COMP 330 - Assignment 3

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Question 1

a) let L=($a^n b^n$ | n=1) This is clearly regular (i.e. can build a DFS for it-trivial)
let L_2=($a^n b^n$ | n>=0) L_2 is know to not be regular.
See that L is contained in L_2
Therefore the statement is false.

b) Assuming AB is regular then there exists an NFA that describes it. Divide this machine 2 parts: one that processes A then uses an ε-move when the B part arrives. Thus, B must have a DFA/NFA to process it because AB is regular.
Therefore, the statement is true.

c) We know that if A_1 and A_2 are regular then A_1 union A_2 is so too. Form A_i union A_{i+1} pairs. These pairs are regular. New pairing can be performed ad infinitum.
Therefore, the statement is true.

d) Let A be the set for L= ($a^n b^n$ | n>=0). Clearly L_2= ($a^n b^n$ | n=1) is contained in A and is regular (because we can make a DFA for it).
Therefore, the statement is false.

Question 2

L= ($a^n b^{2n}$ | n>0)

1) Demon chooses p.
2) I choose $a^p b^{2p}$.
3) Demon picks y consisting only of a's. y = a l s.t. |xy| <= p; say |y| = l > 0
4) I pick i= 5
|xy^5z|= p-l+5l +2p
   - ⇨ $a^{p+4l} b^{2p}$ is not in L.
   - ⇨ Therefore, L is not regular.

Question 3

F= {$a^i b^j c^k$ | i, j, k >=0 and if i= 1 then j=k}

Let L_1= {ab^nc^m | n, m >= 0}. L_1 is regular (proof trivial).
Let L_2= {ab^nc^n | n >= 0}. Clearly L_2 is contained in L_1.
By language closure properties L_1 intersection L_2 if and only if L_2 is regular.
Claim: L_2 is not regular.

Let L_3= {a^n | n=1} Clearly L_3 is regular.
Let L_4= {b^nc^n | n>=0} L_4 is known not to be regular.
Therefore, L= L_3*L_4 is not regular by language closure properties.

Therefore, L_1 intersection L_2 is not regular and neither is F.

Satisfaction of the pumping lemma conditions.

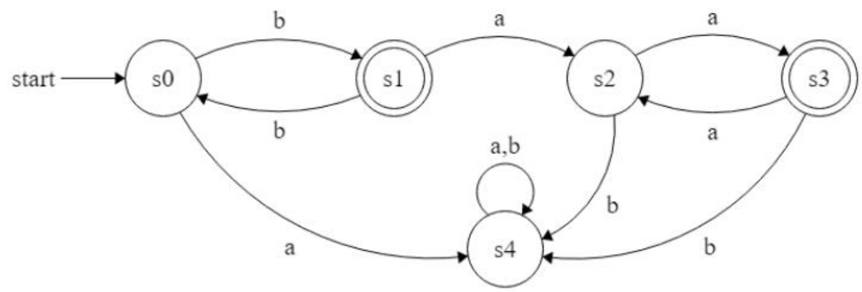
1) Demon chooses p.
2) I choose $a^pb^pc^{2p}$.
3) Demon picks y consisting only of a's. y = a l s.t. |xy| <= p; say |y| = l > 0
4) I choose i= 0
   $|xy^0z|$= |xz|= p-l+3p

⇨ $a^{p-l}b^pc^{2p}$ l, p>0

3 conditions: There exists x, y, z in Σ^* s.t. w= xyz & |xy|<= p & |y|>0
   Let p=5, l=1 then $a^4b^5c^{10}$
   W= xyz as x=a^4; y=a^0; z=b^5c^{10} & |xy|=4<=5 & |y|=1 (before pumping).
All 3 conditions are satisfied, get F is not regular. Why?
The pumping lemma states: L regular => L can be pumped.
This does not mean: L can be pumped => L regular.
The pumping lemma offers a sufficient but not a necessary condition to establish a language's regularity.

Question 4

a)



b) $b(bb)^*(aa)^*$

Question 5

We test right invariance. Let $\Sigma = \{a, b\}$. Let $x= a^nb^{n-1}$, $y= a^nb^{n-2}$, $z=b$ s.t. $n>=2$

$xz= a^nb^{n-1}b= a^nb^n$ which is not in L since n=n

$yz= a^nb^{n-2}b= a^nb^{n-1}$ which is in L.

Therefore no 2 elements of L are in the same equivalence class thus there are infinitely many equivalence classes.