- **Case 2:** when a current source exists between two or more meshes, we create a supermesh by excluding the current source and any elements connected in series with it.

Example : mesh analysis case 1 circuit.



→ $i_1 = i_2 + i_3$
↳ $i_3 = i_1 - i_2$

opposite direction!

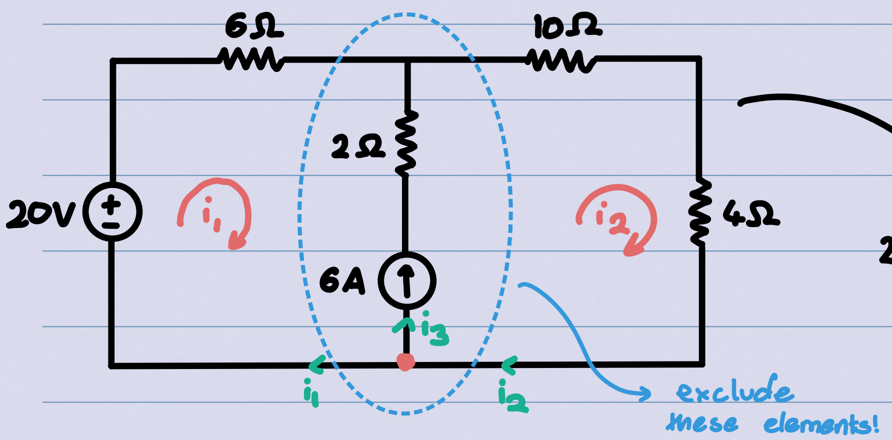
↳ i_2 is set to −5 A, and we perform KVL on mesh 1 as follows : $-10 + 4i_1 + 6i_3 = 0$

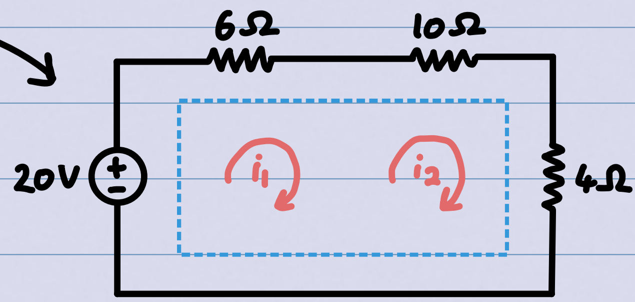
↪ $-10 + 4i_1 + 6i_1 - 6i_2 = 0 \rightarrow 20 = -10i_1 \rightarrow i_1 = -2.$

$\therefore i_1 = -2\,A,\ i_2 = -5\,A,\ i_3 = 3\,A$

- **Note:** be very careful setting the mesh current! negative if it's flowing the other way!

Example: mesh analysis case 2 circuit





performing KVL on the supermesh: $-20 + 6i_1 + 10i_2 + 4i_2 = 0$

↳ $6i_1 + 14i_2 = 20 \rightarrow 3i_1 + 7i_2 = 10$.

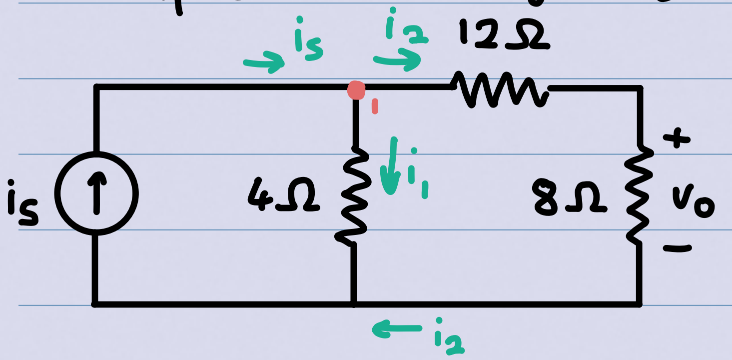
Now, apply KCL to a node in the branch where the two meshes intersect: $i_2 = i_1 + 6$

$\therefore$, $i_1 = -3.2\,A$, and $i_2 = 2.8\,A$.

- **Notes:**
  - ↳ the current source in the supermesh provides the constraint equation necessary to solve for the mesh currents
  - ↳ supermesh has no current of its own.
  - ↳ A supermesh requires the application of both KCL and KVL.

- **Linear Circuit:** output is linearly related (or directly proportional) to its input.

Example: Find V_o when $i_s = 30\text{ A}$ and $i_s = 45\text{ A}$.



KVL: $12i_2 + 8i_2 - 4i_1 = 0$

$\quad \hookrightarrow 5i_2 = i_1$

KCL: $i_s = i_1 + i_2$

$\therefore 6i_2 = i_s \rightarrow i_2 = \frac{i_s}{6}$.

we know that $V_o = 8\,i_2$, so $V_o = \frac{4 i_s}{3}$.

when $i_s = 30$: $V_o = \frac{4(30)}{3} \rightarrow V_o = 40\text{ V}.$

when $i_s = 45$: $V_o = \frac{4(45)}{3} \rightarrow V_o = 60\text{ V}$

Since the difference in these voltages is proportional to the difference in the inputs (i_s), this circuit is linear!

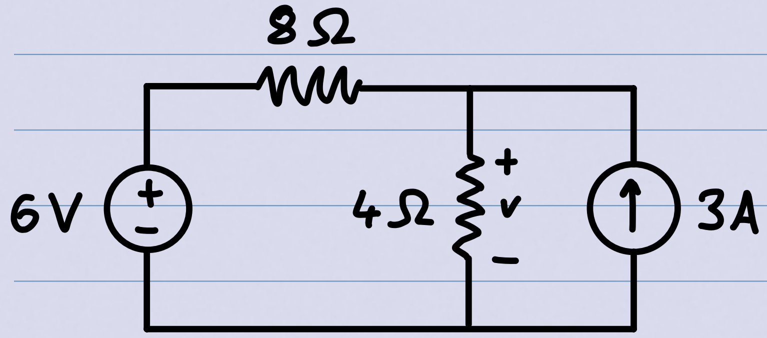
Superposition: the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or current through) that element due to each independent source acting alone.

Steps to Apply Superposition:

1) Turn off all independent sources except one (turn off means set to 0V or 0A). Find the output due to the active source.
2) Repeat step 1 for all the independent sources.
3) Find the total contribution by summing all the contributions due to the independent sources.

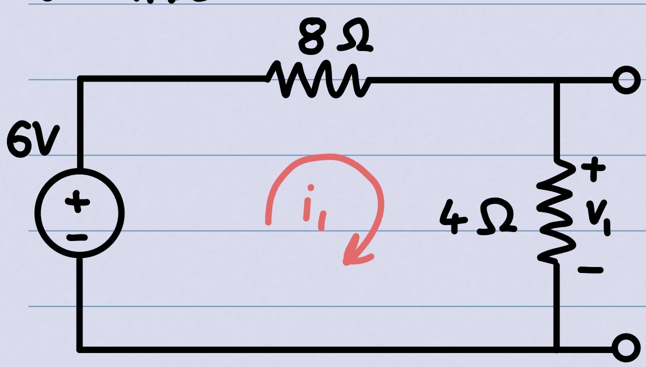
· Note: Superposition only works on linear circuits!

Example: Use superposition to find v:



let $v = v_1 + v_2$, where v_1 is the contribution from the 6V voltage source, and v_2 is the contribution from the 3A current source.

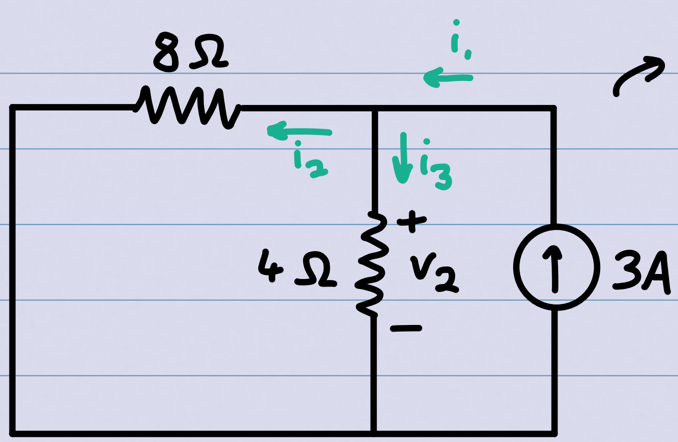
to obtain v_1, we set the current source to 0A and do KVL:



$\to -6 + 8i_1 + 4i_1 = 0$

$\quad \hookrightarrow 12i_1 = 6 \to i_1 = \frac{1}{2}$ A.

to obtain v_2, we set the voltage source to 1A and do KCL:



since $\frac{8}{4} = 2$, $i_3 = 2i_2$.

$\to i_1 = i_2 + i_3$

$\quad 3 = 3i_2 \to i_2 = 1$A.

$\therefore i_3 = 1$A.

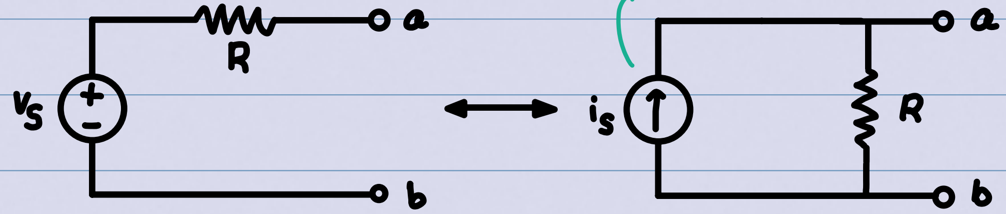
$v_1 = 4\Omega \cdot \frac{1}{2}A = 2V$, $\quad v_2 = 4\Omega \cdot 2A = 8V$.

$\therefore v = v_1 + v_2 = 2 + 8 = 10V$.

Source Transformation: the process of replacing a voltage source V_s in series with a resistor R by a current source in parallel with a resistor R, or vice versa.

↳

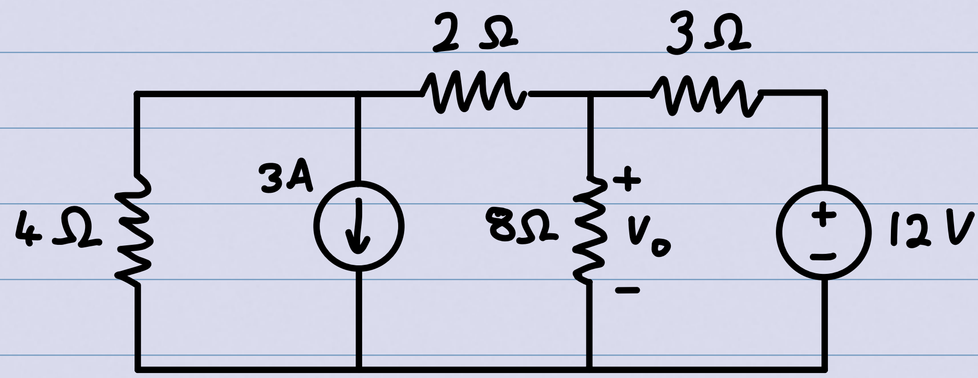
this arrow must be directed in the direction of - to + in V_s!!



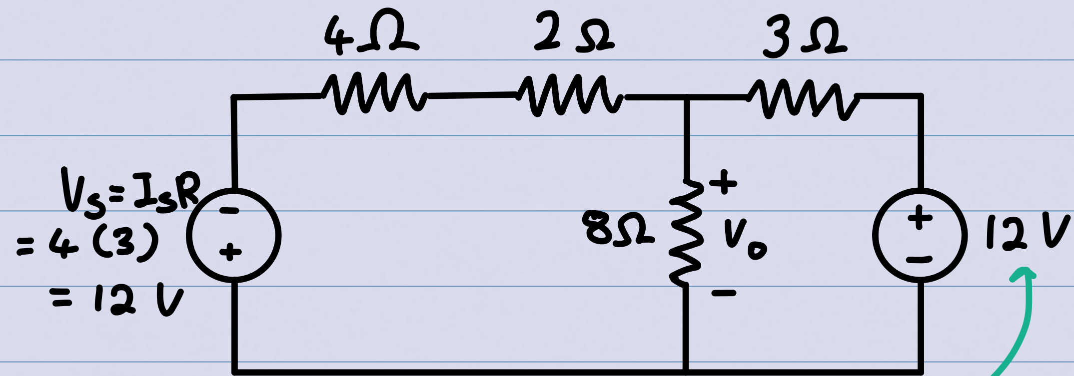
Source Transformations require: $V_s = i_s R$ or $i_s = \frac{V_s}{R}$

- This technique also works for dependent sources!

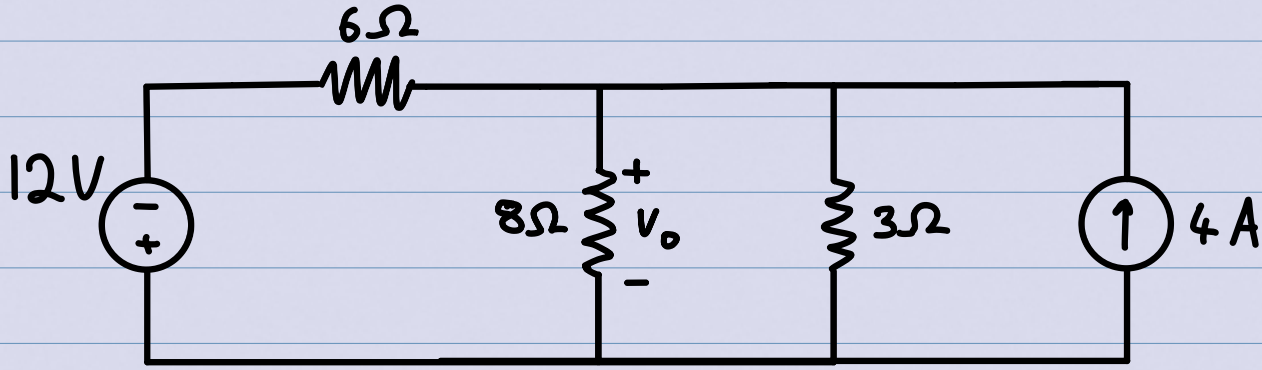
Example: Use source transformation to find V_o:



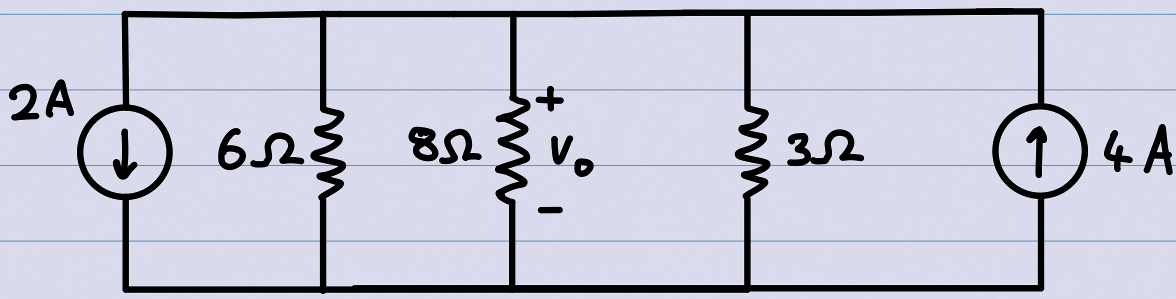
Transforming the 3A current source:



Transforming the 12V (right) voltage source:

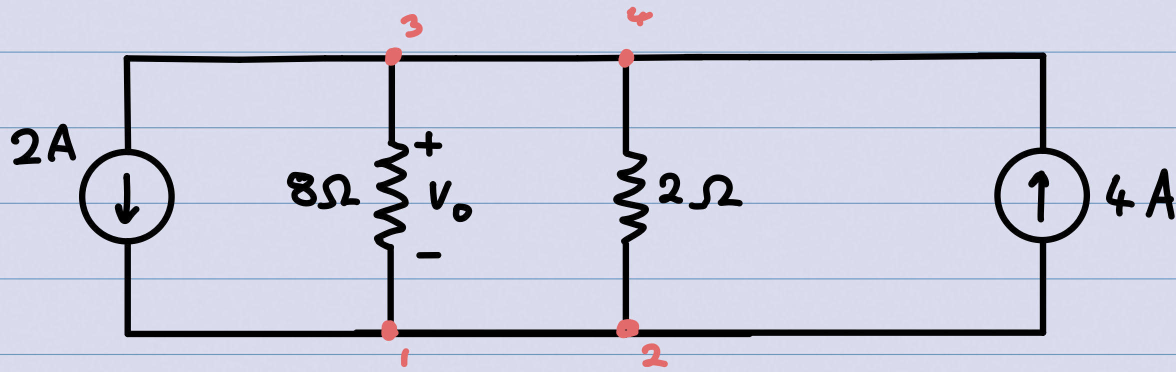


now, transforming the 12V voltage source again! :

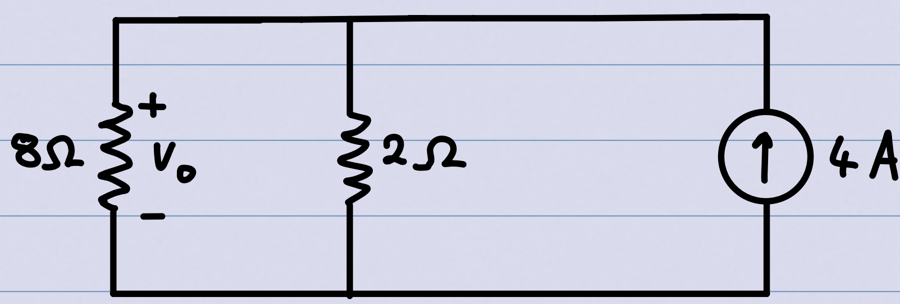


finding the equiv. resistance of the 3Ω and 6 Ω resistors:

$$R = \left(\frac{1}{6}+\frac{1}{3}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2\,\Omega :$$



using superposition, we "turn off" the 2A C source:



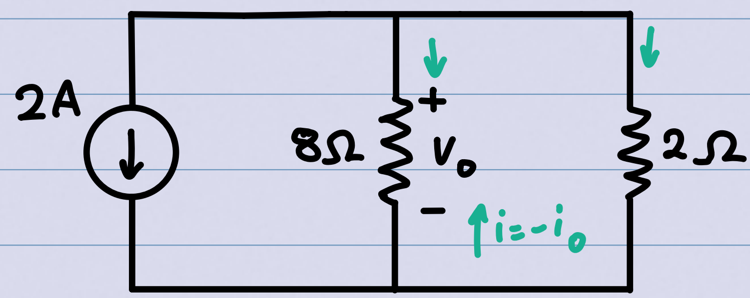
we know the voltage drop accross the 8Ω resistor and the 2Ω resistor must be the same.

$$\left(\frac{1}{8}+\frac{4}{8}\right)^{-1} = \frac{8}{5} = R_{eq}.$$

$\frac{32}{5}$ V is dropped over the resistors

$$V_{drop} = i_o R \rightarrow V_{drop} = 4\left(\frac{8}{5}\right) \rightarrow \frac{32}{5}.$$

now, we "turn off" the 4A C source:



$$R_{eq} = \left(\frac{1}{8}+\frac{4}{8}\right)^{-1} = \frac{8}{5}$$

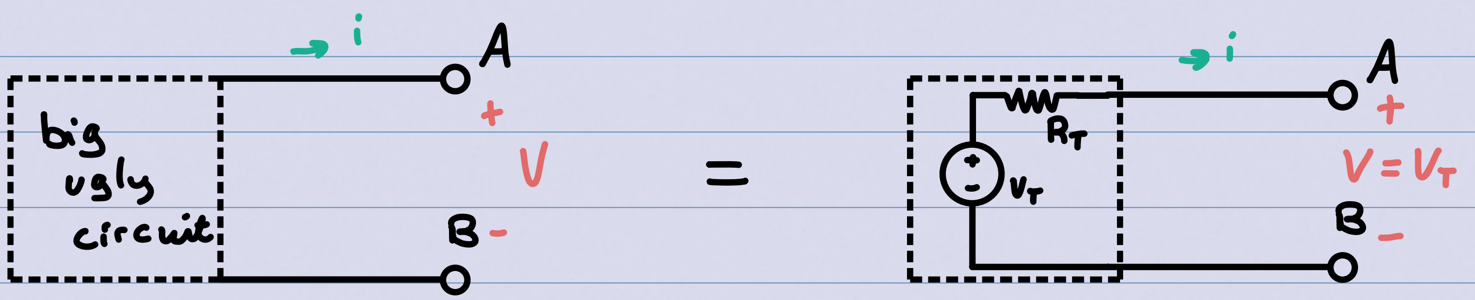
$$V_{drop} = i_o R_{eq} = 2\left(\frac{8}{5}\right) = \frac{16}{5}.$$

since the 2A CS is pushing current up the resistors, we must subtract the V components!

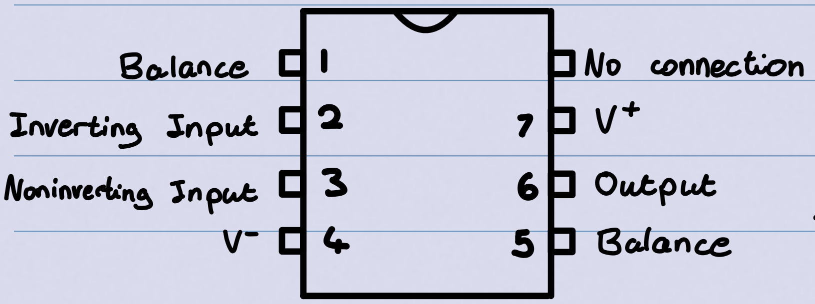
$$\therefore \frac{32}{5} - \frac{16}{5} = \frac{16}{5} V$$

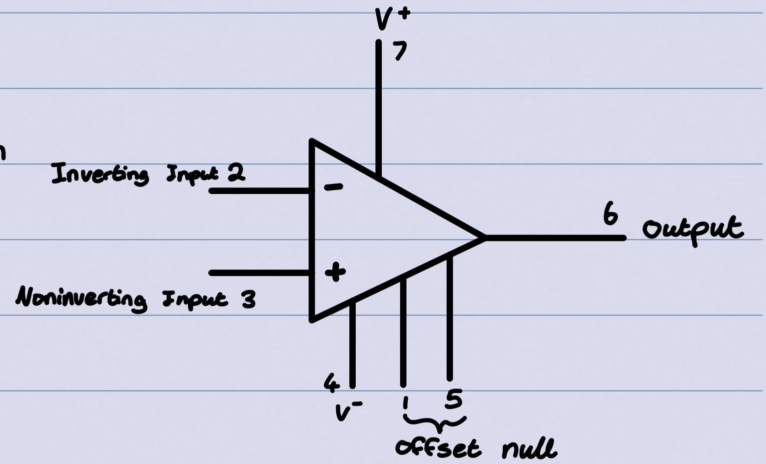
Thevenin's Theorem :

* sorry, my notes don't have thevenin / norton!

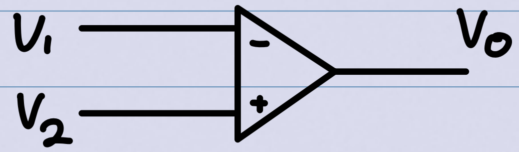


Operational Amplifiers: a circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.





More simply;



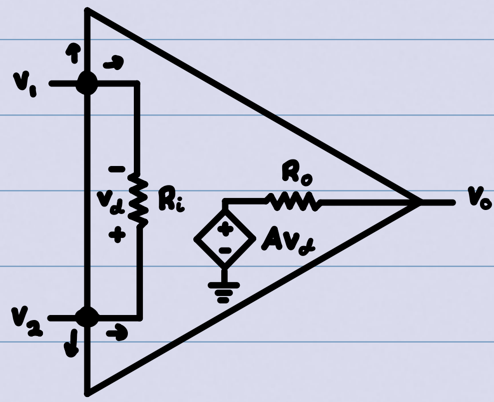
→ $V_o = A(V_1 - V_2)$

op-amp's intrinsic multiplier!

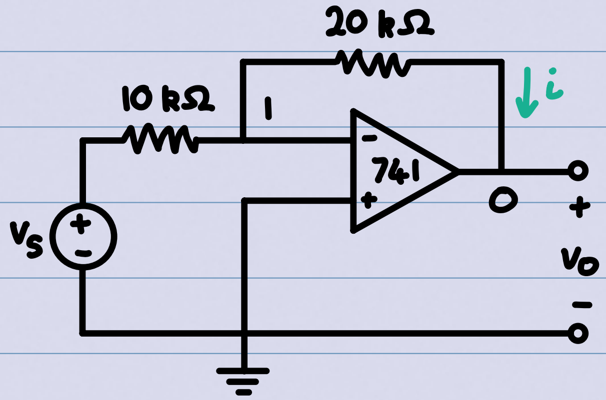
In an ideal op-amp, both inputs should have equal voltage and 0 current!!!

Op-amp Circuit:

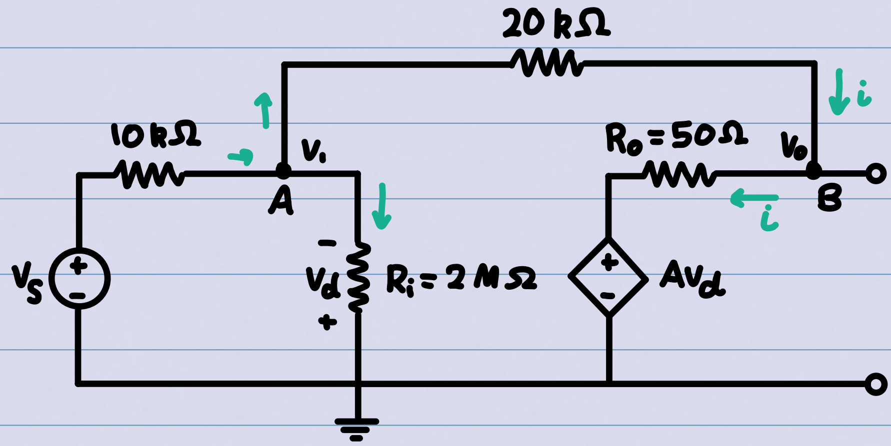
* to solve op-amp questions, just replace the op-amp with it's circuit!



Example: Find the closed-loop gain $\frac{V_o}{V_s}$ if the op-amp has an open-loop voltage gain of 2×10^5, input resistance of $2\ M\Omega$, and output resistance of 50Ω. The circuit is as follows:



The first step is to re-draw the circuit using the op-amp's internal circuit:



Now, we can use Nodal Analysis! At node A:

↳ KCL: $\dfrac{V_s - V_1}{10\ k\Omega} = \dfrac{V_1 - V_o}{20\ k\Omega} + \dfrac{V_1}{2\ M\Omega}$

↳ $\dfrac{V_s - V_1}{10 \cdot 10^3} = \dfrac{V_1 - V_o}{20 \cdot 10^3} + \dfrac{V_1}{2 \cdot 10^6}$

↳ $2 \cdot 10^2 (V_s - V_1) = 10^2 (V_1 - V_o) + V_1$

↳ ∴, $200V_s - 200V_1 = 100V_1 - 100V_o + V_1$

↳ $200V_s = 301V_1 - 100V_o \longrightarrow V_1 = \dfrac{2V_s + V_o}{3}$

At node B: $\dfrac{V_1 - V_o}{20\ k\Omega} = \dfrac{V_o - AV_d}{50}$

$\hookrightarrow \dfrac{V_1 - V_0}{20 \cdot 10^3} = \dfrac{V_0 - AV_d}{50} \rightarrow V_1 - V_0 = 400(V_0 - AV_d)$

From the circuit, we see that $V_d = -V_1$, and from the question, we see that $A = 2 \cdot 10^5 = 200{,}000$.

$\therefore V_1 - V_0 = 400(V_0 - AV_d) \longrightarrow V_1 - V_0 = 400(V_0 + 200000 V_1)$

substituting: $\dfrac{2V_s + V_0}{3} - V_0 = 400\left(V_0 + 200000 \dfrac{2V_s + V_0}{3}\right)$

$\hookrightarrow 2V_s + V_0 - 3V_0 = 1200\left(V_0 + 200000 \dfrac{2V_s + V_0}{3}\right)$

$\hookrightarrow 2V_s - 2V_0 = 1200V_0 + 240000000 \dfrac{2V_s + V_0}{3}$

$\hookrightarrow 2V_s - 2V_0 = 1200V_0 + \dfrac{480000000}{3} V_s + \dfrac{240000000}{3} V_0$

$\hookrightarrow 2V_s - 2V_0 = 1200V_0 + 160000000 V_s + 80000000 V_0$

$\hookrightarrow 2V_s - 2V_0 = 160000000 V_s + 80001200 V_0$

$\hookrightarrow 160000002 V_s = -80001202 V_0$

$\hookrightarrow \dfrac{V_0}{V_s} = -1.9999699.$

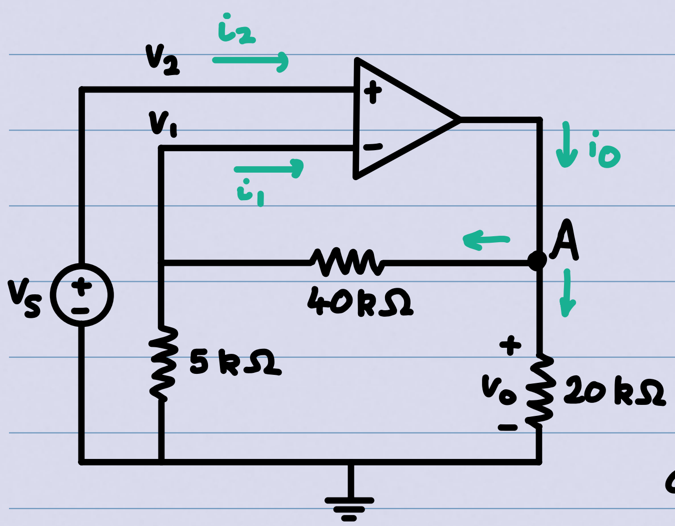
$\therefore$, the closed-loop gain $\dfrac{V_0}{V_s} = -1.9999699$!

Ideal Op-Amp: an amplifier with infinite open-loop gain, infinite input resistance, and zero output resistance.

↳ ie, $A \simeq \infty$, $R_i \simeq \infty$, and $R_o \simeq 0$.

↳ since the input resistance is ∞, there should be no current entering an ideal op-amp!
Also, the inputs should have an equal voltage!

Example: find the closed-loop gain of the ideal op-amp in the following circuit:



- Since the op-amp is ideal, we know that $i_1 = i_2 = 0$!
- we also know that $V_1 = V_2 = V_s$!
- Since $i_1 = 0$, all of the current that goes through the 40 kΩ also goes through the 5kΩ resistor.

∴, the 40 kΩ and 5 kΩ resistors are in series!

from the voltage divider principle, we see that:

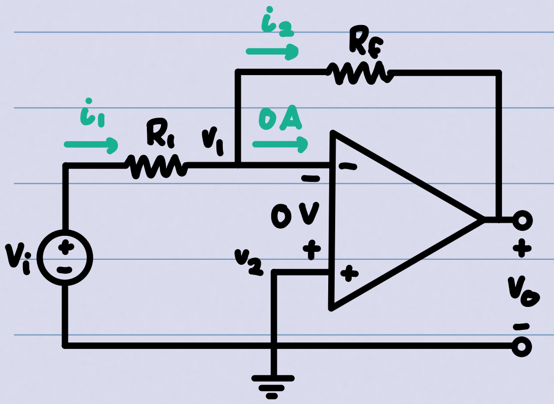
$$V_1 = V_o \frac{5}{40+5}$$

↳ $V_1 = \frac{V_o}{9} \rightarrow V_o = 9V_1$!

since we saw that $V_1 = V_2 = V_s$, $V_o = 9V_s \rightarrow \frac{V_o}{V_s} = 9$.

$\therefore$ the closed-loop gain $\frac{V_o}{V_s}$ is 9.

Inverting Amplifier: noninverting input is grounded, V_i is connected to the inverting input through R_1, and the feedback resistor R_f is connected between the inverting input and output.

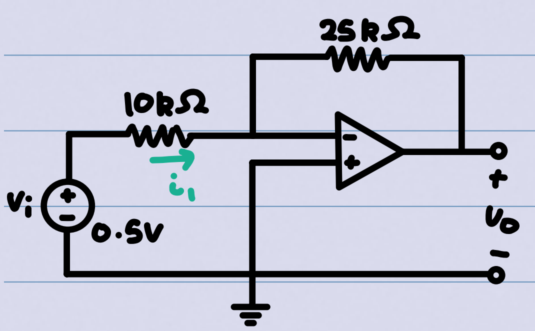


• An inverting amplifier reverses the polarity of the input signal while amplifying it.

$\rightarrow$ * $V_o = -\frac{R_f}{R_1}$

Note: V_o depends only on R_f and R_1, which are external factors to the amplifier!

Example: find the output voltage V_o and the current i_1 in the $10\ k\Omega$ resistor in the circuit:

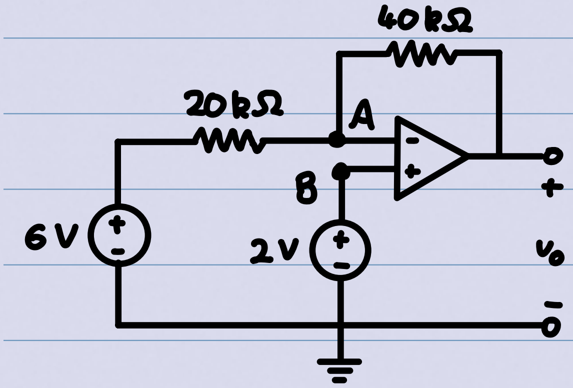


note that $R_f = 25\ k\Omega$ and $R_1 = 10\ k\Omega$. Then, we simply apply the inverting amplifier eq.

$$\rightarrow V_o = -\frac{R_f}{R_1} = -\frac{25\ k\Omega}{10\ k\Omega} \rightarrow V_o = -\frac{5}{2}\ V.$$

$$i_1 = \frac{V_i - 0}{R_1} = \frac{0.5\ V}{10\ k\Omega} \rightarrow i_1 = 50\ \mu A.$$

Example: determine V_o in the op-amp circuit:



KCL at node A:

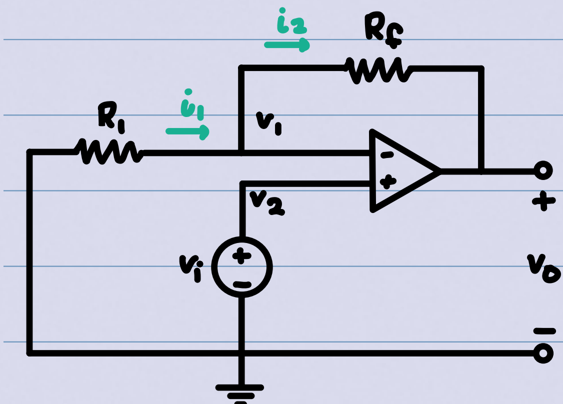
$$\frac{6 - V_A}{20 \cdot 10^3} = \frac{V_A - V_o}{40 \cdot 10^3}$$

$$V_o = 3V_a - 12.$$

Since we are dealing with ideal op-amps, we know $V_A = V_B$. Since $V_B = 2$, $V_A = 2$.

$\therefore$, $V_o = 3(2) - 12 = -6$ V.

Noninverting Amplifier: the input voltage V_i is applied directly at the noninverting input terminal, and R_1 is connected between the ground and the inverting terminal.



- A noninverting amplifier is an op-amp circuit designed to provide a positive voltage gain.

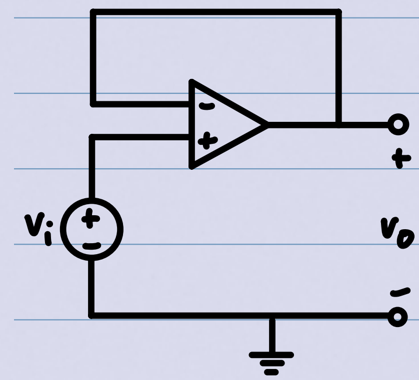
$$* \quad V_o = V_i\left(\frac{R_f}{R_1} + 1\right)$$

Note: like an inverting amplifier, a noninverting amplifier's gain only depends on external factors.

If $R_f = 0$ or $R_1 = \infty$, the gain becomes 1, so

$V_o = V_i$!

↳ this would simplify the circuit as such:

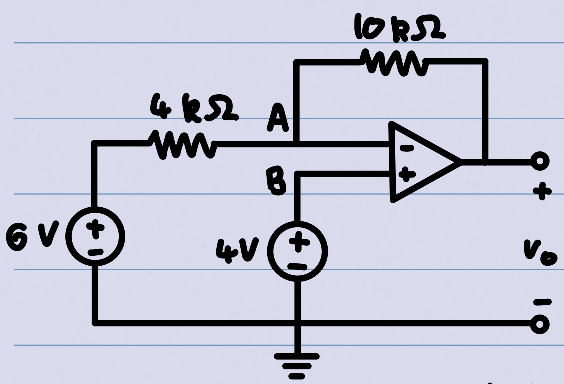


• This is called a voltage follower!

↳ $V_o = V_i$ for a voltage follower.

These are usually just used as buffers.

Example: find the output voltage V_o in the circuit.



KCL at node A :

↳ $\frac{6 - V_A}{4 \cdot 10^3} = \frac{V_A - V_o}{10 \cdot 10^3}$

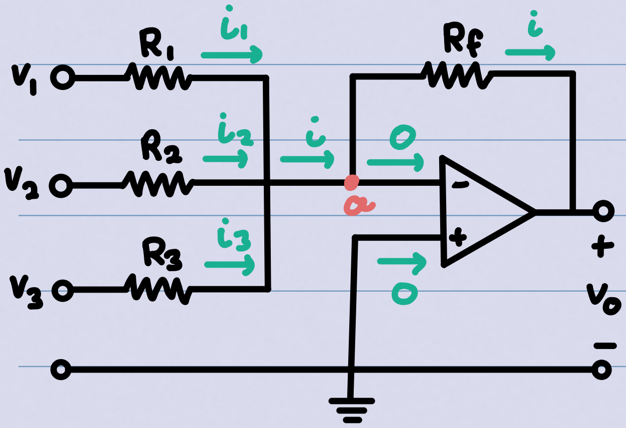
we know $V_A = V_B$ as the op-amp is ideal.

since $V_B = 4$, V_A must be 4. → $V_A = 4$.

$\therefore \frac{6-(4)}{4 \cdot 10^3} = \frac{(4) - V_o}{10 \cdot 10^3} \rightarrow \frac{2}{4} = \frac{4 - V_o}{10} \rightarrow 5 = 4 - V_o$

↳ ∴, $V_o = -1$ V.

Summing Amplifier: an op-amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.



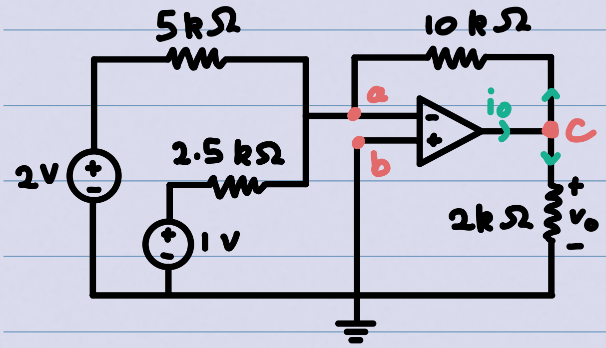
$\rightarrow V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$

$\hookrightarrow V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$

→ output voltage is weighted sum of the inputs!

*can have more than 3 inputs!

Example: Calculate V_o and i_o:



note that this is very similar to the summing circuit above, only it has 2 inputs instead of 3.

from the summing amplifier circuit, $V_o = -10\left(\frac{2}{5} + \frac{1}{2.5}\right)$

$\hookrightarrow V_o = -10\left(\frac{4}{5}\right) = -8\,V.$

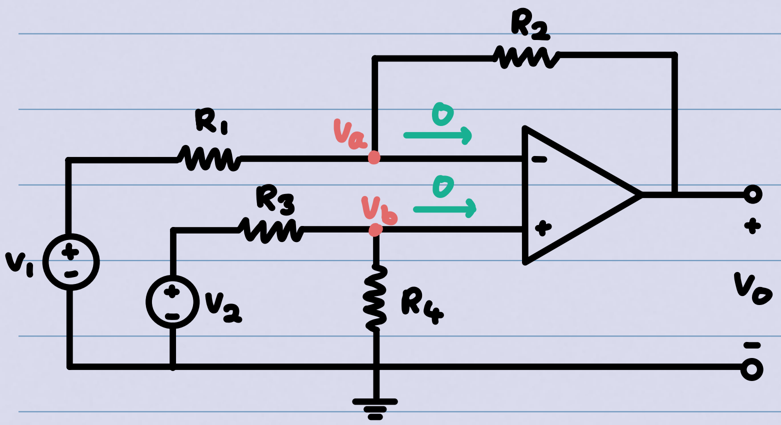
KCL at node c: $i_o = \frac{V_o - V_a}{10} + \frac{V_o - 0}{2}$.

since $V_b = 0$, and since this is an ideal op-amp, we know $V_a = V_b$, so $V_a = 0$.

$\hookrightarrow i_o = \frac{V_o}{10} + \frac{V_o}{2} = -\frac{8}{10} - 4 = -4.8\text{ mA}$

mA bc resistors are kΩ, not Ω.

Difference Amplifier: a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.



$$\rightarrow V_o = \frac{R_2\left(1+\frac{R_1}{R_2}\right)}{R_1\left(1+\frac{R_3}{R_4}\right)} - \frac{R_2}{R_1}V_1$$

if $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, then $V_o = \frac{R_2}{R_1}(V_2 - V_1)$

if $R_1 = R_2$ and $R_3 = R_4$, $V_o = V_2 - V_1$

Example: design a circuit with inputs V_1 and V_2 such that $V_o = 3V_2 - 5V_1$.

from the general difference amplifier equation, we see that $\frac{R_2}{R_1} = 5 \rightarrow R_2 = 5R_1$.

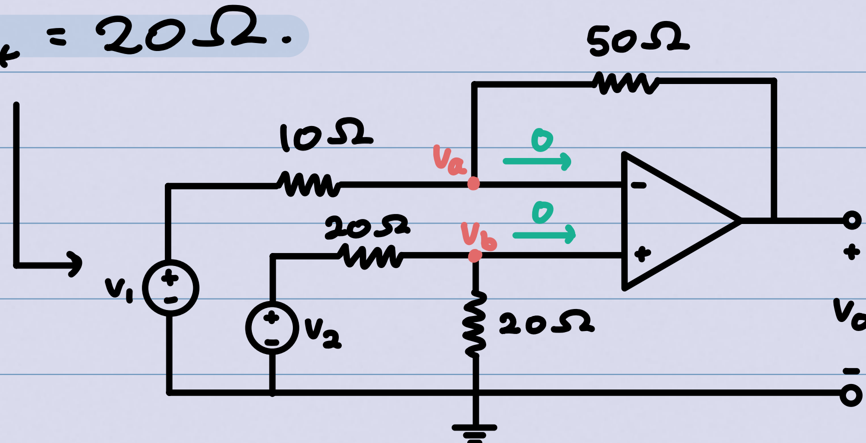
$$\therefore, 5\frac{\left(1+R_1/R_2\right)}{\left(1+R_3/R_4\right)} = 3 \longrightarrow \frac{1+(5)^{-1}}{1+R_3/R_4} = \frac{3}{5}.$$

$$\hookrightarrow \frac{6/5}{1+R_3/R_4} = \frac{3}{5} \longrightarrow 2 = 1 + \frac{R_3}{R_4} \rightarrow \frac{R_3}{R_4} = 1 \rightarrow R_3 = R_4.$$

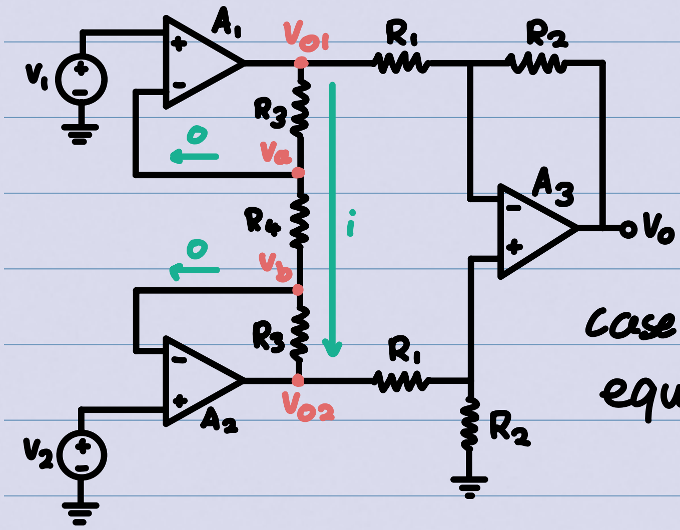
$\therefore$, $R_2 = 5R_1$ and $R_3 = R_4$.

let's choose $R_1 = 10\Omega$ and $R_3 = 20\Omega$, then $R_2 = 50\Omega$ and $R_4 = 20\Omega$.

note: this problem can also be solved using cascading op-amps!



Example: show that $V_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(V_2 - V_1)$ in the following circuit:



- we recognise that A_3 is a difference amplifier!
- this falls into the second case given in the difference amplifier equations, and hence $V_o = \frac{R_2}{R_1}(V_{o2} - V_{o1})$

Since op-amps A_1 and A_2 are ideal, they take in 0 current.

↳ ∴, R_3, R_4, and the other R_3 are in series!

$$i = \frac{V_{o2} - V_{o1}}{R_3 + R_4 + R_3} \rightarrow V_{o2} - V_{o1} = (2R_3 + R_4)i$$

also, $i = \frac{V_a - V_b}{R_4}$, and $V_a = V_1$ and $V_b = V_2$.

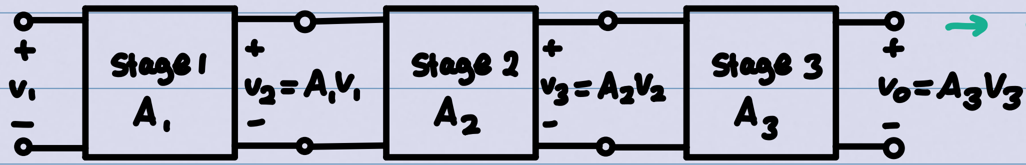
↳ ∴, $i = \frac{V_1 - V_2}{R_4}$!

$$V_o = \frac{R_2}{R_1}(V_{o2} - V_{o1}) \rightarrow V_o = \frac{R_2}{R_1}\, i\,(2R_3 + R_4)$$

↳ $V_o = \frac{R_2}{R_1} \frac{V_1 - V_2}{R_4}(2R_3 + R_4) \rightarrow V_o = \frac{R_2}{R_1}(V_1 - V_2)\left(\frac{2R_3}{R_4} + 1\right)$!

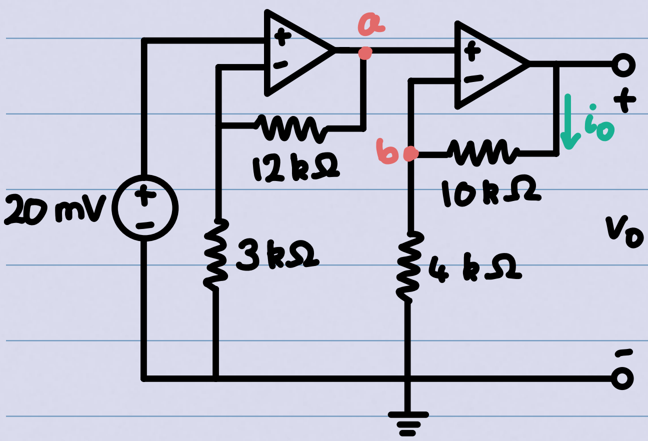
Cascaded Op-Amp Circuits: a head-to-tail arrangement of two or more op-amp circuits such that the output of one is the input of the other

→ when op-amp circuits are cascaded, each circuit in the string is called a stage.

↳ 

→ a 3-stage cascaded connection

Example: find V_o and i_o in the following circuit:

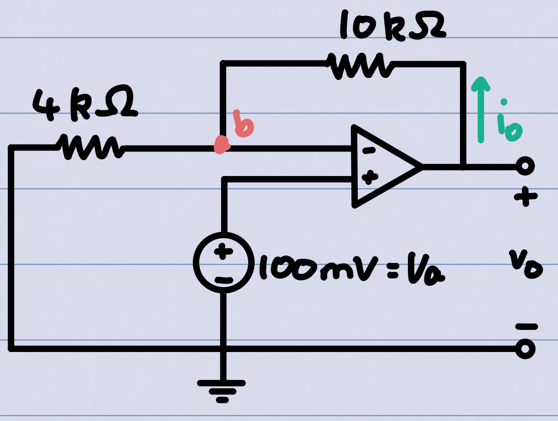


notice that this is just two noninverting amplifiers cascaded!

↳ can use: $V_o = V_i\left(\frac{R_f}{R_1} + 1\right)$

↳ we see that $V_a = 20\left(\frac{12}{3} + 1\right) = 100\,mV$.

now, we can re-draw the circuit to simplify:



→ now, it's clear that

$V_o = V_a\left(\frac{10}{4} + 1\right)$

$V_o = 100(3.5) = 350\,mV.$

we see that i_o goes through the 10 kΩ resistor:

↳ $i_o = \frac{V_o - V_b}{10\,k\Omega} = \frac{V_o - V_a}{10\,k\Omega} = \frac{350\,mV - 100\,mV}{10\,k\Omega} = 25\,\mu A.$

Capacitors: two conducting plates seperated by an insulator (or dielectric).

Capacitance: the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates.

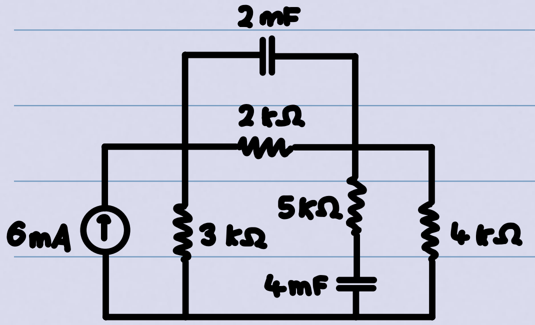
↳ $C = \frac{Q}{V}$ [F]

→ from $i = \frac{dq}{dt}$, we see that $i = C\frac{dv}{dt}$.

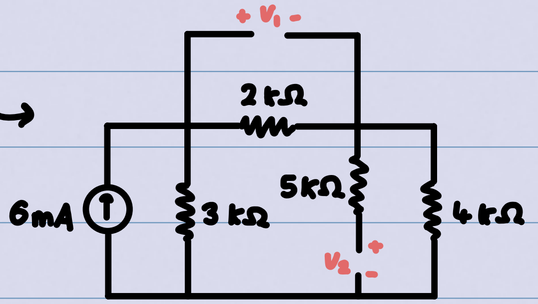
the energy stored on a capacitor is $w = \frac{1}{2}Cv^2$.
↳ stored in the electric field

- A capacitor is an open-circuit to DC
- The voltage on a capacitor cannot change abruptly.

Example: find the energy stored in each capacitor

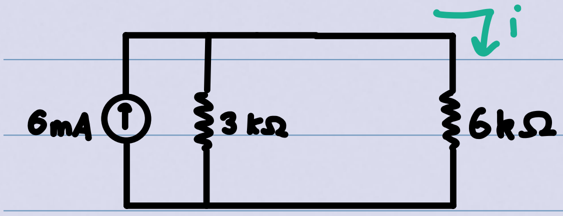


- under dc conditions, we replace each capacitor with an open circuit!

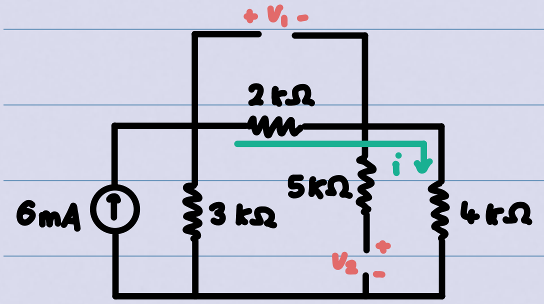
↳


→ we see that no current flows the top loop or the 5kΩ resistor!
∴, the 2 and 4 kΩ resistors are in parallel!

we can re-draw again for clarity:



→ from here, we can see that $i = 2\,mA$.



$\therefore V_1 = 2000i$ and $V_2 = 4000i$

↳ $V_1 = 4\,V$, and $V_2 = 8\,V$.

$w_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(2 \cdot 10^{-3})(4^2) = 16\,mJ$

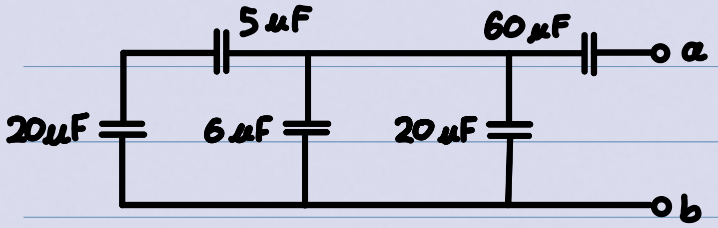
$w_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2}(4 \cdot 10^{-3})(8^2) = 128\,mJ.$

Series and parallel capacitors:

- The equivalent capacitance of N parallel capacitors is the sum of the individual capacitances ⟶ $C_{eq} = C_1 + C_2 + \ldots + C_n$

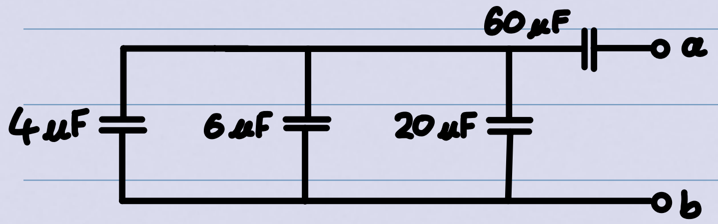
- The equivalent capacitance of N series capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances ⟶ $C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}\right)^{-1}$

Example: find the equivalent capacitance seen between the terminals a and b in the following circuit:



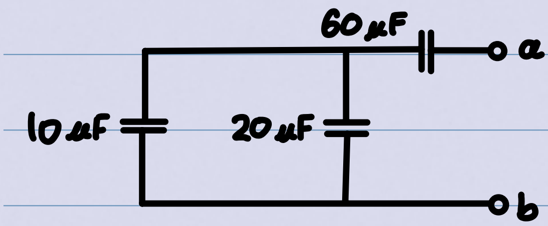
→ firstly, we see that the 5µF and 20µF capacitors are in series.

$$\therefore R_{20,5} = \left(\frac{1}{5} + \frac{1}{20}\right)^{-1} = \left(\frac{5}{20}\right)^{-1} = 4\ \mu F.$$



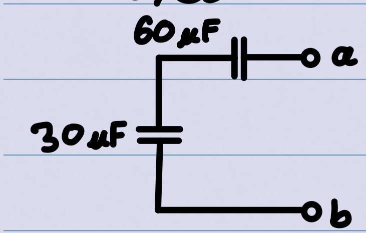
→ now, we see that the 4µF and 6µF are in parallel.

$$\therefore R_{4,6} = 4 + 6 = 10\ \mu F.$$



→ again, the 10 µF and 20µF capacitors are in parallel!

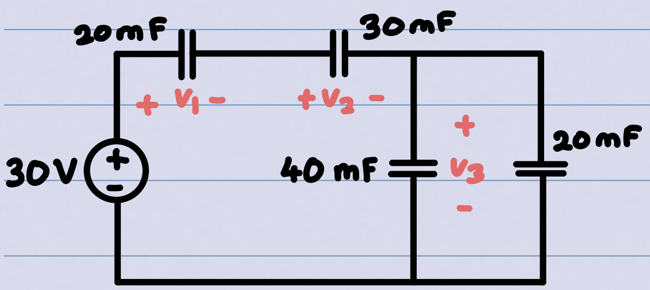
$$R_{10,20} = 10 + 20 = 30\ \mu F.$$



→ the 30µF and 60µF capacitors are in series:

$$R_{eq} = \left(\frac{1}{30} + \frac{1}{60}\right)^{-1} = \left(\frac{3}{60}\right)^{-1} = 20\ \mu F\ !$$

Example: find the voltage across each capacitor:



→ let's first find the equivelent capacitance C_{eq}:

$$C_{eq} = \left(\frac{1}{20} + \frac{1}{30} + \frac{1}{20+40}\right)^{-1}$$

$\hookrightarrow C_{eq} = 10\,mF.$

∴, the total charge is $q = C_{eq} V \rightarrow q = (10 \cdot 10^{-3})(30)$
$\hookrightarrow q = 0.3$ C.

this is the charge on the 20 mF and 30 mF capacitors, as they are in series with the 30 V source:

$$\therefore,\ V_1 = \frac{q}{C_1} = \frac{0.3}{20 \cdot 10^{-3}} = 15\text{ V}$$

$$V_2 = \frac{q}{C_2} = \frac{0.3}{30 \cdot 10^{-3}} = 10\text{V}.$$

we can now perform KVL: $-30 + V_1 + V_2 + V_3 = 0$
$\hookrightarrow V_3 = 30 - V_1 - V_2 = 5\text{V}$.

Inductors: a coil of conducting wire

Inductance: the property whereby an inductor exhibits opposition to the change of current flowing through it.

$\hookrightarrow$ the voltage across an inductor is $v = L\frac{di}{dt}$.

the energy stored by an inductor is $w = \frac{1}{2}Li^2$!

- An inductor acts like a short circuit to dc
- The current flowing through an inductor cannot change instantaneously.

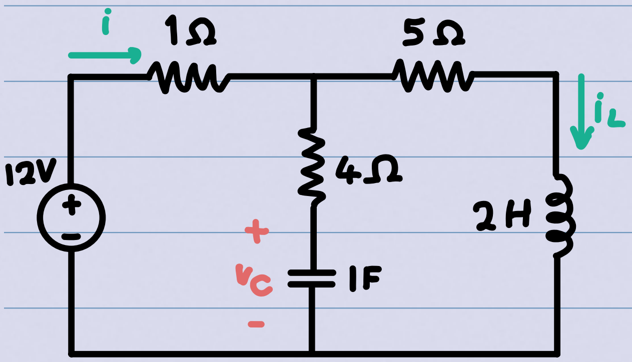
Example: the current through a 0.1H inductor is $i(t) = 10te^{-5t}$. Find the voltage across the inductor and the energy stored by it.

↳ $v = L\frac{di}{dt} = (0.1)\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t}$

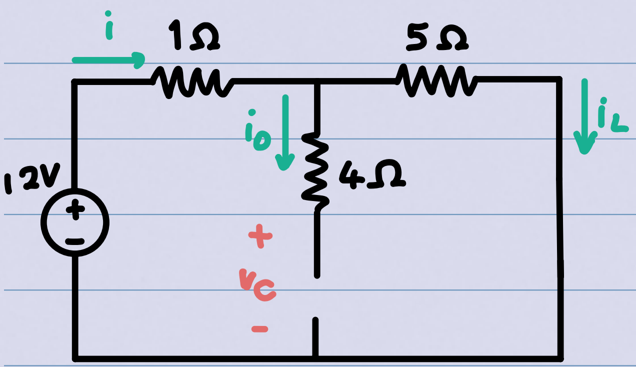
↳ $V = e^{-5t}(1-5t)$ V

$\omega = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)(10te^{-5t})^2 = 5t^2e^{-10t}$ J.

Example: Find (a) i, V_c, i_L, and (b), the energy stored in the capacitor and inductor:



→ we replace the capacitor with an open-circuit and the inductor with a short-circuit as follows:



→ since no current flows through the 4Ω resistor, the 1Ω and 5Ω resistors are in series.

↳ $i = \frac{V}{R} = \frac{12}{5+1} = 2\ A = i_L$

the voltage V_c is the same as the voltage across

the 5Ω resistor
↳ $V_C = 5i = 5(2) = 10\,V$.

$w_c = \frac{1}{2}CV_c^2 = \frac{1}{2}(1)(10)^2 = 50\,J$

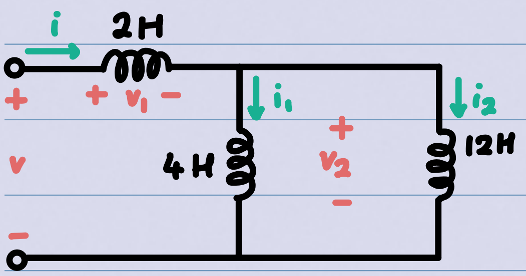
$w_L = \frac{1}{2}Li^2 = \frac{1}{2}(2)(2)^2 = 4\,J$

Series and Parallel Inductors

the equivalent inductance of series inductors is the sum of their individual inductances. → $L_{eq} = L_1 + L_2 + \ldots + L_N$

the equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances. ⟶ $L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}\right)^{-1}$

Example: $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find : (a) $i_1(0)$, (b) $v(t)$, $v_1(t)$, and $V_2(t)$, and (c) $i_1(t)$ and $i_2(t)$:



→ we see that $i = i_1 + i_2$.

$\therefore$, $i(0) = i_1(0) + i_2(0)$

↳ $4 = i_1(0) - 1$ → $i_1(0) = 5\,mA$ (a)

$L_{eq} = 2 + \left(\frac{1}{4} + \frac{1}{12}\right)^{-1} = 2 + \left(\frac{4}{12}\right)^{-1} = 5H$.

$v(t) = L_{eq}\frac{di}{dt} = 5(4)(-1)(-10)e^{-10t}$ mV

↳ $V(t) = 200e^{-10t}$ mV

$$V_1(t) = L\frac{di}{dt} = 2(4)(-1)(-10)e^{-10t} \text{ mV}$$

$$\hookrightarrow V_1(t) = 80e^{-10t} \text{ mV}$$

$$V = V_1 + V_2 \rightarrow V_2(t) = V(t) - V_1(t) = 120e^{-10t} \text{ mV}$$

$$V = L\frac{di}{dt} \rightarrow di_1 = \frac{V_1}{L_1}dt \rightarrow i_1 = \frac{1}{L_1}\int V_1\,dt + i_1(0)$$

$$\hookrightarrow i_1 = \frac{1}{4}\int 80e^{-10t}\,dt + 5 \text{ mA}$$

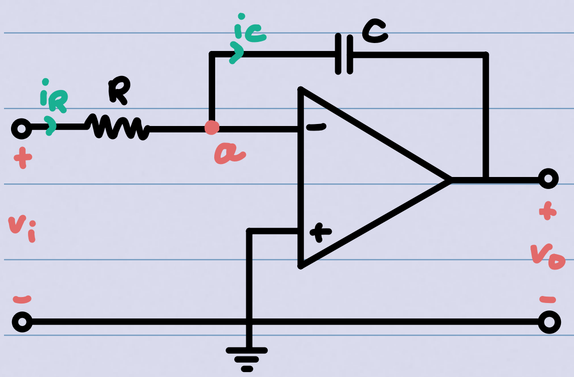
$$\hookrightarrow i_1 = 8 - 3e^{-10t} \text{ mA}$$

$$V = L\frac{di}{dt} \rightarrow di_2 = \frac{V_2}{L_2}dt \rightarrow i_2 = \frac{1}{L_2}\int V_2\,dt + i_2(0)$$

$$\hookrightarrow i_2 = \frac{1}{12}\int 120e^{-10t}\,dt - 1 \text{ mA}$$

$$\hookrightarrow i_2 = -e^{-10t} \text{ mA}$$

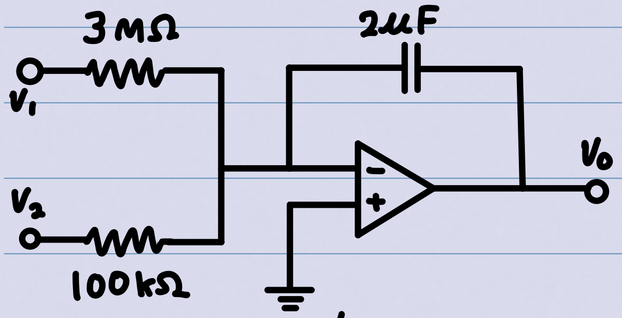
Integrator: an op-amp circuit whose output is proportional to the integral of the input signal.



→ note that this is simply an inverting amplifier, but the feedback resistor is replaced with a capacitor.

$\hookrightarrow$ assuming $V_o(0) = 0$, $V_o = -\frac{1}{RC}\int_0^t V_i(t)\,dt$

Example: if $V_1 = 10\cos(2t)$ mV and $V_2 = 0.5t$ mV, find V_o in the op-amp circuit. Assume the voltage across the capacitor is initially 0.



→ $V_i = V_1 + V_2$.

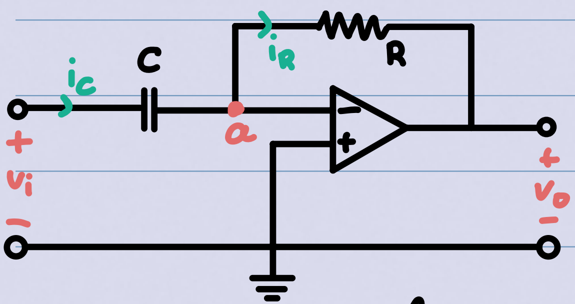
we know $V_o = -\frac{1}{RC}\int_0^t V_1(t)\,dt$

$$\therefore V_o = -\frac{1}{R_1 C}\int_0^t V_1\,dt - \frac{1}{R_2 C}\int_0^t V_2\,dt$$

$$= -\frac{1}{3\cdot 10^6 \cdot 2\cdot 10^{-6}}\int_0^t 10\cos(2t)\,dt - \frac{1}{100\cdot 10^6\cdot 2\cdot 10^{-6}}\int_0^t 0.5t\,dt$$

↳ $V_o = -0.833\sin(2t) - 1.25t^2$ mV

Differentiator: an op-amp circuit whose output is proportional to the rate of change of the input.

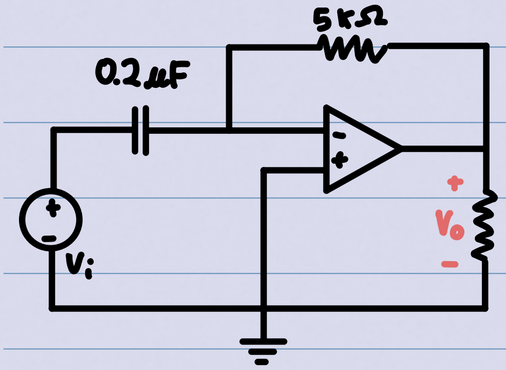


→ note that this is also just the inverting amplifier, but the input resistor is replaced with a capacitor.

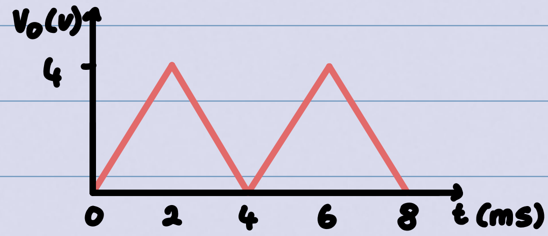
↳ $V_o = -RC\frac{dv_i}{dt}$

↳ The differentiator is rarely used in practice; it's not as accurate as the integrator.

Example: sketch the output voltage for the circuit, given the input voltage. Assume $V_o(0) = 0$.



input voltage V_i :



this is a differentiator with $RC = 5 \cdot 10^3 \cdot \frac{2}{10} \cdot 10^{-6}$

↳ $RC = 10^{-3}$.

we can express V_i as: $\begin{cases} 2000t & 0<t<2 \\ -2000t & 2<t<4 \\ 2000t & 4<t<6 \\ -2000t & 6<t<8 \end{cases}$.

∴, the output is obtained as $V_o = -RC \frac{dV_i}{dt}$

↳ $V_o = -10^{-3} \begin{cases} 2000 & 0<t<2 \\ -2000 & 2<t<4 \\ 2000 & 4<t<6 \\ -2000 & 6<t<8 \end{cases}$.

↳ ∴, $V_o = \begin{cases} -2 & 0<t<2 \\ 2 & 2<t<4 \\ -2 & 4<t<6 \\ 2 & 6<t<8 \end{cases}$ V.

First-order circuit: characterized by a first-order differential equation.

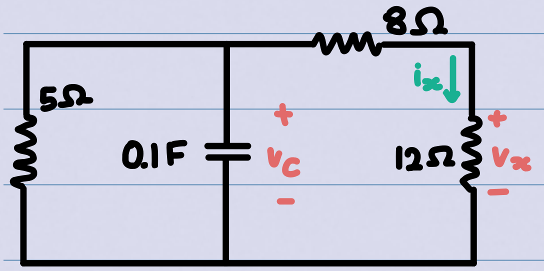
Source-Free RC circuit: occurs when the source of a circuit with a resistor and capacitor is suddenly disconnected.

Time Constant (τ) = the time required for the response to decay a factor of 1/e (or 36.8%)

of its initial value.

For an RC circuit, $V = V_0 e^{-t/\tau}$, and $\tau = RC$

Example: let $V_c(0) = 15V$. Find V_c, V_x, and i_x $\forall$ $t > 0$:



the 8Ω and 12Ω resistor can be combined into a 20Ω resistor, as they're in series.

Then, the 5Ω and 20Ω resistor are parallel, so $R_{eq} = \left(\frac{1}{5} + \frac{1}{20}\right)^{-1} = 4\Omega$.

$\tau = R_{eq} C = 4 \cdot 0.1 = 0.4$.

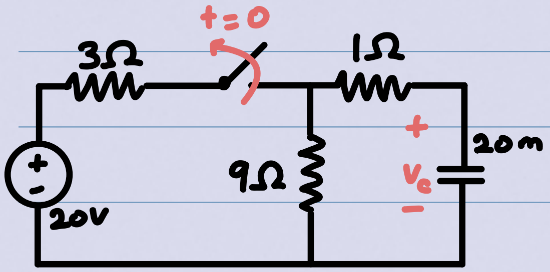
$\hookrightarrow \therefore,\ V_c = V = V_c(0) e^{-t/\tau} = 15 e^{-2.5t}\ V$

We can use voltage division to find V_x:

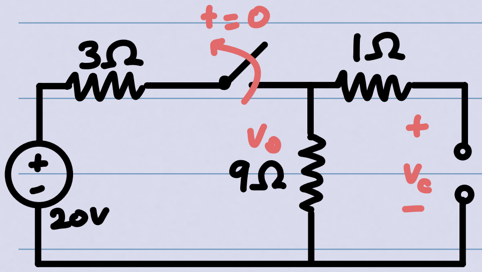
$\hookrightarrow V_x = \frac{12}{12+8} V = \frac{12}{20} V = \frac{3}{5} V = 9e^{-2.5t}\ V.$

$i_x = \frac{V_x}{R_{12}} = \frac{9e^{-2.5t}}{12} = \frac{3}{4} e^{-2.5t}.$

Example: the switch has been closed for a long time and is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Also, calculate the initial energy stored in the capacitor.

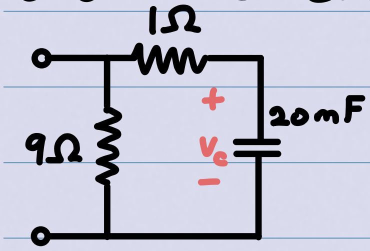


as the capacitor acts like an open circuit after a long time ($t < 0$), we can find $V_c(0)$:



Using voltage division, $V_o = \frac{9}{9+3}(20) = V_c$.
↳ $V_c = 15\ V$ at $t \leq 0$. ∴, $V_c(0) = 15\ V$.

once the switch is opened, we have this RC circuit:



→ $R_{eq} = 9+1 = 10\Omega$.
∴, $\tau = R_{eq}C = 10(20 \cdot 10^{-3}) = 0.2$.

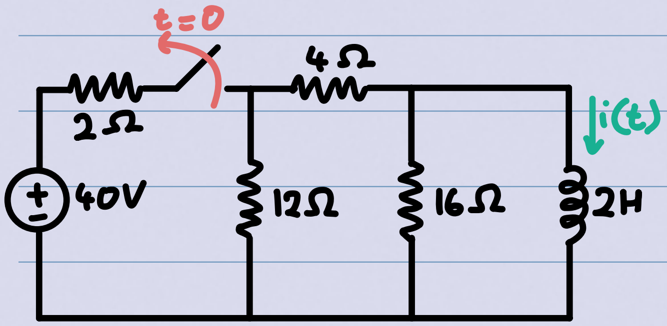
$V_c(t) = V_c(0)e^{-t/\tau} = 15e^{-5t}\ V$.

The initial energy stored in the capacitor is given by:
↳ $w_c(0) = \frac{1}{2}CV_c^2 = \frac{1}{2}(20 \cdot 10^{-3})(15)^2 = 2.25\ J$.

Source-Free RL circuit: occurs when the source of a circuit with a resistor and inductor is suddenly disconnected.

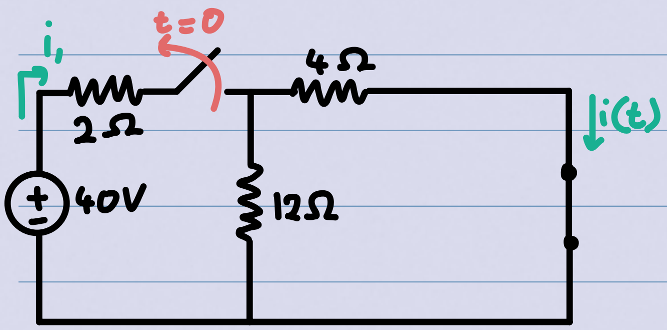
For an LR circuit, $\tau = \frac{L}{R}$ and $i(t) = I_0 e^{-t/\tau}$

Example: the switch in the circuit has been closed for a long time. A $t=0$, the switch is opened. Calculate $i(t)$ for $t > 0$.



after a long time $(t<0)$, the inductor acts as a closed switch (wire).

This would result in the 16Ω resistor being short-circuited:



using the current divider rule,
$i(t) = i_1 \frac{12}{12+4} = \frac{3}{4} i_1$.

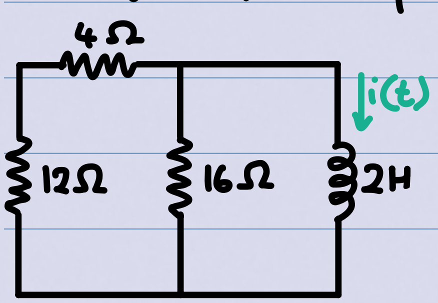
we also see that $R_{eq} = 2 + \frac{48}{16} = 5\Omega$

$\therefore, i_1 = \frac{40V}{5\Omega} = 8A$.

since current cannot instantaneously change in an inductor, $I(0) = I(0^-)$.

$\hookrightarrow i(0) = 8\left(\frac{3}{4}\right) = 6A$.

when the switch is opened, the inductor will act as the new power source:



$R_{eq} = 8\Omega$. $\therefore, \tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}$.

$\therefore, i(t) = i_0 e^{-t/\tau} = 6e^{-4t}$ A.

Singularity Functions: functions that are either discontinuous or have discontinuous derivatives.

the unit step function $u(t)$ is 0 for negative values of t and 1 for positive values of t.

$$u(t) = \begin{cases} 0, & t < 0 \\ 0, & t > 1 \end{cases}$$ → $u(0)$ is undefined!

∴, $V(t) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$ can be expressed as $V(t) = V_0 u(t - t_0)$.

the unit impulse function $\delta(t)$ is 0 everywhere except at $t = 0$, where it is undefined.

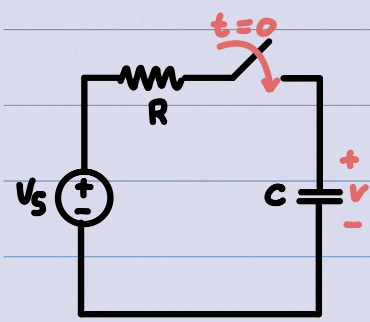
$$\int_{0^-}^{0^+} \delta(t)\,dt = 1 \longrightarrow \therefore, \quad f(t_0) = \int_a^b f(t)\delta(t - t_0)\,dt$$

the unit ramp function $r(t)$ is 0 for negative values of t and has a unit slope for positive t.

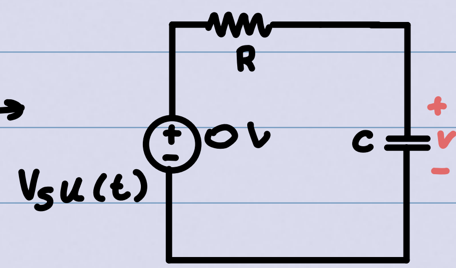
$$r(t) = \begin{Bmatrix} 0, & t \leq 0 \\ t, & t \geq 0 \end{Bmatrix}, \quad r(t - t_0) = \begin{Bmatrix} 0, & t \leq t_0 \\ t - t_0, & t \geq t_0 \end{Bmatrix}$$

$$\frac{dr(t)}{dt} = u(t), \quad \frac{du(t)}{dt} = \delta(t)$$

Step Response of an RC circuit: the behavior when the excitation is the step function, which may be a voltage or current source.



→ can be redrawn as →



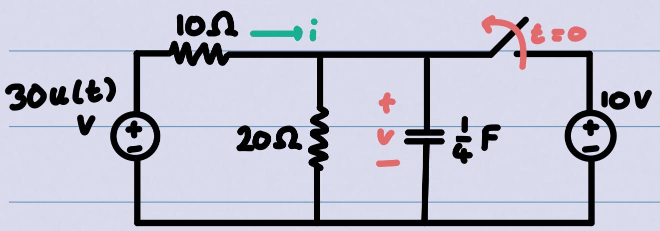
$$\hookrightarrow v(t) = \begin{cases} V_0 & , t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & , t > 0 \end{cases}$$

transient response : the circuit's temporary response that will die out with time.

the steady state response : the circuit's long-term behavior after external excitation is applied.

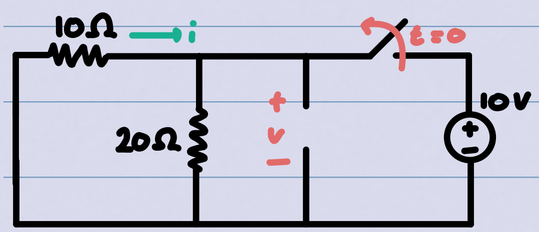
$$\hookrightarrow v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

Example : the switch has been closed for a long time and is opened at $t = 0$. Find i and v for all time.



$$\rightarrow 30u(t) = \begin{cases} 0, & t < 0 \\ 30, & t > 0 \end{cases}$$

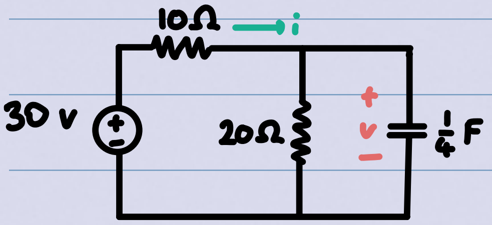
for $t < 0$, $30u(t) = 0$ and the capacitor acts like an open switch :



$$\rightarrow i = -\frac{V}{10} = -\frac{10}{10} = -1\,A.$$

$\hookrightarrow v(0) = 10\,V$

for $t > 0$, $30u(t) = 30$ and the 10V source gets disconnected :



→ using voltage divider, we see $V = \frac{20}{10+20}(30) = 20V. = V(\infty)$

$R_{Th} = 10 \| 20 = \frac{200}{30} = \frac{20}{3}\,\Omega.$

↳ $\tau = R_{Th}C = \left(\frac{20}{3}\right)\left(\frac{1}{4}\right) = \frac{5}{3}$ s.

$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$
$$= 20 + [10 - 20]e^{-3/5t} \rightarrow V(t) = 20 - 10e^{-0.6t}\ V$$

$$i = \frac{V}{20} + C\frac{dv}{dt} = \frac{(20-10e^{-0.6t})}{20} + \frac{1}{4}(-0.6)(-10)e^{-0.6t}\ A$$

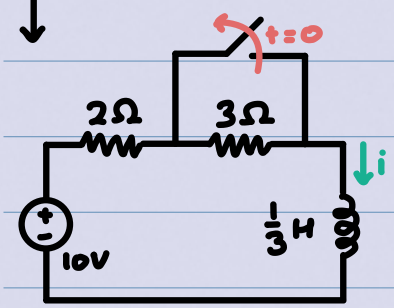
$$i = 1 - \frac{1}{2}e^{-0.6t} + \frac{3}{2}e^{-0.6t} = 1 + e^{-0.6t}\ A.$$

↳ $$\boxed{V = \begin{cases} 10\ V, & t < 0 \\ (20 - 10e^{-0.6t}, & t \geq 0 \end{cases} \qquad i = \begin{cases} -1\ A, & t < 0 \\ 1 + e^{-0.6t}\ A, & t > 0 \end{cases}}$$

Step Response of an RL circuit:

↳ $i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$

Example: find $i(t)$ for $t > 0$. Assume that the switch has been closed for a long time.



when $t<0$, the 3Ω resistor is short-circuited.
↳ $\therefore\ i(0) = 5A$.

when $t>0$, the $R_{eq} = 5\Omega$. → $\tau = \frac{L}{R} = \frac{1/3}{5} = \frac{1}{15}$ s.

$i(\infty) = \frac{V}{R_{eq}} = \frac{10}{5} = 2A$.

$i(t) = 2 + [5-2]e^{-t/1/15} = 2 + 3e^{-15t}$

↳ $i(t) = \begin{cases} 5\ A, & t<0 \\ 2+3e^{-15t}\ A, & t>0 \end{cases}$.

Sinusoid: a signal that has the form of sin / cos

↳ $T = \frac{2\pi}{\omega}$

↳ $A\cos(\omega t) + B\sin(\omega t) = C\cos(\omega t - \theta)$
where $C = \sqrt{A^2+B^2}$, $\theta = \tan^{-1}\frac{b}{a}$

phasors: complex numbers that represent the amplitude and phase of a sinusoid.

• $z = x + yj = r\angle\phi = re^{j\phi}$ where $r = \sqrt{x^2+y^2}$, $\phi = \tan^{-1}\frac{y}{x}$

$\therefore\ z = x + yj = r\angle\phi = r(\cos\theta + j\sin\theta)$

multiplication: $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$

division : $\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$

reciprocal : $\frac{1}{z} = \frac{1}{r} \angle -\phi$

$\sqrt{\ }$: $\sqrt{z} = \sqrt{r} \angle \phi/2$

conjugate : $\bar{z} = r\angle -\phi = x - yj = re^{-j\theta}$

$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$