CS206 Quiz 4 **Nov 6, 2023** **Section 1**

Name: _____

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Section No.: _____

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1. (20%) McWendKing, your favorite math-run fast food chain, offers 20 ingredients that can be added to your burger. You have enough $ to buy a burger with 5 added ingredients. How many distinct burgers, with exactly 5 toppings, can you order if:

   (a) You refuse to use any of the ingredients more than once?

   (b) You refuse repeats but care about the order the ingredients are added (e.g., a tomato slice should be added on top of bacon)?

   (c) You allow yourself multiples of the same ingredients (e.g., the famous blood-clotting burger made with 3 bacons and 2 mayo's)?

   (d) You allow yourself repeats of ingredients, and care about the order that the ingredients are added?

Solution:

(a) $\binom{20}{5}$ ways. Combinations because order does not matter and repeats are not allowed.

(b) P(20,5)=20x19x18x17x16 burgers. Permutations because order matters and repeats are not allowed.

(c) $\binom{24}{5}$ burgers. Pirates problem because order does not matter and repeats are allowed; 5 golden bars and 20 pirates (19 dividers).

(d) 20^5 burgers. Permutations with repetitions because order matters and repeats are allowed; 20 choices, 5 times each.

2. (20%) How many different words(existing and non-existing) can be formed from the letters of the word "ROOFER".

Solution: $\frac{6!}{2!2!} = 180$. Order of letters results in different words, hence order matters. There are 6 letters to arrange total, with 2 letters both repeating twice. So we divide by 2! for each repeating letter, since the swapping of repeating letters does not produce a different word.

3. (30%) How many ways are there to arrange 20 books on a bookcase with 3 shelves? Assume, as in real life, that one shelf can easily fit all 20 books, books are distinguishable and the order of the books on each shelf matters.

Solution: To put 20 books on 3 shelves, you can first put the books in order and then decide how many go on each shelf. There are P(20,20) = 20! ways to put the books in order and, by stars and bars, $\binom{20+3-1}{20}$ ways to choose how many books go on each shelf. So the final answer is 20! $\binom{22}{20}$.

4. (30%) How many integer solutions are there to the equation $x + y + z = 10$ for which

(a) x, y and z are all positive?

(b) x, y and z are all non-negative?

(c) x, y and z are all greater or equal to -3?

Solution:

(a) $\binom{9}{2}$ solutions. After each variable (pirate) gets 1 bar for free, we are left with 7 bars and 2 dividers.

(b) $\binom{12}{2}$ solutions. We have 10 bars and 2 dividers for the 3 pirates. This is the plain vanilla flavor pirates problem.

(c) $\binom{21}{2}$ solutions. This problem is equivalent to finding the number of solutions to $x' + y' + z' = 19$ where x', y' and z' are non-negative. This is the same as if we substitute $x = x' - 3$, $y = y' - 3$, $z = z' - 3$.

5. (Extra Credits - 20%) How many 7 digit phone numbers are there in which the digits are non-increasing? That is, every digit is less than or equal to the

previous one, e.g., 955-4331. (Hint: Use the pirates method).

Solution: To create 7 digit phone numbers, we will use the 7 digits as 7 bars. The dividers will represent a switch from each possible single digit number down the next smaller one. So the phone number 955-4331 is represented as $*||||*|*|*|*||*|$. For each digit, we have 10 choices (0-9) so we use 9 dividers. We have 7 stars (golden bars) and 9 dividers, so the total number of phone numbers is $\binom{16}{9}$.

GOOD LUCK!

CS206 Quiz 4 **Nov 8, 2023** **Section 2**

Name: _____

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1. (20%) MeltingIce, your favorite math-run ice cream store, offers 15 flavors which can be added to your ice cream. You have enough money to buy an ice cream with 3 scoops. How many different ice creams with exactly 3 scoops can you order if:

   (a) You allow yourself multiples of the same flavor (e.g., two strawberry scoops, and one scoop of their new, "wall paint" flavor)?

   (b) You refuse repeated flavors, you care about the order the flavors added, and you definitely want their best-seller, "onion" flavor, on the top?

   (c) You refuse to use any of the flavors more than once, and you dislike their "new car" flavor?

   (d) You allow yourself repeats of flavors, and care about the order that the flavors are added?

Solution:

(a) $\binom{17}{3}$ ice creams. Pirates problem because order does not matter and repeats are allowed; 15 pirates (flavors) and 3 golden bars (scoops).

(b) P(14,2)=14x13 ice creams. Permutations because order matters and repeats are not allowed. On top we have the onion flavor, so we have to fill 2 open positions.

(c) $\binom{14}{3}$ ways. Combinations because order does not matter and repeats are not allowed. The "new car" flavor is removed.

(d) 15^3 ice creams. Permutations with repetitions because order matters and repeats are allowed; 15 choices 3 times.

2. (20%) How many different words (existing and non-existing) can be formed from the letters of the word "SKIER", such that the two vowels "I","E" are always next to each other.

Solution: 4!2! = 48. Since we need to count all arrangements with vowels neighboring each other, we can treat the vowels as 1 letter. This results in 4! arrangements of 4 letters. We must also account for the 2 arrangements of the 2 vowel letter through 2!, since they are different.

3. (30%) How many ways are there to arrange 50 cars on a parking lot with 5 rows? Each row can take more than 50 cars. Assume, as in real life, that cars are distinguishable and that the order of the cars on each row matters.

Solution: To put 50 cars on 5 rows, you can first put the cars in order and then decide how many go on each row. There are P(50,50) = 50! ways to put the cars in order and, by stars and bars, $\binom{50+5-1}{4}$ ways to choose how many cars go on each row. So the final answer is 50! $\binom{52}{4}$.

4. (30%) How many integer solutions are there to the equation $x + y + z = 5$ for which

(a) x, y and z are all positive?

(b) x, y and z are all non-negative?

(c) x, y and z are all greater or equal to -5?

Solution:

(a) $\binom{4}{2}$ solutions. After each variable (pirate) gets 1 bar for free, we are left with 2 bars and 2 dividers.

(b) $\binom{5}{2}$ solutions. We have 5 bars and 2 dividers for the 3 pirates. This is the plain vanilla flavor pirates problem.

(c) $\binom{22}{2}$ solutions. This problem is equivalent to finding the number of solutions to $x' + y' + z' = 20$ where x', y' and z' are non-negative. This is the same as if we do a simple substitution $x = x' - 5$, $y = y' - 5$, $z = z' - 5$.

5. (Extra Credits - 20%) How many 10 digit phone numbers are there in which the digits are non-decreasing? That is, every digit is larger than or equal to the previous one, e.g., 344-566-7899. (Hint: Use the pirates and bars method).

Solution: To create 10 digit phone numbers, we will use the 10 digits as 10 bars . The dividers will represent a switch from each possible single digit number up to the next larger number. So the phone number 344-566-7899 is represented as $|||*|**|*|**|*|*|**$. For each digit, we have 10 choices (0-9) so we use 9 dividers. We have 10 bars and 9 dividers, so the total number of phone numbers is $\binom{19}{9}$.

GOOD LUCK!

CS206 Quiz 4 **Nov 8, 2023** **Section 3**

Name: _____

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1. (20%) BurntPizza, your favorite math-run pizza store, offers 12 toppings which can be added to your pizza. You have enough money to buy a pizza with 4 toppings. How many different pizzas can you order if:

   (a) You allow yourself repeats of toppings, and care about the order that the toppings are added?

   (b) You allow yourself multiples of the same topping (e.g., three cheddar-cheese toppings, and one of their new, "calamari-rings" topping)?

   (c) You refuse to use any of the toppings more than once, and you dislike their "foie gras" topping?

   (d) You refuse repeats, you care about the order the toppings added, and you definitely want their best-seller, "caramelized onion" topping, on the top?

Solution:

(a) 12^4 pizzas. Permutations with repetitions because order matters and repeats are allowed; 12 choices 4 times.

(b) $\binom{15}{4}$ pizzas. Pirates problem because order does not matter and repeats are allowed; 12 pirates (11 dividers) and 4 gold bars.

(c) $\binom{11}{4}$ pizzas. Combinations because order does not matter and repeats are not allowed. The "foie gras" topping is removed.

(d) P(11,3)=11x10x9 pizzas. Permutations because order matters and repeats are not allowed. On top we have the caramelized-onion topping, so we have to fill 3 open positions.

2. (20%) How many different words (existing and non-existing) can be formed from the letters of the word "THEIR", such that the two vowels "E","I" are always next to each other.

Solution: 4!2! = 48. Since we need to count all arrangements with vowels neighboring each other, we can treat the vowels as 1 letter. This results in 4! arrangements of 4 letters. We must also account for the 2 arrangements of the 2 vowel letter through 2!, since they are different.

3. (30%) How many ways are there to arrange 100 boxes on a storage rack with 10 rows? Each row can take more than 100 boxes. Assume, as in real life, that boxes are distinguishable and that the order of the boxes on each row matters.

Solution: To put 100 boxes on 10 rows, you can first put the boxes in order and then decide how many go on each row. There are P(100,100) = 100! ways to put the boxes in order and, by pirates method, $\binom{100+10-1}{9}$ ways to choose how many boxes go on each row. So the final answer is 100! $\binom{109}{9}$.

4. (30%) How many integer solutions are there to the equation $x + y + z = 25$

for which

(a) x, y and z are all positive?

(b) x, y and z are all non-negative?

(c) x, y and z are all greater or equal to -10?

Solution:

(a) $\binom{24}{2}$ solutions. After each variable (pirate) gets 1 bar for free, we are left with 23 bars and 2 dividers.

(b) $\binom{27}{2}$ solutions. We have 25 bars and 2 dividers for the 3 pirates. This is the plain vanilla flavor pirates problem.

(c) $\binom{57}{2}$ solutions. This problem is equivalent to finding the number of solutions to $x' + y' + z' = 55$ where x', y' and z' are non-negative. This is the same as if we do a simple substitution $x = x' - 10$, $y = y' - 10$, $z = z' - 10$.

5. (Extra Credits - 20%) How many 9 digit phone numbers are there in which the digits are non-decreasing? That is, every digit is larger than or equal to

the previous one, e.g., 344-566-789. (Hint: Use the pirates and bars method).

Solution: To create 9 digit phone numbers, we will use the 10 digits as 10 bars . The dividers will represent a switch from each possible single digit number up to the next larger number. So the phone number 344-566-789 is represented as $|||*|**|*|**|*|*|*$. For each digit, we have 10 choices (0-9) so we use 9 dividers. We have 9 bars and 9 dividers, so the total number of phone numbers is $\binom{18}{9}$.

GOOD LUCK!