Recitation 8

Question 1

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- Solution: C(24,20) = 10626
- Explanation: n = 5, r = 20,
- **(n+r-1) choose (r)** = (20+5-1) choose 20

Question 2

- Five batteries will be put on the display and there are twenty different kinds of batteries (AAA, AA, C, D, 9-volt, 5-volt, etc).

- a) How many ways can we choose the five batteries?

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- a) How many ways can we choose the five batteries?
- Solution: C(24, 5) = 42504
- Explanation: n = 20, r = 5,
- **(n+r-1) choose (r)** = (20+5-1) choose 5

Question 3

How many non-negative solutions are there for:

$x1 + x2 + x3 + x4 = 20$

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Solution:

3 bars, 20 stars:

23C3 OR 23C20

Question 3

How many non-negative solutions are there for:

$x1 + x2 + x3 + x4 = 20$, if $x1 >= 3$

Question 3

How many non-negative solutions are there for:

x1 + x2 + x3 + x4 = 20, if x1 >= 3

Solution:

We’ve fixed 3 of the stars, left with 3 bars, 17 stars:

20C3 OR 20C17

Question 3

8. How many integer solutions to $x_1 + x_2 + x_3 + x_4 = 25$ are there for which $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$ and $x_4 \geq 4$?

Question 3

8. How many integer solutions to $x_1 + x_2 + x_3 + x_4 = 25$ are there for which $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$ and $x_4 \geq 4$?

10 stars already allocated

Left with 15 stars and 3 bars:

18C3 or 18C15

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Question 3

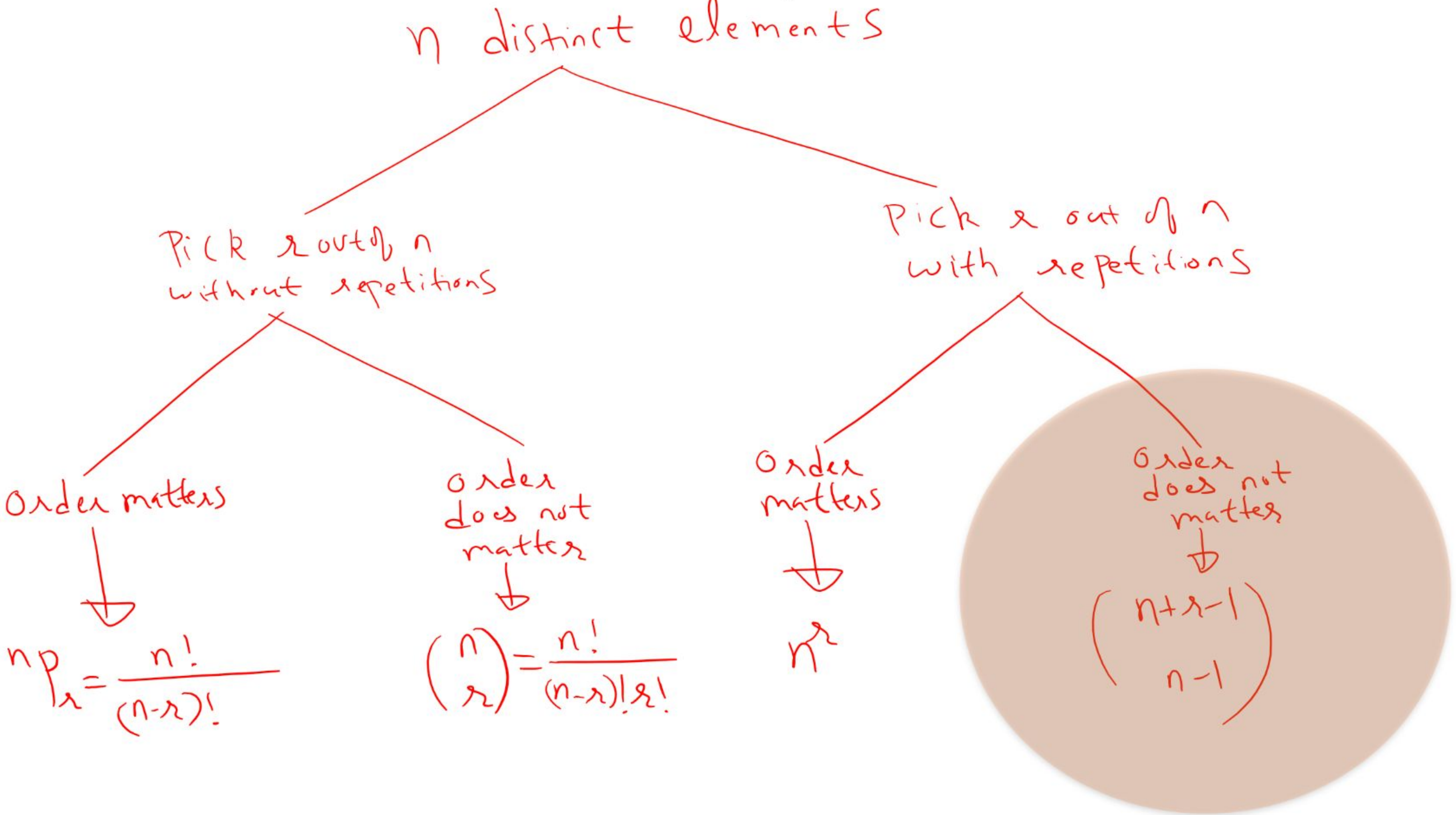
How many non-negative solutions are there for:

$x1 + x2 + x3 + x4 = 20$, if $x1 = 3$

Solution:
We've fixed 3 of the stars AND a bar, left with 2 bars, 17 stars:

19C2 OR 19C17

Flowchart: Permutations/Combinations



How many 7 digit phone numbers are there in which the digits are non-increasing? That is, every digit is less than or equal to the previous one.

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Solution:

We don’t care about order, why?

Consider the number 754-3330 as the following: ||*||*|*|***|||*

We have 7 stars and 9 bars, so the result is 16C7 or 16C9

How many ways can you write down 4 phone numbers in a row, such that each phone number has 7 non-increasing digits, and repeated phone numbers are allowed?

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Solution:

$(16C7)^{(4)}$

New Material

Pigeonhole Principle
Inclusion-Exclusion

Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.

Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.

Solution: Let A be one of the six people. Of the five other people in the group, there are either three or more who are friends of A, or three or more who are enemies of A. This follows from the generalized pigeonhole principle, because when five objects are divided into two sets, one of the sets has at least $\lceil 5/2 \rceil = 3$ elements. In the former case, suppose that B, C, and D are friends of A. If any two of these three individuals are friends, then these two and A form a group of three mutual friends. Otherwise, B, C, and D form a set of three mutual enemies. The proof in the latter case, when there are three or more enemies of A, proceeds in a similar manner. ◀

What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

Solution: The minimum number of students needed to ensure that at least six students receive the same grade is the smallest integer N such that $\lceil N/5 \rceil = 6$. The smallest such integer is $N = 5 \cdot 5 + 1 = 26$. If you have only 25 students, it is possible for there to be five who have received each grade so that no six students have received the same grade. Thus, 26 is the minimum number of students needed to ensure that at least six students will receive the same grade. ◀

1. Among 18 students in a room, 7 study mathematics, 10 study science, and 10 study computer programming. Also, 3 study mathematics and science, 4 study mathematics and computer programming, and 5 study science and computer programming. We know that 1 student studies all three subjects. How many of these students study none of the three subjects?

1. Among 18 students in a room, 7 study mathematics, 10 study science, and 10 study computer programming. Also, 3 study mathematics and science, 4 study mathematics and computer programming, and 5 study science and computer programming. We know that 1 student studies all three subjects. How many of these students study none of the three subjects?

   Solution:

   Solution. Let M, S, and C denote the sets of students who study math, science and computing respectively and let U be the entire set of 18 students. Then $|M| = 7, |S| = 10$, and $|C| = 10$. Also, we have $|MS| = 3, |MC| = 4$, and $|SC| = 5$, where, $|x|$ denotes the number of elements of the set x and juxtoposition of sets means intersection. Finally, $|MCS| = 1$. Then

$$|U|-(|M|+|S|+|C|-|MS|-|MC|-|SC|+|MSC|) = \overline{MSC}| = 18-(27-12+1) = 2.$$