COMP 360 - Winter 2021 - Assignment 1

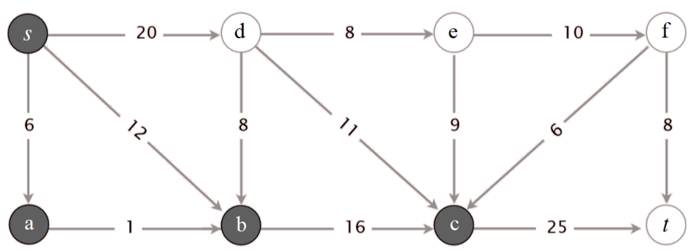
Due: 11:59pm February 5th.

General rules: In solving these questions you may consult books or other available notes, but you need to provide citations in that case. You can discuss high level ideas with each other, but each student must find and write his/her own solution. Copying solutions from any source, completely or partially, allowing others to copy your work, will not be tolerated, and will be reported to the disciplinary office. You should upload the pdf file (either typed, or a clear and readable scan) of your solution to MyCourses.

1. (*10 points*)

a) Run the Ford Fulkerson algorithm on the following network. For each step of the algorithm, give the augmenting path by listing the vertices on the path and state the amount by which the path increases the current value of the flow.
Label each edge of the network with its flow at the end of the execution of the algorithm. State the value of the maximum flow.

b) Check if the black vertices $\{s, a, b, c\}$ form a min-cut.



2. (*10 points*) Describe an algorithm of $O(m + n)$ to determine whether a given flow network contains a *unique* minimum (s, t)-cut. Explain.

3. (*10 points*) Suppose we are given a flow network $G = (V, E)$ in which every edge has capacity 1. Describe and analyze an algorithm to identify one edge in G such that after deleting that edge, the value of the maximum (s, t)-flow in the remaining graph decreases by one.

4. (*10 points*) The edge connectivity of an undirected graph is the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cycle is 2. Show how the edge connectivity of an undirected graph $G = (V, E)$ can be determined by running Ford-Falkerson algorithm on at most $|V|$ flow networks, each having $O(|V|)$ vertices and $O(|E|)$ edges.

5. (*10 points*) A new instructor is claiming that to find the max-flow if we use the shortest augmenting paths, then we do not need to consider the residual network. Prove this idea or present a counterexample.

6. (*10 points*) In this problem we model a simple power grid. As input, we are given n houses, represented by n pairs of positive integers $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$ which are their coordinates in the plane. We are also given a list of m power plants. Each power plant is represented by

a) A pair of positive integers (a_i, b_i), representing their coordinates in the plane.

b) A positive integer r_i, representing the maximum distance to which the power plant can send power (measured by standard Euclidean distance).

c) A positive integer c_i, representing the number of houses that power plant can simultaneously power.

Design an algorithm to solve the following problem: given a list of n houses and m power plants as described above, decide if it is possible to connect each house to each power plant such that all houses receive power and no power plant is over its capacity.