HW5

1) For all $n \in N$, let (P) be the proposition

$P(n): \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ Proof by induction

Basis for induction

when $n = 1$

$P(1): \sum_{i=1}^{1} i^2 = 1^2 = 1$ and $\frac{1(1+1)(2 \times 1 + 1)}{6} = 1$

Induction Hypothesis:

Let's assume that $P(k)$ is true, where $k \geq 1$, then we need to show that it follows that $P(k+1)$ is also true

$P(k): \sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$ is true

So, consider for $n = k+1$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \cancel{k(k+1)(2k+1)} \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)(k(2k+1) + 6(k+1))}{6} = \frac{(k+1)(2k^2 + 7k + 6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+2)(2(k+1)+1)}{6} \Rightarrow P(k+1)$$

2) $\prod_{k=1}^{m} \left(\frac{k+1}{k} \right)$

$$= \frac{1+1}{1} \times \frac{2+1}{2} \times \frac{3+1}{2} \times \cdots\cdots \times \frac{m}{m-1} \times \frac{m+1}{m}$$

$$= \frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \cdots\cdots \times \frac{m}{m-1} \times \frac{m+1}{m}$$

$= 1 \times 1 \times 1 \cdots\cdots m+1$

$= m+1$

3) Pascal's triangle
theorem.
We have to fin
triangle
To do this
initial rows to

Sum at Row
Sum at Row 1
Sum at Row 2
Sum at Row 3 : 1
Sum at Row 4 : 1 +

We can clearly
at 2 and sum

Now we will
From the binom
row of Pascal
to n choose
Now, from bin
$(a+b)^n = \sum_{r=0}^{n} ($
$a = 1 \quad b = 1$
$2^n = \sum_{r=0}^{n} \binom{n}{}$

3) Pascal's Griangle is used in the expansion of bionomial theorem.

We have to find the formule of sum of row n of Pascal's Griangle

To do this first we try to calculate the sum of a few initial rows to see if there any pattern

Sum of Row 0: $1 = 2^0$

Sum of Row 1: $1+1 = 2 = 2^1$

Sum of Row 2: $1+2+1 = 4 = 2^2$

Sum of Row 3: $1+3+3+1 = 8 = 2^3$

Sum of Row 4: $1+4+6+4+1 = 2^4$

We can clearly see the pattern that sum is increasing by a factor of 2 and sum of row n is 2^n

Now we will prove that sum n is 2^n

From the bionomial theorem we can see that n^{th} entry the n^{th} row of Pascal's triangle is $n = n!\,n\,(n-n)!\,n$ which is similar to n choose n $\sum_{n=0}^{n} \binom{n}{n} = 2^n$

Now, from bionomial expansion we know that,

$$(a+b)^n = \sum_{n=0}^{n} \binom{n}{n} a^n b^{n-n}$$

$a = 1 \quad b = 1$

$$2^n = \sum_{n=0}^{n} \binom{n}{n}$$

Proved

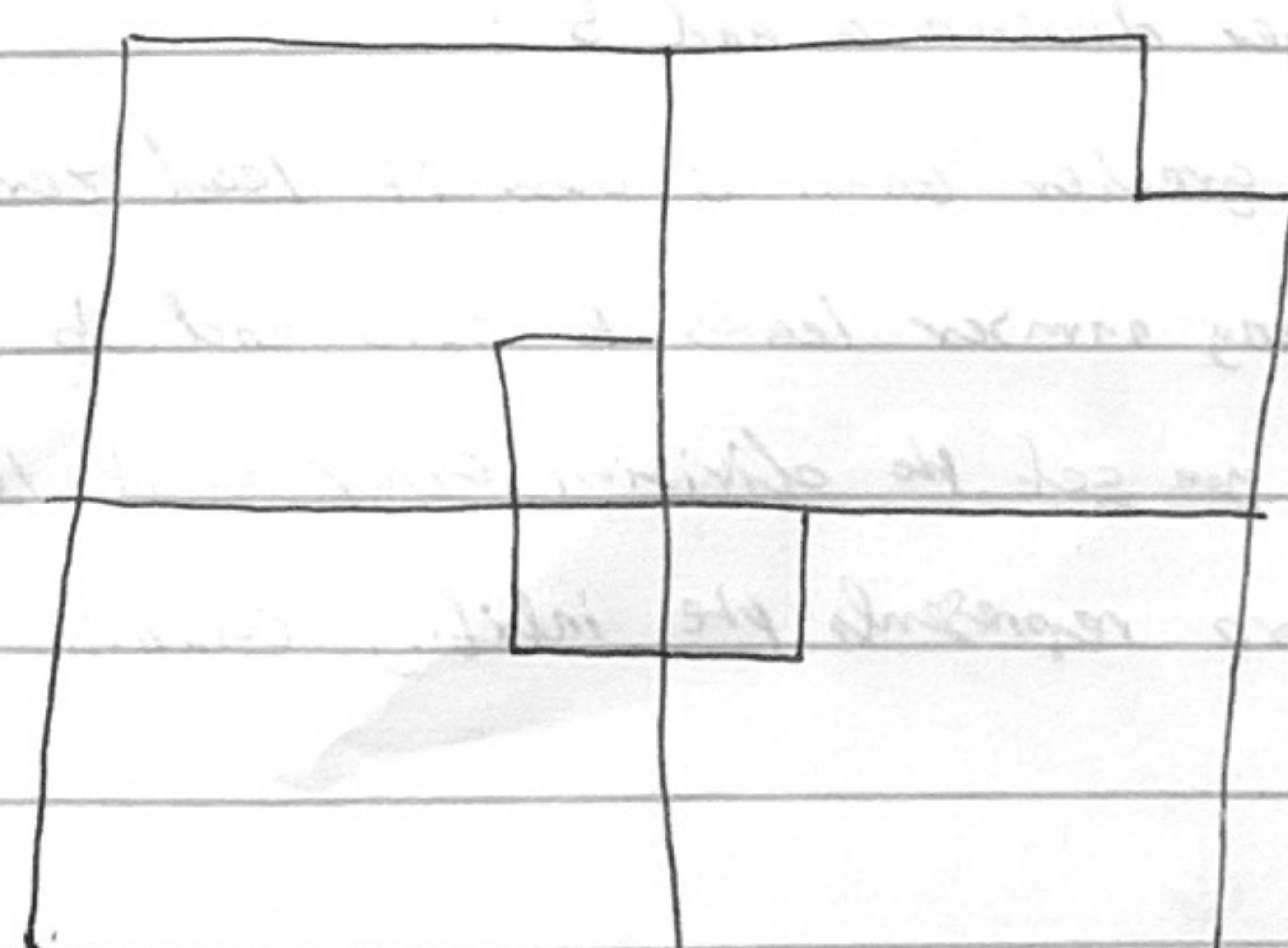
6) The proof follows by induction

If $n=1$, the solution is trivial as it can be covered with one L-shaped tile made up of three squares.

Assume that these L-shaped tiles can cover a $2^{n-1} \times 2^{n-1}$ square with one chosen square left uncovered. Divide the $2^n \times 2^n$ square into $2^{n-1} \times 2^{n-1}$ square quadrants.

$2^{(n-1)} \times 2^{(n-1)}$	$2^{(n-1)} \times 2^{(n-1)}$
$2^{(n-1)} \times 2^{(n-1)}$	$2^{(n-1)} \times 2^{(n-1)}$

One quadrant contains the square we want to leave uncovered by induction, we can cover this quadrants except for the one square, with L-shaped tiles. For the remaining 3 quadrants, cover each of these except for one of it's corner with square L-shaped tiles. Rotate the three quadrants so that their uncovered corners lie together, at the center. These three remaining squares can be then covered with one L-shaped tile



7) The program is correct, since the least value of 'x' and 'y' gets stored in 'z' variable in all possible cases.

Proof:

In this given program the cases to be considered are $x<y$, $x=y$ and $x>y$.

1) If x is less than y ($x<y$) then the value of 'x' gets stored in 'z'. Since x is the least value, so as the minimum value gets in 'z' this is the correct operation.

2) If $x=y$ is one of condition of else, in this the 'y' value gets stored in 'z' where $x=y$, since x and y are equal no problem of assigning any value to the variable 'z'.

3) If $x>y$ is other condition of else statement, in this the 'y' gets stored in 'z' and also 'y' is the smallest value, therefore it is correct assignment.

8) Pre condition:

For any integer division a and b

a should be greater than 0 else it lead zero as a number dividing with any number leads to zero and b should be greater than 0 else you get the division error as it leads to anything divide by zero represents the infinity cannot represented in system.

So $(a>0) \wedge (b>0)$

Post condition:

The quotient remainder theorem

For any given integer a and

solution that unique integers q

$A = b * q + r$ where $0 <=$

This can be represented

a mod $b = r$

q represents the quotient an

9) $m, n \in Z^{+}$. m, n are

Now let $a \equiv b$ (mod

$\Rightarrow (a-b)$ divided by

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$\Rightarrow (a-b) \equiv 0 \pmod{m}$,

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$\Rightarrow a \equiv b \pmod{n}$ -

So $(a>0) \wedge (b>0)$

Post condition:

The quotient remainder theorem represents:

For any given integer a and a positive integer b, there exists a solution that unique integers q and r such that

$A = b * q + r$ where $0 <= r < b$

This can be represented as:

$a \bmod b = r$

q represents the quotient and r is the remainder.

9) $m, n \in Z^+$. m, n are relatively prime

Now let $a \equiv b \pmod{mn}$

$\Rightarrow$ (a-b) divided by mn

$\Rightarrow$ (a-b) divided by m & n

$\Rightarrow (a-b) \equiv 0 \pmod{m}$, $(a-b) \equiv 0 \pmod{n}$

$\Rightarrow a \equiv b \pmod{m}$, $a \equiv b \pmod{n}$

let $a \equiv b \pmod{m}$; $a \equiv b \pmod{n}$

$\Rightarrow$ (a-b) divided by m, n

$\Rightarrow$ (a-b) divided by lcm (m, n)

$\Rightarrow$ (a-b) divided by mn

$\Rightarrow (a-b) \equiv 0 \pmod{n}$

$\Rightarrow a \equiv b \pmod{n}$ -2

$a \equiv b \pmod{mn}$ iff $a \equiv b \pmod{m}$

$a \equiv b \pmod{n}$

1c)

a) Given $x \in h$

$y \in h$

and we have to proof $xy \in h$

So suppose $x = 4k+1$

And $y = 4p+1$

And the product of x and y

$XY = (4k+1)(4p+1)$

$= 16kp + 4k + 4p + 1$

After common 4

$XY = 4(4kp + k + p) + 1$

Suppose $4kp + k + p = k'$

So $xy = 4k+1$

b) if it's just greater than 1 then we can't write as product of two smaller values of h

Eg 5 that is belong to h but we can't written it as the product of smaller values of h

$9 \times 49 = 441$

where 9 and 49 badges.

c) $441 = 4k+1$

So $k = 110$ so 441 belongs to

We can write 441 as $9 * 4$

$49 = 4 \cdot 12 + 1$

and $9 = 4 \cdot 2 + 1$

So 49 and 9 belongs to

So we can write 441 as

c) $441 = 4k+1$

So $k = 110$ so 441 belongs to H

We can write 441 as $9*49$ or $49*9$

$49 = 4 \cdot 12 + 1$

and $9 = 4 \cdot 2 + 1$

So 49 and 9 belongs to H

So we can write 441 as product of H prime number.

4) $P_n = P_{n+1} + (3n-2)$

Mattemetical induction consists of two districts parts

First you must show that formull is true, when $n=1$

Assume

$$P_n = \frac{n(3n-1)}{2}$$

$$P_{n+1} = P_n + [3(n+1)-2]$$

$$= P_n + (3n+1)$$

$$= \frac{n(3n-1)}{2} + (3n+1)$$

$$= \frac{n(3n-1)+2(3n+1)}{2}$$

$$= \frac{3n^2+5n+2}{2}$$

$$= \frac{(n+1)(3n+2)}{2}$$

$$= \frac{(n+1)[3(n+1)-1]}{2}$$

5) The Proofs form a falsidical paradox; it seems to show by valid reasoning something that is manifestly false, but in fact that reasoning is flawed

$n=1$ is meaningless as there is only horse of one color and no other horses in the middle of the set.

For example let two horses in the set be horse A and horse B. When horse A is removed, it is true that the remaining horses in the set are the same color. The same is true when horse B is removed. Therefore the logic is broken.

6) The proof follows by induction

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