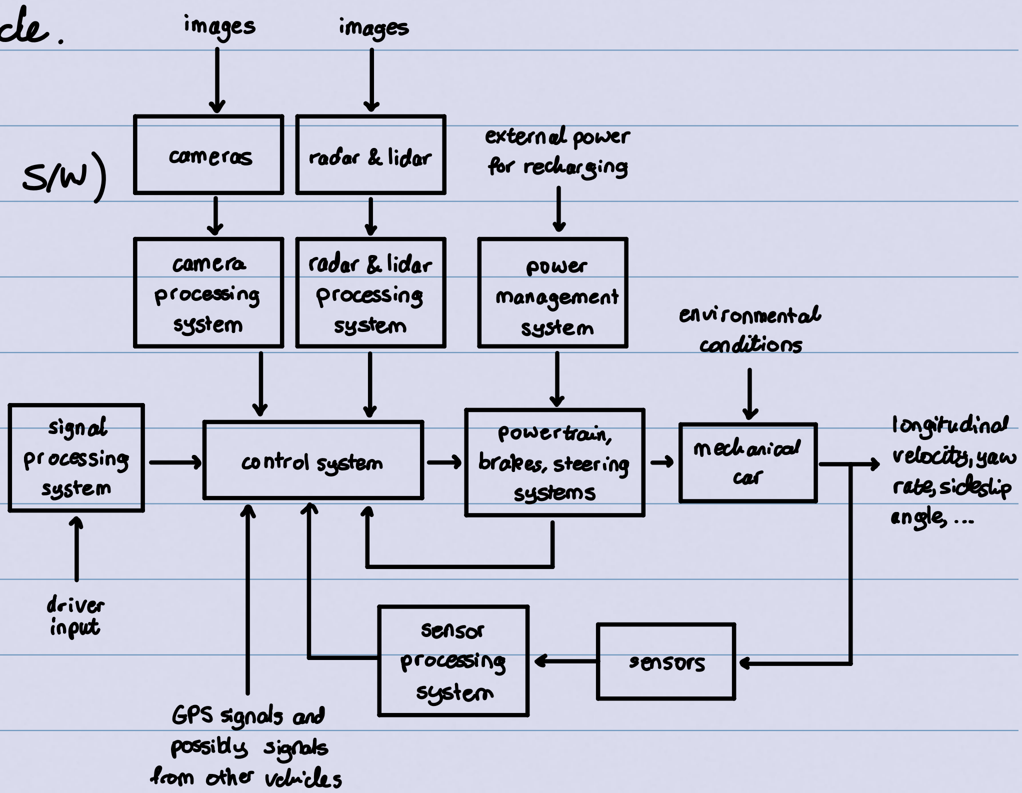
Motivating Example: design of software and hardware used to control an electric vehicle.

boxes = systems (H/W or S/W)

arrows = signals

⇒ this is an example of a block diagram



Can decompose this into **3 types of tasks**:

1. **Modelling**
   - we need some way to describe **how systems process to generate outputs**
2. **Analysis**
   - we need tools to **determine and study the behavior** of the various systems
   - eg, is the control system stable? is it fast or slow? How does it respond in windy conditions?
3. **Design**
   - we need to have a systematic way to **create and tune the various systems** (control system, image processing system, etc)

Signal: a **function of one or more independent variables**, generally containing information about the behavior of some phenomenon of interest.

⇒ **Example:** a signal may represent a force, a torque, an angle, a speed, a stock price, available SSD memory, etc.

- We will deal with only the situation where there is one independent variable, namely time:
  - If time is varying consistently, it's a continuous-time signal.
    ↳ we denote time by t and continuous-time signals as $x(t), u(t), y(t)$, etc
  - If time jumps from one value to the next, it's a discrete-time signal.
    ↳ we denote time by k and discrete-time signals as $x[k], u[k], y[k]$, etc

System: a device, process, or algorithm that takes one or more input signals and generates one or more output signals.

⇒ Example: each of the blocks in the electric vehicle system, a rocket, a heart, a phone, a planet, etc

It's traditional to denote a generic input signal by u (ie, either $u(t)$ or $u[k]$) and a generic output signal by y (ie, either $y(t)$ or $y[k]$)

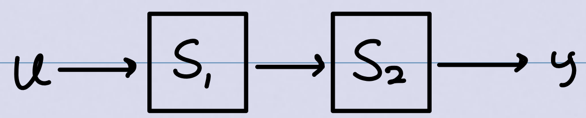
Systems that have one input signal and one output signal are called single-input single-output (SISO). Systems that have multiple inputs and multiple outputs are called multi-input multi-output (MIMO).

The output of the system is also called the response of the system.

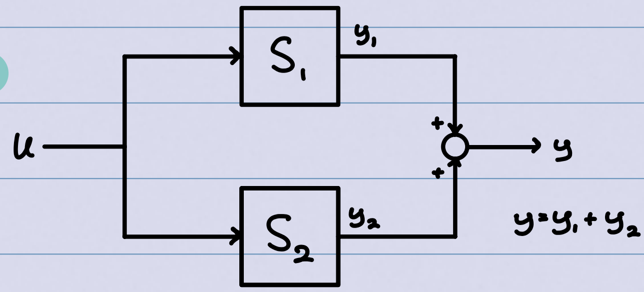
If both the input signal(s) and output signal(s) are continuous-time signals, then we say the system is a continuous-time system. Similarly, if both are discrete-time signals, then we say the system is a discrete-time system. Any other combination results in a hybrid system.

In a block diagram, blocks can be connected in **series** (aka a **cascade connection**) or in **parallel** (with the help of a summer):

series:



parallel:



Differential Equation: any math equation that, in contrast to a purely algebraic equation, **includes the derivatives of one or more dependent variable with respect to one or more independent variables.**

Ordinary Differential Equation (ODE): a differential equation with **only one independent variable.**

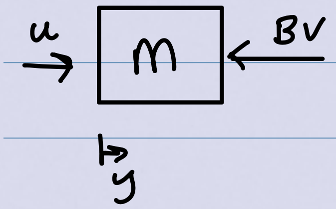
Partial Differential Equation (PDE): a differential equation with **more than one independent variable.**

Order of a Differential Equation: the **order of the highest derivative** in the equation.

⇒ Example: are the following algebraic, ODEs, or PDEs?

- $\frac{d^3y}{dt^3} + 4y = \frac{du}{dt} + 2u$ — ODE, 3^{rd} order
- $F = ma$ — algebraic
- $F = m\frac{d^2y}{dt^2}$ — ODE, 2^{nd} order
- $\dddot{y} + 2(2-\dot{y}^3)\dot{y} + 4y = u$ — ODE, 3^{rd} order
- $\frac{\partial y(x,t)}{\partial t} - K\frac{\partial y^2(x,t)}{\partial^2 x} = u(x,t)$ — PDE, 2^{nd} order

Example: consider the dynamics of a vehicle moving in a straight line. The system is affected mainly by the force applied by the engine and air resistance (friction). Let u = input force due to engine and v = output velocity ($= \dot{y}$)



$F = ma \rightarrow u - Bv = m\ddot{y}$

$\rightarrow u - Bv = m\dot{v}$

$\rightarrow u = m\dot{v} + Bv$ $\therefore$, 1^{st} order ODE