COMP 330 - Assignment 2

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Question 1

1) b* (a b* a)* b*
2) (a* (b a* b)*)* b a*
3) b* a*
4) b* (a* b b b*)* a* b*

Question 2

Proof by induction on the length of the word.

Base Case

let $w = ab$ s.t. $a, b \in \Sigma$

$$\delta^*(q_1, ab) = \delta^*(q_2, ab)$$

$$\delta(\delta(q_1, a), b) = \delta(\delta(q_2, a), b)$$

Recall $\delta(q_1, a) = \delta(q_2, a) = q'$ (I)

thus: $\delta(q', b) = \delta(q', b)$

Trivially true $\square$

I.A: Assume δ^* works for words of length $= K$
(We'll prove it for $l = K+1$)

let w_K be a word of length K

let $w_{K+1} = a w_K$ s.t $a \in \Sigma$

By I.A. $\delta^*(s_1, w_K) = \delta^*(s_2, w_K)$ holds true.

I.H $\quad \delta^*(s_1, a w_K) = \delta^*(s_2, a w_K)$

$\longrightarrow$ Apply def of δ^*

$$\delta^*(\delta(s_1, a), w_K) = \delta^*(\delta(s_2, a), w_K)$$

By I) $\quad \delta(s_1, a) = \delta(s_2, a) = q$

$$\delta^*(q, w_K) = \delta^*(q, w_K)$$

Trivially true in a DFA $\square$

Question 3

I)

1) Demon picks p.
2) I choose $a^p\ b^p\ a^{2p}$.
3) Demon has to pick x, y, z but $|xy| <= p$ so this part must consist only of a's.
4) I pick $I = 2$ s.t. $w = |xyyz| = (p + l) +3p = 4p +l$
   There are $3p +l$ a's in w.
   There are p b's in w.

Se that $a^{p+l}\ b^p\ a^{2p}$ is not in L. Therefore, L is not regular.

II)

1) Demon picks p.
2) I pick $a^p\ b\ a^p$.
3) Demon picks y consisting only of a's. $y = a^l$ s.t. $|xy| <= p$; say $|y| = l > 0$
4) I pick $i = 0$ s.t. $|xy^0z| = |xz| = a^{p-l}\ b\ a^p$ which is clearly not in L.
   Therefore, L is not regular.

Question 4

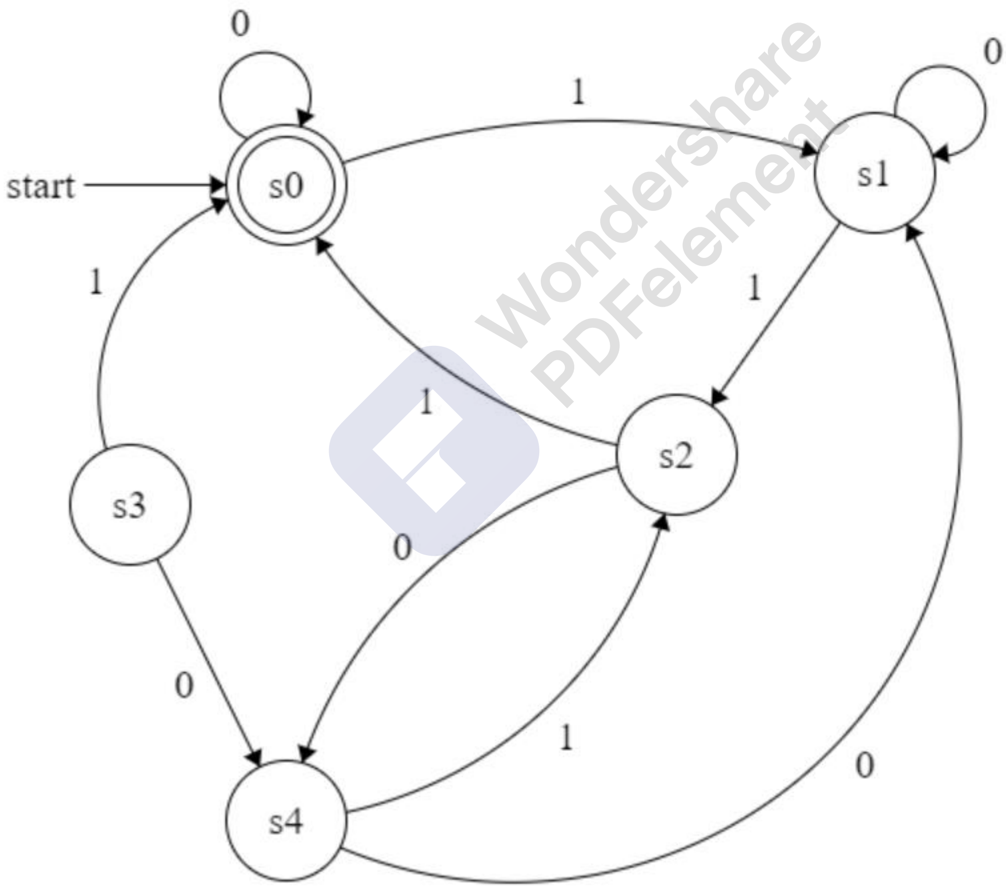
I)

1) Demon picks p.
2) I pick a^{p*p}.
3) Demon picks y consisting only of a's. $y = a^l$ s.t. $|xy| <= p$; say $|y| = l > 0$
4) I pick $i = 2$ s.t. $|xyyz| = a^{p-l}\ a^{2l}\ a^{p*p-p} = a^{p*p+l}$
   $(p+1)^2 = p^2 +2p +1 > p^2+ l$ *because* $l<=p$
   See that $p^2 +2p +1 > p^2+ p$
   Thus $p^2+ l$ is not a square number.
   Therefore $a^{p*p +l}$ is not in L and L is not regular.

II)

1) Demon picks p.
2) I choose $a^{2p}\ b^{p}$.
3) Demon must pick x, y, z but $|xy| <= p$ so this part must consist only of a's
   $|y| > 0$; let $|y| = l > 0$
4) I pick $i = 0$ s.t. $|xy^0z| = |xz| = 2p - l + p = 3p - l$
   Thus $a^{2p-l}\, b^{\,p}$ is not in L.
   Therefore, L is not regular.

Question 5



Step 0: Note that s3 is unreachable thus we will not take it into account in the table.

Step 1: Define an SxS array. s0 is an accept state, fill the rest of the row with 0s.

s4				X
s2			X	
s1		X		
s0	X	0	0	0
	s0	s1	s2	s4

Step 2: For every pair (p,q) in the array s.t. p ∈ F and q !∈ F put a 0 in the corresponding entry. Repeat until a zero is filled or all the letters in ∑ are tried.

s4				X
s2			X	0
s1		X	0	
s0	X	0	0	0
	s0	s1	s2	s4

$(\delta(s_1,1), \delta(s_2,1)) = (s_2, s_0)$

$(\delta(s_1,1), \delta(s_4,1)) = (s_2, s_2)$

$(\delta(s_1,0), \delta(s_4,0)) = (s_1, s_4)$

$(\delta(s_2,1), \delta(s_4,1)) = (s_0, s_2)$

Step 3: Fill the remaining entries with 1s.

s4				X
s2			X	0
s1		X	0	1
s0	X	0	0	0
	s0	s1	s2	s4

s1 is equivalent to s4.

Minimum automaton: