CS206 Quiz 3 **Oct 23, 2023** **Section 1**

Name: ____________________________

NetID: ___________(Please **PRINT**)

Section No.: ___________

1. (10%) True or False (explain briefly): Given a set X, its permutation set has less elements than its combination set.

   Solution: False. Permutations take into account ordering of the elements, while combinations do not. Hence, multiple permutations count as a single combination, so the set of permutations must be larger than set of combinations.

2. (10%) How many different words (existing and non-existing) can be formed from the letters of the word "JERSEY".

   Solution: $6!/2!$

3. (20%) How many different words (existing and non-existing) can be formed from the letters of the word "PSYCHE", such that the two vowels "Y","E" are *always* next to each other.

   Solution: $5!2! = 240$. Since we need to count all arrangements with vowels neighboring each other, we can treat the vowels as 1 letter. This results in 5! arrangements of 5 letters. We must also account for the 2 arrangements of the 2 vowels as 2!, since they are distinct.

4. (20%) How many different words (existing and non-existing) can be formed from the letters of the word "PEACE", such that the two consonants "P","C" are *never* next to each other.

   Solution: Difference Method. Total number of words are $5!/2! = 60$, because we need to divide the permutations of the 5 letters by 2! to remove the overcounting of the 2 identical E's. Negative argument: We count all arrangements with consonants neighboring each other, and treat them as 1 letter. This results in $4!/2! = 12$ arrangements of 4 letters, multiplied by 2! to include both cased (P,C) and (C,P). Difference Method – $60 - 12x2 = 36$ words.

5. (20%) There are 300 airline passengers waiting to board a plane.

(a) In how many ways can I arrange 50 of them in the 50 seats of the Business class?

> **Solution:** $P(300, 50) = \frac{300!}{(300-50)!}$. In this case order of seating arrangement matters. Hence, we want to obtain the number of ways we can permute 300 people into 50 slots.

(b) From the remaining passengers (i.e., after the business class passengers boarded the plane), in how many ways can I create the first boarding group of 50 people, with no particular order?

> **Solution:** $\binom{250}{50} = \frac{250!}{50!(250-50)!}$. In this case the order of the group doesn't matter, hence we get the number of combinations of 50 people from group of 250 people.

6. (20%) In a wine contest, there are 10 white wine samples and 15 red wine samples. Give an example question related to picking some of the wine samples, for which the answer is:
a. 25
b. 150

> **Solution:** 25 is the number of choices you have if you want to pick one wine sample, either a red or a white wine. 150 is the number of choices you have if you want to pick two wine samples, first a red and then a white wine.

7. (extra credits - 20%) How many ways are there to arrange the letters a, b, c, d, e, and f such that a is **not** directly followed by either b or c? For example, "$abdefc$" and "$acdefb$" are both invalid, but "$adbcef$" is valid.

> **Solution:** We will use the Difference and the Partition Methods. We will first count the number of ways to arrange the six letters so that a is followed directly by either b or c. To find the number of arrangements where a is directly followed by b, we can consider ab as a single letter. Thus there are 5! such arrangements (since we can just put the 5 letters ab, c, d, e, f in any order). And by the same reasoning, the number of arrangements where a is directly followed by c is the same: 5!. Since it is impossible for a to be directly followed by b and c at the same time, there are $2 \cdot 5!$ arrangements where a is directly followed by either b or c. Since there are 6! total ways to arrange the six letters when there are no restrictions, the answer to the question is $6! - 2 * 5!$

GOOD LUCK!

CS206 Quiz 3 **Oct 25, 2023** **Section 2**

Name: ____________________________

NetID: ____________(Please **PRINT**)

Section No.: ____________

1. (10%) True or False (explain briefly): Given a set X, its permutation set has less elements than its combination set.

   Solution: False. Permutations take into account ordering of the elements, while combinations do not. Hence, multiple permutations count as a single combination, so the set of permutations must be larger than set of combinations.

2. (10%) How many different words (existing and non-existing) can be formed from the letters of the word "RUTGERS".

   Solution: 7!/2!

3. (20%) How many different possible words (existing and non-existing) can be made from the word "TRAPPER" such that the vowels A and E are *always* together?

   Solution: Permutations with vowels together is (6!/(2!*2!))*2!=360, because vowels are assumed to be one letter, resulting in arrangement of only 6 letters with two letters repeating twice. We also taking into account vowel arrangements by multiplying by 2!

4. (20%) How many different possible words (existing and non-existing) can be made from the word "WALLET" such that the vowels are *never* together?

   Solution: Difference Method. Total number of words are 6!/2! = 360, because we need to divide the permutations of the 6 letters by 2! to remove the overcounting of the 2 identical L's. Negative argument: We count all arrangements with vowels neighboring each other, and treat them as 1 letter. This results in 5!/2! = 60 arrangements of 5 letters, multiplied by 2! to include both cases (A,E) and (E,A). Difference Method – 360 − 120 = 240 words.

5. (20%) There are 100 airline passengers waiting to board a plane.

(a) In how many ways can I arrange 40 of them in Business class?

Solution: $P(100, 40) = \frac{100!}{(100-40)!}$. In this case order of seating arrangement matters. Hence, we want to obtain the number of ways we can permute 100 people into 40 slots.

(b) In how many ways can I create a boarding group of 40 people, with no particular order?

Solution: $C(100, 40) = \frac{100!}{40!(100-40)!}$. In this case the order of the group doesn't matter, hence we get the number of combinations of 40 people from group of 100 people.

6. (20%) In a song contest, there are 10 rock songs and 5 ballads. Give a question related to picking some of the songs, for which the answer is:
a. 15
b. 50

Solution: 15 is the number of choices you have if you want to pick one song, either a rock or a ballad. 50 is the number of choices you have if you want to pick two songs, first a rock and then a ballad.

7. (extra credits - 20%) How many ways are there to arrange the letters $a, b, c, d, e,$ and f such that a is directly followed by either b or c? For example, "$abdefc$" and "$acdefb$" are both valid, but "$adbcef$" is invalid.

Solution: Partition Method. We will first count the number of ways to arrange the six letters so that a is followed directly by either b or c. To find the number of arrangements where a is directly followed by b, we can consider ab as a single letter. Thus there are 5! such arrangements (since we can just put the 5 letters ab, c, d, e, f in any order). And by the same reasoning, the number of arrangements where a is directly followed by c is the same: 5!. Since it is impossible for a to be directly followed by b and c at the same time, there are $2 \cdot 5!$ arrangements where a is directly followed by either b or c.

GOOD LUCK!

CS206 Quiz 3 **Oct 25, 2023** **Section 3**

Name: ____________________________

NetID: ____________(Please **PRINT**)

Section No.: ____________

1. (10%) True or False (explain briefly): Given a set X, its permutation set has less elements than its combination set.

   Solution: False. Permutations take into account ordering of the elements, while combinations do not. Hence, multiple permutations count as a single combination, so the set of permutations must be larger than set of combinations.

2. (10%) How many different words (existing and non-existing) can be formed from the letters of the word "ROOFER".

   Solution: $\frac{6!}{2!2!} = 180$. Order of letters results in different words, hence order matters. There are 6 letters to arrange in total, with 2 letters both repeating twice (R and O). So we divide by 2! for each repeating letter to correct for overcounting, since the swapping of repeating letters does not result in a different word.

3. (20%) How many different words (existing and non-existing) can be formed from the letters of the word "SKIER", such that the two vowels "I","E" are *always* next to each other.

   Solution: $4!2! = 48$. Since we need to count all arrangements with vowels neighboring each other, we can treat the vowels as 1 letter. This results in 4! arrangements of 4 letters. We must also account for the 2 arrangements of the 2 vowel letter through 2!, since they are different.

4. (20%) How many different possible words (existing and not-existing) can be made from the word "TRAPPER" such that the vowels are *never* together?

Solution: Total number of permutations of TRAPPER is 7!/(2!*2!)=1260, because we have 7 letters with two letters repeating twice.
Permutations with vowels together is (6!/(2!*2!))*2!=360, because vowels are assumed to be one letter, resulting in arrangement of only 6 letters with two letters repeating twice. We also taking into account vowel arrangements by multiplying by 2!
Thus, permutations with vowels never together is 1260-360=900, by difference method

5. (20%) There are 100 airline passengers waiting to board a plane.

   (a) In how many ways can I arrange 40 of them in Business class?

   Solution: $P(100, 40) = \frac{100!}{(100-40)!}$. In this case order of seating arrangement matters. Hence, we want to obtain the number of ways we can permute 100 people into 40 slots.

   (b) In how many ways can I create the first boarding group of 40 people, without caring about putting these 40 people in any particular order?

   Solution: $C(100, 40) = \frac{100!}{40!(100-40)!}$. In this case the order of the group doesn't matter, hence we get the number of combinations of 40 people from group of 100 people.

6. (20%) In a dog contest, there are 30 Labrador Retrievers and 15 Cocker Spaniels. Give an example question related to picking some of the dogs, for which the answer is:
   a. 45
   b. 450

   Solution: 45 is the number of choices you have if you want to pick one dog, either a Labrador Retriever or a Cocker Spaniel. 450 is the number of choices you have if you want to pick two dogs, first a Labrador Retriever and then a Cocker Spaniel.

7. (Extra credits - 20%) How many ways are there to arrange the letters a, b, c, d, e, and f such that c is **not** directly followed by either d or e? For example, "$abcfed$" is valid, but "$abcdef$" and "$cedfab$" are invalid.

Solution: We will use the Difference and the Partition Methods. We will first count the number of ways to arrange the six letters so that c is followed directly by either d or e. To find the number of arrangements where c is directly followed by d, we can consider cd as a single letter. Thus there are 5! such arrangements (since we can just put the 5 letters a, b, cd, e, f in any order). And by the same reasoning, the number of arrangements where c is directly followed by e is the same: 5!. Since it is impossible for c to be directly followed by d and e at the same time, there are $2 \cdot 5!$ arrangements where c is directly followed by either d or e. Since there are 6! total ways to arrange the six letters when there are no restrictions, the answer to the question is $6! - 2 * 5!$.