CS206 Quiz 5 **Nov 29, 2023** **A**

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Section No.: ____________

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1. (20%) Consider the following random experiment. We roll two standard dice and sum their face values.

   (a) Define the *sample space* for the experiment and count its elements.

   Solution: $E = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$. Total 11 elements.

   (b) Calculate the probability for the event E={*1*}.

   Solution: P(E)=0. The event E is not present in the sample space.

   (c) Calculate the probability for the event E={*the sum is an even number*}.

   Solution: $E = 2, 4, 6, 8, 10, 12$. Therefore, $P(E) = 6/11 = 0.54$

2. (15%) Using the axioms of probability, prove that for any experiment and associated sample space Ω, $P(\emptyset) = 0$. In other words, prove that the probability of the event corresponding to the empty set is 0.

   Solution: We know that $\emptyset = \Omega'$. We also know that $P(A) + P(A') = 1$ for any event A. Furthermore, $P(\Omega) = 1$. Hence, $P(\emptyset) = 0$.

3. (15%) Three fair 6-sided dice colored red, blue and green are rolled. What is the probability that **at least** two of them roll the same number?

Solution: The size of $\Omega = 6^3$. Difference Method: P(A)=1-P(A'), where A'={all dice have different numbers}. In this case, $|A'| = 6 \times 5 \times 4 = 120$. Hence, $P(A) = 1 - 0.55 = 0.45$. Alternatively, we may use the sum method. Specifically, we are looking at all outcomes where all three have the same number or exactly two of them have the same number. There are 6 outcomes where they all have the same number and $\binom{3}{2} * 6 * 5$ outcomes where exactly two of them have the same number. Hence the probability that at least two have the same number $= \frac{16}{36} = \frac{4}{9}$.

4. (15%) Three fair 6-sided dice colored red, blue and green are rolled. What is the probability that **exactly** two of them roll the same number?

Solution: The size of $\Omega = 6^3$. There are $\binom{3}{2} * 6 * 5$ outcomes where exactly two of them have the same number. Hence, the probability that at least two have the same number $= \frac{15}{36} = \frac{5}{12}$

5. (15%) 8 identical chocolates are randomly divided among 3 kids. Assume that each possible way to divide is equally likely. What is the probability that kid

1 gets at least 3 chocolates?

Solution: Total outcomes $\binom{10}{2}$. Outcomes where kid 1 gets at least 3 $= \binom{7}{2}$. Hence, probability is $\frac{21}{45} = \frac{7}{15}$.

6. (20%) From an unlimited supply of blocks that are blue, red, and yellow, we select 7 blocks. What is the probability that there are more blue than red and more red than yellow? *Hint: How many yellow blocks can we really choose?*

Solution: Sample space is $x_1 + x_2 + x_3 = 7$. Pirates method: $\binom{7+2}{2}$. Sample space has 36 elements. We then find the event space. To do this, note that the number of yellow blocks can be either 0 or 1. When number of yellow blocks is 0 we are looking at the number of non-negative solutions to $x_1 + x_2 = 7$ with $x_1 > x_2$ and $x_2 \geq 1$. This equals 3.
When number of yellow blocks is 1 we are looking at the number of non-negative solutions to $x_1 + x_2 = 6$ with $x_1 > x_2$ and $x_2 \geq 2$. This equals 1. Therefore, we have 3+1 = 4 total solutions. Hence, $P = 4/36 = 1/9$.

7. (extra credits - 20%) Two cards are dealt from a standard, well-shuffled, pack. What is the probability that both are hearts?
Hint. The standard pack has 52 cards and 13 hearts.

   (a) Treat the sample space as consisting of the pair of cards dealt *in order*.

Solution: The sample space has size $|S| = 52 \times 51$. The event space has size $|E| = 13 \times 12$. Hence, $P = |E|/|S| = 1/17$.

(b) Treat the sample space as consisting of the pair of cards in *no* order.
Did you find the same probability? Why? Why not?
(This is very interesting...)

Solution: The sample space has a size of $|S| = \binom{52}{2} = 1326$ elements. The event space has a size of $\binom{|E|=13}{2=78}$ elements. Hence, $P = |E|/|S| = 1/17$. Of course, we got the same probability as before.

CS206 Quiz 5 **Nov 29, 2023** **B**

Name: ____________________________

NetID: ____________(Please **PRINT**)

Section No.: ____________

1. (20%) Consider the following random experiment. We roll two standard dice and take the difference of their face values (i.e., the difference might be negative).

   (a) Define the *sample space* for the experiment and count its elements.

   Solution: $E = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$. Total 11 elements. Note that if you assume that you will always subtract from the larger number, the sample space is different. $E = 0, 1, 2, 3, 4, 5$

   (b) Calculate the probability for the event E={*6*}.

   Solution: P(E)=0. The event E is not present in the sample space.

   (c) Calculate the probability for the event E={*the difference is even*}.

   Solution: $E = -4, -2, 2, 4$. Therefore, $P(E) = 4/11 = 0.36$

2. (15%) Using the axioms of probability, prove that for any experiment and associated sample space Ω, $P(\emptyset) = 0$. In other words, prove that the probability of the event corresponding to the empty set is 0.

Solution: We know that $\emptyset = \Omega'$. We also know that $P(A) + P(A') = 1$ for any event A. Furthermore, $P(\Omega) = 1$. Hence, $P(\emptyset) = 0$.

3. (15%) Four fair 6-sided dice colored red, blue, yellow and green are rolled. What is the probability that **at least** two of them roll the same number?

Solution: The size of $\Omega = 6^4$. Difference Method. P(A)=1-P(A'), where A'={all dice have different numbers}. In this case, $|A'| = 6 \times 5 \times 4 \times 3 = 360$. Hence, $P(A) = 1 - 0.27 = 0.73$. Alternatively, we may use the sum method. We are looking at all outcomes where all four have the same number or exactly three have the same number or exactly two have the same number. There are 6 outcomes when they all have the same number, $\binom{4}{3} \times 6 \times 5$ outcomes where exactly three of them have the same number, and $\binom{4}{2} \times 6 \times 5 \times 4$ outcomes where exactly two of them have the same number. Hence the probability that at least three have the same number $= \frac{6+4\times6\times5+6\times6\times5\times4}{6^4}$.

4. (15%) Four fair 6-sided dice colored red, blue, yellow and green are rolled. What is the probability that **exactly** three of them roll the same number?

Solution: The size of $\Omega = 6^4$. There are $\binom{4}{3} \times 6 \times 5 = 120$ outcomes where exactly three dice roll the same number. Hence, the probability $= \frac{120}{1296} = 0.092$

5. (15%) 9 identical chocolates are randomly divided among 3 kids. Assume that each possible way to divide is equally likely. What is the probability that kid-3 gets at least 3 chocolates?

Solution: Total outcomes $\binom{11}{2}$. Outcomes where kid 3 gets at least 3 $= \binom{8}{2}$. Hence, probability is $\frac{28}{55}$.

6. (20%) From an unlimited supply of blocks that are blue, red, and yellow, we select 7 blocks. What is the probability that there are more blue than red and more red than yellow? *Hint: How many yellow blocks can we really choose?*

Solution: Sample space is $x_1 + x_2 + x_3 = 7$. Pirates method: $\binom{7+2}{2}$. Sample space has 36 elements. We then find the event space. To do this, note that the number of yellow blocks can be either 0 or 1. When number of yellow blocks is 0 we are looking at the number of non-negative solutions to $x_1 + x_2 = 7$ with $x_1 > x_2$ and $x_2 \geq 1$. This equals 3.
When number of yellow blocks is 1 we are looking at the number of non-negative solutions to $x_1 + x_2 = 6$ with $x_1 > x_2$ and $x_2 \geq 2$. This equals 1. Therefore, we have 3+1 = 4 total solutions. Hence, $P = 4/36 = 1/9$.

7. (extra credits - 20%) Three cards are dealt from a standard, well-shuffled, pack. What is the probability that all of them are spades?
Hint. The standard pack has 52 cards and 13 spades.

   (a) Treat the sample space as consisting of the pair of cards dealt *in order*.

   Solution: The sample space has size $|S| = 52 \times 51 \times 50$. The event space has size $|E| = 13 \times 12 \times 11$. Hence, $P = |E|/|S| = 0.01294$.

   (b) Treat the sample space as consisting of the pair of cards in *no* order. Did you find the same probability? Why? Why not?
   (This is very interesting...)

   Solution: The sample space has a size of $|S| = \binom{52}{3} = 22100$ elements. The event space has a size of $\binom{|E|=13}{3=286}$ elements. Hence, $P = |E|/|S| = 0.01294$. Of course, we got the same probability as before.

CS206 Quiz 5 **Dec 2, 2023** **C**

Name: ______________________________

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Section No.: ____________

1. (20%) Consider the following random experiment. We roll three standard dice and sum their face values.

   (a) Define the *sample space* for the experiment and count its elements.

   Solution: $E = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18$. Total 16 elements.

   (b) Calculate the probability for the event E={*2*}.

   Solution: P(E)=0. The event E is not present in the sample space.

   (c) Calculate the probability for the event E={*the sum is an even number*}.

   Solution: $E = 4, 6, 8, 10, 12, 14, 16, 18$. Therefore, $P(E) = 8/16 = 0.5$

2. (15%) Using the axioms of probability, prove that for any experiment and associated sample space Ω, $P(\emptyset) = 0$. In other words, prove that the probability of the event corresponding to the empty set is 0.

Solution: We know that $\emptyset = \Omega'$. We also know that $P(A) + P(A') = 1$ for any event A. Furthermore, $P(\Omega) = 1$. Hence, $P(\emptyset) = 0$.

3. (15%) Four fair 6-sided dice colored red, blue, yellow and green are rolled. What is the probability that **at least** two of them roll the same number?

Solution: The size of $\Omega = 6^4$. Difference Method. P(A)=1-P(A'), where A'={all dice have different numbers}. In this case, $|A'| = 6 \times 5 \times 4 \times 3 = 360$. Hence, $P(A) = 1 - 0.27 = 0.73$. Alternatively, we may use the sum method. We are looking at all outcomes where all four have the same number or exactly three have the same number or exactly two have the same number. There are 6 outcomes when they all have the same number, $\binom{4}{3} \times 6 \times 5$ outcomes where exactly three of them have the same number, and $\binom{4}{2} \times 6 \times 5 \times 4$ outcomes where exactly two of them have the same number. Hence the probability that at least three have the same number $= \frac{6+4\times6\times5+6\times6\times5\times4}{6^4}$.

4. (15%) Four fair 6-sided dice colored red, blue, yellow and green are rolled. What is the probability that **exactly** three of them roll the same number?

Solution: The size of $\Omega = 6^4$. There are $\binom{4}{3} \times 6 \times 5 = 120$ outcomes where exactly three dice roll the same number. Hence, the probability $= \frac{120}{1296} = 0.092$

5. (15%) 15 identical chocolates are randomly divided among 4 kids. Assume that each possible way to divide is equally likely. What is the probability that kid-1 gets at least 8 chocolates?

Solution: Total outcomes $\binom{18}{3=816}$. Outcomes where kid 1 gets at least 8 $= \binom{10}{3=120}$. Hence, probability is $\frac{120}{816}$.

6. (20%) From an unlimited supply of blocks that are blue, red, and yellow, we select 7 blocks. What is the probability that there are more blue than red and more red than yellow? *Hint: How many yellow blocks can we really choose?*

Solution: Sample space is $x_1 + x_2 + x_3 = 7$. Pirates method: $\binom{7+2}{2}$. Sample space has 36 elements. We then find the event space. To do this, note that the number of yellow blocks can be either 0 or 1. When number of yellow blocks is 0 we are looking at the number of non-negative solutions to $x_1 + x_2 = 7$ with $x_1 > x_2$ and $x_2 \geq 1$. This equals 3.
When number of yellow blocks is 1 we are looking at the number of non-negative solutions to $x_1 + x_2 = 6$ with $x_1 > x_2$ and $x_2 \geq 2$. This equals 1. Therefore, we have 3+1 = 4 total solutions. Hence, $P = 4/36 = 1/9$.

7. (extra credits - 20%) Four cards are dealt from a standard, well-shuffled, pack. What is the probability that all of them are diamonds?
Hint. The standard pack has 52 cards and 13 diamonds.

   (a) Treat the sample space as consisting of the pair of cards dealt *in order*.

   Solution: The sample space has size $|S| = 52 \times 51 \times 50 \times 49$. The event space has size $|E| = 13 \times 12 \times 11 \times 10$. Hence, $P = |E|/|S| = 0.0026$.

   (b) Treat the sample space as consisting of the pair of cards in *no* order. Did you find the same probability? Why? Why not?
   (This is very interesting...)

   Solution: The sample space has a size of $|S| = \binom{52}{4} = 270725$ elements. The event space has a size of $\binom{|E|=13}{4=715}$ elements. Hence, $P = |E|/|S| = 0.0026$. Of course, we got the same probability as before.