2) $\neg(A \land \neg A)$

a)

$$\cfrac{\cfrac{\overline{A} \qquad \overline{\neg A}}{A \land \neg A}}{\cfrac{\bot}{\neg(A \land \neg A)}} \qquad \cfrac{\cfrac{\cfrac{\overline{A \land \neg A}}{A} \qquad \cfrac{\overline{A \land \neg A}}{\neg A}}{\bot}}{\neg(A \land \neg A)}$$

b) $\neg A \rightarrow (A \rightarrow B)$

$$\cfrac{\cfrac{\cfrac{\cfrac{\overline{\neg A} \qquad \overline{A}}{\bot}}{B}}{A \rightarrow B}}{\neg A \rightarrow (A \rightarrow B)}$$

c) $A \lor \neg A$

$$\cfrac{\overline{A}}{A \lor \neg A} \qquad \cfrac{\overline{\neg A}}{A \lor \neg A}$$

$$\overline{\qquad A \lor \neg A \qquad}$$

d) $(A \to (C \lor D)) \to ((A \to C) \lor (A \to D))$

$$\frac{\dfrac{C}{A \to D}}{(A \to C) \lor (A \to D)} \qquad \frac{\dfrac{D}{A \to D}}{}$$

$$\frac{D}{\dfrac{A \to D}{\dfrac{((A \to C) \lor (A \to D))}{(A \to (C \lor D)) \to ((A \to C) \lor (A \to D))}}}$$

e)

$$\frac{\overline{A \wedge B}^{\,2}}{B} \wedge E_r$$

$$\frac{\overline{A \wedge C}^{\,2}}{C} \wedge E_r$$

$$\cfrac{\overline{(A \wedge B) \vee (A \wedge C)}^{\,1} \quad \cfrac{\dfrac{\overline{A \wedge B}^{\,2}}{B}\wedge E_r}{B \vee C}\vee I_6 \quad \cfrac{\dfrac{\overline{A \wedge C}^{\,2}}{C}\wedge E_r}{B \vee C}\vee I_r}{B \vee C} \,\vee E \qquad \frac{\overline{A \wedge B}^{\,2}}{A}$$

$$\frac{A \wedge (B \vee C)}{(A \wedge B) \vee (A \wedge C) \rightarrow A \wedge (B \vee C)}\,1 \wedge I$$

$$\frac{A \wedge (B \vee C) \rightarrow (A \wedge B) \vee (A \wedge C) \qquad (A \wedge B) \vee (A \wedge C) \rightarrow A \wedge (B \vee C)}{A \wedge (B \vee C) \longleftrightarrow (A \wedge B) \vee (A \wedge C)}\longleftrightarrow I$$

f) Given $A \longleftrightarrow B$, show $\neg A \longleftrightarrow \neg B$

$$\frac{\dfrac{A \longleftrightarrow B \quad \overline{\neg A}^{\,1}}{\neg B}\longleftrightarrow E_1}{\neg A \rightarrow \neg B}\,1 \rightarrow I \qquad \frac{\dfrac{A \longleftrightarrow B \quad \overline{\neg B}^{\,2}}{\neg A}}{\neg B \rightarrow \neg A}$$

$$\neg A \longleftrightarrow \neg B$$

9) Given $A \to C$ and $B \to D$, show $A \lor B \to (C \lor D)$

$$\cfrac{\cfrac{\cfrac{\overline{A}^1 \quad A \to C}{C} \to E}{C \lor D} \lor I_1 \qquad \cfrac{\cfrac{\overline{B}^1 \quad B \to D}{D} \to E}{C \lor D} \lor I_r}{\cfrac{\cfrac{\overline{A \lor B}^2 \qquad C \lor D}{C \lor D} \lor E}{\cfrac{C \lor D}{A \lor B \to C \lor D} 2 \to I}}$$