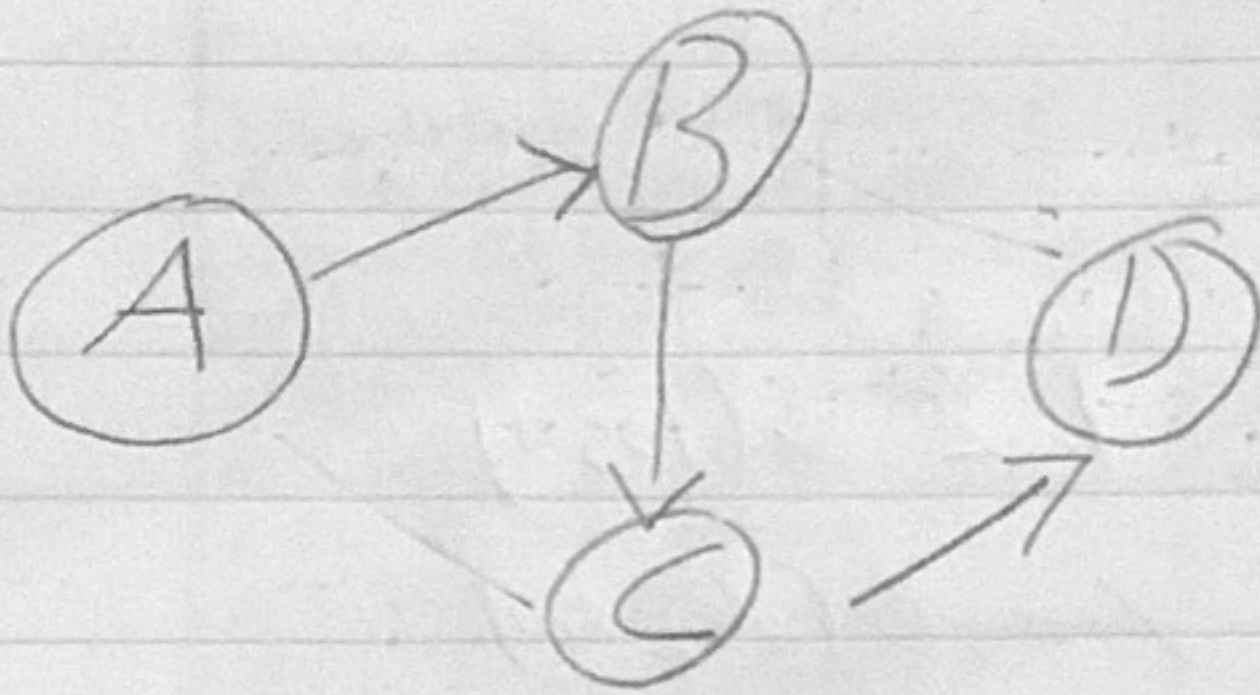


# HW3

1)

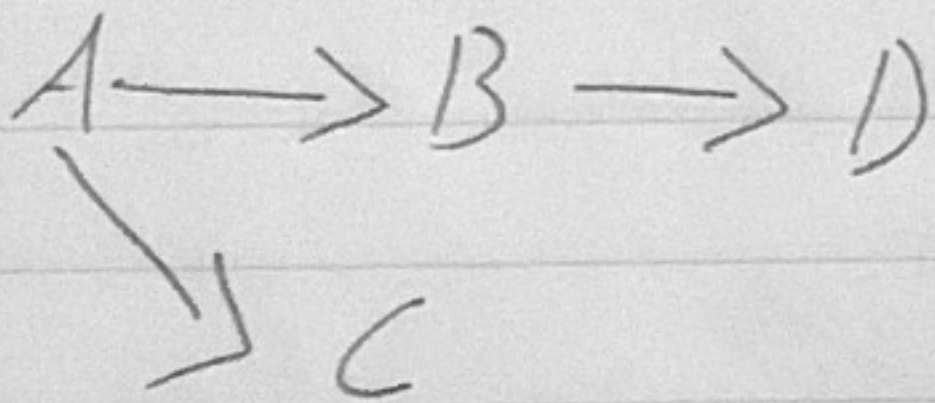
1-



DFS: A, B, C, D

BFS: A, B, C, D

2



DFS: A, B, D, C

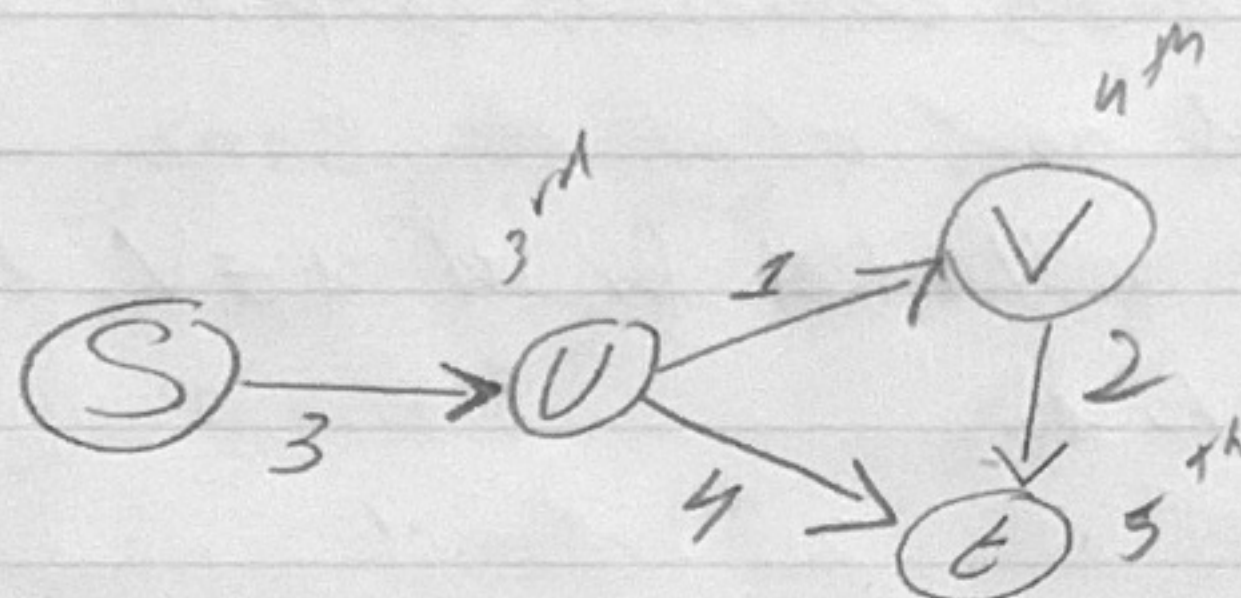
BFS: A, B, C, D



2)

|   | $d[s]$ | $d[u]$   | $d[v]$   | $d[t]$   | $P[s]$ | $P[u]$ | $P[v]$ | $P[t]$ |
|---|--------|----------|----------|----------|--------|--------|--------|--------|
| When entering the first while loop for the first time, the state is | 0      | $\infty$ | $\infty$ | $\infty$ | None   | None   | None   | None   |
| Immediately after the first element of D is added, the state is:    | 0      | 3        | $\infty$ | $\infty$ | None   | S      | None   | None   |
| After the second element of D is added, the state is:               | 0      | 3        | 4        | 7        | None   | S      | u      | u      |
| After the third element of D is added, the state is:                | 0      | 3        | 4        | 6        | None   | S      | u      | v      |
| After the fourth element of D is added, the state is:               | 0      | 3        | 4        | 6        | None   | S      | u      | v      |

The path should return  $s \rightarrow u \rightarrow v \rightarrow t$   
 $d[t] = 6$



3) Algo: Build the complete graph with weights that are equal to the fees  ~~$G(V, E, w)$~~   $G(V, E, w)$ .  $V = \{c_1, \dots, c_n\}$   $E = \{(c_i, c_j) : i \neq j\}$ , and  $w(c_i, c_j) = f_{ij}$ . Use Dijkstra's algorithm from  $c_s$ ; return shortest  $c_s \rightarrow c_t$  path.

Correctness: Any path from  $c_s \rightarrow c_t$  in  $G$  indicates a sequence of exchanges and that weight total is the sum of the fees needed to perform these exchanges. This makes it ideal to find path  $c_s \rightarrow c_t$  in min weight.  $f_{ij}$ s are positive, so this is a valid input for Dijkstra's algo.



Running Time:  $O(n^2)$  total. All path pairs are possible we have  $m = \binom{n}{2} = \Theta(n^2)$  edges - this is also how long it takes to build  $G$ . Dijkstra's also takes  $O(n^2 + n \log n) = O(n^2)$  if we use the Fibonacci heap implementation.

4)

a. if  $v$  and  $v'$  are influential, there is a path from  $v$  to  $v'$  from  $v'$  to  $v$ . Thus  $v$  and  $v'$  are in the same and strongly connected component. Also if  $v$  is influential and  $v'$  is in the same SCC as  $v$ , then  $v'$  is influential for all  $u \in V$ , there's a path from  $v'$  to  $v$  to  $u$ .

b. findInfluentialPerson( $G$ ):

$L = V$

while  $L$  is not empty:

$v = \text{any vertex in } L$

$VISTED = []$

$DFS(VISTED, L, v)$

$L.pop(v)$

return  $v$

$DFS(VISTED, L, S)$ :

if  $S == \text{NULL}$  or  $S$

is not in  $L$ :

return

$VISTED.push(S)$

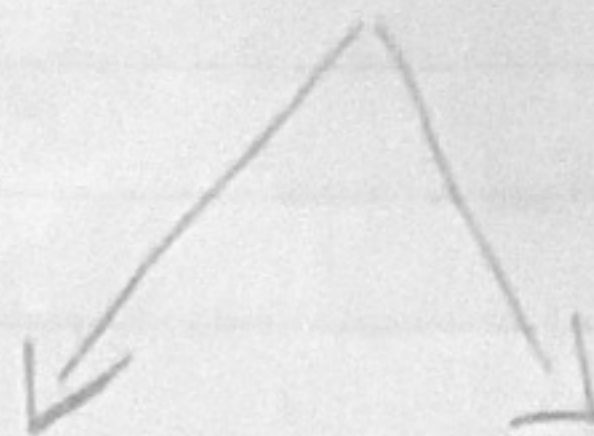
for  $v$  in neighbors:

$DFS(VISTED, L, v)$



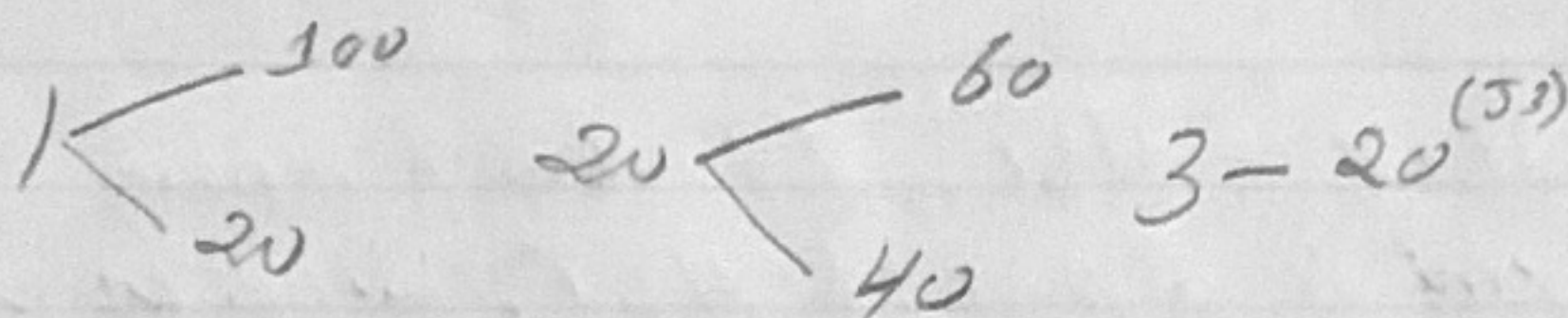
5)

|       |       |       |       |
|-------|-------|-------|-------|
| $J_0$ | $J_1$ | $J_2$ | $J_3$ |
| 2     | 2     | 3     | 3     |
| 60    | 40    | 100   | 80    |



$J_6$   $J_0$   
 (4)  
 150  
 x

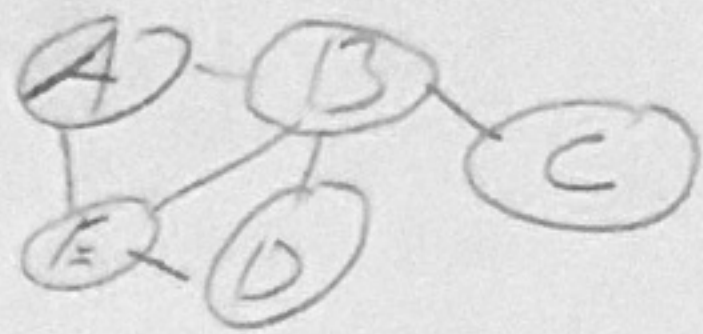
|       |       |       |       |       |
|-------|-------|-------|-------|-------|
| $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ |
| 2     | 0     | 3     | 2     | 0     |
| 60    | 100   | 20    | 40    | 20    |



1<sup>st</sup> sec  $\rightarrow J_1$  100,  $J_5$  20  
 2<sup>nd</sup> sec  $\rightarrow J_1$  60, 40  
 3<sup>rd</sup> sec  $\rightarrow J_3$  20

$\downarrow$   
 $J_2 \rightarrow J_1 \rightarrow J_3$

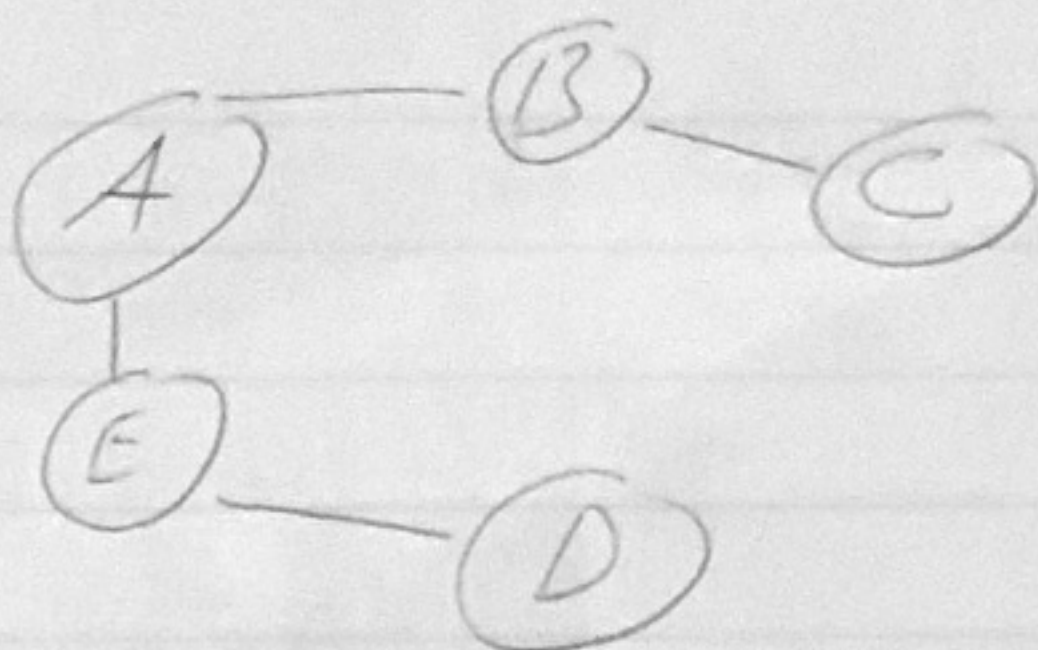




If I remove  
 $B-D$  it still  
 $B-E$

6)

a. yes MST



$A \rightarrow B$  ✓

$A \rightarrow E$  ✓

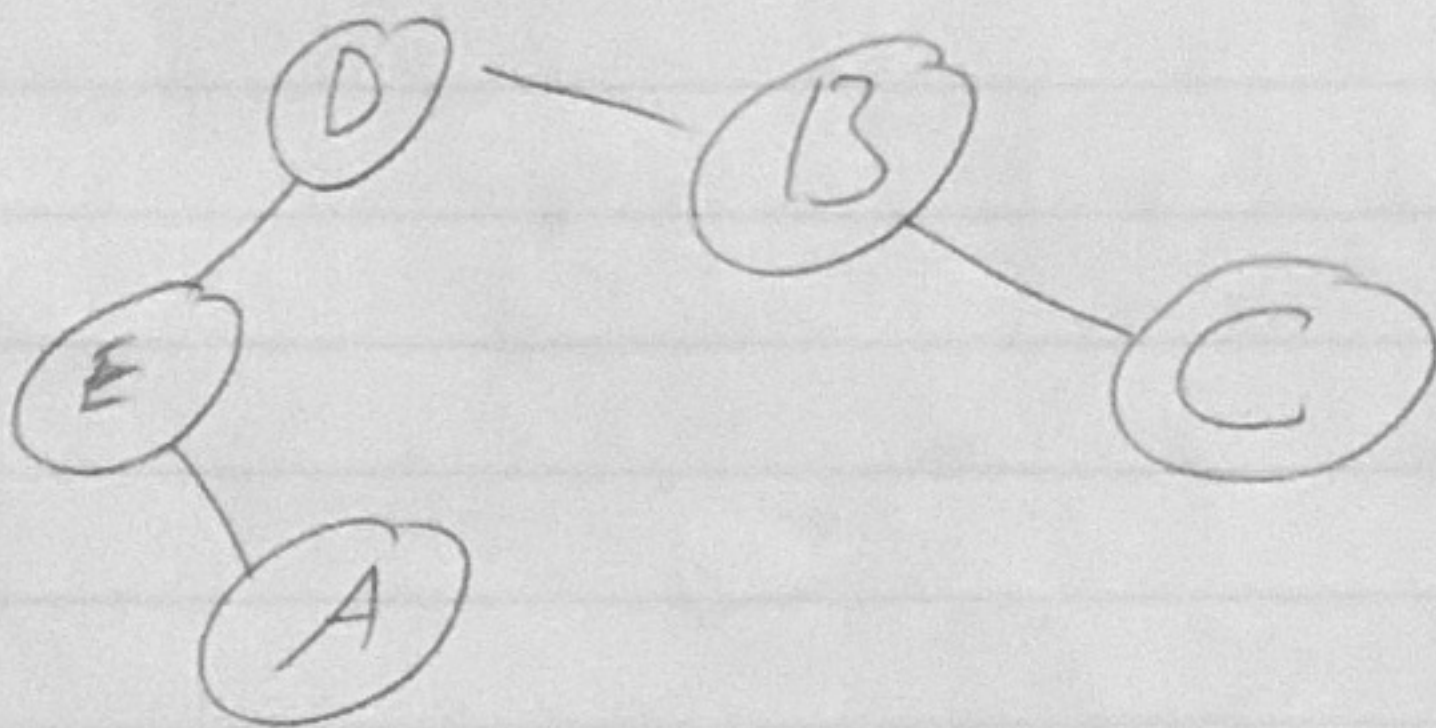
$B \rightarrow E$  ✗

$E \rightarrow D$  ✓

$B \rightarrow D$  ✗

$B \rightarrow C$  ✓

b. No MST



a) It does return a MST. It doesn't remove an edge that is a part of MST. Since we don't remove edges in an increasing order, the weight of every edge on the cycle must be less than or equal to  $e$ .

b) It doesn't return a MST, It doesn't spare the edges in a cycle.