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Design concept evaluation based on cloud rough model and modified AHP-VIKOR: An application to lithography tool manufacturing process

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ABSTRACT

Design concept evaluation is a wide domain of research as the production process of new product development is mainly based on the evaluations of experts. As different types of uncertainties like vagueness, randomness, and diversity essentially exist in the evaluation information, the manipulation models that deal with just one type of uncertainty are not appropriate for ranking the design alternatives. The integration of multiple approaches into a single mathematical model to create an effective framework is more useful for uncertain information manipulation. This paper presents an innovative group decision-making approach to evaluate design concepts by integrating cloud rough numbers, analytic hierarchy process (AHP) and VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method to introduce a novel cloud rough AHP-VIKOR model. The concept of cloud rough numbers is utilized by fusing the merits of the cloud model theory and rough approximations in addressing randomness in experts' judgments and intrapersonal uncertainty for handling consensus inconsistency. Some new modified algebraic operations are introduced to remove the errors in already existing operations of cloud rough numbers. The cloud rough AHP technique is introduced to compute weights of design criteria by computing the cloud rough numbers of linguistic values and their reciprocals. Based on cloud rough values of positive and negative ideals, the individual regret, group utility values and evaluation indices are determined. Using the distance of evaluation indices from minimized cloud rough value, the design alternatives are ranked using different formulae. The significance of the proposed approach is illustrated with a real-world application in lithography tool manufacturing process. The out-performance and reliability of the proposed model is studied by a sensitivity analysis of different parameters in the method, and a comparison analysis with existing approaches.

1. Introduction

New product development (NPD) has a broad scope of real-world applications and plays an important role in the enhancement and development of manufacturing enterprises, and numerous research scholars have improved their cognition of the need to manage it strategically [1]. Design concept evaluation (DCE) is one of the most important assessment problem in decision analysis. In the process of evaluating a new product concept, several new designs are compared based on various criteria in order to identify the most satisfactory product concept [2]. Making the incorrect decision during the concept evaluation may force you to subsequently modify or rebuild the system, which raises costs unnecessarily. Thus, one of the most important challenges in NPD is evaluating the design concept at a very early design phase. It is well acknowledged that conceptual design determines up to 70%–80% of the product life cycle cost. Moreover, the drawbacks at this phase can rarely be compensated in the upcoming design process. As a result, due to its significant impact on the design activities, the evaluation of the final design concept is probably the most important step among NPD in the early phases of product development.

DCE is a challenging multi-criteria decision making (MCDM) problem under uncertain environment that takes into account a variety of factors ranging from customer needs to design constraints and develops an appropriate evaluation framework for assessing the risk priority ranking of design alternatives (DAs) [3,4]. At the same time, it is a group decision making (GDM) problem. In this problem, evaluation data are obtained from DCE knowledge and experiences at the early design phase, as well as subjective assessment of experts. In the early design phase, information is usually incomplete and ambiguous. The judgments made by the experts often lack precision and accuracy which results in certain degrees of uncertainty. Therefore, the efficiency of decision-making depends on one's ability to deal with the ambiguous and uncertain nature of the

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information. The aggregation of individual judgments and alternative ranking techniques in the evaluation model is crucial to the accuracy and efficiency of the evaluation of design concepts.

In order to deal with both objective and subjective uncertainties in the DCE model, this research uses a systematic approach of cloud rough (CR) numbers based of GDM framework by integrating the concept of CR numbers with AHP (analytical hierarch process) and VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) approaches.

1.1. Motivation and objectives

The main motives of current research study are highlighted as follows:

1. The existing DCE models based approaches usually deal with single uncertainty or multiple uncertainties using pre-defined parameters and membership functions. Ayağ [2] only dealt with intrapersonal uncertainty by executing fuzzy set theory or its derivatives, while Geng et al. [5], Qi et al. [6] only considered interpersonal uncertainty using rough approximations [7,8] and their extensions. It is not enough to consider only one of the multiple uncertainties inherent in MAGDM problems.
2. To deal with interpersonal uncertainty and intrapersonal uncertainty simultaneously, an uncertain manipulation model (that is, CR number) is used which combines the advantages of the CM theory and rough approximations. Although the CM theory concentrates on representing the linguistic terms in a stochastic and fuzzy manner, whereas rough numbers concentrate on describing uncertain data via interval-based approximations. And this technique helps to strengthen the objectivity of the evaluation outcomes in a subjective process and better capture the genuine perception of experts.
3. The traditional AHP and VIKOR approaches have their strengths in determining criteria weights and ranking different DAs, respectively. But they seem unable to handle the uncertainty that arises throughout the concept evaluation. In situations where data is uncertain or vague, these methods may not provide robust results. The objective of this research lies in three folds:

   a. In order to fill the gap in the evaluation of design concepts, a new MCDM model is being developed.
   b. It aims to present a CR model to address various uncertainties, better describe the true perceptions of DMs, and improve the objectivity of evaluation outcomes in the subjective processes.
   c. In this paper, we propose a CR-based AHP as well as a CR-based VIKOR in order to enhance the performance of MCDM algorithms and to emphasize the benefits of CR numbers.

1.2. Innovative contribution

adequately identify the criteria weights and prioritize the DAs while accounting for various uncertainties, we build a CR-AHP-VIKOR model. The most significant innovative contributions of the research are given below:

1. This paper integrated the concept of the CM theory into rough number theory that can measure the vague information in the traditional cloud model and deal with the interpersonal uncertainty in the group assessment process. The main characteristic of this model is that some of the advantages of certain approaches are used to overcome the weaknesses of others, therefore combining the benefits of both approaches into one strategy that effectively addresses the diverse sources of uncertainty under the complex group decision environment.
2. For the calculation of criteria weights, the CR-AHP method reduces the influence of multiple factors of uncertainty by extending the AHP method into the CR environment. It provides a systematic and structured approach to complex decision-making problems, making it easier to reach rational and accurate decisions.
3. A new method known as CR-VIKOR has been presented to evaluate different design concepts based on multiple conflicting criteria and selected the most suitable design concept from a set of alternatives.
4. A real-world application of the lithography tool design concept based on DMs' ideas is employed to show the relevance and importance of the proposed model.
5. Furthermore, sensitivity analysis as well as comparative analysis with other earlier DCE models are also performed to demonstrate the proposed GDM technique's efficiency and benefits. As a result, the proposed model provides more accurate assessment results and deals effectively with various uncertainties.

1.3. Framework of the paper

The structure of this paper is executed in the following manner:

1. The background and brief review are described in Section 2.
2. In Section 3, the basic concepts of CM theory, rough numbers, and CR numbers are discussed.
3. Operational rules, and aggregating operator for CR numbers are outlined in Section 4.
4. In Section 5, a novel CR-AHP-VIKOR is created to determine criterion weights and ranking of the design concepts.
5. A case study is analyzed using the developed model and sensitivity analysis is also conducted in Section 6.
6. In Section 7, comparisons of the developed method are performed.
7. Section 8 contains conclusion and future directions.

The list of acronyms used in this research study are illustrated in Table 1.

Table 1
Notations used in this paper.

Abbreviation	Brief description	Abbreviation	Brief description
NPD	New product development	DAs	Design alternatives
DCE	Design concept evaluation	CM	Cloud model
TFS	Triangular fuzzy set	DAs	Design alternatives
DMs	Decision makers	CR	Cloud rough
GDM	Group decision making		
MAGDM	Multi-attribute group decision making		

2. Related works

Group decision-making techniques can evaluate the DAs under multiple criterions in different problems such as supplier evaluation [9,10], financial performance of companies [11], innovation performance evaluation and entrepreneurship, whereas evaluation information manipulation is provided by decision-makers (DMs) [6,12]. In the field of product design, the evaluation of design concepts is unquestionably the most important step, as it determines the direction of the upcoming design phase. A team of experts as DMs first give their preferences on different design concepts subject to a set of evaluation criteria on the basis of their experience and perceptions. The main focus is to select the optimal design concept to improve the rationality and accuracy of alternative selection in the evaluation process. However, most of the information used in the DCE process is determined by the DMs personal estimation which can often be inaccurate, uncertain, or even inconsistent in nature. Due to the lack of information, selection of DAs, and uncertainty in the evaluation data, there is a big gap between the objective assessment and the uncertain subjective environment.

It is too hard for the DMs to provide precise quantitative values in the evaluation process and to deal with these uncertainties in a reasonable manner is difficult due to multiple uncertainties (such as fuzziness and randomness) present in DMs linguistic terms. Fuzzy logic-based techniques are commonly used to deal with vagueness and uncertainty given by DMs in the evaluation processes [2,4] including triangular fuzzy sets (TFSs) and intuitionistic fuzzy sets. Fuzzy logic-based methods are primarily able to address intrapersonal uncertainty (uncertainties arising from individual understandings about judgments), but ignore interpersonal uncertainty (the differences or randomness among opinions of various DMs). Some researchers use the notions of rough sets or rough numbers (RNs) to deal with interpersonal uncertainty [6,13,14]. However, rough numbers based approaches lack the mechanism to deal with intrapersonal uncertainty. Taking into account the negative impacts of the two types of uncertainties, manipulating a single interpersonal information interaction or a single intrapersonal uncertainty effected the accuracy and efficacy of the evaluation results. The fuzzy rough numbers based approaches have been used to address both intrapersonal and interpersonal uncertainties simultaneously by integrating triangular fuzzy sets with rough approximations [14,15]. However, the shortcomings of triangular fuzzy sets can restrict the efficiency and accuracy of the outcomes obtained by the fuzzy rough numbers such as fixed membership distribution [10,16] and ignoring the uncertainty of interval boundary (referred to as randomness).

In contrast to fuzzy based theories and their extensions, which use crisp numbers or interval numbers to represent the membership degree of an object, cloud model (CM) utilizes a large number of discrete points to depict the fuzziness and randomness of the assessment object. There is a lot of evidence to suggest that CM is able to reliably and accurately express the judgment information of DMs compared to a TFSs [10,12]. As a result, we can use CM theory to describe and capture experts' uncertain preferences. Thus, this paper introduces an improved DCE model by simultaneously considering the positive features of CM theory and rough approximations for the manipulation of various types of uncertainties, which is important for uncertain characterization and subjectivity elimination. [17] applied modified VIKOR method to classify watershed vulnerability in southern Taiwan. Akram et al. [18] proposed VIKOR method under complex Pythagorean fuzzy N-soft information and illustrated its application in intelligent manufacturing. The researchers are actively working on VIKOR and AHP approaches, for instance, intuitionistic fuzzy extended VIKOR [19], VIKOR method based on extended fuzzy based approaches [20–23] medical service evaluation in Taiwan [24], group decision analysis based on spherical fuzzy VIKOR method [18], decision making framework for the selection of energy project using complex Pythagorean fuzzy information [25], VIKOR method based on trapezoidal bipolar fuzzy information [26], multi-objective decision making [27], intuitionistic fuzzy rough Dombi aggregation operations [28].

In these MAGDM techniques, AHP and VIKOR are quite popular methods for the evaluation of design concepts. An important feature of AHP [29] is that it is a well-known technique for weighting criteria and offers a systematic approach to examine complicated decision issues and deal with tangible and non-tangible factors. In multi-level hierarchical structures, this technique is utilized to create ratio scales from pairwise comparisons. While, VIKOR was first developed by Opricovic and Tzeng [30] and it is derived from the L_p-metric in compromise programming. This tool has been found to be especially appropriate in situations where DMs have difficulty in expressing their preferences in alternative ranking. However, the conventional AHP and VIKOR techniques cannot handle intrapersonal and interpersonal uncertainty and only employs crisp numbers to describe DMs evaluations, which may result in losing data during complex decision-making environments. Kahraman et al. [31] utilized fuzzy AHP to select the best supplier firm providing the most satisfaction for the criteria determined. Yu et al. [32] discussed the research evolution on AHP from 1982 to 2018. Other researches on AHP under fuzzy based approaches and their extensions include fuzzy AHP mthods for subjective judgments [33]. Zhu [34] proposed a rough Z-numbers based MABAC approach for DCE approach in complex and uncertain decision-making environments. Jing et al. [35] studied conceptual design evaluation by integrating intuitionistic fuzzy sets with VIKOR method and Dempster-Shafer evidence theory.

The important step is to prioritize the design concepts in DCE process that perform well according to the established evaluation criteria. It is a multi-attribute GDM (MAGDM) problem in which criteria are weighted and ranking of the design alternatives is based on their performance. The most common techniques for calculating criterion weights are ANP (analytic network process) [36], AHP (analytic hierarchy process) [2,37,38], BWM (best worst method) [39], etc. Moreover, several decision-making approaches are applied to perform the ranking of design alternatives, such as VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) [40–42], grey relational analysis [3], TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [14,43], DEA (data envelopment analysis) [44], DEMATEL (decision-making trial and evaluation laboratory) [45,46], and PROMETHEE (Preference Ranking Organization Method for Enrichment of Evaluations) [47]. [48] proposed interval rough integrated SWARA-ELECTRE model for the evaluation of manufacturing machine tools. Akram et al. [49] introduced the powerful notion of Pythagorean fuzzy rough numbers and applied it to enhanced CRITIC-REGIME method in decision making approach and the same model is also applied on a DCE problem in [50]. Akram and Ashraf [51] proposed a new model by integrating spherical fuzzy numbers with rough approximations. Jiang et al. [52] proposed a hybrid model for a new similarity calculation method for cloud model. Zhou et al. [53] proposed a novel hesitant fuzzy linguistic cloud DEMATEL method for smart vehicle service system.

The researchers are actively working on rough approximations, clouds models and their various integrations and extensions, for instance, risk assessment based on rough fuzzy number clouds with cloud weights computed using subjective and objective aspects of uncertainty [54], the application of ELECTRE-II and CODAS methods in business intelligence under 2-tuple linguistic Fermatean fuzzy information [55], multi-criteria group decision making using MARCOS method under 2-tuple linguistic q-rung picture fuzzy environment [55], decision analysis based on CRITIC-EDAS method under Pythagorean fuzzy environment [56], parameter influencing testing based on AHP-TOPSIS [57], dual 2-tuple linguistic rough number clouds for health care waste management [58], risk assessment based on rough ELECTRE II [59], application of rough approximations in agriculture and criminal networks [60], distance measures and δ-approximations in rough complex fuzzy environment [61], design assessment scheme based on Z-cloud rough numbers [62], machine tool remanufacturing based on dual interval rough integrated clouds [63], risk assessment based on MULTIMOORA method and rough cloud model [64], decision analysis based on dual clouds extracted from dual Z-numbers [65], decision making based on dual trapezium clouds [66], Logarithmic Percentage Change-driven Objective Weighting (LOPCOW) and Evaluation based on Distance from Average Solution (EDAS) approaches for decision making framework in sales and emerging market [67], interval valued intuitionistic fuzzy sets based CODAS method, decision making based on q-rung othopair fuzz sets [68], rough AHP and MABAC [69], spherical fuzzy AHP-ARAS [70], complex spherical fuzzy VIKOR approach [71], CODAS technique based on interval-valued intuitionistic sets [72], VIKOR method for pesonnel training problem [73], rough cloud AHP [74]. The importance of dual interval rough number clouds is studied in intelligent manufacturing process in Sarwar et al. [75]

2.1. Knowledge gaps

There are several MCDM approaches based on the integration of cloud theory and rough approximation to handle different types of uncertainties in DCE models.

The rough integrated cloud model first converts the initial data into rough numbers and then cloud evaluations. The role of upper and lower approximations is only on the initial step when the initial decision matrix is converted into cloud values. In the interval rough number clouds and dual interval rough integrated cloud model, the initial data is converted into interval rough numbers and then clouds and interval cloud values, respectively. Similarly, in this case, the notion of rough approximations is used only at the first. However, in case of CR number, the whole method is based on rough approximations as well as cloud model. A CR number consists of three components expectation, entropy and hyper entropy in which each component is a rough number. This model is more rational and validated to handle uncertainty and randomness in decision analysis.

Zhu et al. [37] studied DCE approach by integrating AHP-VIKOR with rough approximations. However, the randomness in the experts' evaluations is ignored. This research study overcomes this limitation and deals with interpersonal vagueness and randomness simultaneously in DCE models by converting the data of different domains into CR numbers. Moreover, certain new techniques, operations and operators are introduced, their validity and important properties are proved theoretically and numerically. Huang et al. [76] initiated the notion of CR numbers and applied it on TOPSIS method and computed the weight values using aggregation operators. This research study is based on the implementation of CR numbers into AHP and VIKOR approaches. The CR numbers are applied on AHP approach and computed the weight values implementing the concept of consistency ratio. Moreover, the CR numbers are computed not only for integral values but also for reciprocal initial data at the same time. The significance of the approach is then discussed with an application example by converting the CR numbers in different domains.

3. Basic notions

The aim of this section is to briefly describe some basic concepts regarding the CM theory, rough numbers, and uncertainty manipulation model (CR numbers).

3.1. Cloud model theory

To tackle imprecise and uncertain situations in reality, Li [77] developed the theory of cloud model based on fuzziness and probability theory. In contrast to crisp or interval numbers utilized in fuzzy based theories and their variants, it employs a huge number of discrete points to indicate the fuzziness of the given component. It can effectively deal with the vagueness of components and the randomness of interval boundaries. In recent years, CM has been employed with great success in many areas, such as low-speed wind farm site selection [78], green supplier evaluation and selection [79], safety risk perception [80], and hotel selection [81], etc. The most common sort of CM is the normal cloud, written as $C = (Ex, En, HEn)$. Where Ex expectation depict the component's qualitative concept, En entropy describes the randomness and fuzziness of the qualitative concept, and HEn hyper-entropy demonstrates the membership's level of uncertainty.

Definition 1 (*[82]*)**.** Let LT is a qualitative concept related to a universe U, suppose $y \in U$ is an arbitrary realization of LT, satisfying the $y \sim N(Ex, {En'}^2)$ and $En' \sim N(En, HEn^2)$. $\mu_{LT}(y) \in [0,1]$ is the membership degree of y on LT, which satisfies Eq. (1).

$$\mu_{LT}(y) = e^{-\frac{(y-Ex)^2}{2{En'}^2}} \tag{1}$$

Definition 2 (*[16,83]*)**.** Assume that the effective domain $U = [x_{\min}, x_{\max}]$ and let $LT = \{LT^k | k = 0, 1, \dots, g\}$ be a linguistic terms set given by decision makers in which LT^k is a possible value of linguistic variables provided by experts E_k. Then, g basic clouds $C_0, C_1, \dots, C_{g-1}$ can be generated by using the golden segmentation method. For example, the numerical characters of the seventeen cloud are illustrated as follows:

$$C_0 = (Ex_0, En_0, HEn_0) = \left(x_{min} + 3En_0, \frac{En_1}{0.618}, \frac{HEn_1}{0.618}\right); C_1 = (Ex_1, En_1, HEn_1) = \left(Ex_2 - 0.382(Ex_2 - Ex_0), \frac{En_2}{0.618}, \frac{HEn_2}{0.618}\right);$$

$$C_2 = (Ex_2, En_2, HEn_2) = \left(Ex_3 - 0.382(Ex_3 - Ex_0), \frac{En_3}{0.618}, \frac{HEn_3}{0.618}\right); C_3 = (Ex_3, En_3, HEn_3) = \left(Ex_4 - 0.382(Ex_4 - Ex_0), \frac{En_4}{0.618}, \frac{HEn_4}{0.618}\right);$$

$$C_4 = (Ex_4, En_4, HEn_4) = \left(Ex_5 - 0.382(Ex_5 - Ex_0), \frac{En_5}{0.618}, \frac{HEn_5}{0.618}\right); C_5 = (Ex_5, En_5, HEn_5) = \left(Ex_6 - 0.382(Ex_6 - Ex_0), \frac{En_6}{0.618}, \frac{HEn_6}{0.618}\right);$$
$$C_6 = (Ex_6, En_6, HEn_6) = \left(Ex_7 - 0.382(Ex_7 - Ex_0), \frac{En_7}{0.618}, \frac{HEn_7}{0.618}\right); C_7 = (Ex_7, En_7, HEn_7) = \left(Ex_8 - 0.382(Ex_8 - Ex_0), \frac{En_8}{0.618}, \frac{HEn_8}{0.618}\right);$$
$$C_8 = (Ex_8, En_8, HEn_8) = \left(\frac{x_{min} + x_{max}}{2}, 0.382\frac{(x_{max} - x_{min})}{3(g+1)}, HEn_8\right); \left(\begin{array}{c} g = 17 \\ HEn_8 = 0.02 \end{array}\right)$$
$$C_9 = (Ex_9, En_9, HEn_9) = \left(Ex_8 + 0.382(Ex_{16} - Ex_8), \frac{En_1}{0.618}, \frac{HEn_8}{0.618}\right);$$
$$C_{10} = (Ex_{10}, En_{10}, HEn_{10}) = \left(Ex_9 + 0.382(Ex_{16} - Ex_9), \frac{En_2}{0.618}, \frac{HEn_9}{0.618}\right);$$
$$C_{11} = (Ex_{11}, En_{11}, HEn_{11}) = \left(Ex_{10} + 0.382(Ex_{16} - Ex_{10}), \frac{En_3}{0.618}, \frac{HEn_{10}}{0.618}\right);$$
$$C_{12} = (Ex_{12}, En_{12}, HEn_{12}) = \left(Ex_{11} + 0.382(Ex_{16} - Ex_{11}), \frac{En_{11}}{0.618}, \frac{HEn_{11}}{0.618}\right);$$
$$C_{13} = (Ex_{13}, En_{13}, HEn_{13}) = \left(Ex_{12} + 0.382(Ex_{16} - Ex_{12}), \frac{En_{12}}{0.618}, \frac{HEn_{12}}{0.618}\right);$$
$$C_{14} = (Ex_{14}, En_{14}, HEn_{14}) = \left(Ex_{13} + 0.382(Ex_{16} - Ex_{13}), \frac{En_{13}}{0.618}, \frac{HEn_{13}}{0.618}\right);$$
$$C_{15} = (Ex_{15}, En_{15}, HEn_{15}) = \left(Ex_{14} + 0.382(Ex_{16} - Ex_{14}), \frac{En_{14}}{0.618}, \frac{HEn_{14}}{0.618}\right);$$
$$C_{16} = (Ex_{16}, En_{16}, HEn_{16}) = \left(x_{max} - 3En_{16}, \frac{En_{15}}{0.618}, \frac{HEn_{15}}{0.618}\right).$$

The linguistic information can be quantitatively analyzed by converting it into the corresponding cloud values. Inspired by the studies of [76], these seventeen cloud values $(C_0, C_1, \dots, C_{16})$ can be determined using the golden segmentation method by presetting the effective domain $U = [x_{\min}, x_{\max}]$ and the value of HEn_8 in the midterm cloud.

Definition 3 ([84]). Assume that $C = (Ex, En, HEn)$ and $C' = (Ex', En', HEn')$ are two normal clouds in the domain U and γ be a real number, the main operations of normal clouds are listed below:

1. $C \pm C' = \left(Ex \pm Ex', \sqrt{(En)^2 + (En')^2}, \sqrt{(HEn)^2 + (HEn')^2}\right)$
2. $C \times C' = \left(ExEx', |ExEx'|\sqrt{(\frac{En}{Ex})^2 + (\frac{En'}{Ex'})^2}, |ExEx'|\sqrt{(\frac{HEn}{Ex})^2 + (\frac{HEn'}{Ex'})^2}\right)$
3. $\frac{C}{C'} = \left(\frac{Ex}{Ex'}, |\frac{Ex}{Ex'}|\sqrt{(\frac{En}{Ex})^2 + (\frac{En'}{Ex'})^2}, |\frac{Ex}{Ex'}|\sqrt{(\frac{HEn}{Ex})^2 + (\frac{HEn'}{Ex'})^2}\right)$
4. $\gamma C = \left(\gamma Ex, \sqrt{\gamma} En, \sqrt{\gamma} HEn\right)$

3.2. Rough numbers

To manipulate uncertain data in the GDM process, Pawlak [7] proposed the notions of lower and upper approximations. Using the concept of Pawlak rough approximations, the notion of a rough number was first proposed by Zhai et al. [85] for establishing boundary intervals and handling personal decisions of customers. Additionally, rough number shares the beneficial aspects of upper and lower approximations, such as the complete reliance on the original dataset, the absence of auxiliary data, subjectivity and variation in group perception can be efficiently managed. It has been widely used to combine with different MCDM approaches and utilized in many fields, such as QFD (quality function deployment) [85,86], FMEA (failure mode and effects analysis) [16,64], DCE (design concept evaluation) [13,37], and supply chain evaluation [87].

Definition 4 ([85]). Let $Y = \{y_1, y_2, \dots, y_n\}$ be a collection of assessment, which is sorted in the manner $y_1 \le y_2 \le \cdots \le y_n$ and y_i is an arbitrary value in collection Y. Then, its lower approximation $\underline{Apr}(y_i)$, upper approximation $\overline{Apr}(y_i)$ and boundary region $Bnd(y_i)$ can be determined as:

$$\underline{Apr}(y_i) = \cup\{y_j \in Y \mid y_j \le y_i\},$$
$$\overline{Apr}(y_i) = \cup\{y_j \in Y \mid y_j \ge y_i\},$$
$$Bnd(y_i) = \cup\{y_j \in Y \mid y_j \ne y_i\}.$$

Then y_i can be represented by a rough number $RN(y_i)$, which is determined by its corresponding lower and upper limit given as

$$\underline{Lim}(y_i) = \frac{1}{M_L}\sum_{j=1}^{M_L} y_j | y_j \in \underline{Apr}(y_i) \quad \text{and} \quad \overline{Lim}(y_i) = \frac{1}{M_U}\sum_{j=1}^{M_U} y_j | y_j \in \overline{Apr}(y_i),$$

where $\underline{Lim}(y_i)$ and $\overline{Lim}(y_i)$ are lower limit and upper limit of the rough interval and M_L and M_U are the numbers of elements that contained in $\underline{Apr}(y_i)$ and $\overline{Apr}(y_i)$, respectively, and their difference $IRBnd(y_i) = \overline{Lim}(y_i) - \underline{Lim}(y_i)$ is defined as rough boundary interval.

Definition 5 ([85]). Let $RN(x) = \lceil \underline{lim}(x), \overline{lim}(x) \rfloor$ and $RN(y) = \lceil \underline{lim}(y), \overline{lim}(y) \rfloor$ be the rough numbers and k be a constant, then the binary operations and scalar multiplication of rough numbers can be defined as:

1. $RN(x) + RN(y) = \lceil \underline{lim}(x), \overline{lim}(x) \rfloor + \lceil \underline{lim}(y), \overline{lim}(y) \rfloor = \lceil \underline{lim}(x) + \underline{lim}(y), \overline{lim}(x) + \overline{lim}(y) \rfloor$
2. $RN(x) \times RN(y) = \lceil \underline{lim}(x), \overline{lim}(x) \rfloor \times \lceil \underline{lim}(y), \overline{lim}(y) \rfloor = \lceil \underline{lim}(x) \times \underline{lim}(y), \overline{lim}(x) \times \overline{lim}(y) \rfloor$
3. $k \times RN(x) = k \times \lceil \underline{lim}(x), \overline{lim}(x) \rfloor = \lceil k \times \underline{lim}(x), k \times \overline{lim}(x) \rfloor$

3.3. Cloud rough numbers

As discussed in previous sections, it can be seen that the CM theory can manage intrapersonal uncertainty well, but it is unable to manage interpersonal uncertainty brought on by the diversity of human perspectives, ideas, traditions, and demands. In this situation, rough approximations are a useful tool for manipulating interpersonal uncertainty. To simultaneously manage intrapersonal and interpersonal uncertainty in GDM environments, CM theory with rough approximations are put together to form the integrated uncertain manipulation model, known as CR model [76]. This model integrates the notion of CM in manipulating the randomness and uncertainty of subjective assessments and the worth of the rough approximations in handling the vagueness and consensus of GDM information.

The integration of CM theory and rough approximations has been studied in different ways including rough integrated clouds, interval rough number clouds, dual interval rough integrated clouds and CR numbers. In all the integrated models except CR numbers, the initial data is first converted into rough numbers or interval rough numbers and then cloud values or interval cloud values. Rough approximations are applied on initial fixed values and then transformed into cloud evaluations. However, in CR numbers, the initial data is first converted into cloud evaluations and then rough approximations are applied on cloud values. The resulting model is basically a cloud value with its components in the form of rough numbers. It is more rational approach than existing models as rough approximations, expectation, entropy and hyper entropy are studied simultaneously in the resulting model.

The rough boundary, upper limit, and lower limit are the standard three features of rough approximations. Following Definitions 2 and 4, linguistic variables are classified into n classes, that is, they can be ranked in the increasing order as $A_1 < A_2 < \cdots < A_n$. A_i $(1 \leq i \leq n)$ represents a class of cloud value and $C_i = (Ex_i, En_i, HEn_i)$ is the associated cloud value of A_i. Suppose $C_j = (Ex_j, En_j, HEn_j)$ denotes any specific cloud value in U, then the lower approximation of the class A_i is defined in Eq. (2), (3), (4).

$$\underline{Apr}(Ex_i) = \cup\{Ex_j \in U | Ex_j \leq Ex_i\} \tag{2}$$

$$\underline{Apr}(En_i) = \cup\{En_j \in U | En_j \leq En_i\} \tag{3}$$

$$\underline{Apr}(HEn_i) = \cup\{HEn_j \in U | HEn_j \leq HEn_i\} \tag{4}$$

where $\underline{Apr}(Ex_i)$, $\underline{Apr}(En_i)$ and $\underline{Apr}(HEn_i)$ denote the lower approximation of Ex_i, En_i and HEn_i, respectively. The upper approximation of the class A_i is calculated in Eq. (5), (6), (7).

$$\overline{Apr}(Ex_i) = \cup\{Ex_j \in U \mid Ex_j \geq Ex_i\} \tag{5}$$

$$\overline{Apr}(En_i) = \cup\{En_j \in U \mid En_j \geq En_i\} \tag{6}$$

$$\overline{Apr}(HEn_i) = \cup\{HEn_j \in U \mid HEn_j \geq HEn_i\} \tag{7}$$

where $\overline{Apr}(Ex_i)$, $\overline{Apr}(En_i)$ and $\overline{Apr}(HEn_i)$ denote the upper approximation of Ex_i, En_i, and HEn_i of class A_i, respectively. The lower limit of class A_i is described in Eq. (8), (9), (10).

$$\underline{Lim}(Ex_i) = \frac{1}{M_L^{Ex}} \sum_{j=1}^{M_L^{Ex}} Ex_j | Ex_j \in \underline{Apr}(Ex_i) \tag{8}$$

$$\underline{Lim}(En_i) = \frac{1}{M_L^{En}} \sum_{j=1}^{M_L^{En}} En_j | En_j \in \underline{Apr}(En_i) \tag{9}$$

$$\underline{Lim}(HEn_i) = \frac{1}{M_L^{HEn}} \sum_{j=1}^{M_L^{HEn}} HEn_j | HEn_j \in \underline{Apr}(HEn_i) \tag{10}$$

where $M_L^E x$, $M_L^E n$, $M_L^H En$ denote the total number of elements in $\underline{Apr}(Ex_i)$, $\underline{Apr}(En_i)$ and $\underline{Apr}(HEn_i)$. $\underline{Lim}(Ex_i)$, $\underline{Lim}(En_i)$ and $\underline{Lim}(HEn_i)$ denote the lower limit of Ex_i, En_i, and HEn_i, respectively. Similarly, the upper limit of class A_i is defined in Eq. (11), (12), (13).

$$\overline{Lim}(Ex_i) = \frac{1}{M_U^{Ex}} \sum_{j=1}^{M_U^{Ex}} Ex_j | Ex_j \in \overline{Apr}(Ex_i) \tag{11}$$

$$\overline{Lim}(En_i) = \frac{1}{M_U^{En}} \sum_{j=1}^{M_U^{En}} En_j | En_j \in \overline{Apr}(En_i) \tag{12}$$

$$\overline{Lim}(HEn_i) = \frac{1}{M_U^{HEn}} \sum_{j=1}^{M_U^{HEn}} HEn_j | HEn_j \in \overline{Apr}(HEn_i) \tag{13}$$

where M_U^{Ex}, M_U^{En}, and M_U^{HEn} represent the total number of elements in $\overline{Apr}(Ex_i)$, $\overline{Apr}(En_i)$ and $\overline{Apr}(HEn_i)$. $\overline{Lim}(Ex_i)$, $\overline{Lim}(En_i)$ and $\overline{Lim}(HEn_i)$ denote the upper limit of Ex_i, En_i, and HEn_i, respectively.

The rough boundary interval of Ex_i, En_i, HEn_i, and class A_i is determined in Eq. (14).

$$IRBnd(\tilde{A}_i) = \left(Ex_i^U + 3En_i^U\right) - \left(Ex_i^L + 3En_i^L\right) \tag{14}$$

$IRBnd(Ex_i) = Ex_i^U - Ex_i^L$, $IRBnd(En_i) = En_i^U - En_i^L$, $IRBnd(HEn_i) = HEn_i^U - HEn_i^L$. The CR numbers of class A_i can be expressed in Eq. (15).

$$\begin{aligned} CR(A_i) &= \left(CR(Ex_i), CR(En_i), CR(HEn_i)\right) \\ &= \left(\left[Ex_i^L, Ex_i^U\right], \left[En_i^L, En_i^U\right], \left[HEn_i^L, HEn_i^U\right]\right) \\ &= \left(\left[\underline{Lim}(Ex_i), \overline{Lim}(Ex_i)\right], \left[\underline{Lim}(En_i), \overline{Lim}(En_i)\right], \left[\underline{Lim}(HEn_i), \overline{Lim}(HEn_i)\right]\right) \end{aligned} \tag{15}$$

For convenience, $CR(Ex_i)$, $CR(En_i)$, and $CR(HEn_i)$ are expressed as Ex_i^L, Ex_i^U, En_i^L, En_i^U, HEn^i_L, and HEn_i^U in subsequent contents, respectively. In this study, we define the $\tilde{A}_i$ to represent the $CR(A_i)$ for easier representation.

4. The proposed new algebraic operations of CR numbers

To effectively handle the substantial evaluating parameters, some important basic operations should be first established. Based on the study by [42,62], several new operational laws for CR numbers are presented.

Let $\tilde{A} = ([Ex^L, Ex^U], [En^L, En^U], [HEn^L, HEn^U])$ and $\tilde{B} = \left(\left[Ex'^L, Ex'^U\right], \left[En'^L, En'^U\right], \left[HEn'^L, HEn'^U\right]\right)$ be two arbitrary CR numbers, and $\lambda > 0$ is a constant. In order to define the operations of CR numbers, we must combine the arithmetic operations of cloud model and rough numbers, which are as follows:

1.
$$\begin{aligned}\tilde{A}+\tilde{B} &= \left(\tilde{E}x+\tilde{E}x', \tilde{E}n+\tilde{E}n', \widetilde{HEn}+\widetilde{HEn}'\right)\\ &= \left(\left[Ex^L+Ex'^L, Ex^U+Ex'^U\right], \left[\sqrt{(En^L)^2+(En'^L)^2}, \sqrt{(En^U)^2+(En'^U)^2}\right],\right.\\ &\qquad \left.\left[\sqrt{(HEn^L)^2+(HEn'^L)^2}, \sqrt{(HEn^U)^2+(HEn'^U)^2}\right]\right)\end{aligned}$$

2.
$$\begin{aligned}\tilde{A}-\tilde{B} &= \left(\tilde{E}x-\tilde{E}x', \tilde{E}n-\tilde{E}n', \widetilde{HEn}-\widetilde{HEn}'\right)\\ &= \left(\left[Ex^L-Ex'^U, Ex^U-Ex'^L\right], \left[\sqrt{(En^L)^2+(En'^L)^2}, \sqrt{(En^U)^2+(En'^U)^2}\right],\right.\\ &\qquad \left.\left[\sqrt{(HEn^L)^2+(HEn'^L)^2}, \sqrt{(HEn^U)^2+(HEn'^U)^2}\right]\right)\end{aligned}$$

3.
$$\begin{aligned}\tilde{A}\times\tilde{B} &= \left(\tilde{E}x\times\tilde{E}x', \tilde{E}n\times\tilde{E}n', \widetilde{HEn}\times\widetilde{HEn}'\right)\\ &= \left(\left[Ex^L Ex'^L, Ex^U Ex'^U\right], \left[|Ex^L Ex'^L|\sqrt{\left(\frac{En^L}{Ex^L}\right)^2+\left(\frac{En'^L}{Ex'^L}\right)^2}, |Ex^U Ex'^U|\sqrt{\left(\frac{En^U}{Ex^U}\right)^2+\left(\frac{En'^U}{Ex'^U}\right)^2}\right],\right.\\ &\qquad \left.\left[|Ex^L Ex'^L|\sqrt{\left(\frac{HEn^L}{Ex^L}\right)^2+\left(\frac{HEn'^L}{Ex'^L}\right)^2}, |Ex^U Ex'^U|\sqrt{\left(\frac{HEn^U}{Ex^U}\right)^2+\left(\frac{HEn'^U}{Ex'^U}\right)^2}\right]\right)\end{aligned}$$

4.
$$\begin{aligned}\lambda B &= \left(\lambda\tilde{E}x', \lambda\tilde{E}n', \lambda\widetilde{HEn}'\right)\\ &= \left(\left[\lambda Ex'^L, \lambda Ex'^U\right], \left[\sqrt{\lambda}En'^L, \sqrt{\lambda}En'^U\right], \left[\sqrt{\lambda}HEn'^L, \sqrt{\lambda}HEn'^U\right]\right)\end{aligned}$$

5.
$$\begin{aligned}\tilde{A}\div\tilde{B} &= \left(\tilde{E}x\div\tilde{E}x', \tilde{E}n\div\tilde{E}n', \widetilde{HEn}\div\widetilde{HEn}'\right)\\ &= \left(\left[\frac{Ex^L}{Ex'^U}, \frac{Ex^U}{Ex'^L}\right], \left[\left|\frac{Ex^L}{Ex'^U}\right|\sqrt{\left(\frac{En^L}{Ex^L}\right)^2+\left(\frac{En'^L}{Ex'^U}\right)^2}, \left|\frac{Ex^U}{Ex'^L}\right|\sqrt{\left(\frac{En^U}{Ex^U}\right)^2+\left(\frac{En'^U}{Ex'^L}\right)^2}\right],\right.\\ &\qquad \left.\left[\left|\frac{Ex^L}{Ex'^U}\right|\sqrt{\left(\frac{HEn^L}{Ex^L}\right)^2+\left(\frac{HEn'^L}{Ex'^U}\right)^2}, \left|\frac{Ex^U}{Ex'^L}\right|\sqrt{\left(\frac{HEn^U}{Ex^U}\right)^2+\left(\frac{HEn'^U}{Ex'^L}\right)^2}\right]\right)\end{aligned}$$

6.
$$\begin{aligned}d(\tilde{A},\tilde{B}) &= \sqrt{d_1+d_2}\\ d_1 &= |Ex^L-Ex'^L|^2+|En^L-En'^L|^2+|HEn^L-HEn'^L|^2\\ d_2 &= |Ex^U-Ex'^U|^2+|En^U-En'^U|^2+|HEn^U-HEn'^U|^2\end{aligned} \tag{16}$$

The fusion evaluation information operators are crucial for aggregating preferences or perceptions of different experts for MAGDM problems. Based on [88,89], we define CR weighted average operator (CRW_{avg}) and CR average operation (CR_{avg}).

Definition 6. Suppose $\tilde{A}_i = \left(\left[Ex_i^L, Ex_i^U\right], \left[En_i^L, En_i^U\right], \left[HEn_i^L, HEn_i^U\right]\right)$ $(i = 1, 2, \ldots, n)$ are n CR numbers, and $W = (W_1, W_2, \ldots, W_n)$ denotes the weight vector of assessment criteria, with the condition $W_i \in [0, 1]$ and $\sum_{i=1}^n W_i = 1$. The CR weighted average (CRW_{avg}) operator is defined in Eq. (17).

$$CRW_{avg}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \sum_{i=1}^n W_i\tilde{A}_i = \left(\left[Ex^L_{W_{avg}}, Ex^U_{W_{avg}}\right], \left[En^L_{W_{avg}}, En^U_{W_{avg}}\right], \left[HEn^L_{W_{avg}}, HEn^U_{W_{avg}}\right]\right)$$

$$= \left(\left[W_i \sum_{j=1}^{n} Ex_i^L, \frac{1}{n} \sum_{i=1}^{n} Ex_i^U \right], \left[\sqrt{\sum_{i=1}^{n} W_i (En_i^L)^2}, \sqrt{\sum_{i=1}^{n} W_i (En_i^U)^2} \right], \left[\sqrt{\sum_{i=1}^{n} W_i (HEn_i^L)^2}, \sqrt{\sum_{j=1}^{n} W_i (HEn_i^U)^2} \right] \right) \tag{17}$$

Particularly, if $W_i = \frac{1}{n}$, then the CRW_{avg} becomes into the CR average (CR_{avg}) operation is defined in Eq. (18).

$$CR_{avg}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \frac{1}{n} \sum_{i=1}^{n} \tilde{A}_i$$
$$= \left(\left[\frac{1}{n} \sum_{i=1}^{n} Ex_i^L, \frac{1}{n} \sum_{i=1}^{n} Ex_i^U \right], \left[\sqrt{\frac{1}{n} \sum_{i=1}^{n} (En_i^L)^2}, \sqrt{\frac{1}{n} \sum_{i=1}^{n} (En_i^U)^2} \right], \left[\sqrt{\frac{1}{n} \sum_{i=1}^{n} (HEn_i^L)^2}, \sqrt{\frac{1}{n} \sum_{i=1}^{n} (HEn_i^U)^2} \right] \right) \tag{18}$$

Theorem 1. *Assume that $\tilde{A}_i = \left(\left[Ex_i^L, Ex_i^U \right], \left[En_i^L, En_i^U \right], \left[HEn_i^L, HEn_i^U \right] \right)$ $(i = 1, 2, \dots, n)$ are n CR numbers, and $W = (W_1, W_2, \dots, W_n)$ denotes the weight vector of assessment criteria, with the condition $W_i \in [0, 1]$ and $\sum_{i=1}^{n} W_i = 1$. Then, the $CRW_{avg}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n)$ is a CR number*

Proof. Using Eq. (17), it can be easily observed that $CRW_{avg}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n)$ are CR numbers which is presented in the form of rough numbers as the components of cloud evaluations.

Furthermore, it should be proved that the lower limit of rough boundary interval ($Ex_{W_{avg}}^L + 3En_{W_{avg}}^L$) of $\tilde{A}_i$ is less than or equal to the upper limit of rough boundary interval ($Ex_{W_{avg}}^U + 3En_{W_{avg}}^U$), i.e., $Ex_{W_{avg}}^L + 3En_{W_{avg}}^L \le Ex_{W_{avg}}^U + 3En_{W_{avg}}^U$. Obviously, according to the previous construction processes of CR numbers in Section 3.3, for $IRBnd(\tilde{A}_1)$, it holds that $Ex_1^L + 3En_1^L = Ex_1^U + 3En_1^U$ if and only if the initial cloud values are equal; or else $Ex_1^L + 3En_1^L < Ex_1^U + 3En_1^U$. Thus, we have either $Ex_{W_{avg}}^L + 3En_{W_{avg}}^L = Ex_{W_{avg}}^U + 3En_{W_{avg}}^U$ or $Ex_{W_{avg}}^L + 3En_{W_{avg}}^L < Ex_{W_{avg}}^U + 3En_{W_{avg}}^U$. Based on the above analysis, it has been proven that $CRW_{avg}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n)$ is also a CR numbers. □

Definition 7. Suppose $\tilde{A}_i = \left(\left[Ex_i^L, Ex_i^U \right], \left[En_i^L, En_i^U \right], \left[HEn_i^L, HEn_i^U \right] \right)$ $(i = 1, 2, \dots, n)$ are n CRNs, and $W = (W_1, W_2, \dots, W_n)$ denotes the weight vector of assessment criteria, satisfying $W_i \in [0, 1]$ and $\sum_{i=1}^{n} W_i = 1$. The cloud rough weighted geometric average ($CRWG_{avg}$) operator is defined as:

$$CRWG_{avg}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \prod_{i=1}^{n} \tilde{A}_i^{W_i}$$
$$= \left(\left[\left(\prod_{i=1}^{n} Ex_i^L \right)^{W_i}, \left(\prod_{i=1}^{n} Ex_i^U \right)^{W_i} \right], \left[\left(\prod_{i=1}^{n} Ex_i^L \right)^{W_i} \sqrt{W_i \sum_{i=1}^{n} \left(\frac{En_i^L}{Ex_i^L} \right)^2}, \left(\prod_{i=1}^{n} Ex_i^U \right)^{W_i} \sqrt{W_i \sum_{i=1}^{n} \left(\frac{En_i^U}{Ex_i^U} \right)^2} \right], \right.$$
$$\left. \left[\left(\prod_{i=1}^{n} Ex_i^L \right)^{W_i} \sqrt{W_i \sum_{i=1}^{n} \left(\frac{HEn_i^L}{Ex_i^L} \right)^2}, \left(\prod_{i=1}^{n} Ex_i^U \right)^{W_i} \sqrt{W_i \sum_{i=1}^{n} \left(\frac{HEn_i^U}{Ex_i^U} \right)^2} \right] \right) \tag{19}$$

Particularly if $W_i = \frac{1}{n}$, then Eq. (19) reduces to the CR average operator (CRG_{avg}), as shown in Theorem 2.

$$W_i = CRG_{avg}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \prod_{i=1}^{n} \tilde{A}_i^{\frac{1}{n}}$$
$$= \left(\left[\left(\prod_{i=1}^{n} Ex_i^L \right)^{\frac{1}{n}}, \left(\prod_{i=1}^{n} Ex_i^U \right)^{\frac{1}{n}} \right], \left[\left(\prod_{i=1}^{n} Ex_i^L \right)^{\frac{1}{n}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{En_i^L}{Ex_i^L} \right)^2}, \left(\prod_{i=1}^{n} Ex_i^U \right)^{\frac{1}{n}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{En_i^U}{Ex_i^U} \right)^2} \right], \right.$$
$$\left. \left[\left(\prod_{i=1}^{n} Ex_i^L \right)^{\frac{1}{n}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{HEn_i^L}{Ex_i^L} \right)^2}, \left(\prod_{i=1}^{n} Ex_i^U \right)^{\frac{1}{n}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{HEn_i^U}{Ex_i^U} \right)^2} \right] \right) \tag{20}$$

Definition 8. Let that $\tilde{A}_i = \left(\left[Ex_i^L, Ex_i^U \right], \left[En_i^L, En_i^U \right], \left[HEn_i^L, HEn_i^U \right] \right)$ $(i = 1, 2)$ be two CR numbers, then $\tilde{A}_1$ and $\tilde{A}_2$ can be compared as

1. If $0.5(Ex_1^L + Ex_1^U) > 0.5(Ex_2^L + Ex_2^U)$, then $\tilde{A}_1 > \tilde{A}_2$.
2. If $0.5(Ex_1^L + Ex_1^U) = 0.5(Ex_2^L + Ex_2^U)$, $0.5(En_1^L + En_1^U) < 0.5(En_2^L + En_2^U)$, then $\tilde{A}_1 > \tilde{A}_2$.
3. If $0.5(Ex_1^L + Ex_1^U) = 0.5(Ex_2^L + Ex_2^U)$, $0.5(En_1^L + En_1^U) = 0.5(En_2^L + En_2^U)$, $0.5(HEn_1^L + HEn_1^U) < 0.5(HEn_2^L + HEn_2^U)$, then $\tilde{A}_1 > \tilde{A}_2$.
4. If $0.5(Ex_1^L + Ex_1^U) = 0.5(Ex_2^L + Ex_2^U)$, $0.5(En_1^L + En_1^U) = 0.5(En_2^L + En_2^U)$, $0.5(HEn_1^L + HEn_1^U) = 0.5(HEn_2^L + HEn_2^U)$, then $\tilde{A}_1 = \tilde{A}_2$.

Theorem 2. *Assume that $\tilde{A}_i = \left(\left[Ex_i^L, Ex_i^U \right], \left[En_i^L, En_i^U \right], \left[HEn_i^L, HEn_i^U \right] \right)$ $(i = 1, 2, \dots, n)$ are n CR numbers, and $W = (W_1, W_2, \dots, W_n)$ denotes the weight vector of assessment criteria, with the condition $W_i \in [0, 1]$ and $\sum_{i=1}^{n} W_i = 1$. Then, the properties of the CRG_{avg} operators are as follows:*

1. *(Idempotency) Suppose $\tilde{A}_i = \tilde{A} = \left(\left[Ex_i^L, Ex_i^U \right], \left[En_i^L, En_i^U \right], \left[HEn_i^L, HEn_i^U \right] \right)$ $(i = 1, 2, 3, \dots, n)$ then $CRG_{avg}(A_1, A_2, \dots, A_n) = \tilde{A}$*
2. *(Boundedness) The CRG_{avg} holds that $A^- \le CRG_{avg}(A_1, A_2, \dots, A_n) \le A^+$, where*

$$\tilde{A}^- = \left(\left[\min_i Ex_i^L, \min_i Ex_i^U \right], \left[\max_i En_i^L, \max_i En_i^U \right], \left[\max_i HEn_i^L, \max_i HEn_i^U \right] \right)$$

$$\tilde{A}^+ = \left(\left[\max_i Ex_i^L, \max_i Ex_i^U \right], \left[\min_i En_i^L, \min_i En_i^U \right], \left[\min_i HEn_i^L, \min_i HEn_i^U \right] \right)$$

3. *(Monotonicity) Suppose $A_1, A_2, \ldots, A_n$ are another n number of CRNs, where $\hat{A}_i = \left(\left\lceil \hat{E}x_i^L, \hat{E}x_i^U \right\rfloor, \left\lceil \hat{E}n_i^L, \hat{E}n_i^U \right\rfloor, \left\lceil \widehat{HEn}_i^L, \widehat{HEn}_i^U \right\rfloor \right)$ $(i = 1, 2, 3, \ldots, n)$, satisfying*
   a. $\tilde{E}x_i^L \geq \hat{E}x_i^L, \tilde{E}n_i^L \leq \hat{E}n_i^L, \widetilde{HEn}_i^L \leq \widehat{HEn}_i^L$
   b. $\tilde{E}x_i^U \geq \hat{E}x_i^U, \tilde{E}n_i^U \leq \hat{E}n_i^U, \widetilde{HEn}_i^U \leq \widehat{HEn}_i^U$.

   Then it holds that $CRG_{avg}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) \geq CRG_{avg}(\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_n)$.

Proof. 1. Following Definition 7, the CR average operator of n CR numbers $A_1, A_2, \ldots, A_n$ can be written as:

$$CRG_{avg}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \left(\left\lceil \left(\prod_{i=1}^{n} Ex_i^L \right)^{\frac{1}{n}}, \left(\prod_{i=1}^{n} Ex_i^U \right)^{\frac{1}{n}} \right\rfloor, \left\lceil \left(\prod_{i=1}^{n} Ex_i^L \right)^{\frac{1}{n}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{En_i^L}{Ex_i^L} \right)^2}, \left(\prod_{i=1}^{n} Ex_i^U \right)^{\frac{1}{n}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{En_i^U}{Ex_i^U} \right)^2} \right\rfloor, \right.$$
$$\left. \left\lceil \left(\prod_{i=1}^{n} Ex_i^L \right)^{\frac{1}{n}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{HEn_i^L}{Ex_i^L} \right)^2}, \left(\prod_{i=1}^{n} Ex_i^U \right)^{\frac{1}{n}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{HEn_i^U}{Ex_i^U} \right)^2} \right\rfloor \right)$$

Taking $\tilde{A}_i = \tilde{A}$, we obtained

$$= \left(\left\lceil (Ex^L)^{n\times\frac{1}{n}}, (Ex^U)^{n\times\frac{1}{n}} \right\rfloor, \left\lceil (Ex^L)^{n\times\frac{1}{n}} \sqrt{\frac{1}{n} \times n \left(\frac{En^L}{Ex^L} \right)^2}, (Ex^U)^{n\times\frac{1}{n}} \sqrt{\frac{1}{n} \times n \left(\frac{En^U}{Ex^U} \right)^2} \right\rfloor, \right.$$
$$\left. \left\lceil (Ex_i^L)^{n\times\frac{1}{n}} \sqrt{\frac{1}{n} \times n \left(\frac{HEn^L}{Ex^L} \right)^2}, (Ex^U)^{n\times\frac{1}{n}} \sqrt{\frac{1}{n} \times n \left(\frac{HEn^U}{Ex^U} \right)^2} \right\rfloor \right)$$
$$= \left(\lceil Ex^L, Ex^U \rfloor, \lceil En^L, En^U \rfloor, \lceil HEn^L, HEn^U \rfloor \right) = A$$

2. It is trivial to mention that the geometric average of n values always lies between the minimum and maximum of these values. Therefore,

$$0.5 \left(\min_{i=1}^{n} Ex_i^L + \min_{i=1}^{n} Ex_i^U \right) \leq 0.5 \left(\left(\prod_{i=1}^{n} Ex_i^L \right)^{\frac{1}{n}} + \left(\prod_{i=1}^{n} Ex_i^U \right)^{\frac{1}{n}} \right) \leq 0.5 \left(\max_{i=1}^{n} Ex_i^L + \max_{i=1}^{n} Ex_i^U \right)$$
$$\Rightarrow \quad A^- \leq CRG_{avg}(A_1, A_2, \ldots, A_n) \leq A^+$$

3. Since $\tilde{E}x_i^L \geq \hat{E}x_i^L$, $\tilde{E}x_i^U \geq \hat{E}x_i^U$, therefore following Definition 7, it is easy to write Eq. (21).

$$0.5 \left(\left(\prod_{i=1}^{n} \tilde{E}x_i^L \right)^{\frac{1}{n}} + \left(\prod_{i=1}^{n} \tilde{E}x_i^U \right)^{\frac{1}{n}} \right) \geq 0.5 \left(\left(\prod_{i=1}^{n} \hat{E}x_i^L \right)^{\frac{1}{n}} + \left(\prod_{i=1}^{n} \hat{E}x_i^U \right)^{\frac{1}{n}} \right) \tag{21}$$
$$\Rightarrow \quad CRG_{avg}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) \geq CRG_{avg}(\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_n) \quad \square \quad \square$$

5. The proposed cloud rough AHP-VIKOR design concept evaluation model

This section establishes an improved framework for evaluating and choosing design concepts to assure the validity and rationality of the assessment results, where the inherent various uncertainty is adequately taken into consideration, and the relative importance of each evaluation criterion is also calculated. The evaluation of design concepts is divided into two stages: (1) determining subjective criterion weights using the CR-AHP and (2) identifying the ranking orders of design alternatives using the CR-VIKOR. Initially, The AHP concept is implemented into the CR environment to determine the subjective weights of evaluation criteria. And then the VIKOR approach is implemented into the CR environment to obtain the final performance of various DAs. The framework of the proposed approach is illustrated in Fig. 1. The proposed MAGDM approach uses the following notations: The collection of design alternatives/design concept is denoted by $\mathbf{A} = \{A_1, A_2, \ldots, A_i, \ldots, A_n\}$. The collection of design criteria is given by $\mathbf{C} = \{C_1, C_2, \ldots, C_g, C_h, \ldots, C_j, \ldots, C_m\}$. The collection of experts is indicated as $\mathbf{E} = \{E_1, E_2, \ldots, E_k, \ldots, E_s\}$. The set of weights assigned to assessment criteria is denoted as $\mathbf{W} = \{W_1, W_2, \ldots, W_m\}$.

5.1. Cloud rough AHP for design criteria weights

An expert group is examined to determine the assessment components at the start of the DCE process. This is accomplished by taking into account the significant capabilities of AHP approaches in evaluating complicated judgments, managing numerous experts, measuring consistency, and modifying tangible and non-tangible criteria. To calculate criterion weights, CR numbers based on AHP is suggested and executed as follows:

Construct individual pairwise comparison matrices (PCMs) using AHP surveys

While conducting AHP surveys, the experts are asked for their impartial opinions on the pairwise comparison of design criteria. Compile these comparison values, then create a set of pairwise comparison matrices. The PCM provided by the expert E_k is described as given in Eq. (22).

$$M^k = \begin{bmatrix} 1 & \phi_{12}^k & \cdots & \phi_{1m}^k \\ \phi_{21}^k & 1 & \cdots & \phi_{2m}^k \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{m1}^k & \phi_{m2}^k & \cdots & 1 \end{bmatrix} \tag{22}$$

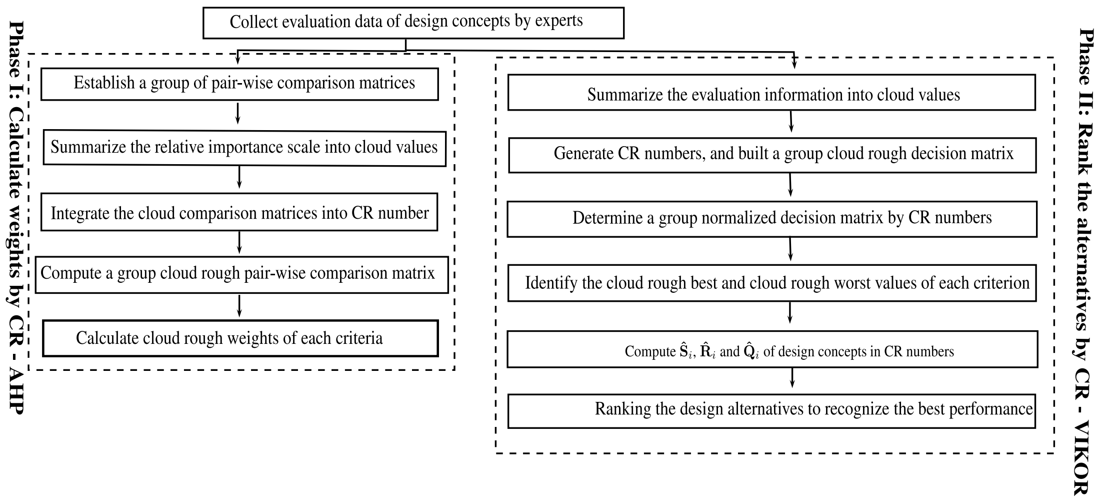

Fig. 1. Developed framework for the proposed CR AHP-VIKOR model.

Table 2
Random consistency index [29].

m	3	4	5	6	7	8	9	10
RI	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

where $\phi^k_{gh}(1 \le g \le m, 1 \le h \le m, 1 \le k \le s)$ is criterion's relative importance C_g on criterion C_h offered by experts E_k, m is the total number of assessment criteria, and s represented the number of experts. After establishment of $M^k = (\phi^k_{gh})_{m\times m}$, consistency test is applied on each PCMs. The consistency ratio $(\mathtt{CR})$ is represented in Eq. (23).

$$\mathtt{CR} = \frac{\mathtt{CI}}{\mathtt{RI}} \tag{23}$$

where $\mathtt{CI}$ and $\mathtt{RI}$ denoted consistency index and random consistency index. Calculate consistency index by utilizing $\mathtt{CI} = \frac{\lambda^k_{\max}-m}{m-1}$, where $\lambda^k_{\max}$ is the maximum eigenvalue of M^k and determine random consistency index as shown in Table 2 based on m. Conduct consistency test. If $\mathtt{CR} \le 0.1$, comparison matrix is acceptable. Alternatively, experts' opinions must be modified until $\mathtt{CR} \le 0.1$.

Calculate the consistency ratio (CR) of each cloud rough PCM

Transform the all these comparison matrices given in Eqs. (22) into CR numbers using Eqs. (2)–(13). The result of the CR pairwise comparison on m criteria can be summarized in an $(m \times m)$ evaluation matrix N^k as shown in Eq. (24).

$$N^k = \begin{bmatrix} \varrho^k_{11} & \varrho^k_{12} & \cdots & \varrho^k_{1m} \\ \varrho^k_{21} & \varrho^k_{22} & \cdots & \varrho^k_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \varrho^k_{m1} & \varrho^k_{m2} & \cdots & \varrho^k_{mm} \end{bmatrix} \tag{24}$$

where $\varrho_{gh} = \left(\left[Ex^L_{gh}, Ex^U_{gh}\right], \left[En^L_{gh}, En^U_{gh}\right], \left[HEn^L_{gh}, HEn^U_{gh}\right]\right)$ is a CR number. In the process of determining the largest eigenvalues in cloud rough numbers, these eigenvalues are obtained by solving the equation $N^k \times \omega^k = \lambda^k_{Max} \times \omega^k$, where N^k represents the CR-PCM provided by expert E_k $(1 \le k \le s)$, ω^k is the specific vector of weights, and λ^k_{Max} corresponds to the largest eigenvalue of N^k. Calculate the consistency index by utilizing $\mathtt{CI}^k = \frac{\lambda^k_{\max}-m}{m-1}$, where the final value of $\lambda^k_{\max}$ is calculated in rough boundary as $\left(Ex^U_{\lambda_{\max}} + 3En^U_{\lambda_{\max}}\right) - \left(Ex^L_{\lambda_{\max}} + 3En^L_{\lambda_{\max}}\right)$. In the context of determining criteria weights through the CRG_{avg} operator, the eigenvector corresponding to the largest eigenvalue (λ_{Max}) serves as the vector of criteria weights. The consistency ratio $(\mathtt{CR}^k)$ is represented in Eq. (25).

$$\mathtt{CR}^k = \frac{\mathtt{CI}^k}{\mathtt{RI}^k} = \frac{\left(Ex^U_{\lambda_{\max}} + 3En^U_{\lambda_{\max}}\right) - \left(Ex^L_{\lambda_{\max}} + 3En^L_{\lambda_{\max}}\right) - m}{(m-1)\mathtt{RI}^k} \tag{25}$$

where $\mathtt{CI}^k$ and $\mathtt{RI}^k$ denote consistency index and random index, for each k. If $\mathtt{CR}^k \le 0.1$, CR comparison matrix is acceptable. Otherwise, experts' opinions must be modified until $\mathtt{CR}^k \le 0.1$. Alternatively, In Eq. (25), if $\lambda^k_{\max}$ is not converted into rough boundary, then $\mathtt{CI}^k$ can be computed by considering $m = (\lceil m, m \rfloor, \lceil 0, 0 \rfloor, \lceil 0, 0 \rfloor)$, and then the final $\mathtt{CR}^k$ value is computed by applying the notion of rough boundary.

Construction of cloud comparison matrix and generate CR numbers

Convert the pairwise comparison matrices into cloud values by using Definition 2. This paper presents the effective domain $[0, 12] \cup [0, 1]$for the linguistic term sets $LT = \{\{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cup \{1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9\}\}$ and the hyper entropy of their mid value is $HEn_5 = HEn_{\frac{1}{5}} = 0.02$. The correlation between linguistic scale and cloud values are indicated in Table 3. The transformation between linguistic scale and cloud values is illustrated in Table 3.

Table 3
Relative importance scale and their corresponding quantitative values.

Linguistic values	Cloud values	Reciprocal values	Cloud values
1	(3.1426, 1.0475, 0.1371)	1	(0.2619, 0.0873, 0.1371)
2	(3.8170, 0.6474, 0.0847)	$\frac{1}{2}$	(0.3181, 0.0539, 0.0847)
3	(4.2339, 0.4001, 0.0524)	$\frac{1}{3}$	(0.3528, 0.0333, 0.0524)
4	(4.9085, 0.2472, 0.0324)	$\frac{1}{4}$	(0.4090, 0.0206, 0.0324)
5	(6, 0.1528, 0.02)	$\frac{1}{5}$	(0.5, 0.0127, 0.02)
6	(7.0915, 0.2472, 0.0324)	$\frac{1}{6}$	(0.5910, 0.0206, 0.0324)
7	(7.7661, 0.4001, 0.0524)	$\frac{1}{7}$	(0.6472, 0.0333, 0.0524)
8	(8.1830, 0.6474, 0.0847)	$\frac{1}{8}$	(0.6819, 0.0539, 0.0847)
9	(8.8574, 1.0475, 0.1371)	$\frac{1}{9}$	(0.7381, 0.0873, 0.1371)

The cloud comparison matrix provided by the expert E_k is described as given in Eq. (26).

$$\hat{M}^k = \begin{bmatrix} \hat{\phi}^k_{11} & \hat{\phi}^k_{12} & \cdots & \hat{\phi}^k_{1m} \\ \hat{\phi}^k_{21} & \hat{\phi}^k_{22} & \cdots & \hat{\phi}^k_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\phi}^k_{m1} & \hat{\phi}^k_{m2} & \cdots & \hat{\phi}^k_{mm} \end{bmatrix} \tag{26}$$

Here $\hat{\phi}^k_{gh} = (\hat{E}x^k_{gh}, \hat{E}n^k_{gh}, \widehat{HEn}^k_{gh})$ is cloud comparison value of criterion C_g on criterion C_h. When all cloud values are converted to CR numbers, the CR comparison matrix can be built as $\tilde{M}^k = (\tilde{\phi}^k_{gh})_{m\times m}$ where both intrapersonal and interpersonal uncertainties of expert team evaluation are effectively addressed. As a result, quantitative assessment data can convey expert opinions more accurately and rationally. The procedure for constructing CR numbers is described in Section 5.1. The CR comparison matrix $\tilde{M}^k$ of expert $E_k{}^{th}$ is written in Eq. (27).

$$\tilde{M}^k = \begin{bmatrix} \tilde{\phi}^k_{11} & \tilde{\phi}^k_{12} & \cdots & \tilde{\phi}^k_{1m} \\ \tilde{\phi}^k_{21} & \tilde{\phi}^k_{22} & \cdots & \tilde{\phi}^k_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\phi}^k_{m1} & \tilde{\phi}^k_{m2} & \cdots & \tilde{\phi}^K_{mm} \end{bmatrix} \tag{27}$$

$\tilde{\phi}^k_{gh} = (\lceil \tilde{E}x^{kL}_{gh}, \tilde{E}x^{kU}_{gh} \rfloor, \lceil \tilde{E}n^{kL}_{gh}, \tilde{E}n^{kU}_{gh} \rfloor, \lceil \widetilde{HEn}^{kL}_{gh}, \widetilde{HEn}^{kU}_{gh} \rfloor)$ and $\tilde{\phi}^k_{gh}(1 \le g \le m, 1 \le h \le m, 1 \le k \le s)$ is criterion C_g on criterion C_h given by expert E_k.

Computation of group aggregated evaluation data

Utilizing the CR average (CR_{avg}) operator, to aggregate all CR comparison matrices $\tilde{M}^k = (\tilde{\phi}^k_{gh})_{m\times m}$ and transformed into group CR comparison matrix $\tilde{\mathbf{M}} = (\tilde{\boldsymbol{\Phi}}_{gh})_{m\times m}$, which can be determined using Eq. (28).

$$\begin{aligned} \tilde{\boldsymbol{\Phi}}_{gh} &= CR_{avg}(\tilde{\phi}^1_{gh}, \tilde{\phi}^2_{gh}, \ldots, \tilde{\phi}^s_{gh}) = \frac{1}{s}\sum_{k=1}^{s} \tilde{\phi}^k_{gh} \\ &= \left(\left[\frac{1}{s}\sum_{k=1}^{s} Ex^{kL}_{gh}, \frac{1}{s}\sum_{k=1}^{s} Ex^{kU}_{gh} \right], \left[\sqrt{\frac{1}{s}\sum_{k=1}^{s} (En^{kL}_{gh})^2}, \sqrt{\frac{1}{s}\sum_{k=1}^{s} (En^{kU}_{gh})^2} \right], \left[\sqrt{\frac{1}{s}\sum_{k=1}^{s} (HEn^{kL}_{gh})^2}, \sqrt{\frac{1}{s}\sum_{k=1}^{s} (HEn^{kU}_{gh})^2} \right] \right) \end{aligned} \tag{28}$$

The group CR comparison matrix $\tilde{\mathbf{M}}$ is constructed in Eq. (29).

$$\tilde{\mathbf{M}} = \begin{bmatrix} \tilde{\boldsymbol{\Phi}}_{11} & \tilde{\boldsymbol{\Phi}}_{12} & \cdots & \tilde{\boldsymbol{\Phi}}_{1m} \\ \tilde{\boldsymbol{\Phi}}_{21} & \tilde{\boldsymbol{\Phi}}_{22} & \cdots & \tilde{\boldsymbol{\Phi}}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\boldsymbol{\Phi}}_{m1} & \tilde{\boldsymbol{\Phi}}_{m2} & \cdots & \tilde{\boldsymbol{\Phi}}_{mm} \end{bmatrix} \tag{29}$$

where $\tilde{\boldsymbol{\Phi}}_{gh} = \left(\left\lceil \tilde{E}x^L_{gh}, \tilde{E}x^U_{gh} \right\rfloor, \left\lceil \tilde{E}n^L_{gh}, \tilde{E}n^U_{gh} \right\rfloor, \left\lceil \widetilde{HEn}^L_{gh}, \widetilde{HEn}^U_{gh} \right\rfloor \right)$.

Calculation of CR criteria weights [62]

Based on group CR comparison matrix $\tilde{\mathbf{M}}$, the CR weights for each criterion is computed using Eq. (30).

$$\begin{aligned} W_g = \Bigg(&\left[\left(\prod_{h=1}^{m} Ex^L_{gh}\right)^{\frac{1}{m}}, \left(\prod_{h=1}^{m} Ex^U_{gh}\right)^{\frac{1}{m}} \right], \left[\left(\prod_{h=1}^{m} Ex_g h^L\right)^{\frac{1}{m}} \sqrt{\frac{1}{m}\sum_{h=1}^{m}\left(\frac{En^L_{gh}}{Ex^L_{gh}}\right)^2}, \left(\prod_{h=1}^{m} Ex^U_{gh}\right)^{\frac{1}{m}} \sqrt{\frac{1}{m}\sum_{h=1}^{m}\left(\frac{En^U_{gh}}{Ex^U_{gh}}\right)^2} \right], \\ &\left[\left(\prod_{h=1}^{m} Ex^L_{gh}\right)^{\frac{1}{m}} \sqrt{\frac{1}{m}\sum_{h=1}^{m}\left(\frac{HEn^L_{gh}}{Ex^L_{gh}}\right)^2}, \left(\prod_{h=1}^{m} Ex^U_{gh}\right)^{\frac{1}{m}} \sqrt{\frac{1}{m}\sum_{h=1}^{m}\left(\frac{HEn^U_{gh}}{Ex^U_{gh}}\right)^2} \right] \Bigg) \end{aligned} \tag{30}$$

It can also be normalized using Eq. (31).

$$\grave{W}_g=\begin{cases}\dfrac{\left(\left\lceil \tilde{E}x^L_{gh},\tilde{E}x^U_{gh}\right\rfloor,\left\lceil \tilde{E}n^L_{gh},\tilde{E}n^U_{gh}\right\rfloor,\left\lceil \widetilde{HEn}^L_{gh},\widetilde{HEn}^U_{gh}\right\rfloor\right)}{\left(\left\lceil \max_g\left(\tilde{E}x^U_{gh}\right),\max_g\left(\tilde{E}x^U_{gh}\right)\right\rfloor,\left\lceil \min_g\left(\tilde{E}n^U_{gh}\right),\min_g\left(\tilde{E}n^U_{gh}\right)\right\rfloor,\left\lceil \min_g\left(\widetilde{HEn}^U_{gh}\right),\min_g\left(\widetilde{HEn}^U_{gh}\right)\right\rfloor\right)}, & For\ h\in\mathcal{B}\\[2ex] \dfrac{\left(\left\lceil \min_g\left(\tilde{E}x^L_{gh}\right),\min_g\left(\tilde{E}x^L_{gh}\right)\right\rfloor,\left\lceil \max_g\left(\tilde{E}n^L_{gh}\right),\max_g\left(\tilde{E}n^L_{gh}\right)\right\rfloor,\left\lceil \max_g\left(\widetilde{HEn}^L_{gh}\right),\max_g\left(\widetilde{HEn}^L_{gh}\right)\right\rfloor\right)}{\left(\left\lceil \tilde{E}x^L_{gh},\tilde{E}x^U_{gh}\right\rfloor,\left\lceil \tilde{E}n^L_{gh},\tilde{E}n^U_{gh}\right\rfloor,\left\lceil \widetilde{HEn}^L_{gh},\widetilde{HEn}^U_{gh}\right\rfloor\right)}, & For\ h\in C\end{cases} \tag{31}$$

where $\mathcal{B}$ and C indicate the benefit criteria and the cost criteria, respectively.

Theorem 3. *Prove the validity of $\grave{W}_g$ given in Eq. (31).*

Proof. Inspired by the studies of [12], this paper extends the normalization method of the cloud based MCDM methods into cloud rough numbers. In the extended cloud normalization method, the denominator and numerator values are expressed using cloud rough numbers. It is our contention that cloud rough numbers are capable of preserving the characteristics of cloud normalization method throughout mathematical operations. Thus, the results obtained within this framework consistently maintain their representation in cloud rough numbers. □

5.2. Cloud rough VIKOR for ranking of design alternatives

The novel CR-VIKOR method is developed to aggregate individual perceptions of decision makers and evaluate DAs using the relative weights of each design criteria obtained by CR-AHP approach.

Construct a group CR decision matrix using cloud evaluation data

Collect the evaluation information of DAs that represent the judgment of experts and then construct a group of decision matrices and transform them into a group CR decision matrix $\tilde{\mathbf{D}}=(\tilde{\Psi}_{ij})_{n\times m}$ where $\tilde{\Psi}_{ij}=CR_{avg}\left(\psi^1_{ij},\psi^2_{ij},\ldots,\psi^s_{ij}\right)$, ψ^k_{ij} indicates the CR evaluation information of the design alternative A_i under the criterion C_j given by expert E_k, the construction of matrix $\tilde{\mathbf{D}}$ is similar as $\tilde{\mathbf{M}}$ as stated in Section 5.1. The CR decision matrix is shown in Eq. (32).

$$\tilde{\mathbf{D}}=\begin{bmatrix}\tilde{\Psi}_{11} & \tilde{\Psi}_{12} & \cdots & \tilde{\Psi}_{1m}\\ \tilde{\Psi}_{21} & \tilde{\Psi}_{22} & \cdots & \tilde{\Psi}_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\Psi}_{n1} & \tilde{\Psi}_{n2} & \cdots & \tilde{\Psi}_{nm}\end{bmatrix} \tag{32}$$

where $\tilde{\Psi}_{ij}=(\lceil \tilde{E}x^L_{ij},\tilde{E}x^U_{ij}\rfloor,\lceil \tilde{E}n^L_{ij},\tilde{E}n^U_{ij}\rfloor,\lceil \widetilde{HEn}^L_{ij},\widetilde{HEn}^U_{ij}\rfloor)$.

Calculate the normalized decision matrix by CR numbers

Due to the multiple dimensions and magnitudes of the evaluation of conceptual design, the group CR evaluation information must be normalized to make it into a comparable one. In this paper, inspired by the research of [42,62] we discuss the normalizing procedure of the CR value $\tilde{\Psi}_{ij}$, which is shown in Eqs. (33), (34), (35).

$$\hat{\Psi}_{ij}=\begin{cases}\dfrac{\left(\left\lceil \tilde{E}x^L_{ij},\tilde{E}x^U_{ij}\right\rfloor,\left\lceil \tilde{E}n^L_{ij},\tilde{E}n^U_{ij}\right\rfloor,\left\lceil \widetilde{HEn}^L_{ij},\widetilde{HEn}^U_{ij}\right\rfloor\right)}{\left(\left\lceil \max_i\left(\tilde{E}x^U_{ij}\right),\max_i\left(\tilde{E}x^U_{ij}\right)\right\rfloor,\left\lceil \min_i\left(\tilde{E}n^U_{ij}\right),\min_i\left(\tilde{E}n^U_{ij}\right)\right\rfloor,\left\lceil \min_i\left(\widetilde{HEn}^U_{ij}\right),\min_i\left(\widetilde{HEn}^U_{ij}\right)\right\rfloor\right)}, & For\ j\in\mathcal{B}\\[2ex] \dfrac{\left(\left\lceil \min_i\left(\tilde{E}x^L_{ij}\right),\min_i\left(\tilde{E}x^L_{ij}\right)\right\rfloor,\left\lceil \max_i\left(\tilde{E}n^L_{ij}\right),\max_i\left(\tilde{E}n^L_{ij}\right)\right\rfloor,\left\lceil \max_i\left(\widetilde{HEn}^L_{ij}\right),\max_i\left(\widetilde{HEn}^L_{ij}\right)\right\rfloor\right)}{\left(\left\lceil \tilde{E}x^L_{ij},\tilde{E}x^U_{ij}\right\rfloor,\left\lceil \tilde{E}n^L_{ij},\tilde{E}n^U_{ij}\right\rfloor,\left\lceil \widetilde{HEn}^L_{ij},\widetilde{HEn}^U_{ij}\right\rfloor\right)}, & For\ j\in C\end{cases} \tag{33}$$

$$\begin{aligned}\hat{\Psi}_{ij}=\Bigg(&\left[\frac{\tilde{E}x^L_{ij}}{\max_i\left(\tilde{E}x^U_{ij}\right)},\frac{\tilde{E}x^U_{ij}}{\max_i \tilde{E}x^U_{ij}}\right],\left[\left|\frac{\tilde{E}x^L_{ij}}{\max_i\left(\tilde{E}x^U_{ij}\right)}\right|\sqrt{\left(\frac{\tilde{E}n^L_{ij}}{\tilde{E}x^L_{ij}}\right)^2+\left(\frac{\min_i \tilde{E}n^U_{ij}}{\max_i \tilde{E}x^U_{ij}}\right)^2},\left|\frac{\tilde{E}x^U_{ij}}{\max_i \tilde{E}x^U_{ij}}\right|\sqrt{\left(\frac{\tilde{E}n^U_{ij}}{\tilde{E}x^U_{ij}}\right)^2+\left(\frac{\min_i \tilde{E}n^U_{ij}}{\max_i \tilde{E}x^U_{ij}}\right)^2}\right],\\ &\left[\left|\frac{\tilde{E}x^L_{ij}}{\max_i\left(\tilde{E}x^U_{ij}\right)}\right|\sqrt{\left(\frac{\widetilde{HEn}^L_{ij}}{\tilde{E}x^L_{ij}}\right)^2+\left(\frac{\min_i \widetilde{HEn}^U_{ij}}{\max_i \tilde{E}x^U_{ij}}\right)^2},\left|\frac{\tilde{E}x^U_{ij}}{\max_i \tilde{E}x^U_{ij}}\right|\sqrt{\left(\frac{\widetilde{HEn}^U_{ij}}{\tilde{E}x^U_{ij}}\right)^2+\left(\frac{\min_i \widetilde{HEn}^U_{ij}}{\max_i \tilde{E}x^U_{ij}}\right)^2}\right]\Bigg),\quad For\ j\in\mathcal{B}\end{aligned} \tag{34}$$

$$\begin{aligned}\hat{\Psi}_{ij}=\Bigg(&\left[\frac{\min_i \tilde{E}x^L_{ij}}{\tilde{E}x^U_{ij}},\frac{\min_i \tilde{E}x^L_{ij}}{\tilde{E}x^L_{ij}}\right],\left[\left|\frac{\min_i \tilde{E}x^L_{ij}}{\tilde{E}x^U_{ij}}\right|\sqrt{\left(\frac{\max_i \tilde{E}n^L_{ij}}{\min_i \tilde{E}x^L_{ij}}\right)^2+\left(\frac{\tilde{E}n^L_{ij}}{\tilde{E}x^U_{ij}}\right)^2},\left|\frac{\min_i \tilde{E}x^L_{ij}}{\tilde{E}x^L_{ij}}\right|\sqrt{\left(\frac{\max_i \tilde{E}n^L_{ij}}{\min_i \tilde{E}x^L_{ij}}\right)^2+\left(\frac{\tilde{E}n^U_{ij}}{\tilde{E}x^L_{ij}}\right)^2}\right],\\ &\left[\left|\frac{\min_i \tilde{E}x^L_{ij}}{\tilde{E}x^U_{ij}}\right|\sqrt{\left(\frac{\max_i \widetilde{HEn}^L_{ij}}{\min_i \tilde{E}x^L_{ij}}\right)^2+\left(\frac{\widetilde{HEn}^L_{ij}}{\tilde{E}x^U_{ij}}\right)^2},\left|\frac{\min_i \tilde{E}x^L_{ij}}{\tilde{E}x^L_{ij}}\right|\sqrt{\left(\frac{\max_i \widetilde{HEn}^L_{ij}}{\min_i \tilde{E}x^L_{ij}}\right)^2+\left(\frac{\widetilde{HEn}^U_{ij}}{\tilde{E}x^L_{ij}}\right)^2}\right]\Bigg),\quad For\ j\in C\end{aligned} \tag{35}$$

where $\mathcal{B}$ represents the benefit criteria (larger-the-better) and $\mathcal{C}$ indicates the cost criterion (smaller-the-better). Thus, the group CR normalized matrix $\hat{\mathbf{D}} = (\hat{\Psi}_{ij})_{n\times m}$ is written in Eq. (36), where

$$\hat{\Psi}_{ij} = \left(\left\lceil \hat{E}x^{L}_{ij}, \hat{E}x^{U}_{ij}\right\rfloor, \left\lceil \hat{E}n^{L}_{ij}, \hat{E}n^{U}_{ij}\right\rfloor, \left\lceil \widehat{HEn}^{L}_{ij}, \widehat{HEn}^{U}_{ij}\right\rfloor\right)$$

represents the group normalized evaluations in CR numbers of design concept A_i under evaluation criteria C_j.

$$\hat{\mathbf{D}} = \begin{bmatrix} \hat{\Psi}_{11} & \hat{\Psi}_{12} & \cdots & \hat{\Psi}_{1m} \\ \hat{\Psi}_{21} & \hat{\Psi}_{22} & \cdots & \hat{\Psi}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\Psi}_{n1} & \hat{\Psi}_{n2} & \cdots & \hat{\Psi}_{nm} \end{bmatrix} \tag{36}$$

Identify the best and worst values in CR numbers

Based on benefit and cost criterion, find the CR best and CR worst values of each criterion. For any design criteria C_j, the CR best and CR worst values represented by Ψ^*_j and Ψ^-_j can be computed using Eqs. (37), (38).

$$\Psi^*_j = \begin{cases} \left(\left\lceil \max_i \hat{E}x^{L}_{ij}, \max_i \hat{E}x^{U}_{ij}\right\rfloor, \left\lceil \min_i \hat{E}n^{L}_{ij}, \min_i \hat{E}n^{U}_{ij}\right\rfloor, \left\lceil \min_i \widehat{HEn}^{L}_{ij}, \min_i \widehat{HEn}^{U}_{ij}\right\rfloor\right), & For\ j \in \mathcal{B} \\ \left(\left\lceil \min_i \hat{E}x^{L}_{ij}, \min_i \hat{E}x^{U}_{ij}\right\rfloor, \left\lceil \max_i \hat{E}n^{L}_{ij}, \max_i \hat{E}n^{U}_{ij}\right\rfloor, \left\lceil \max_i \widehat{HEn}^{L}_{ij}, \max_i \widehat{HEn}^{U}_{ij}\right\rfloor\right), & For\ j \in \mathcal{C} \end{cases} \tag{37}$$

$$\Psi^-_j = \begin{cases} \left(\left\lceil \min_i \hat{E}x^{L}_{ij}, \min_i \hat{E}x^{U}_{ij}\right\rfloor, \left\lceil \max_i \hat{E}n^{L}_{ij}, \max_i \hat{E}n^{U}_{ij}\right\rfloor, \left\lceil \max_i \widehat{HEn}^{L}_{ij}, \max_i \widehat{HEn}^{U}_{ij}\right\rfloor\right), & For\ j \in \mathcal{B} \\ \left(\left\lceil \max_i \hat{E}x^{L}_{ij}, \max_i \hat{E}x^{U}_{ij}\right\rfloor, \left\lceil \min_i \hat{E}n^{L}_{ij}, \min_i \hat{E}n^{U}_{ij}\right\rfloor, \left\lceil \min_i \widehat{HEn}^{L}_{ij}, \min_i \widehat{HEn}^{U}_{ij}\right\rfloor\right), & For\ j \in \mathcal{C} \end{cases} \tag{38}$$

CR group utility $\hat{\mathbf{S}}_i$ and CR regret $\hat{\mathbf{R}}_i$ values

Using the Ψ^*_j and Ψ^-_j values of each criterion, CR group utility $\hat{\mathbf{S}}_i$ and CR regret $\hat{\mathbf{R}}_i$ values for design alternative A_i, which can be determined as illustrated in Eqs. (39), (40).

$$\hat{\mathbf{S}}_i = \sum_{j=1}^{m} \grave{W}_j \frac{d(\Psi^*_j, \hat{\Psi}_{ij})}{d(\Psi^*_j, \Psi^-_j)} \tag{39}$$

$$\hat{\mathbf{R}}_i = \max_i \grave{W}_j \frac{d(\Psi^*_j, \hat{\Psi}_{ij})}{d(\Psi^*_j, \Psi^-_j)} \tag{40}$$

Note that $\grave{W}_j$ is each criterion's weight, which are computed using the CR-AHP method.

Identify the best performance by ranking the various design alternatives

For each design alternative A_i, calculate the evaluation index $\hat{\mathbf{Q}}_i$ in CR numbers as specified in Eq. (41).

$$\hat{\mathbf{Q}}_i = \eta\left(\frac{\hat{\mathbf{S}}_i - \min_i \hat{\mathbf{S}}_i}{\max_i \hat{\mathbf{S}}_i - \min_i \hat{\mathbf{S}}_i}\right) + (1-\eta)\left(\frac{\hat{\mathbf{R}}_i - \min_i \hat{\mathbf{R}}_i}{\max_i \hat{\mathbf{R}}_i - \min_i \hat{\mathbf{R}}_i}\right) \tag{41}$$

Here $\eta \in [0, 1]$ is the importance for group utility, that usually take $\eta = 0.5$. Ranking the alternatives of $\hat{\mathbf{S}}_i$, $\hat{\mathbf{R}}_i$, and $\hat{\mathbf{Q}}_i$ in ascending order by using the rough boundary interval. Using $\hat{\mathbf{Q}}_i$ values, the evaluated DAs are compared, and the DA with the lowest CR evaluation index is taken into consideration as the best design concept.

6. Case study: DCE model for upgrading lithography tool manufacturing process

In this section, we will discuss how the proposed model is utilized to choose an appropriate lithography tool, a critical step in evaluating alternative designs, particularly in the context of integrated circuit production. Lithography is a fundamental and indispensable technique in the manufacturing of integrated circuits, as it plays a central role in defining the intricate patterns on semiconductor wafers. Among the numerous tools and equipment used in the integrated circuits industry, the lithography tool stands out as one of the most essential. Although lithography tool is a complex and high-cost piece of equipment, primarily due to its exceptional level of precision and accuracy. As a result, conceptual design evaluation becomes essential. One of the lithography tool manufacturing companies developed six design concepts in conceptual design, including A_1, A_2, A_3, A_4, A_5 and A_6. The features of the alternatives are described in Table 4. The purpose of this model is to identify the best design concept from given DAs for further development [37].

The evaluation criteria are based on the expert opinions and technical features of the lithography tool design, which include the following seven indices: linewidth (C_1), field size (C_2), throughput (C_3), overlay (C_4), illumination uniformity (C_5), manufacturing cost (C_6) and power consumption (C_7). As mentioned in Table 5, power consumption, overlay, manufacturing cost and linewidth are classified as cost criterion, while other three are categorized as benefit criterion. In addition, Five professional experts (E_1, E_2, E_3, E_4, E_5) are encouraged to express their own perceptions and assessments in the evaluation of lithography tool design. It is possible to evaluate the design concept of the lithography tool using the framework shown in Fig. 1 after determining the DMs, evaluation criteria, and DAs. Initially, each DM is asked to provide his individual pairwise comparison values of the criteria and the evaluations of the DAs with respect to these criteria. Then criteria weighting and alternative ranking are executed by CR-based AHP and CR-based VIKOR, respectively.

Table 4
Description of the design alternatives for the lithography tool in DCE.

Design	Corresponding performances alternatives
A_1	A1 is an automated step-and-repeat lithography system which utilizes a 4X reduction ArF projection lens, an ArF excimer laser with a wavelength of (193 nm), and an ultra-precision screw rail stage.
A_2	A2 is a step-and-repeat lithographic system with a 4X reduction KrF projection lens, an excimer laser with a wavelength of (248 nm), and a screw rail dual-stage with self-adaptive focusing.
A_3	A3 is a step-and-scan lithography system that uses a 4X reduction KrF projection lens with off-axis illumination and resolution improvement methods, a KrF excimer laser emitting at wavelength (248 nm), and an aerostatic bearing stage.
A_4	A4 is a step-and-scan lithography system that employs a 4X reduction ArF projection lens with optional off-axis illumination and resolution crucial factors, an ultra-precision aerostatic bearing dual-stage and an ArF excimer laser emitting at wavelength (193 nm).
A_5	A5 is a step-and-repeat lithography system that uses a KrF excimer laser with a wavelength (248 nm), a 4X reduction KrF projection lens with off-axis illumination and resolution improvement methods, and magnetic levitation.
A_6	A6 is a step-and-scan lithographic system that uses a 4X reduction ArF projection lens with conventional illumination, an ArF excimer laser with a wavelength of (193 nm), and magnetic levitation with self-adaptive focusing and leveling technology.

Table 5
An overview of the evaluation criteria.

Criteria	Brief description	Category
C_1	Linewidth	Cost
C_2	Field size	Benefit
C_3	Throughput	Benefit
C_4	Overlay	Cost
C_5	Illumination uniformity	Benefit
C_6	Manufacturing cost	Cost
C_7	Power consumption	Cost

6.1. *Calculation of criteria weights using CR-AHP*

The CR numbers based AHP method is used to collect the independent decisions of experts and display the criterion weights after identifying the evaluation criteria for the lithography tool design concept evaluation.

1. Information collection and construction of individual PCMs
   Collect independent decisions of experts and construct a group of PCMs. Continue performing the consistency test until all comparison matrices are passed. The PCMs for the five experts are given in the following manner:

$$M^1 = \begin{bmatrix} 1 & 5 & 4 & 2 & 3 & 5 & 9 \\ \frac{1}{5} & 1 & \frac{1}{3} & \frac{1}{5} & \frac{1}{4} & \frac{1}{2} & 5 \\ \frac{1}{4} & 3 & 1 & \frac{1}{5} & \frac{1}{3} & 2 & 5 \\ \frac{1}{2} & 5 & 5 & 1 & 3 & 5 & 9 \\ \frac{1}{3} & 4 & 3 & \frac{1}{3} & 1 & 3 & 9 \\ \frac{1}{5} & 2 & \frac{1}{2} & \frac{1}{5} & \frac{1}{3} & 1 & 5 \\ \frac{1}{9} & \frac{1}{5} & \frac{1}{5} & \frac{1}{9} & \frac{1}{9} & \frac{1}{5} & 1 \end{bmatrix}, \quad M^2 = \begin{bmatrix} 1 & 5 & 3 & 1 & 1 & 4 & 9 \\ \frac{1}{5} & 1 & \frac{1}{2} & \frac{1}{5} & \frac{1}{5} & \frac{1}{3} & 5 \\ \frac{1}{3} & 2 & 1 & \frac{1}{5} & \frac{1}{2} & 3 & 7 \\ 1 & 5 & 5 & 1 & 2 & 4 & 9 \\ 1 & 5 & 2 & \frac{1}{2} & 1 & 3 & 7 \\ \frac{1}{4} & 3 & \frac{1}{3} & \frac{1}{4} & \frac{1}{3} & 1 & 3 \\ \frac{1}{9} & \frac{1}{5} & \frac{1}{7} & \frac{1}{9} & \frac{1}{7} & \frac{1}{3} & 1 \end{bmatrix}, \quad M^3 = \begin{bmatrix} 1 & 7 & 3 & 1 & 3 & 5 & 9 \\ \frac{1}{7} & 1 & \frac{1}{2} & \frac{1}{7} & \frac{1}{3} & \frac{1}{3} & 3 \\ \frac{1}{3} & 2 & 1 & \frac{1}{5} & \frac{1}{2} & 3 & 5 \\ 1 & 7 & 5 & 1 & 2 & 4 & 9 \\ \frac{1}{3} & 3 & 2 & \frac{1}{2} & 1 & 3 & 5 \\ \frac{1}{5} & 3 & \frac{1}{3} & \frac{1}{4} & \frac{1}{3} & 1 & 3 \\ \frac{1}{9} & \frac{1}{3} & \frac{1}{5} & \frac{1}{9} & \frac{1}{5} & \frac{1}{3} & 1 \end{bmatrix}$$

$$M^4 = \begin{bmatrix} 1 & 7 & 5 & 2 & 3 & 7 & 9 \\ \frac{1}{7} & 1 & \frac{1}{3} & \frac{1}{6} & \frac{1}{5} & \frac{1}{2} & 3 \\ \frac{1}{5} & 3 & 1 & \frac{1}{3} & \frac{1}{3} & 2 & 5 \\ \frac{1}{2} & 6 & 3 & 1 & 3 & 5 & 7 \\ \frac{1}{3} & 5 & 3 & \frac{1}{3} & 1 & 4 & 7 \\ \frac{1}{7} & 2 & \frac{1}{2} & \frac{1}{5} & \frac{1}{4} & 1 & 4 \\ \frac{1}{9} & \frac{1}{3} & \frac{1}{5} & \frac{1}{7} & \frac{1}{7} & \frac{1}{4} & 1 \end{bmatrix}, \quad M^5 = \begin{bmatrix} 1 & 7 & 5 & 1 & 2 & 5 & 7 \\ \frac{1}{7} & 1 & \frac{1}{3} & \frac{1}{5} & \frac{1}{4} & \frac{1}{2} & 3 \\ \frac{1}{5} & 3 & 1 & \frac{1}{3} & \frac{1}{3} & 2 & 5 \\ 1 & 5 & 3 & 1 & 2 & 5 & 7 \\ \frac{1}{2} & 4 & 3 & \frac{1}{2} & 1 & 3 & 5 \\ \frac{1}{5} & 2 & \frac{1}{2} & \frac{1}{5} & \frac{1}{3} & 1 & 2 \\ \frac{1}{7} & \frac{1}{3} & \frac{1}{5} & \frac{1}{7} & \frac{1}{5} & \frac{1}{2} & 1 \end{bmatrix}$$

   Based on Eq. (23), the CR is obtained as: $\mathtt{CR}^1 = 0.0585$, $\mathtt{CR}^2 = 0.0508$, $\mathtt{CR}^3 = 0.0388$, $\mathtt{CR}^4 = 0.0458$, $\mathtt{CR}^5 = 0.0329$. Clearly $\mathtt{CR}^k \leq 0.10 (k = 1, 2, \ldots, 5)$. Thus, all these given matrices are acceptable.

2. Check the consistency of the CR-PCM
   Using this formula $N^k \times weight^k = \lambda_{\max}^k \times weight^k$ the maximum eigenvalue ($\lambda_{\max}$) is determined and for the consistency of cloud rough pairwise comparisons the consistency index is also calculated. To ensure the accuracy and reliability of our calculations, we selected a random index (RI) is 2.35 as a critical parameter in our analysis. Then, compare the consistency index to the random index, the CR is obtained as: $\mathrm{CR}^1 = 0.0459$, $\mathrm{CR}^2 = 0.0990$, $\mathrm{CR}^3 = 0.0035$, $\mathrm{CR}^4 = 0.1070$, $\mathrm{CR}^5 = 0.0227$. Clearly $\mathrm{CR}^k \le 0.10 (k = 1, 2, \dots, 5)$. Thus, all these given cloud rough comparison matrices are acceptable.

$$\tilde{N}^1 = \begin{bmatrix} (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) & \dots & (\lceil 8.6391, 8.8574\rfloor, \lceil 0.9180, 1.0475\rfloor, \lceil 0.1202, 0.1371\rfloor) \\ (\lceil 0.50000.5883\rfloor, \lceil 0.0127, 0.0251\rfloor, \lceil 0.0200, 0.0394\rfloor) & \dots & (\lceil 4.9404, 6.0000\rfloor, \lceil 0.1528, 0.3012\rfloor, \lceil 0.0200, 0.0394\rfloor) \\ (\lceil 0.3716, 0.4697\rfloor, \lceil 0.0154, 0.0291\rfloor, \lceil 0.0241, 0.0457\rfloor) & \dots & (\lceil 6.0000, 6.3532\rfloor, \lceil 0.1528, 0.2023\rfloor, \lceil 0.0200, 0.0265\rfloor) \\ (\lceil 0.2844, 0.3181\rfloor, \lceil 0.0539, 0.0740\rfloor, \lceil 0.0847, 0.1162\rfloor) & \dots & (\lceil 8.4209, 8.8574\rfloor, \lceil 0.7886, 1.0475\rfloor, \lceil 0.1032, 0.1371\rfloor) \\ (\lceil 0.3277, 0.3528\rfloor, \lceil 0.0333, 0.0483\rfloor, \lceil 0.0524, 0.0758\rfloor) & \dots & (\lceil 7.2779, 8.8574\rfloor, \lceil 0.4307, 1.0475\rfloor, \lceil 0.0564, 0.1371\rfloor) \\ (\lceil 0.47730.5368\rfloor, \lceil 0.0127, 0.0184\rfloor, \lceil 0.0200, 0.0289\rfloor) & \dots & (\lceil 4.6387, 6.0000\rfloor, \lceil 0.1528, 0.3695\rfloor, \lceil 0.0200, 0.0484\rfloor) \\ (\lceil 0.7199, 0.7381\rfloor, \lceil 0.0765, 0.0873\rfloor, \lceil 0.1202, 0.1371\rfloor) & \dots & (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) \end{bmatrix}$$

$$\tilde{N}^2 = \begin{bmatrix} (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) & \dots & (\lceil 8.6391, 8.8574\rfloor, \lceil 0.9180, 1.0475\rfloor, \lceil 0.1202, 0.1371\rfloor) \\ (\lceil 0.5000, 0.5883\rfloor, \lceil 0.0127, 0.0251\rfloor, \lceil 0.0200, 0.0394\rfloor) & \dots & (\lceil 4.9404, 6.0000\rfloor, \lceil 0.1528, 0.3012\rfloor, \lceil 0.0200, 0.0394\rfloor) \\ (\lceil 0.3528, 0.4229\rfloor, \lceil 0.0226, 0.0333\rfloor, \lceil 0.0354, 0.0524\rfloor) & \dots & (\lceil 6.3532, 7.7661\rfloor, \lceil 0.2023, 0.4001\rfloor, \lceil 0.0265, 0.0524\rfloor) \\ (\lceil 0.2619, 0.2844\rfloor, \lceil 0.0740, 0.0873\rfloor, \lceil 0.1162, 0.1371\rfloor) & \dots & (\lceil 8.4209, 8.8574\rfloor, \lceil 0.7886, 1.0475\rfloor, \lceil 0.1032, 0.1371\rfloor) \\ (\lceil 0.2619, 0.3277\rfloor, \lceil 0.0483, 0.0873\rfloor, \lceil 0.0758, 0.1371\rfloor) & \dots & (\lceil 6.8830, 8.1299\rfloor, \lceil 0.2764, 0.6159\rfloor, \lceil 0.0362, 0.0806\rfloor) \\ (\lceil 0.4090, 0.5112\rfloor, \lceil 0.0147, 0.0270\rfloor, \lceil 0.0231, 0.0424\rfloor) & \dots & (\lceil 4.0950, 4.8441\rfloor, \lceil 0.3001, 0.4825\rfloor, \lceil 0.0393, 0.0632\rfloor) \\ (\lceil 0.7199, 0.7381\rfloor, \lceil 0.0765, 0.0873\rfloor, \lceil 0.1202, 0.1371\rfloor) & \dots & (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) \end{bmatrix}$$

$$\tilde{N}^3 = \begin{bmatrix} (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) & \dots & (\lceil 8.6391, 8.8574\rfloor, \lceil 0.9180, 1.0475\rfloor, \lceil 0.1202, 0.1371\rfloor) \\ (\lceil 0.5883, 0.6472\rfloor, \lceil 0.0251, 0.0333\rfloor, \lceil 0.0394, 0.0524\rfloor) & \dots & (\lceil 4.2339, 4.9404\rfloor, \lceil 0.3012, 0.4001\rfloor, \lceil 0.0394, 0.0524\rfloor) \\ (\lceil 0.3528, 0.4229\rfloor, \lceil 0.0226, 0.0333\rfloor, \lceil 0.0354, 0.0524\rfloor) & \dots & (\lceil 6.0000, 6.3532\rfloor, \lceil 0.1528, 0.2023\rfloor, \lceil 0.0200, 0.0265\rfloor) \\ (\lceil 0.2619, 0.2844\rfloor, \lceil 0.0740, 0.0873\rfloor, \lceil 0.1162, 0.1371\rfloor) & \dots & (\lceil 8.4209, 8.8574\rfloor, \lceil 0.7886, 1.0475\rfloor, \lceil 0.1032, 0.1371\rfloor) \\ (\lceil 0.3277, 0.3528\rfloor, \lceil 0.0333, 0.0483\rfloor, \lceil 0.0524, 0.0758\rfloor) & \dots & (\lceil 6.0000, 7.2779\rfloor, \lceil 0.1528, 0.4307\rfloor, \lceil 0.0200, 0.0564\rfloor) \\ (\lceil 0.4773, 0.5368\rfloor, \lceil 0.0127, 0.0184\rfloor, \lceil 0.0200, 0.0289\rfloor) & \dots & (\lceil 4.0950, 4.8441\rfloor, \lceil 0.3001, 0.4825\rfloor, \lceil 0.0393, 0.0632\rfloor) \\ (\lceil 0.7199, 0.7381\rfloor, \lceil 0.0765, 0.0873\rfloor, \lceil 0.1202, 0.1371\rfloor) & \dots & (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) \end{bmatrix}$$

$$\tilde{N}^4 = \begin{bmatrix} (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) & \dots & (\lceil 8.6391, 8.8574\rfloor, \lceil 0.9180, 1.0475\rfloor, \lceil 0.1202, 0.1371\rfloor) \\ (\lceil 0.5883, 0.6472\rfloor, \lceil 0.0251, 0.0333\rfloor, \lceil 0.0394, 0.0524\rfloor) & \dots & (\lceil 4.2339, 4.9404\rfloor, \lceil 0.3012, 0.4001\rfloor, \lceil 0.0394, 0.0524\rfloor) \\ (\lceil 0.4229, 0.5000\rfloor, \lceil 0.0127, 0.0226\rfloor, \lceil 0.0200, 0.0354\rfloor) & \dots & (\lceil 6.0000, 6.3532\rfloor, \lceil 0.1528, 0.2023\rfloor, \lceil 0.0200, 0.0265\rfloor) \\ (\lceil 0.2844, 0.3181\rfloor, \lceil 0.0539, 0.0740\rfloor, \lceil 0.0847, 0.1162\rfloor) & \dots & (\lceil 7.7661, 8.4209\rfloor, \lceil 0.4001, 0.7886\rfloor, \lceil 0.0524, 0.1032\rfloor) \\ (\lceil 0.3277, 0.3528\rfloor, \lceil 0.0333, 0.0483\rfloor, \lceil 0.0524, 0.0758\rfloor) & \dots & (\lceil 6.8830, 8.1299\rfloor, \lceil 0.2764, 0.6159\rfloor, \lceil 0.0362, 0.0806\rfloor) \\ (\lceil 0.5112, 0.6472\rfloor, \lceil 0.0184, 0.0333\rfloor, \lceil 0.0289, 0.0524\rfloor) & \dots & (\lceil 4.2983, 5.4542\rfloor, \lceil 0.2000, 0.4237\rfloor, \lceil 0.0262, 0.0555\rfloor) \\ (\lceil 0.7199, 0.7381\rfloor, \lceil 0.0765, 0.0873\rfloor, \lceil 0.1202, 0.1371\rfloor) & \dots & (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) \end{bmatrix}$$

$$\tilde{N}^5 = \begin{bmatrix} (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) & \dots & (\lceil 7.7661, 8.6391\rfloor, \lceil 0.4001, 0.9180\rfloor, \lceil 0.0524, 0.1202\rfloor) \\ (\lceil 0.5883, 0.6472\rfloor, \lceil 0.0251, 0.0333\rfloor, \lceil 0.0394, 0.0524\rfloor) & \dots & (\lceil 4.2339, 4.9404\rfloor, \lceil 0.3012, 0.4001\rfloor, \lceil 0.0394, 0.0524\rfloor) \\ (\lceil 0.4229, 0.5000\rfloor, \lceil 0.0127, 0.0226\rfloor, \lceil 0.0200, 0.0354\rfloor) & \dots & (\lceil 6.0000, 6.3532\rfloor, \lceil 0.1528, 0.2023\rfloor, \lceil 0.0200, 0.0265\rfloor) \\ (\lceil 0.2619, 0.2844\rfloor, \lceil 0.0740, 0.0873\rfloor, \lceil 0.1162, 0.1371\rfloor) & \dots & (\lceil 7.7661, 8.4209\rfloor, \lceil 0.4001, 0.7886\rfloor, \lceil 0.0524, 0.1032\rfloor) \\ (\lceil 0.2900, 0.3441\rfloor, \lceil 0.0385, 0.0706\rfloor, \lceil 0.0605, 0.1109\rfloor) & \dots & (\lceil 6.0000, 7.2779\rfloor, \lceil 0.1528, 0.4307\rfloor, \lceil 0.0200, 0.0564\rfloor) \\ (\lceil 0.4773, 0.5368\rfloor, \lceil 0.0127, 0.0184\rfloor, \lceil 0.0200, 0.0289\rfloor) & \dots & (\lceil 3.8170, 4.6387\rfloor, \lceil 0.3695, 0.6474\rfloor, \lceil 0.0484, 0.0847\rfloor) \\ (\lceil 0.6472, 0.7199\rfloor, \lceil 0.0333, 0.0765\rfloor, \lceil 0.0524, 0.1202\rfloor) & \dots & (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) \end{bmatrix}$$

3. Generate CR numbers and develop an integrated CR-PCM
   Convert the above five individual comparison matrices into corresponding cloud values using Table 3. According to the construction procedure of the CR numbers discussed previously, the cloud data can be further transformed into the CR-PCM. An example is provided to demonstrate how to transform the data into CR numbers.
   Suppose there is a collection of cloud values $\{(6, 0.1528, 0.02), (6, 0.1528, 0.02), (7.7661, 0.4001, 0.0524), (7.7661, 0.4001, 0.0524), (7.7661, 0.4001, 0.0524)\}$ that represent the evaluation result of a scheme according to five experts'criteria i.e. $\{\phi_{12}^1, \phi_{12}^2, \phi_{12}^3, \phi_{12}^4, \phi_{12}^5\} = \{5, 5, 7, 7, 7\}$. Then, employing the above methodology of cloud rough numbers, we can derive the CR form from the cloud values. For $\hat{\phi}_{12}^1 = (6, 0.1528, 0.02)$ the set of all expectations Ex would be $(6, 6, 7.7661, 7.7661, 7.7661)$. Then, using Eq. (2), we have $\underline{Apr}(Ex_{12}^1) = \underline{Apr}(6) = \{6, 6\}$ because the lower approximation of 6 contains the all values of Ex in the domain that should be ≤ 6. Likewise, utilizing Eqs. (3), (4), (5), (6), (7), the following calculations are obtained:

$$\underline{Apr}(En_{12}^1) = \{0.1528, 0.1528\}$$

$$\underline{Apr}(HEn_{12}^1) = \{0.02, 0.02\}$$

$$\overline{Apr}(Ex_{12}^1) = \{6, 6, 7.7661, 7.7661, 7.7661\}$$

$$\overline{Apr}(En_{12}^1) = \{0.1528, 0.1528, 0.4001, 0.4001, 0.4001\}$$

$$\overline{Apr}(HEn_{12}^1) = \{0.02, 0.02, 0.0524, 0.0524, 0.0524\}$$

   Then, using Eqs. (8), (9), (10), (11), (12), (13), the lower limit and upper limit of $\hat{\phi}_{12}^1$ are determined as:

$$\underline{Lim}(Ex_{12}^1) = \frac{1}{2}(6 + 6) = 6$$

Table 6
Weight information of seven criterion.

	W_g	$\dot{W}_g$
C_1	$(\lceil 4.7563, 5.2748\rfloor, \lceil 0.8065, 0.9798\rfloor, \lceil 0.0179, 0.0238\rfloor)$	$(\lceil 0.1253, 0.1390\rfloor, \lceil 0.1541, 0.1720\rfloor, \lceil 0.0155, 0.0225\rfloor)$
C_2	$(\lceil 0.77820.8567\rfloor, \lceil 0.1127, 0.1308\rfloor, \lceil 0.0122, 0.0171\rfloor)$	$(\lceil 0.1475, 0.1624\rfloor, \lceil 0.0216, 0.0250\rfloor, \lceil 0.0023, 0.0033\rfloor)$
C_3	$(\lceil 1.48871.6181\rfloor, \lceil 0.1783, 0.2136\rfloor, \lceil 0.0138, 0.0205\rfloor)$	$(\lceil 0.2822, 0.3068\rfloor, \lceil 0.0343, 0.0410\rfloor, \lceil 0.0027, 0.0040\rfloor)$
C_4	$(\lceil 3.30373.5922\rfloor, \lceil 0.5556, 0.6480\rfloor, \lceil 0.0818, 0.1069\rfloor)$	$(\lceil 0.1840, 0.2001\rfloor, \lceil 0.2263, 0.2472\rfloor, \lceil 0.0232, 0.0256\rfloor)$
C_5	$(\lceil 2.07852.2674\rfloor, \lceil 0.3191, 0.3852\rfloor, \lceil 0.0340, 0.0541\rfloor)$	$(\lceil 0.3940, 0.4299\rfloor, \lceil 0.0611, 0.0736\rfloor, \lceil 0.0065, 0.0103\rfloor)$
C_6	$(\lceil 1.02891.1234\rfloor, \lceil 0.1539, 0.1780\rfloor, \lceil 0.0147, 0.0196\rfloor)$	$(\lceil 0.5884, 0.6425\rfloor, \lceil 0.7224, 0.7916\rfloor, \lceil 0.0732, 0.0805\rfloor)$
C_7	$(\lceil 0.66110.7363\rfloor, \lceil 0.0926, 0.1109\rfloor, \lceil 0.0091, 0.0145\rfloor)$	$(\lceil 0.8978, 1.0000\rfloor, \lceil 1.1011, 1.2314\rfloor, \lceil 0.1117, 0.1257\rfloor)$

$$\underline{Lim}(En_{12}^1) = \frac{1}{2}(0.1528 + 0.1528) = 0.1528$$
$$\underline{Lim}(HEn_{12}^1) = \frac{1}{2}(0.02 + 0.02) = 0.02$$
$$\overline{Lim}(Ex_{12}^1) = \frac{1}{5}(6 + 6 + 7.7661 + 7.7661 + 7.7661) = 7.0597$$
$$\overline{Lim}(En_{12}^1) = \frac{1}{5}(0.1528 + 0.1528 + 0.4001 + 0.4001 + 0.4001) = 0.3012$$
$$\overline{Lim}(HEn_{12}^1) = \frac{1}{5}(0.02 + 0.02 + 0.0524 + 0.0524 + 0.0524) = 0.0394$$

Hence, the CR form of $\hat{\phi}_{12}^1$ can be determined as $\tilde{\phi}_{12}^1 = (\lceil 6, 7.0596\rfloor, \lceil 0.1528, 0.3012\rfloor\ , \lceil 0.02, 0.0394\rfloor)$.
In a similar manner, CR numbers for the remaining cloud values are calculated as follows:

$$\tilde{\phi}_{12}^2 = (\lceil 6, 7.0596\rfloor, \lceil 0.1528, 0.3012\rfloor, \lceil 0.02, 0.0394\rfloor)$$
$$\tilde{\phi}_{12}^3 = (\lceil 7.0596, 7.7661\rfloor, \lceil 0.3012, 0.4001\rfloor, \lceil 0.0394, 0.0524\rfloor)$$
$$\tilde{\phi}_{12}^4 = (\lceil 7.0596, 7.7661\rfloor, \lceil 0.3012, 0.4001\rfloor, \lceil 0.0394, 0.0524\rfloor)$$
$$\tilde{\phi}_{12}^5 = (\lceil 7.0596, 7.7661\rfloor, \lceil 0.3012, 0.4001\rfloor, \lceil 0.0394, 0.0524\rfloor)$$

Then, the overall evaluation data of each experts can be determined by using the CR_{avg} operation. Using Eq. (29), $\tilde{\Phi}_{12}$ is computed as:

$$\tilde{\Phi}_{12} = \left(\left\lceil \frac{1}{5}\sum_{k=1}^{5} Ex_{12}^{kL}, \frac{1}{5}\sum_{k=1}^{5} Ex_{12}^{kU} \right\rfloor, \left\lceil \sqrt{\frac{1}{5}\sum_{k=1}^{5} (En_{12}^{kL})^2}, \sqrt{\frac{1}{5}\sum_{k=1}^{5} (En_{12}^{kU})^2} \right\rfloor, \left\lceil \sqrt{\frac{1}{5}\sum_{k=1}^{5} (HEn_{12}^{kL})^2}, \sqrt{\frac{1}{5}\sum_{k=1}^{5} (HEn_{12}^{kU})^2} \right\rfloor \right)$$
$$= (\lceil 6.6358, 7.4835\rfloor\ , \lceil 0.2525, 0.3638\rfloor\ , \lceil 0.0331, 0.0476\rfloor)$$

All these five cloud values and their collection are transformed into CR numbers. The importance of this conversion is that the cloud information provided by single expert ignores the interaction between different experts and did not accurately reflect the group's preferences. In contrast, CR numbers are established using an inclusive perception and the integrative nature of diverse opinions under GDM environments. Furthermore, these CR numbers are developed on the original dataset. The transformation of other elements in $M^k, (k = 1, 2, 3, 4, 5)$ are implemented in the same way. The integrated CR comparison values is given in matrix $\tilde{\mathbf{M}}$.

$$\tilde{\mathbf{M}} = \begin{bmatrix} (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) & \ldots & (\lceil 8.4645, 8.8137\rfloor, \lceil 0.8404, 1.0230\rfloor, \lceil 0.1100, 0.1339\rfloor) \\ (\lceil 0.5530, 0.6236\rfloor, \lceil 0.0210, 0.0303\rfloor, \lceil 0.0331, 0.0476\rfloor) & \ldots & (\lceil 4.5165, 5.3642\rfloor, \lceil 0.2525, 0.3638\rfloor, \lceil 0.0331, 0.0476\rfloor) \\ (\lceil 0.3846, 0.4631\rfloor, \lceil 0.0178, 0.0286\rfloor, \lceil 0.0279, 0.0449\rfloor) & \ldots & (\lceil 6.0706, 6.6358\rfloor, \lceil 0.1639, 0.2544\rfloor, \lceil 0.0215, 0.0333\rfloor) \\ (\lceil 0.2709, 0.2979\rfloor, \lceil 0.0667, 0.0822\rfloor, \lceil 0.1047, 0.1291\rfloor) & \ldots & (\lceil 8.1590, 8.6828\rfloor, \lceil 0.6611, 0.9524\rfloor, \lceil 0.0865, 0.1247\rfloor) \\ (\lceil 0.3070, 0.3461\rfloor, \lceil 0.0378, 0.0626\rfloor, \lceil 0.0594, 0.0983\rfloor) & \ldots & (\lceil 6.6088, 7.9346\rfloor, \lceil 0.2775, 0.6674\rfloor, \lceil 0.0363, 0.0874\rfloor) \\ (\lceil 0.4704, 0.5538\rfloor, \lceil 0.0144, 0.0239\rfloor, \lceil 0.0227, 0.0376\rfloor) & \ldots & (\lceil 4.1888, 5.1562\rfloor, \lceil 0.2757, 0.4901\rfloor, \lceil 0.0361, 0.0641\rfloor) \\ (\lceil 0.7054, 0.7345\rfloor, \lceil 0.0700, 0.0852\rfloor, \lceil 0.1100, 0.1339\rfloor) & \ldots & (\lceil 3.1426, 3.1426\rfloor, \lceil 1.0475, 1.0475\rfloor, \lceil 0.1371, 0.1371\rfloor) \end{bmatrix}$$

4. Determine the weights of the criteria: Based on $\tilde{\mathbf{M}}$, relative weights and then its normalization form are determined using Eqs. (30) and (31), which are displayed in Table 6. Thus, all criteria weights are derived from the CR-AHP.

6.2. Prioritization of design concepts using CR-VIKOR

Based on criterion weights which is determined in Section 6.1, CR-based VIKOR is conducted to determine the final ranking of design concepts. The performance of a design concept can be represented by a set of such values from experts' estimation. Table 7 shows the experts' evaluation for alternative ranking.

1. Summarize the evaluation information into cloud values: Using Table 8, we can convert the numerical information in Table 7 into cloud values which are displayed in Table 9.
2. Translate the cloud evaluation data into CR numbers of each design alternatives: Based on Section 3.3, Table 9 further converted into CR decision matrix. Tables 10–12 shown the CR evaluation information for the alternatives A_1, A_2 and A_6, respectively.
3. Compute the group decision matrix by CR numbers: The CR values given by six design concepts are aggregated by using Eq. (18), which is presented in Table 13. Then, normalized the group decision matrix using Eqs. (33) and (34), which is presented in Table 14.
4. Identify the best and worst values in CR numbers: Using Table 14 and Eqs. (37) and (38), best value (Ψ_j^*) and worst value (Ψ_j^-) of each criterion are computed as shown in Table 15.

Table 7
Evaluation information for design concepts.

DAs	Experts	Evaluation criteria						
		C_1	C_2	C_3	C_4	C_5	C_6	C_7
	E_1	95	852	208	12	97	5	7
	E_2	99	851	210	13	98	7	7
A_1	E_3	98	850	206	11	98	5	7
	E_4	97	857	205	12	97	7	5
	E_5	93	855	207	13	99	3	7
	E_1	93	853	208	11	98	7	5
	E_2	95	857	209	11	98	5	7
A_2	E_3	90	850	203	12	97	7	3
	E_4	91	857	206	12	98	5	5
	E_5	90	854	205	11	99	7	5
	E_1	88	852	201	10	98	7	9
	E_2	93	850	208	11	98	7	9
A_3	E_3	92	852	205	10	98	7	9
	E_4	86	858	207	10	97	9	7
	E_5	91	857	202	11	98	7	9
	E_1	85	855	200	9	99	7	9
	E_2	90	856	205	8	98	9	7
A_4	E_3	87	858	198	7	99	9	7
	E_4	85	854	204	8	98	7	7
	E_5	88	855	203	8	99	7	9
	E_1	89	855	204	11	98	5	7
	E_2	93	857	207	11	98	5	9
A_5	E_3	90	855	206	12	98	7	7
	E_4	88	862	207	10	99	5	7
	E_5	92	857	203	11	98	7	7
	E_1	88	853	204	10	98	9	7
	E_2	92	856	203	9	99	9	5
A_6	E_3	89	855	208	10	98	7	5
	E_4	90	854	207	11	99	7	5
	E_5	91	856	202	10	98	9	7

Table 8
Conversion between evaluation information and Clouds values.

Scale (C_1)	Cloud values	Scale (C_2)	Cloud values	Scale (C_3)	Cloud values
85	(92.347, 2.4489, 0.3590)	850	(855.30, 1.7657, 0.2219)	198	(9.9743, 1.2698, 0.3590)
86	(92.361, 1.5134, 0.2219)	851	(855.62, 1.0912, 0.1371)	200	(9.5259, 0.7847, 0.2219)
87	(92.369, 0.9353, 0.1371)	852	(855.82, 0.6744, 0.0847)	201	(9.2487, 0.4850, 0.1371)
88	(92.383, 0.5780, 0.0847)	853	(856.14, 0.4167, 0.0524)	202	(8.8002, 0.2997, 0.0847)
89	(92.405, 0.3572, 0.0524)	854	(856.66, 0.2576, 0.0324)	203	(8.0745, 0.1852, 0.0524)
90	(92.442, 0.2208, 0.0324)	854.5	(857.5, 0.1592, 0.02)	204	(6.9002, 0.1145, 0.0324)
91	(92.5, 0.1364, 0.02)	855	(858.34, 0.2576, 0.0324)	204.5	(5, 0.0707, 0.02)
92	(92.559, 0.2208, 0.0324)	856	(858.86, 0.4167, 0.0524)	205	(3.0998, 0.1145, 0.0324)
93	(92.595, 0.3572, 0.0524)	857	(859.18, 0.6744, 0.0847)	206	(1.9255, 0.1852, 0.0524)
95	(92.617, 0.5780, 0.0847)	858	(859.38, 1.0912, 0.1371)	207	(1.1998, 0.2997, 0.0847)
97	(92.631, 0.9353, 0.1371)	862	(859.70, 1.7657, 0.2219)	208	(0.7513, 0.4850, 0.1371)
98	(92.639, 1.5134, 0.2219)			209	(0.4741, 0.7847, 0.2219)
99	(92.653, 2.4489, 0.3590)			210	(0.0257, 1.2698, 0.3590)
Scale (C_4)	Cloud values	Scale (C_5)	Cloud values	Scale (C_6, C_7)	Cloud values
7	(8.6184, 0.5395, 0.0847)	97	(97.464, 0.1545, 0.0324)	3	(4.1669, 0.3890, 0.0524)
8	(9.5280, 0.3334, 0.0524)	98	(98.5, 0.0955, 0.02)	5	(5.6088, 0.2404, 0.0324)
9	(10.090, 0.2060, 0.0324)	99	(99.536, 0.1545, 0.0324)	6	(6.5, 0.1486, 0.02)
10	(11, 0.1273, 0.02)			7	(7.3912, 0.2404, 0.0324)
11	(11.910, 0.2060, 0.0323)			9	(8.8331, 0.3890, 0.0524)
12	(12.472, 0.3334, 0.0524)				
13	(13.382, 0.5395, 0.0847)				

5. Ranking the DAs in ascending order based on $\hat{\mathbf{S}}_i$, $\hat{\mathbf{R}}_i$, $\hat{\mathbf{Q}}_i$: Using Eqs. (39), (40) and (41), the CR values of group utility ($\hat{\mathbf{S}}_i$), individual regret ($\hat{\mathbf{R}}_i$) and evaluation index ($\hat{\mathbf{Q}}_i$) are determined as shown in Table 16. To determine the ranking, we can use the rough boundary interval of $\hat{\mathbf{S}}_i$, $\hat{\mathbf{R}}_i$, $\hat{\mathbf{Q}}_i$ for each DA that are also listed in Table 16. A_4 is the best design alternative in $\hat{\mathbf{Q}}_i$.

6.3. Sensitivity analysis

To further analyze the evaluation procedure of the proposed CR-AHP-VIKOR, a sensitivity analysis is carried out for different values of η in Eq. (41). Moreover, we can measure the effectiveness of experts' risks on the final product ranking. Each design alternatives can be ranked on the basis of rough boundary interval of evaluation index (IRBnd($\hat{\mathbf{Q}}_i$)), rough boundary interval of expectation in evaluation index (IRBnd($\hat{E}x_i$)) and

Table 9
The design alternatives' performances in forms of clouds values.

DAs	Experts	Evaluation criteria						
		C_1	C_2	C_3	C_4	C_5	C_6	C_7
	E_1	(92.617, 0.5780, 0.0847)	(855.82, 0.6744, 0.0847)	(0.7513, 0.4850, 0.1371)	(12.472, 0.3334, 0.0524)	(97.464, 0.1545, 0.0324)	(5.6088, 0.2404, 0.0324)	(7.3912, 0.2404, 0.0324)
A_1	E_2	(2.6532, 2.4489, 0.3590)	(855.62, 1.0912, 0.1371)	(0.0257, 1.2698, 0.3590)	(13.382, 0.5395, 0.0847)	(98.500, 0.0955, 0.02)	(7.3912, 0.2404, 0.0324)	(7.3912, 0.2404, 0.0324)
	E_3	(92.639, 1.5134, 0.2219)	(855.30, 1.7657, 0.2219)	(1.9254, 0.1852, 0.0524)	(11.000, 0.1273, 0.02)	(98.500, 0.0955, 0.02)	(5.6088, 0.2404, 0.0324)	(7.3912, 0.2404, 0.0324)
	E_4	(92.631, 0.9353, 0.1371)	(859.18, 0.6744, 0.0847)	(3.1000, 0.1145, 0.0324)	(12.472, 0.3334, 0.0523)	(97.464, 0.1545, 0.0324)	(7.3912, 0.2404, 0.0324)	(5.6088, 0.2404, 0.0324)
	E_5	(92.595, 0.3572, 0.0524)	(858.34, 0.2576, 0.0324)	(1.1998, 0.2997, 0.0847)	(13.382, 0.5395, 0.0847)	(99.536, 0.1545, 0.0324)	(4.1669, 0.3890, 0.0524)	(7.3912, 0.2404, 0.0324)
	E_1	(92.595, 0.3572, 0.0524)	(856.14, 0.4167, 0.0524)	(0.7513, 0.4850, 0.1371)	(11.910, 0.2060, 0.0323)	(98.500, 0.0955, 0.02)	(7.3912, 0.2404, 0.0324)	(5.6087, 0.2404, 0.0324)
A_2	E_2	(92.617, 0.5780, 0.0847)	(859.18, 0.6744, 0.0847)	(0.4741, 0.7847, 0.2219)	(11.910, 0.2060, 0.0324)	(98.500, 0.0955 0.02)	(5.6088, 0.2404, 0.0324)	(7.3912, 0.2404, 0.0324)
	E_3	(92.441, 0.2208, 0.0324)	(855.30, 1.7657, 0.2218)	(8.0745, 0.1852, 0.0524)	(12.472, 0.3334, 0.0524)	(97.464, 0.1545, 0.0324)	(7.3912, 0.2404, 0.0324,)	(4.1669, 0.3890, 0.0524)
	E_4	(92.500, 0.13642, 0.02)	(859.18, 0.6744, 0.0847)	(1.9255, 0.1852, 0.0524)	(12.472, 0.3334, 0.0524)	(98.500, 0.0955, 0.02)	(5.6088, 0.2404, 0.0324)	(5.6088, 0.2404, 0.0324)
	E_5	(92.441, 0.2208, 0.0324)	(856.66, 0.2576, 0.0324)	(3.0998, 0.1145, 0.0324)	(11.910, 0.2060, 0.0324)	(99.536, 0.1545, 0.0324)	(7.3912, 0.2404, 0.0324)	(5.6087, 0.2404, 0.0324)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	E_1	(92.383, 0.5780, 0.0847)	(856.14, 0.4167, 0.0524)	(6.9002, 0.1145, 0.0324)	(11.000, 0.1273, 0.02)	(98.500, 0.0955, 0.02)	(8.8331, 0.3890, 0.0524)	(7.3912, 0.2404, 0.0324)
A_6	E_2	(92.559, 0.2208, 0.0324)	(858.862, 0.4167, 0.0524)	(8.0745, 0.1852, 0.0524)	(10.090, 0.2060, 0.0324)	(99.536, 0.1545, 0.0324)	(8.8331, 0.3890, 0.0524)	(5.6088, 0.2404, 0.0324)
	E_3	(92.405, 0.3572, 0.0524)	(858.34, 0.2576, 0.0324)	(0.7513, 0.4850, 0.1371)	(11.000, 0.1273, 0.02)	(98.500, 0.0955, 0.02)	(7.3912, 0.2404, 0.0324)	(5.6088, 0.2404, 0.0324)
	E_4	(92.442, 0.2208, 0.0324)	(856.66, 0.2576, 0.0324)	(1.1998, 0.2997, 0.0847)	(11.9098, 0.2060, 0.0324)	(99.536, 0.1545, 0.0324)	(7.3912, 0.2404, 0.0324)	(5.6088, 0.2404, 0.0324)
	E_5	(92.500, 0.1364, 0.02)	(858.8616, 0.4167, 0.0524)	(8.8002, 0.2997, 0.0847)	(11.000, 0.1273, 0.02)	(98.500, 0.0955, 0.02)	(8.8331, 0.3890, 0.0524)	(7.3912, 0.2404, 0.0324)

Table 10
Transform the cloud assessment matrix into CR numbers for alternative A_1.

Expert	Evaluation criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
E_1	$(\lceil 92.606, 92.635\rfloor, \lceil 0.4676, 1.3689\rfloor, \lceil 0.0686, 0.2007\rfloor)$	$(\lceil 855.58, 857.78\rfloor, \lceil 0.5354, 1.0514\rfloor, \lceil 0.0673, 0.1321\rfloor)$	$(\lceil 0.3885, 1.7441\rfloor, \lceil 0.2711, 0.8774\rfloor, \lceil 0.0766, 0.2481\rfloor)$	$(\lceil 11.981, 12.927\rfloor, \lceil 0.2647, 0.4364\rfloor, \lceil 0.0416, 0.0686\rfloor)$	$(\lceil 97.464, 98.293\rfloor, \lceil 0.1309, 0.1545\rfloor, \lceil 0.0274, 0.0324\rfloor)$	$(\lceil 5.1281, 6.5000\rfloor, \lceil 0.2404, 0.2701\rfloor, \lceil 0.0324, 0.0364\rfloor)$	$(\lceil 7.0347, 7.3912\rfloor, \lceil 0.2404, 0.2404\rfloor, \lceil 0.0324, 0.0324\rfloor)$
E_2	$(\lceil 92.627, 92.653\rfloor, \lceil 1.1666, 2.4489\rfloor, \lceil 0.1710, 0.3590\rfloor)$	$(\lceil 855.46, 857.24\rfloor, \lceil 0.6744, 1.4284\rfloor, \lceil 0.0847, 0.1795\rfloor)$	$(\lceil 0.0257, 1.4004\rfloor, \lceil 0.4708, 1.2698\rfloor, \lceil 0.1331, 0.3590\rfloor)$	$(\lceil 12.541, 13.382\rfloor, \lceil 0.3746, 0.5395\rfloor, \lceil 0.0588, 0.0847\rfloor)$	$(\lceil 97.982, 98.845\rfloor, \lceil 0.0955, 0.1309\rfloor, \lceil 0.0200, 0.0274\rfloor)$	$(\lceil 6.0334, 7.3912\rfloor, \lceil 0.2404, 0.2701\rfloor, \lceil 0.0324, 0.0364\rfloor)$	$(\lceil 7.0347, 7.3912\rfloor, \lceil 0.2404, 0.2404\rfloor, \lceil 0.0324, 0.0324\rfloor)$
E_3	$(\lceil 92.621, 92.646\rfloor, \lceil 0.8460, 1.9812\rfloor, \lceil 0.1240, 0.2904\rfloor)$	$(\lceil 855.30, 856.85\rfloor, \lceil 0.8926, 1.7657\rfloor, \lceil 0.1122, 0.2219\rfloor)$	$(\lceil 0.9756, 2.5127\rfloor, \lceil 0.1498, 0.5599\rfloor, \lceil 0.0424, 0.1583\rfloor)$	$(\lceil 11.000, 12.541\rfloor, \lceil 0.1273, 0.3746\rfloor, \lceil 0.0200, 0.0588\rfloor)$	$(\lceil 97.982, 98.845\rfloor, \lceil 0.0955, 0.1309\rfloor, \lceil 0.0200, 0.0274\rfloor)$	$(\lceil 5.1281, 6.5000\rfloor, \lceil 0.2404, 0.2701\rfloor, \lceil 0.0324, 0.0364\rfloor)$	$(\lceil 7.0347, 7.3912\rfloor, \lceil 0.2404, 0.2404\rfloor, \lceil 0.0324, 0.0324\rfloor)$
E_4	$(\lceil 92.614, 92.641\rfloor, \lceil 0.6235, 1.6326\rfloor, \lceil 0.0914, 0.2393\rfloor)$	$(\lceil 856.85, 859.18\rfloor, \lceil 0.5354, 1.0514\rfloor, \lceil 0.0673, 0.1321\rfloor)$	$(\lceil 1.4004, 3.0998\rfloor, \lceil 0.1145, 0.4708\rfloor, \lceil 0.0324, 0.1331\rfloor)$	$(\lceil 11.981, 12.927\rfloor, \lceil 0.2647, 0.4364\rfloor, \lceil 0.0416, 0.0686\rfloor)$	$(\lceil 97.464, 98.293\rfloor, \lceil 0.13090.1545\rfloor, \lceil 0.0274, 0.0324\rfloor)$	$(\lceil 6.0334, 7.3912\rfloor, \lceil 0.2404, 0.2701\rfloor, \lceil 0.0324, 0.0364\rfloor)$	$(\lceil 5.6088, 7.0347\rfloor, \lceil 0.2404, 0.2404\rfloor, \lceil 0.0324, 0.0324\rfloor)$
E_5	$(\lceil 92.595, 92.627\rfloor, \lceil 0.3572, 1.1666\rfloor, \lceil 0.0524, 0.1710\rfloor)$	$(\lceil 856.27, 858.76\rfloor, \lceil 0.2576, 0.8926\rfloor, \lceil 0.0324, 0.1122\rfloor)$	$(\lceil 0.6589, 2.0750\rfloor, \lceil 0.1998, 0.6848\rfloor, \lceil 0.0565, 0.1936\rfloor)$	$(\lceil 12.541, 13.382\rfloor, \lceil 0.3746, 0.5395\rfloor, \lceil 0.0588, 0.0847\rfloor)$	$(\lceil 98.293, 99.536\rfloor, \lceil 0.1309, 0.1545\rfloor, \lceil 0.0274, 0.0324\rfloor)$	$(\lceil 4.1669, 6.0334\rfloor, \lceil 0.2701, 0.3890\rfloor, \lceil 0.0364, 0.0524\rfloor)$	$(\lceil 7.0347, 7.3912\rfloor, \lceil 0.2404, 0.2404\rfloor, \lceil 0.0324, 0.0324\rfloor)$

distance of evaluation index from minimum value ($d(\hat{\mathbf{Q}}_i, \min_i \hat{\mathbf{Q}}_i)$) as seen in Eq. (14), (16). When $\eta \geq 0.5$, final ranking in all cases is obtained as $A_4 < A_3 < A_5 < A_6 < A_1 < A_2$; otherwise, the final ranking (using $\mathrm{IRBnd}(\hat{\mathbf{Q}}_i)$) is obtained as $A_4 < A_3 < A_5 < A_1 < A_6 < A_2$. It can be seen that the design concept A_4 has a maximum priority in each situation. In each round of sensitivity analysis, design concept A_1 and design concept A_6 are replaced, while others remain the same. But in the remaining cases i.e. $\mathrm{IRBnd}(\hat{E}x_i)$ and $d(\hat{\mathbf{Q}}_i, \min_i \hat{\mathbf{Q}}_i)$ may be ranked vary when $\eta < 0.5$. Due to the space limitation, we have listed the results of the sensitivity analysis for some values of η as shown in Table 17 and Fig. 2, respectively.

7. Comparison analysis

In this section, the evaluation procedure of the CR numbers-based MAGDM framework, the traditional crisp numbers, fuzzy logic, and rough approximations are presented to assess the effectiveness of the proposed model. In order to compare the different approaches, crisp AHP-VIKOR, fuzzy AHP-VIKOR, and rough AHP-VIKOR are used. All experiments are implemented under the GDM environment.

7.1. Comparison of alternative ranking information

The improved DCE model utilizes the CM theory to explain the vagueness and randomness of experts' linguistic judgment, and rough numbers to deal with ambiguity and inconsistency in a flexible and objective manner. CR values come from the initial evaluation data offered by experts,

Table 11
Transform the cloud assessment matrix into CR numbers for alternative A_2.

Expert	Evaluation criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
E_1	([92.494, 92.606], [0.2338, 0.4676], [0.0343, 0.0686])	([855.72, 857.79], [0.3372, 0.8828], [0.0424, 0.1109])	([0.6127, 3.4628], [0.2425, 0.6349], [0.0686, 0.1795])	([11.910, 12.135], [0.2060, 0.2570], [0.0324, 0.0404])	([98.241, 98.759], [0.0955, 0.1191], [0.0200, 0.0249])	([6.6782, 7.3912], [0.2404, 0.2404], [0.0324, 0.0324])	([5.2483, 6.0544], [0.2404, 0.2701], [0.0324, 0.0364])
E_2	([92.519, 92.617], [0.3026, 0.5780], [0.0444, 0.0847])	([857.29, 859.18], [0.5058, 1.0381], [0.0635, 0.1304])	([0.4741, 2.8650], [0.3509, 0.7847], [0.0992, 0.2219])	([11.910, 12.135], [0.2060, 0.2570], [0.0324, 0.0404])	([98.241, 98.759], [0.0955, 0.1191], [0.0200, 0.0249])	([5.6088, 6.6782], [0.2404, 0.2404], [0.0324, 0.0324])	([5.6769, 7.3912], [0.2404, 0.2701], [0.0324, 0.0364])
E_3	([92.441, 92.519], [0.1926, 0.3442], [0.0282, 0.0505])	([855.30, 857.29], [0.7577, 1.7657], [0.0952, 0.2219])	([2.8650, 8.0745], [0.1616, 0.4100], [0.0457, 0.1159])	([12.135, 12.472], [0.2570, 0.3334], [0.0404, 0.0524])	([97.464, 98.500], [0.1191, 0.1545], [0.0249, 0.0324])	([6.6782, 7.3912], [0.2404, 0.2404], [0.0324, 0.0324])	([4.1669, 5.6769], [0.2701, 0.3890], [0.0364, 0.0524])
E_4	([92.461, 92.571], [0.1364, 0.3026], [0.0200, 0.0444])	([857.29, 859.18], [0.5058, 1.0381], [0.0635, 0.1304])	([1.0503, 4.3666], [0.1616, 0.4100], [0.0457, 0.1159])	([12.135, 12.472], [0.2570, 0.3334], [0.0404, 0.0524])	([98.241, 98.759], [0.0955, 0.1191], [0.0200, 0.0249])	([5.6088, 6.6782], [0.2404, 0.2404], [0.0324, 0.0324])	([5.2483, 6.0544], [0.2404, 0.2701], [0.0324, 0.0364])
E_5	([92.44192.519], [0.19260.3442], [0.02820.0505])	([856.03858.34], [0.25760.7577], [0.03240.0952])	([1.56275.5872], [0.11450.3509], [0.03240.0992])	([11.91012.135], [0.20600.2570], [0.03240.0404])	([98.50099.536], [0.11910.1545], [0.02490.0324])	([6.67827.3912], [0.24040.2404], [0.03240.0324])	([5.24836.0544], [0.24040.2701], [0.03240.0364])

Table 12
Transform the cloud assessment matrix into CR numbers for alternative A_6.

Expert	Evaluation criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
E_1	([92.383, 92.458], [0.3026, 0.5780], [0.0444, 0.0847])	([856.14, 857.77], [0.3531, 0.4167], [0.0444, 0.0524])	([2.9504, 7.9250], [0.1145, 0.2768], [0.0324, 0.0783])	([10.773, 11.227], [0.1273, 0.1588], [0.0200, 0.0249])	([98.500, 98.915], [0.0955, 0.1191], [0.0200, 0.0249])	([8.2564, 8.8331], [0.3295, 0.3890], [0.0444, 0.0524])	([6.3218, 7.3912], [0.2404, 0.2404], [0.0324, 0.0324])
E_2	([92.458, 92.559], [0.1926, 0.3442], [0.0282, 0.0505])	([857.77, 858.86], [0.3531, 1.0381], [0.0444, 0.0524])	([4.2314, 8.4374], [0.1498, 0.3174], [0.0424, 0.0897])	([10.090, 11.000], [0.1588, 0.2060], [0.0249, 0.0324])	([98.915, 99.536], [0.1191, 0.1545], [0.0249, 0.0324])	([8.2564, 8.8331], [0.3295, 0.3890], [0.0444, 0.0524])	([5.6088, 6.3218], [0.2404, 0.2404], [0.0324, 0.0324])
E_3	([92.394, 92.476], [0.2338, 0.4676], [0.0343, 0.0686])	([857.05, 858.69], [0.2576, 0.7259], [0.0324, 0.0444])	([0.7513, 5.1452], [0.2768, 0.4850], [0.0783, 0.1371])	([10.773, 11.227], [0.1273, 0.1588], [0.0200, 0.0249])	([98.500, 98.915], [0.0955, 0.1191], [0.0200, 0.0249])	([7.3912, 8.2564], [0.2404, 0.3295], [0.0324, 0.0444])	([5.6088, 6.3218], [0.2404, 0.2404], [0.0324, 0.0324])
E_4	([92.410, 92.500], [0.1926, 0.3442], [0.0282, 0.0505])	([856.40, 858.18], [0.2576, 0.7259], [0.0324, 0.0444])	([0.9755, 6.2437], [0.2248, 0.3615], [0.0635, 0.1022])	([11.000, 11.910], [0.1588, 0.2060], [0.0249, 0.0324])	([98.915, 99.536], [0.1191, 0.1545], [0.0249, 0.0324])	([7.3912, 8.2564], [0.2404, 0.3295], [0.0324, 0.0444])	([5.6088, 6.3218], [0.2404, 0.2404], [0.0324, 0.0324])
E_5	([92.432, 92.529], [0.1364, 0.3026], [0.0200, 0.0444])	([857.77, 858.86], [0.3531, 1.0381], [0.0444, 0.0524])	([5.1452, 8.8002], [0.2248, 0.3615], [0.0635, 0.1022])	([10.773, 11.227], [0.1273, 0.1588], [0.0200, 0.0249])	([98.500, 98.915], [0.0955, 0.1191], [0.0200, 0.0249])	([8.2564, 8.8331], [0.3295, 0.3890], [0.0444, 0.0524])	([6.3218, 7.3912], [0.2404, 0.2404], [0.0324, 0.0324])

Table 13
Aggregated CR decision matrix.

DAs	Evaluation criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	([92.612, 92.641], [0.7499, 1.7789], [0.1099, 0.2608])	([855.89, 857.96], [0.6150, 1.2779], [0.0773, 0.1606])	([0.8365, 2.2464], [0.2723, 0.8230], [0.0770, 0.2327])	([12.022, 13.036], [0.2956, 0.4698], [0.0464, 0.0738])	([97.837, 98.764], [0.1180, 0.1455], [0.0247, 0.0305])	([5.3434, 6.7847], [0.2466, 0.2977], [0.0332, 0.0401])	([6.7736, 7.3213], [0.2404, 0.2404], [0.0324, 0.0324])
A_2	([92.471, 92.566], [0.2187, 0.4198], [0.0321, 0.0615])	([856.33, 858.36], [0.5031, 1.1512], [0.0632, 0.1447])	([1.5719, 5.2092], [0.2224, 0.5437], [0.0629, 0.1537])	([12.000, 12.271], [0.2278, 0.2900], [0.0358, 0.0455])	([98.13898.863], [0.1056, 0.1344], [0.0221, 0.0281])	([6.2724, 7.1146], [0.2404, 0.2404], [0.0324, 0.0324])	([5.1424, 6.2741], [0.2466, 0.2977], [0.0332, 0.0401])
A_3	([92.415, 92.542], [0.3245, 0.9809], [0.0476, 0.1438])	([856.04, 858.17], [0.7646, 1.2661], [0.0961, 0.1591])	([2.7454, 7.3381], [0.2658, 0.4223], [0.0751, 0.1194])	([11.147, 11.585], [0.1408, 0.1792], [0.0221, 0.0281])	([98.127, 98.459], [0.0980, 0.1183], [0.0205, 0.0248])	([7.4498, 7.9238], [0.2466, 0.2977], [0.0332, 0.0401])	([8.3268, 8.7762], [0.3388, 0.3832], [0.0456, 0.0516])
A_4	([92.357, 92.401], [0.9012, 2.0052], [0.1321, 0.2939])	([857.72, 858.86], [0.3148, 0.6904], [0.0396, 0.0868])	([5.9160, 9.0531], [0.2753, 0.8617], [0.0778, 0.2436])	([9.2005, 9.7131], [0.2957, 0.4141], [0.0464, 0.0650])	([98.874, 99.371], [0.1180, 0.1455], [0.0247, 0.0305])	([7.6272, 8.3248], [0.2658, 0.3383], [0.0358, 0.0455])	([7.6272, 8.3248], [0.2658, 0.3383], [0.0358, 0.0455])
A_5	([92.422, 92.536], [0.2774, 0.4349], [0.0407, 0.0638])	([858.63, 859.26], [0.4677, 1.1249], [0.0588, 0.1413])	([2.3551, 6.0463], [0.1786, 0.2624], [0.0505, 0.0742])	([11.581, 12.094], [0.1827, 0.2559], [0.0287, 0.0402])	([98.541, 98.874], [0.0980, 0.1183], [0.02050.0248])	([5.9043, 6.7699], [0.2404, 0.2404], [0.0324, 0.0324])	([7.4498, 7.9238], [0.2466, 0.2977], [0.0332, 0.0401])
	([92.415, 92.504], [0.2187, 0.4198], [0.0321, 0.0615])	([857.03, 858.47], [0.3183, 0.8225], [0.0400, 0.0493])	([3.3045, 7.4417], [0.2065, 0.3671], [0.0584, 0.1038])	([10.686, 11.323], [0.1408, 0.1792], [0.0221, 0.0281])	([98.666, 99.164], [0.1056, 0.1344], [0.0221, 0.0281])	([7.92178.6070], [0.2971, 0.3664], [0.0400, 0.0493])	([5.90436.7699], [0.2404, 0.2404], [0.0324, 0.0324])

which does not need any additional assumptions or data distributions. This linguistic manipulation model enhances the objectivity of the evaluation results in subjective procedure and better reflects the true perceptions of experts. Additionally, it selects to use the CR_{avg} operation for the fusion computation of the evaluation CR values. The AHP approach is used to calculate the evaluation criteria's weights. Subsequently, the VIKOR technique is used to determine each DA's ranking. Based on the expert analysis data offered in the framework of this study, the evaluation of DAs and their ranking scores determined by the CR AHP-VIKOR method are presented in Table 16. The strengths of the CR AHP-VIKOR method are demonstrated as compared to previous DCE approaches based on crisp AHP-VIKOR, crisp AHP-TOPSIS, fuzzy AHP-VIKOR, rough AHP-VIKOR which is shown in Fig. 3. According to the ranking results for all the approaches, A_4 is the best design concept among all the candidate DAs. Although it demonstrates the proposed scheme's validity, but there are still fluctuations between the various approaches.

Table 14
Normalized CR decision matrix.

DAs	Evaluation criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	(⌈0.9969, 0.9972⌋, ⌈0.0126, 0.0215⌋, ⌈0.0019, 0.0031⌋)	(⌈0.9961, 0.9985⌋, ⌈0.0011, 0.0017⌋, ⌈.00011, .00020⌋)	(⌈0.0924, 0.2481⌋, ⌈0.0302, 0.0912⌋, ⌈0.0085, 0.0258⌋)	(⌈0.7058, 0.7653⌋, ⌈0.0278, 0.0387⌋, ⌈0.0044, 0.0061⌋)	(⌈0.9846, 0.9939⌋, ⌈0.0017, 0.0019⌋, ⌈.00035, .00040⌋)	(⌈0.7876, 1.0000⌋, ⌈0.0523, 0.0787⌋, ⌈0.0070, 0.0106⌋)	(⌈0.7024, 0.7592⌋, ⌈0.0517, 0.0568⌋, ⌈0.0070, 0.0076⌋)
A_2	(⌈0.9977, 0.9988⌋, ⌈0.0100, 0.0107⌋, ⌈0.0015, 0.0016⌋)	(⌈0.9966, 0.9989⌋, ⌈0.0010, 0.0016⌋, ⌈.00009, .00018⌋)	(⌈0.1736, 0.5754⌋, ⌈0.0251, 0.0623⌋, ⌈0.0071, 0.0176⌋)	(⌈0.7498, 0.7667⌋, ⌈0.0278, 0.0308⌋, ⌈0.0044, 0.0048⌋)	(⌈0.9876, 0.9949⌋, ⌈0.0016, 0.0018⌋, ⌈.00033, .00038⌋)	(⌈0.7511, 0.8519⌋, ⌈0.0489, 0.0575⌋, ⌈0.0066, 0.0077⌋)	(⌈0.8196, 1.0000⌋, ⌈0.0629, 0.0877⌋, ⌈0.0085, 0.0118⌋)
A_3	(⌈0.9980, 0.9994⌋, ⌈0.0103, 0.0144⌋, ⌈0.0015, 0.0021⌋)	(⌈0.9963, 0.9987⌋, ⌈0.0012, 0.0017⌋, ⌈0.000130.00019⌋)	(⌈0.3033, 0.8106⌋, ⌈0.0306, 0.0522⌋, ⌈0.0087, 0.0148⌋)	(⌈0.7942, 0.8254⌋, ⌈0.0273, 0.0297⌋, ⌈0.0043, 0.0047⌋)	(⌈0.9875, 0.9908⌋, ⌈0.0015, 0.0017⌋, ⌈.00032, .00035⌋)	(⌈0.6744, 0.7173⌋, ⌈0.0430, 0.0491⌋, ⌈0.0058, 0.0066⌋)	(⌈0.5860, 0.6176⌋, ⌈0.0447, 0.0496⌋, ⌈0.0060, 0.0067⌋)
A_4	(⌈0.9995, 1.0000⌋, ⌈0.0138, 0.0238⌋, ⌈0.0020, 0.0035⌋)	(⌈0.9982, 0.9995⌋, ⌈0.0009, 0.0011⌋, ⌈.00007, .00012⌋)	(⌈0.6535, 1.0000⌋, ⌈0.0358, 0.0995⌋, ⌈0.0101, 0.0281⌋)	(⌈0.9472, 1.0000⌋, ⌈0.0419, 0.0553⌋, ⌈0.0066, 0.0087⌋)	(⌈0.9950, 1.0000⌋, ⌈0.0017, 0.0019⌋, ⌈.00035, .00039⌋)	(⌈0.6419, 0.7006⌋, ⌈0.0412, 0.0498⌋, ⌈0.0055, 0.0067⌋)	(⌈0.6177, 0.6742⌋, ⌈0.0452, 0.0536⌋, ⌈0.0061, 0.0072⌋)
A_5	(⌈0.9981, 0.9993⌋, ⌈0.0102, 0.0108⌋, ⌈0.0015, 0.0016⌋)	(⌈0.9993, 1.0000⌋, ⌈0.0010, 0.0015⌋, ⌈.00009, .00017⌋)	(⌈0.2601, 0.6679⌋, ⌈0.0211, 0.0349⌋, ⌈0.0060, 0.0099⌋)	(⌈0.7607, 0.7944⌋, ⌈0.0270, 0.0310⌋, ⌈0.0042, 0.0049⌋)	(⌈0.9917, 0.9950⌋, ⌈0.0015, 0.0017⌋, ⌈.00032, .00035⌋)	(⌈0.7893, 0.9050⌋, ⌈0.0521, 0.0624⌋, ⌈0.0070, 0.0084⌋)	(⌈0.6490, 0.6903⌋, ⌈0.0473, 0.0532⌋, ⌈0.0064, 0.0072⌋)
A_6	(⌈0.9984, 0.9994⌋, ⌈0.0100, 0.0108⌋, ⌈0.0015, 0.0016⌋)	(⌈0.9974, 0.9991⌋, ⌈0.0009, 0.0012⌋, ⌈.00007, .00008⌋)	(⌈0.3650, 0.8220⌋, ⌈0.0251, 0.0470⌋, ⌈0.0071, 0.0133⌋)	(⌈0.8126, 0.8610⌋, ⌈0.0280, 0.0312⌋, ⌈0.0044, 0.0049⌋)	(⌈0.9929, 0.9979⌋, ⌈0.0016, 0.0018⌋, ⌈.00033, .00038⌋)	(⌈0.6208, 0.6745⌋, ⌈0.0406, 0.0488⌋, ⌈0.0055, 0.0066⌋)	(⌈0.7596, 0.8710⌋, ⌈0.0569, 0.0675⌋, ⌈0.0077, 0.0091⌋)

Table 15
Best and worst CR values of design criteria.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Ψ_j^*	(⌈0.9969, 0.9972⌋, ⌈0.0138, 0.0238⌋, ⌈0.0020, 0.0035⌋)	(⌈0.9993, 1.0000⌋, ⌈0.0009, 0.0011⌋, ⌈0.0001, 0.0001⌋)	(⌈0.6535, 1.0000⌋, ⌈0.0211, 0.0349⌋, ⌈0.0060, 0.0099⌋)	(⌈0.7058, 0.7653⌋, ⌈0.0419, 0.0553⌋, ⌈0.0066, 0.0087⌋)	(⌈0.9950, 1.0000⌋, ⌈0.0015, 0.0017⌋, ⌈0.0003, 0.0004⌋)	(⌈0.6208, 0.6745⌋, ⌈0.0523, 0.0787⌋, ⌈0.0070, 0.0106⌋)	(⌈0.5860, 0.6176⌋, ⌈0.0629, 0.0877⌋, ⌈0.0085, 0.0118⌋)
Ψ_j^-	(⌈0.9995, 1.0000⌋, ⌈0.0100, 0.0107⌋, ⌈0.0015, 0.0016⌋)	(⌈0.9961, 0.9985⌋, ⌈0.0012, 0.0017⌋, ⌈0.0001, 0.0002⌋)	(⌈0.0924, 0.2481⌋, ⌈0.0358, 0.0995⌋, ⌈0.0101, 0.0281⌋)	(⌈0.9472, 1.0000⌋, ⌈0.0270, 0.0297⌋, ⌈0.0042, 0.0047⌋)	(⌈0.9846, 0.9908⌋, ⌈0.0017, 0.0019⌋, ⌈0.00035, 0.0004⌋)	(⌈0.7893, 1.0000⌋, ⌈0.0406, 0.0488⌋, ⌈0.0055, 0.0066⌋)	(⌈0.8196, 1.0000⌋, ⌈0.0447, 0.0496⌋, ⌈0.0060, 0.0067⌋)

Table 16
Ranking of design alternatives.

DAs	$\hat{\mathbf{S}}_i$	$\hat{\mathbf{R}}_i$	$\hat{\mathbf{Q}}_i$ ($\eta = 0.5$)	$IRBnd$ ($\hat{\mathbf{S}}_i$)	$IRBnd$ ($\hat{\mathbf{R}}_i$)	$IRBnd$ ($\hat{\mathbf{Q}}_i$)	Rank ($\hat{\mathbf{S}}_i$)	Rank ($\hat{\mathbf{R}}_i$)	Rank ($\hat{\mathbf{Q}}_i$)
A_1	(⌈1.7635, 1.9336⌋, ⌈1.0164, 1.1256⌋, ⌈0.1031, 0.1150⌋)	(⌈0.5849, 0.6386⌋, ⌈0.7202, 0.7922⌋, ⌈0.0730, 0.0809⌋)	(⌈0.5997, 0.8346⌋, ⌈1.0573, 1.5915⌋, ⌈0.1072, 0.1629⌋)	0.4976	0.2698	1.8375	5	4	5
A_2	(⌈1.9652, 2.1671⌋, ⌈1.2471, 1.3891⌋, ⌈0.1265, 0.1425⌋)	(⌈0.8938, 0.9955⌋, ⌈1.0987, 1.2287⌋, ⌈0.1114, 0.1254⌋)	(⌈0.8510, 1.1752⌋, ⌈1.5186, 2.3313⌋, ⌈0.1540, 0.2385⌋)	0.6280	0.4917	2.7622	6	6	6
A_3	(⌈0.9499, 1.0407⌋, ⌈0.5096, 0.5649⌋, ⌈0.0517, 0.0592⌋)	(⌈0.3363, 0.3669⌋, ⌈0.3385, 0.3786⌋, ⌈0.0343, 0.0386⌋)	(⌈0.2002, 0.3196⌋, ⌈0.5616, 0.8131⌋, ⌈0.0570, 0.0836⌋)	0.2569	0.1507	0.8738	2	1	2
A_4	(⌈0.5166, 0.5674⌋, ⌈0.5713, 0.6340⌋, ⌈0.0580, 0.0652⌋)	(⌈0.1833, 0.1993⌋, ⌈0.4514, 0.5048⌋, ⌈0.0458, 0.0515⌋)	(⌈−0.0252, 0.0297⌋ , ⌈0.5912, 0.7739⌋, ⌈0.0600, 0.0794⌋)	0.2392	0.1762	0.6030	1	2	1
A_5	(⌈1.1650, 1.2777⌋, ⌈0.8486, 0.9382⌋, ⌈0.0860, 0.0967⌋)	(⌈0.4574, 0.4994⌋, ⌈0.6369, 0.6979⌋, ⌈0.0646, 0.0709⌋)	(⌈0.3399, 0.4998⌋, ⌈0.8479, 1.2190⌋, ⌈0.0860, 0.1247⌋)	0.3815	0.2252	1.2733	3	3	3
A_6	(⌈1.1397, 1.2597⌋, ⌈0.9603, 1.0724⌋, ⌈0.0974, 0.1104⌋)	(⌈0.6141, 0.6840⌋, ⌈0.9107, 1.0185⌋, ⌈0.0924, 0.1040⌋)	(⌈0.4287, 0.6263⌋, ⌈1.1107, 1.6288⌋, ⌈0.1127, 0.1666⌋)	0.4562	0.3931	1.7518	4	5	4

A number of comparisons are made here to positively illustrate the proposed model's performance and effectiveness. The proposed CR-AHP-VIKOR effectively addresses multiple types of uncertainties in experts' decision analysis. Firstly, a comparison is made between the proposed CR-AHP-VIKOR algorithm and the crisp AHP-VIKOR algorithm. As can be seen in Fig. 3, the ranking results yielded by the crisp AHP-VIKOR ($A_4 < A_6 < A_5 < A_3 < A_2 < A_1$) is slightly compatible with the results of the CR-AHP-VIKOR ($A_4 < A_3 < A_5 < A_6 < A_1 < A_2$) in the DAs. However, the rankings of A_1, A_2, A_3, and A_6 are differ from each another. The main reason for these differences in product ranking between the two DCE techniques is that the crisp AHP-VIKOR method is unable to show the ambiguity and subjectivity in experts' attitudes and preferences under the GDM environment. In contrast, the proposed DCE model deals with both objective and subjective ambiguities that are being analyzed by fusing the CM theory into rough approximations. As a result, the presented DCE model's ranking results are more precise than those of the crisp DCE approach.

In the second comparison, we compare the AHP-VIKOR method based on CR numbers with the fuzzy AHP-VIKOR approach. In both DCE approaches shown in Fig. 3, all ranking results differ except A_4 and A_5. There is a difference in rankings because of uncertainty-coping techniques. Two distinct approaches are incorporated into the proposed DCE model to handle interpersonal and intrapersonal uncertainties. However, the TFNs are employed in the fuzzy DCE approach to cope with uncertainty, which can alter the original information's structure if different TFNs are implemented. In recent years, DCE models with fuzzy numbers have been successfully applied to talk about uncertainty in DM analysis. However, a definite predefined interval boundary, several fuzzy rules, and data distribution pose certain limits to all of these structures.

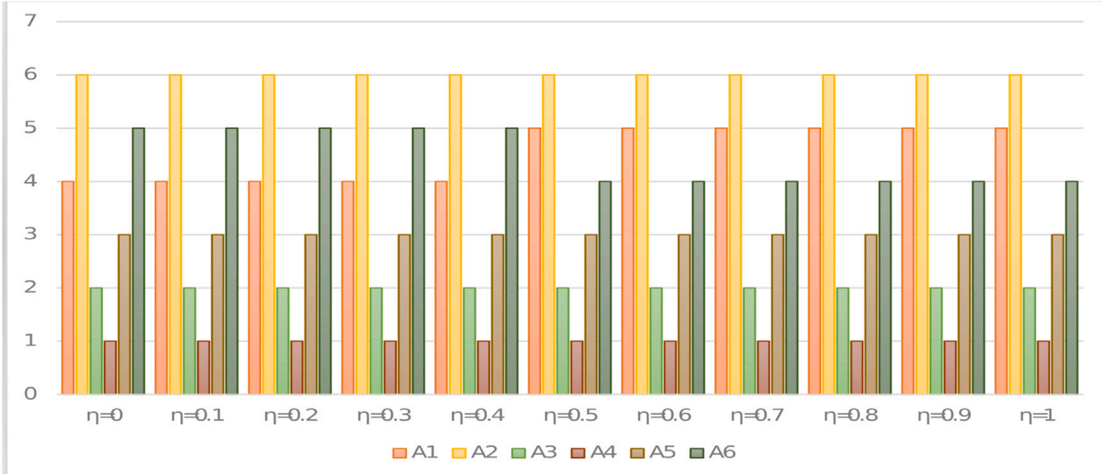

Fig. 2. Ranking results of the sensitivity analysis.

Table 17
Sensitivity analysis.

DAs	A_1	A_2	A_3	A_4	A_5	A_6
$\hat{\mathbf{Q}}_i$ ($\eta = 0$)	$(\lceil 0.4747, 0.6555\rfloor$, $\lceil 1.1880, 1.7524\rfloor$, $\lceil 0.1205, 0.1789\rfloor)$	$(\lceil 0.8551, 1.1695\rfloor$, $\lceil 1.8624, 2.8484\rfloor$, $\lceil 0.1889, 0.2908\rfloor)$	$(\lceil 0.1687, 0.2643\rfloor$, $\lceil 0.6360, 0.9131\rfloor$, $\lceil 0.0645, 0.0932\rfloor)$	$(\lceil -0.0197, 0.0230\rfloor$, $\lceil 0.6953, 0.9096\rfloor$, $\lceil 0.0705, 0.0928\rfloor)$	$(\lceil 0.3177, 0.4551\rfloor$, $\lceil 0.9954, 1.4201\rfloor$, $\lceil 0.1009, 0.1446\rfloor)$	$(\lceil 0.5107, 0.7209\rfloor$, $\lceil 1.3977, 2.0564\rfloor$, $\lceil 0.1418, 0.2099\rfloor)$
$Rank^{IRBnd(\hat{\mathbf{Q}}_i)}$	4	6	2	1	3	5
$Rank^{IRBnd(Ex_i)}$	4	6	2	1	3	5
$Rank^{d(\hat{\mathbf{Q}}_i,\min\hat{\mathbf{Q}}_i)}$	4	6	2	1	3	5
$\hat{\mathbf{Q}}_i$ ($\eta = 0.2$)	$(\lceil 0.5247, 0.7272\rfloor$, $\lceil 1.1375, 1.6899\rfloor$, $\lceil 0.1153, 0.1727\rfloor)$	$(\lceil 0.8534, 1.1717\rfloor$, $\lceil 1.7331, 2.6537\rfloor$, $\lceil 0.1758, 0.2710\rfloor)$	$(\lceil 0.1813, 0.2864\rfloor$, $\lceil 0.6073, 0.8744\rfloor$, $\lceil 0.0616, 0.0895\rfloor)$	$(\lceil -0.0219, 0.0257\rfloor$, $\lceil 0.6556, 0.8579\rfloor$, $\lceil 0.0665, 0.0877\rfloor)$	$(\lceil 0.3266, 0.4730\rfloor$, $\lceil 0.9392, 1.3433\rfloor$, $\lceil 0.0952, 0.1370\rfloor)$	$(\lceil 0.4779, 0.6831\rfloor$, $\lceil 1.2906, 1.8969\rfloor$, $\lceil 0.1309, 0.1938\rfloor)$
$Rank^{IRBnd(\hat{\mathbf{Q}}_i)}$	4	6	2	1	3	5
$Rank^{IRBnd(Ex_i)}$	4	6	2	1	3	5
$Rank^{d(\hat{\mathbf{Q}}_i,\min\hat{\mathbf{Q}}_i)}$	4	6	2	1	3	5
$\hat{\mathbf{Q}}_i$ ($\eta = 0.7$)	$(\lceil 0.6497, 0.9063\rfloor$, $\lceil 1.0002, 1.5224\rfloor$, $\lceil 0.1014, 0.1561\rfloor)$	$(\lceil 0.8493, 1.1774\rfloor$, $\lceil 1.3569, 2.0889\rfloor$, $\lceil 0.1376, 0.2140\rfloor)$	$(\lceil 0.2128, 0.3417\rfloor$, $\lceil 0.5290, 0.7695\rfloor$, $\lceil 0.0537, 0.0795\rfloor)$	$(\lceil -0.0275, 0.0324\rfloor$, $\lceil 0.5441, 0.7125\rfloor$, $\lceil 0.0552, 0.0733\rfloor)$	$(\lceil 0.3488, 0.5177\rfloor$, $\lceil 0.7811, 1.1286\rfloor$, $\lceil 0.0792, 0.1158\rfloor)$	$(\lceil 0.3959, 0.5884\rfloor$, $\lceil 0.9725, 1.4222\rfloor$, $\lceil 0.0986, 0.1458\rfloor)$
$Rank^{IRBnd(\hat{\mathbf{Q}}_i)}$	5	6	2	1	3	4
$Rank^{IRBnd(Ex_i)}$	5	6	2	1	3	4
$Rank^{d(\hat{\mathbf{Q}}_i,\min\hat{\mathbf{Q}}_i)}$	5	6	2	1	3	4
$\hat{\mathbf{Q}}_i$ ($\eta = 1$)	$(\lceil 0.7246, 1.0138\rfloor$, $\lceil 0.9079, 1.4124\rfloor$, $\lceil 0.0921, 0.1452\rfloor)$	$(\lceil 0.8468, 1.1809\rfloor$, $\lceil 1.0696, 1.6601\rfloor$, $\lceil 0.1085, 0.1708\rfloor)$	$(\lceil 0.2317, 0.3749\rfloor$, $\lceil 0.4758, 0.6990\rfloor$, $\lceil 0.0483, 0.0728\rfloor)$	$(\lceil -0.0308, 0.0364\rfloor$, $\lceil 0.4645, 0.6088\rfloor$, $\lceil 0.0471, 0.0632\rfloor)$	$(\lceil 0.3621, 0.5445\rfloor$, $\lceil 0.6686, 0.9773\rfloor$, $\lceil 0.0678, 0.1010\rfloor)$	$(\lceil 0.3467, 0.5317\rfloor$, $\lceil 0.7169, 1.0380\rfloor$, $\lceil 0.0727, 0.1072\rfloor)$
$Rank^{IRBnd(\hat{\mathbf{Q}}_i)}$	5	6	2	1	3	4
$Rank^{IRBnd(Ex_i)}$	5	6	2	1	3	4
$Rank^{d(\hat{\mathbf{Q}}_i,\min\hat{\mathbf{Q}}_i)}$	5	6	2	1	3	4

The third comparison is performed with rough based AHP-VIKOR method. Each of the DAs, except A_4, has a different rank in both DCE models shown in Fig. 3. In the rough AHP-VIKOR approach, uncertainty can be evaluated by using the original data collection without predefined parameters and other assumptions. However, the rough DCE model unable to address intrapersonal uncertainty given in expert judgments. To manipulate the intrapersonal uncertainty in the given evaluation, the initial data is converted into cloud values and then CR numbers in the proposed (CR-AHP-VIKOR) model.

To compare with the rough AHP-VIKOR and proposed CR-AHP-VIKOR, crisp AHP-TOPSIS is also introduced. The crisp AHP-TOPSIS, rough AHP-VIKOR, and CR-AHP-VIKOR's final rankings are displayed in Table 18. If we normalized the given evaluation information by using this Eq. (42), then the final ranking results of the proposed model will also shown in Tables 18 and 19.

$$\grave{W}_g = \frac{\left(\left\lceil \tilde{E}x^L_{gh}, \tilde{E}x^U_{gh}\right\rfloor, \left\lceil \tilde{E}n^L_{gh}, \tilde{E}n^U_{gh}\right\rfloor, \left\lceil \widetilde{HEn}^L_{gh}, \widetilde{HEn}^U_{gh}\right\rfloor\right)}{\left(\left\lceil \max_g\left(\tilde{E}x^U_{gh}\right), \max_g\left(\tilde{E}x^U_{gh}\right)\right\rfloor, \left\lceil \min_g\left(\tilde{E}n^U_{gh}\right), \min_g\left(\tilde{E}n^U_{gh}\right)\right\rfloor, \left\lceil \min_g\left(\widetilde{HEn}^U_{gh}\right), \min_g\left(\widetilde{HEn}^U_{gh}\right)\right\rfloor\right)} \tag{42}$$

7.2. Comparison of criteria weight information

In this subsection, the three distinct approaches (crisp AHP, fuzzy AHP, and rough AHP) are presented for the determination of criteria weights and compared with CR-based AHP. In fuzzy AHP, a symmetrical triangular membership degree is used. The proposed algorithm ranks the criteria weights as ($C_7 > C_6 > C_4 > C_5 > C_1 > C_3 > C_2$), whereas the three approaches produce an identical sequence of criteria weights as ($C_1 > C_4 > C_5 > C_3 > C_6 > C_2 > C_7$). The crisp AHP method evaluates the weights by crisp numbers without taking subjectivity and uncertainty into

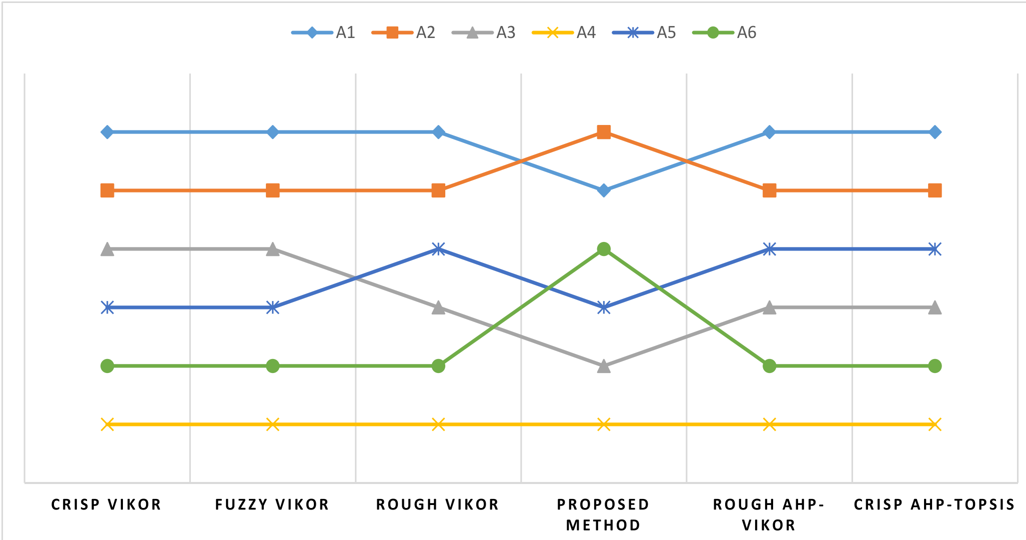

Fig. 3. Comparison of the DAs ranking using various models.

Table 18
Comparison of proposed method with existing approaches.

Design alternatives	Proposed method $IRBnd$ $(\hat{\mathbf{Q}}_i)$ ($\eta = 0.5$)	Proposed method Ranking	Crisp AHP-TOPSIS Closeness coefficient	Crisp AHP-TOPSIS Ranking	Rough AHP-VIKOR $\hat{\mathbf{Q}}_i$ ($\eta = 0.5$)	Rough AHP-VIKOR Ranking	Crisp VIKOR	Fuzzy VIKOR	Rough VIKOR
A_1	1.8375	5	0.190	6	$\lceil 0.511, 1 \rfloor$	6	6	6	6
A_2	2.7622	6	0.273	5	$\lceil 0.385, 0.694 \rfloor$	5	5	5	5
A_3	0.8738	2	0.438	3	$\lceil 0.285, 0.569 \rfloor$	3	4	4	3
A_4	0.6030	1	0.827	1	$\lceil 0, 0.254 \rfloor$	1	1	1	1
A_5	1.2733	3	0.353	4	$\lceil 0.306, 0.593 \rfloor$	4	3	3	4
A_6	1.7518	4	0.512	2	$\lceil 0.194, 0.466 \rfloor$	2	2	2	2

Table 19
Comparison of proposed method for different ranking formulae.

Design alternatives	Proposed method $IRBnd$ $(\hat{\mathbf{Q}}_i)$ ($\eta = 0.5$)	Proposed method Ranking	Proposed method $IRBnd(\hat{\mathbf{Q}}_i)$ (using Equation (42))	Proposed method Ranking
A_1	1.8375	5	0.4425	2
A_2	2.7622	6	0.6562	6
A_3	0.8738	2	0.4998	3
A_4	0.6030	1	0.0808	1
A_5	1.2733	3	0.6255	5
A_6	1.7518	4	0.6189	4

consideration while the CR based AHP, rough AHP, and fuzzy AHP express the weights by interval numbers. In accordance with the membership function, rough numbers have a flexible interval boundary, while a fuzzy set has a fixed boundary.

7.3. Simulation analysis

In this subsection, to further illustrate the efficiency of the developed model, the deviation degrees between different design concepts generated by different approaches are compared. The deviation degree can be determined by the equation $\left|\frac{\hat{\mathbf{Q}}_i - \max_i \hat{\mathbf{Q}}_i}{\max_i \hat{\mathbf{Q}}_i}\right|$, where $\hat{\mathbf{Q}}_i$ and $\max_i \hat{\mathbf{Q}}_i$ separately represent the performance priority value of ith alternative and the final performance priority value of the alternative with the highest performance. The resultant deviations among different design alternatives are given in Table 20 and illustrated in Fig. 4.

It is clear that the deviation values obtained using the proposed CR-VIKOR are higher than those obtained by the other techniques, which illustrates that the developed model produces bigger discrimination for different design alternatives. Apparently, the developed model provides more smoothness and fluctuations among design alternatives than the other approaches. The deviation analysis also illustrates the strength and effectiveness of the proposed model. This facilitates engineers or managers to make a better choice in the alternative decision process.

Advantages and limitations of the proposed model

In contrast, the cloud rough AHP-VIKOR method uses both the CM theory and rough approximations during the criteria weighting and alternative ranking process, making its evaluation outcomes more rational and reliable in comparison to other approaches. This model can effectively deal with interpersonal uncertainty and randomness simultaneously with a single mathematical approach. The CR numbers are more rational as compared

Table 20
Result deviations of design alternatives.

DAs	Crisp AHP-VIKOR	Rough AHP-VIKOR	Proposed method
A_1	0.770	0	0.335
A_2	0.670	0.368	0
A_3	0.470	0.419	0.684
A_4	0	0.481	0.782
A_5	0.573	0.413	0.539
A_6	0.381	0.444	0.366

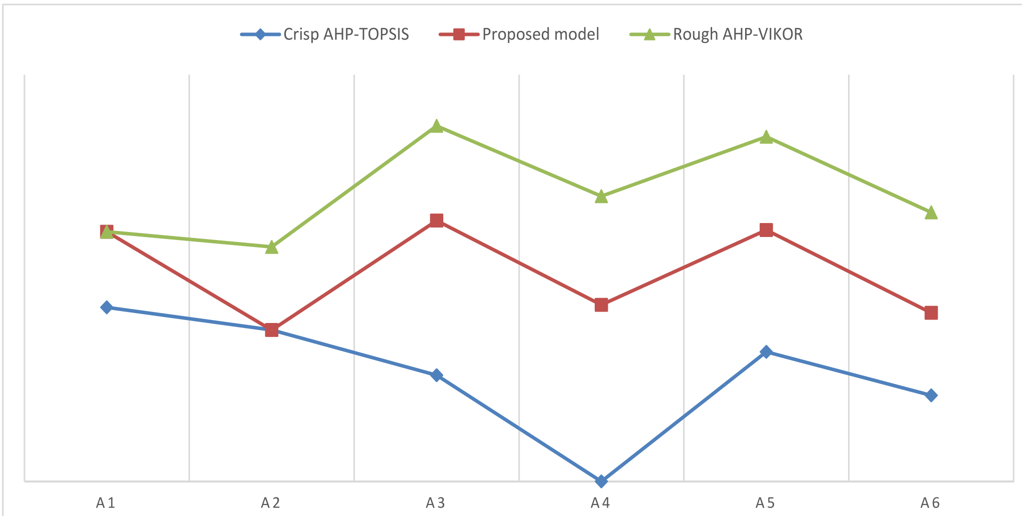

Fig. 4. Deviations of design alternatives generated by different DCE approaches.

to other integrated models of cloud theory and rough numbers as the rough approximations are involved till the final ranking of the results. A CR number consists of three components which are again rough numbers. So, to apply any operation or operator on CR numbers, the properties of both rough numbers and cloud model need to be considered.

The CR numbers are integrated with AHP and VIKOR approaches by computing the consistency ratio and converting the integral as well as reciprocal values in different domains into CR numbers. However, with any decision-making method, the quality of the results depends on the quality of the input data and the validity of the chosen parameters and criteria.

Although the proposed DCE model offers advantages in handling uncertainty and randomness, it also comes with its own set of drawbacks: (1) CR-AHP-VIKOR is a complex method due to the integration of CM theory, rough approximation theory, AHP, and VIKOR. This complexity can make it challenging to apply and understand, especially for decision-makers who are not familiar with these methodologies. (2) Similar to AHP-VIKOR, this model requires a significant amount of data for criteria and alternatives. Gathering and managing this data can be resource-intensive and time-consuming. (3) It may face limitations when dealing with a large number of alternatives and criteria. The computational burden can become prohibitive in such cases, making it less suitable for complex decision problems.

8. Conclusion and future directions

In this paper, an improved design evaluation model is proposed to evaluate and prioritize the DAs under uncertain environments. The concept of CR numbers is utilized to create an efficient way to deal with uncertainty by incorporating the advantages of CM theory and rough approximations simultaneously. The integrated uncertainty manipulation model in design concept evaluation deals with both intrapersonal and interpersonal uncertainties simultaneously, which overcomes the limitations of fuzzy based approaches depending on pre-defined parameters. Some new significant arithmetic operating rules and aggregating formulae of CR numbers are also described. Then, the CR-AHP and the CR-VIKOR techniques are presented to obtain relative importance for each criterion and the final performance ranking of all DAs. To validate the rationality and out-performance of the proposed approach, the importance of this research study is studied with an empirical case study of design concept evaluation problem for lithography tools. To evaluate the risks posed by decision-makers' perceptions, a sensitivity analysis is conducted on the case study. A comparison analysis with existing approaches is presented to show the efficiency and rationality of the proposed model. The ranking results are more objective and the uncertainties are appropriately assessed and addressed well. The proposed method is also applicable to various other decision making scenarios. In the future, other approaches including ANP, BWM, and DEMATEL, and their integrated models will be incorporated with CR numbers to broaden the application domains and address multiple factors of uncertainty in the decision making environment.

Ethical approval

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CRediT authorship contribution statement

Musavarah Sarwar: Conceptualization, Methodology, Supervision, Validation, Writing – review & editing. **Faiqa Bashir:** Writing – original draft.

Declaration of competing interest

No conflict of interest exists. We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Data availability

No data was used for the research described in the article.

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