COMP 360 - Winter 2021 - Assignment 2

Due: 11:59pm March 5th.

General rules: In solving these questions you may consult books or other available notes, but you need to provide citations in that case. You can discuss high level ideas with each other, but each student must find and write his/her own solution. Copying solutions from any source, completely or partially, allowing others to copy your work, will not be tolerated, and will be reported to the disciplinary office. You should upload the pdf file (either typed, or a clear and readable scan) of your solution to MyCourses.

1. (*5 points*) Explain how the following optimization problem can be solved using linear programming. Here the variables are $x_1, \ldots, x_n$, and furthermore a_{ij} for $i = 1, \ldots, m$ and $j = 1, \ldots, n$ are given constants.

$$\begin{array}{lll} \max & \frac{x_1+2x_2+3x_3+\ldots+nx_n}{x_1+\ldots+x_n} & \\ \text{s.t.} & x_1 + \ldots + x_n & > 0 \\ & \sum_{j=1}^n a_{ij}x_j & \geq 0 \quad \forall i = 1, \ldots, m \\ & x_i & \geq 0 \quad \forall i = 1, \ldots, n \end{array}$$

Hint: Note that if $x_1, \ldots, x_n$ is a feasible solution to this optimization problem, then so is $\lambda x_1, \ldots, \lambda x_n$ for any $\lambda > 0$. Moreover these solutions all have the same cost.

2. (*5 points*) Consider the following linear program:

$$\begin{array}{lrl} \max & 4x - 3y & \\ \text{subject to} & 4x - y & \leq 11 \\ & x + 4y & \leq 7 \\ & -2x + 5y & \leq 12 \\ & 10x + 4y & \geq -31 \\ & 3x + 9y & \geq -21 \end{array}$$

- Draw the feasible region of this linear program, and describe the coordinates of each vertex of the feasible region.
- Construct the dual for the above linear program (do not convert the linear program to standard form first; note that the dual should have 5 variables).
- Give an optimal solution for both the linear program and its dual, and then verify that strong duality holds.

3. (*5 points*) Show that $x_1 = x_2 = 10$ and $x_3 = 20$ is the optimal solution of the following LP:

$$
\begin{array}{lrl}
\max & z = 2x_1 + 3x_2 + x_3 & \\
\text{s.t.} & x_1 + x_2 + x_3 & \leq 40 \\
& 2x_1 + x_2 - x_3 & \geq 10 \\
& -x_2 + x_3 & \geq 10 \\
& x_1, x_2, x_3 & \geq 0
\end{array}
$$

4. (*5 points*) Consider the following linear program with a missing objective function.

$$
\begin{array}{lrl}
\max & ? & \\
\text{s.t.} & x_1 + x_2 & \leq 6 \\
& x_1 + x_2 & \geq -6 \\
& x_1 - x_2 & \leq 6 \\
& x_1 - x_2 & \geq -6
\end{array}
$$

- Draw the feasible region.
- Describe all the points that can be an optimal solution of this problem.
- Describe all the points that can be an optimal solution when the objective function is not constant.
- Describe all the points that can be the unique optimal solution of this problem.

5. (*5 points*) A company is producing a product which requires, at the final assembly stage, three parts. These three parts can be produced by two different departments as detailed below.

	Production rate (units/hr)			
	Part 1	Part 2	Part 3	Cost ($/hr)
Department 1	7	6	9	25.0
Department 2	6	11	5	12.5

One week, 1050 finished (assembled) products are needed but up to 1200 can be produced if necessary. If department 1 has 100 working hours available, but department 2 has 110 working hours available, formulate the problem of minimising the cost of producing the finished (assembled) products needed this week as an LP, subject to the constraint that limited storage space means that a total of only 200 unassembled parts (of all types) can be stored at the end of the week.
Note: because of the way production is organised in the two departments it is not possible to produce, for example, only one or two parts in each department, e.g. one hour of working in department 1 produces 7 part 1 units, 6 part 2 units and 9 part 3 units and this cannot be altered.