

$$(1) \int \frac{t^5 - 3t^2 + 1}{t^3} dt$$

$$= \int t^2 - \frac{3}{t} + \frac{1}{t^3} dt$$

$$= \frac{1}{3}t^3 - 3\ln|t| - \frac{1}{2t^2} + C$$

(Cは積分定数)

$$(2) \int \frac{t+1}{t^2+1} dt$$

$$= \int \frac{t}{t^2+1} + \frac{1}{t^2+1} dt$$

① ②

①について、

$$\int \frac{t}{t^2+1} dt$$

$$= \frac{1}{2} \ln(t^2+1)$$

②について、

$$\int \frac{1}{t^2+1} dt$$

$t = \tan \theta$ とおくと、

$$= \int \frac{1}{1+\tan^2 \theta} \cdot \frac{1}{\cos^2 \theta} d\theta$$

$$= \int d\theta = \theta$$

おと、② = $\tan^{-1} t$

ゆえに、

$$(与式) = \frac{1}{2} \ln(t^2+1) + \tan^{-1} t + C$$

(Cは積分定数)

$$(3) \int \frac{dt}{t(1-\frac{t}{a})}$$

$$= \int \frac{1}{t} + \frac{\frac{1}{a}}{1-\frac{t}{a}} dt$$

$$= \ln|t| - \ln|1-\frac{t}{a}| + C$$

$$= \ln \left| \frac{t}{1-\frac{t}{a}} \right| + C$$

(Cは積分定数)