

COMP 360 - Winter 2021 - Assignment 3

Due: 11:59pm March 26th.

General rules: In solving these questions you may consult books or other available notes, but you need to provide citations in that case. You can discuss high level ideas with each other, but each student must find and write his/her own solution. Copying solutions from any source, completely or partially, allowing others to copy your work, will not be tolerated, and will be reported to the disciplinary office. You should upload the pdf file (either typed, or a clear and readable scan) of your solution to MyCourses.

1. (5 points) Either prove that the following problem is NP-complete, or show that it belongs to P:

- Input: A CNF Φ .
- Question: Is there a truth assignment that satisfies none of the clauses in Φ .

2. (5 points) Prove that the following problem is NP-complete.

- Input: A CNF Φ .
- Question: Does Φ have a truth assignment that assigns True to exactly half the terms in each clause?

(For example $x_1 = T, x_2 = F, x_3 = F, x_4 = F$ assigns True to half the terms in each clause of $\Phi = (x_1 \vee x_2 \vee x_3 \vee x_4) \wedge (x_1 \vee x_2)$.)

3. (5 points) Prove that following problem (called the strong matching problem) is NP-complete.

- Input: A graph G and an integer k .
- Question: Are there k edges in G that form a matching and are joined by no other edges of G ?

4. (5 points) Either prove that the following problem is NP-complete or prove that it belongs to P by giving a polynomial time algorithm.

- Input: An undirected graph H .
- Question: Does H have a proper colouring with three colours R, G, B that assigns the colour B to at most 10 vertices?

5. (5 points) Prove that the following problem belongs to P:

- Input: A graph G .
- Question: Does G have an independent set of size 100.

6. (5 points) Show that the following problem is NP-complete:

- Input: An undirected graph G and an edge e .
- Question: Does G have a Hamiltonian cycle that passes through the edge e .