Recitation 3

- Counting
    - Sum Rule
    - Partition Method
    - Product Rule

Counting

- Assume we have a set of objects with certain properties
- Counting is used to determine the number of these objects
- Examples:

  •Number of available phone numbers with 7 digits in the local calling area
  •Number of possible match starters (football, basketball) given the number of team members and their positions

Sum Rule

If a first task can be performed in m ways, while a second task can be performed in n ways, and the two tasks cannot be performed simultaneously, then performing either task can be done in $m+n$ ways.

1. Suppose that you are in a restaurant, and are going to have either soup or salad but not both. There are two soups and four salads on the menu. How many choices do you have?

1. Suppose that you are in a restaurant, and are going to have either soup or salad but not both. There are two soups and four salads on the menu. How many choices do you have?

By the Sum Rule, you have $2+4 = 6$ choices

Partition Method

- To find the size of a set A
- Partition it into a union of disjoint sets A1, A2, ...An
- Use sum rule

Difference Method

- To find the size of a set A
- Find a larger set S such that $S = A \cup B$ and
- A and B are disjoint
- $|A| = |S| - |B|$

Product Rule for counting

1. **Identify the number of sets to be selected from.**

2. **Identify the number of items to select from each set.**

3. **Multiply the number of items in each set.**
4. **If selecting two items from a set, calculate $n*(n-1)$**

2.An ice cream parlor offers sundaes with two scoops of ice cream, where each scoop can be any one of 31 flavors, plus a choice of topping from walnuts, raisins, coconut, m&m’s, chocolate, or fruit, and a choice of hot fudge, caramel, or butterscotch syrup.
Summary: 2 choices of 31 flavors, a choice of 6 toppings (or no topping), a choice of 3 syrups (or no syrup)
How many possible sundaes are there?

2. An ice cream parlor offers sundaes with two scoops of ice cream, where each scoop can be any one of 31 flavors, plus a choice of topping from walnuts, raisins, coconut, m&m's, chocolate, or fruit, and a choice of hot fudge, caramel, or butterscotch syrup.
Summary: 2 choices of 31 flavors, a choice of 6 toppings (or no topping), a choice of 3 syrups (or no syrup)
How many possible sundaes are there?

Adding a choice of no topping and no syrup, by the product rule, we get

31*31*(6+1)*(3+1)

3.Suppose four cards are chosen at random from a standard 52-card deck, with replacement. And we wish to determine the number of four-card sequences where all four cards are from the same suit.

$$\underbrace{HHHH}_{\text{all Hearts}} \text{ or } \underbrace{DDDD}_{\text{all Diamonds}} \text{ or } \underbrace{CCCC}_{\text{all Clubs}} \text{ or } \underbrace{SSSS}_{\text{all Spades}}$$

$$\underbrace{13 \bullet 13 \bullet 13 \bullet 13}_{\text{all Hearts}} + \underbrace{13 \bullet 13 \bullet 13 \bullet 13}_{\text{all Diamonds}} + \underbrace{13 \bullet 13 \bullet 13 \bullet 13}_{\text{all Clubs}} + \underbrace{13 \bullet 13 \bullet 13 \bullet 13}_{\text{all Spades}}$$

4. How many integer numbers less than 500 ends with 0?

How many ways to get a one-digit number that ends in zero?

$$\underset{\text{digit value}}{\underline{0}}$$

1 way

How many ways can we get a two-digit number that ends in zero?

$$\left(\underset{\text{digit value}}{\underline{\#1\text{–}9}}\right)\left(\underset{\text{digit value}}{\underline{0}}\right)$$

$$(9\ ways)(1\ way)$$

How many ways can we get a three-digit number that ends in zero and is less than 500?

$$\left(\underset{\text{digit value}}{\underline{\#1\text{–}4}}\right)\left(\underset{\text{digit value}}{\underline{\#0\text{–}9}}\right)\left(\underset{\text{digit value}}{\underline{0}}\right)$$

$$(4\ ways)(10\ ways)(1\ way)$$

$$1+(9)(1)+(4)(10)(1)=50$$

5.Suppose you need to come up with a password that uses only the letters A, B, and C and which must use each letter at least once. How many such passwords of length 8 are there?

5.Suppose you need to come up with a password that uses only the letters A, B, and C and which must use each letter at least once. How many such passwords of length 8 are there?

Let X be the set of passwords that doesn't contain A;

Let Y be the set of passwords that doesn't contain B;

Let Z be the set of passwords that doesn't contain C.

$|X| = 2^8 = |Y| = |Z|$

$|X \cap Y| = 1$('CCCCCCCC')

$|X \cap Z| = |Y \cap Z| = 1$

$|X \cup Y \cup Z| = 3*2^8-1-1-1-0$

Subtract it from universal Set

$=3^8-3*2^8-3$