Recitation 2

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Marble Problem Review from Last Time

You have 6 marbles: 3 green, 2 red, 1 orange, that you want to give away to your 6 friends in sequence as you encounter them through out the day. For each color, the marbles not distinguishable. How many different ways can you give out the 6 marbles? Use product rule.

Solution:

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Solution:

Break up the sequence into subsets by marble color. The first subset is the number of ways you can give out the single orange marble to any of your 6 friends: 6. Assume orange the marble has been given to a friend, and now you have 5 friends left for green/red marbles. The second subset is the number of ways you can give out 3 green marbles to your 5 remaining friends: 10 (by enumeration). Last set has size 1 since you have 2 friends left and 2 marbles of the same color. Since these sets are independent, then by product rule the total number of sequences is 6*10*1=60.

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g,g,g,r,r/g,g,r,g,r/g,g,r,r,g/g,r,g,r,g/g,r,r,g,g/g,r,g,g,r/r,g,g,g,r/r,g,g,r,g/r,g,r,g,g/r,r,g,g,g

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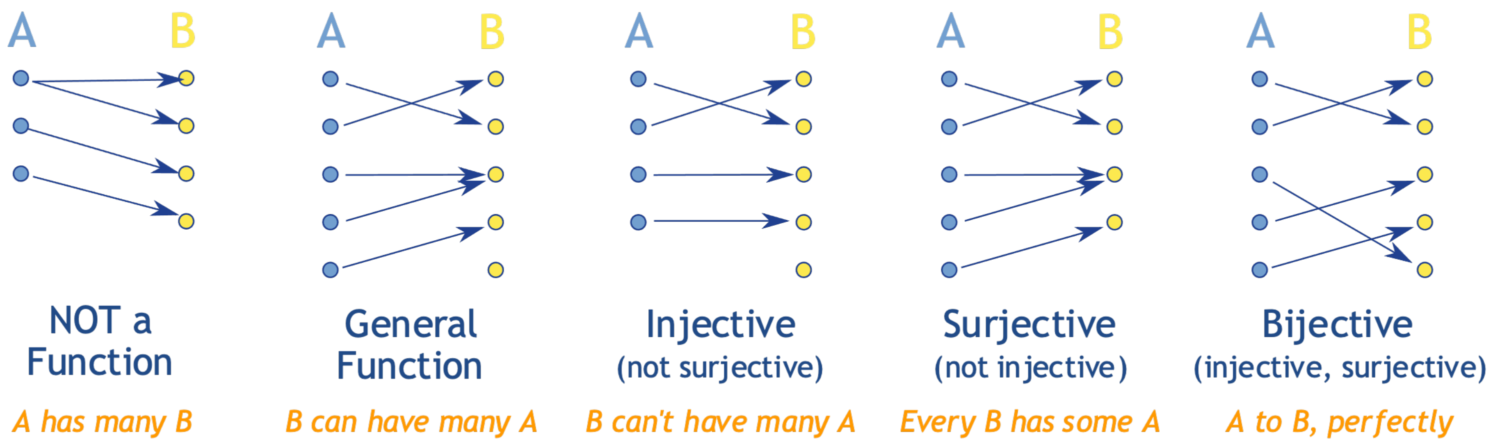
Section 1: Functions

Overview

Injection (1-to-1): $(\forall x_1, x_2 \in D)[(f(x_1) = f(x_2)) \Rightarrow (x_1 = x_2)]$

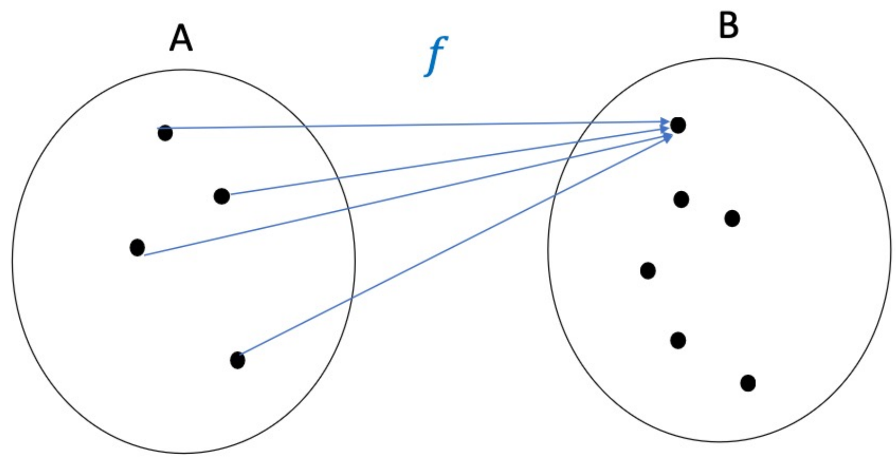
Surjection (onto): $(\forall y \in C)(\exists x \in D)[f(x) = y]$

Bijection: Inj(f) ^ Sur(f)



Problem 1:

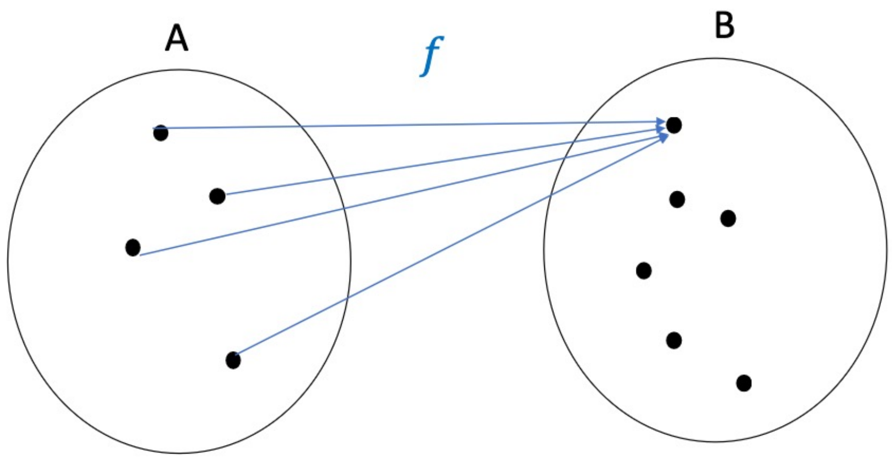
Is this a function?



Problem 1:

Is this a function?

YES



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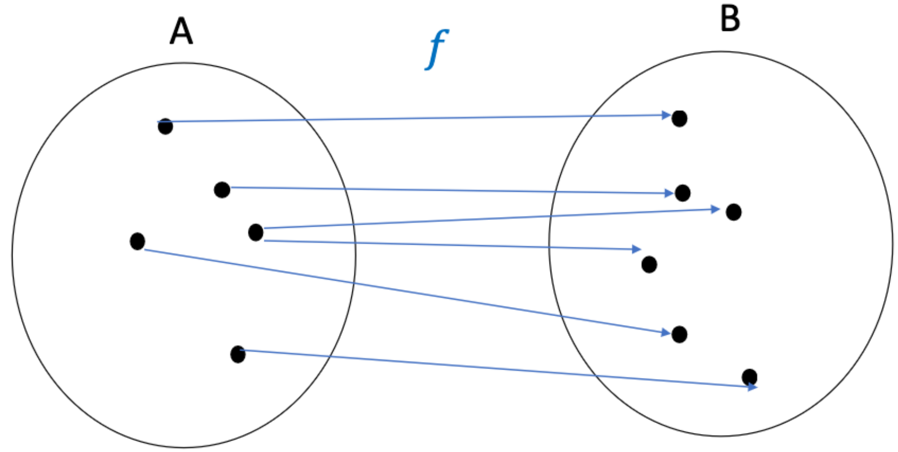
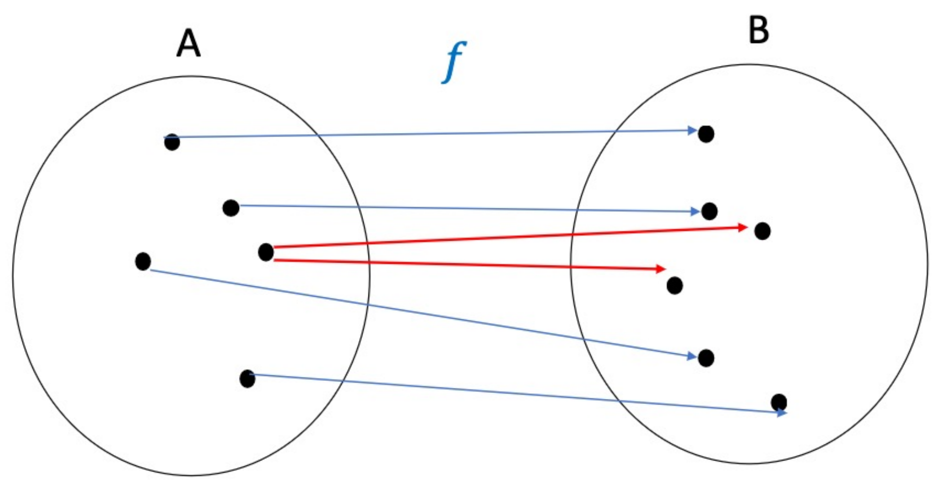


Problem 1:

Is this a function?

NO



Problem 1:

Is this a function?

$$F: N \rightarrow N, f(x) = x/2$$

Problem 1:

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$$F: N \longrightarrow N, f(x) = x/2$$

NO

Problem 1:

Function:

$F: R \rightarrow R, f(x) = 2x+1$

Is it injective?

Is it surjective?

Is it bijective?

Problem 1:

Function:

$$F: R \rightarrow R,\ f(x) = 2x+1$$

Is it injective? YES

Is it surjective? YES

Is it bijective? YES

Problem 1:

Function:

$F: R \rightarrow Z, f(x) = \lceil x \rceil$

Is it injective?

Is it surjective?

Is it bijective?

Problem 1:

Function:

$F: R \rightarrow Z, f(x) = \lceil x \rceil$

Is it injective? NO

Is it surjective? YES

Is it bijective? NO

Problem 1:

Function:

$$F: Z \to Z,\ f(x) = x^4$$

Is it injective?

Is it surjective?

Is it bijective?

Problem 1:

Function:

$$F: Z \longrightarrow Z,\ f(x) = x^4$$

Is it injective? NO

Is it surjective? NO

Is it bijective? NO

Problem 1:

Function:

$F: R \rightarrow R,\ f(x) = 5$

Is it injective?

Is it surjective?

Is it bijective?

Problem 1:

Function:

$F: R \rightarrow R, f(x) = 5$

Is it injective? NO

Is it surjective? NO

Is it bijective? NO

Problem 1:

Function:

$F: R \rightarrow R, f(x) = e^x$

Is it injective?

Is it surjective?

Is it bijective?

Problem 1:

Function:

$$F: R \rightarrow R, f(x) = e^x$$

Is it injective? YES

Is it surjective? NO

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Is it injective?

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$$F: R \rightarrow R, f(x) = x^2$$

Is it injective? NO

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Problem 1:

Function:

$$F: R \longrightarrow R^{\geq 0},\ f(x) = x^2$$

Is it injective?

Is it surjective?

Is it bijective?

Problem 1:

Function:

$$F: R \longrightarrow R^{\geq 0},\ f(x) = x^2$$

Is it injective? NO

Is it surjective? YES

Is it bijective? NO

Section 2: Proofs

Overview (for now...):

Direct Proof:

- $P \Rightarrow Q$, show that if P is assumed to be true, then Q follows

Indirect Proofs:

- Proof by Contradiction: To prove $P \Rightarrow Q$, Assume $\neg(P \Rightarrow Q)$ = P ^ ¬Q, reach a contradiction (r ^ ¬r)

- Proof by Contrapositive: To prove $P \Rightarrow Q$, ***directly*** prove $\neg Q \Rightarrow \neg P$ (assume not Q, then not P follows)

Overview of Proofs (for now...):

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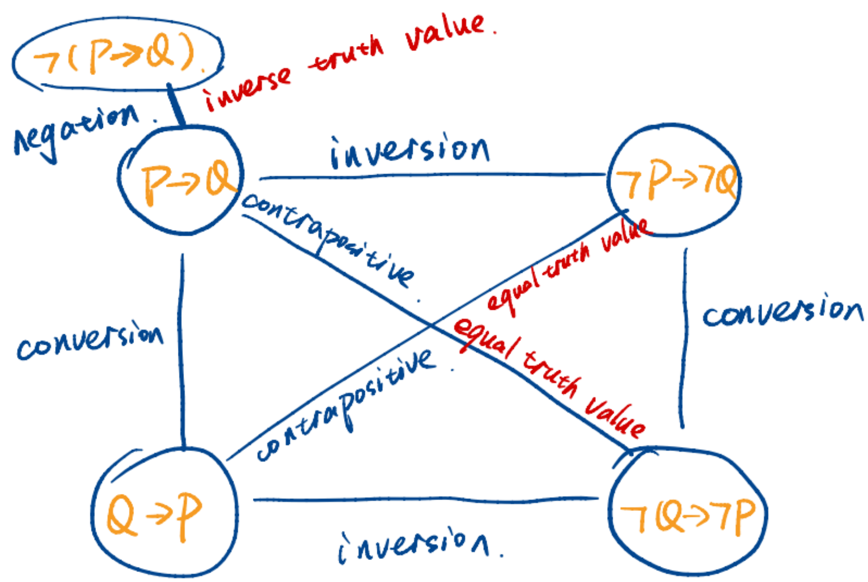

Problem 1:

Claim: The difference of squares of any two consecutive integers is odd

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Question re-written formally: (given n, n+1) $\Rightarrow$ $((n+1)^2 – n^2)$ is odd

$(n+1)^2$ – n^2 = n^2 +2n +1 – n^2 = 2n+1 which is odd by definition of parity.

Problem 2:

Claim: Prove that either $2*10^{500} + 15$ or $2*10^{500} + 16$ is not a perfect square

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Question rewritten formally: $\sqrt{(2*10^{500}+ 15)} \in N \Leftrightarrow \sqrt{(2*10^{500}+ 16)} \notin N$

Suppose $2*10^{500} + 15$ is a perfect square, then by definition we have $n^2 = 2*10^{500} + 15$
By Algebra we have $2*10^{500} + 16 = (2*10^{500} + 15) + 1 = n^2 + 1$

By way of contradiction, suppose $n^2 + 1$ is <u>**also**</u> a perfect square, then by definition $m^2 = n^2 +1$. $m^2 - n^2 = 1$. $(m-n)(m+n) = 1$. Since m and n are integers, this equation cannot hold so we have reached a contradiction, $n^2 + 1$ is not a perfect square.

Problem 3:

Claim: for a, b N if (a*b) % 6 = 0, a % 6 = 0 or b % 6 = 0

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Proof by Counter-example: a = 4, b = 9

6 divides into 36 but it does not divide into 4 or 9

Problem 4:

Claim: if n is an integer and $n^3 + 5$ is odd, then n is even

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$n^3 + 5$ is odd$\Rightarrow$n, even
$n^2 = (n \times n)$, even
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$n^3 + 5 = \text{even} + \text{odd} = \text{odd}$

Problem 5 (Challenge):

Claim: If x and y are integers and $x^2 + y^2$ is even, prove that x+y is even

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Claim: If x and y are integers and $x^2 + y^2$ is even, prove that $x+y$ is even

given $(x^2 + y^2)$ is even, let $(x^2 + y^2) = 2k$ for some integer k,
then for $(x + y)^2 = x^2 + 2xy + y^2 = 2k + 2xy = 2(k + xy)$. Let $k + xy$ = some integer g, then $(x+y)^2$ is even since it equals $2g$ (definition of parity)

Since $(x+y)^2$ is even, $(x+y)$ must also be even since squaring an even number gives an even number and squaring an odd number gives an odd number:

Case 1: $x+y$ is even

- If $x + y$ is even then $x+y = 2m$. $(x + y)^2 = (x + y)(x + y) = (2m)(2m) = 2(2m^2)$ which is even by parity, so $(x + y)^2$ is even

Case 2: $x+y$ is odd

- If $x + y$ is odd then $x+y = 2n + 1$. $(x + y)^2 = (x + y)(x + y) = (2n+1)(2n+1) = 2(2n^2 + 2n) + 1$ which is odd by parity, so $(x + y)^2$ is odd

Coming Soon:
Induction & Counting