CS 205 Homework 1

Spring 2022

1. Prove or disprove each of the following proposed equivalences. For proofs, you can use either truth tables or natural deduction. A disproof should give an assignment as a counterexample.

   (a) $(A \to C) \lor (B \to C) \stackrel{?}{\equiv} (A \lor B) \to C$

   (b) $(A \to C) \lor (B \to C) \stackrel{?}{\equiv} (A \land B) \to C$

   (c) $(A \to C) \land (B \to C) \stackrel{?}{\equiv} (A \lor B) \to C$

   (d) $(A \to C) \land (B \to C) \stackrel{?}{\equiv} (A \land B) \to C$

   (e) $A \to (B \to C) \stackrel{?}{\equiv} (A \to B) \to C$

   (f) $A \to (B \to C) \stackrel{?}{\equiv} A \land B \to C$

   (g) $A \to (B \to C) \stackrel{?}{\equiv} A \to B \land C$

2. Prove the following using natural deduction.

   (a) $\neg(A \land \neg A)$

   (b) $\neg A \to (A \to B)$

   (c) $A \lor \neg A$

   (d) $(A \to C \lor D) \to ((A \to C) \lor (A \to D))$

   (e) $A \land (B \lor C) \leftrightarrow (A \land B) \lor (A \land C)$

   (f) Given $A \leftrightarrow B$, show $\neg A \leftrightarrow \neg B$

   (g) Given $A \to C$ and $B \to D$, show $A \lor B \to C \lor D$

3. Using Lean, prove the six theorems in `hw1.lean`.