Recitation 10

Basic Probability

- Probability theory begins with three basic components. The set of all possible outcomes, denoted S. The collection of all sets of outcomes (events), denoted A. And a probability measure P. Specification of the triple (S, A, P) defines the probability space which models a real-world measurement or experimental process.

Probability Measures

- Probability measures must satisfy the following properties:

  1. $P(A) \geq 0$, $\forall A \in S$
  2. $P(S) = 1$
  3. if $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$

Conditional Probability

- Consider two events $A, B \in S$. The (conditional) probability that A occurs given B occurs is $P(A|B) := P(A \cap B)/P(B)$

Question 1

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- Sample space = [HHH, HHT, HTH, THH, TTH, THT, HTT, TTT]
  Total number of ways = $2 \times 2 \times 2 = 8$. Fav. Cases = 7
  P (A) = 7/8
  OR
  P (of getting at least one head) = 1 – P (no head) $\Rightarrow$ 1 – (1/8) = 7/8

Question 3

- What is the probability of getting a sum of 7 when two dice are thrown?

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- Probability math - Total number of ways = 6 × 6 = 36 ways.
  Favorable cases = (1, 6) (6, 1) (2, 5) (5, 2) (3, 4) (4, 3) --- 6 ways.
  P (A) = 6/36 = 1/6

Question 4

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- Total no. of ways = $^{52}C_2$
  Case I: Both are diamonds = $^{13}C_2$
  Case II: Both are kings = $^{4}C_2$
  P (both are diamonds or both are kings) = $(^{13}C_2 + {}^{4}C_2) / {}^{52}C_2$

Question 5

- Three dice are rolled together. What is the probability as getting at least one '4'?

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- Three dice are rolled together. What is the probability as getting at least one '4’?
- Total number of ways = 6 × 6 × 6 = 216. Probability of getting number ‘4’ at least one time
  = 1 – (Probability of getting no number 4) = 1 – (5/6) x (5/6) x (5/6) = 91/216

Question 6

- From a pack of cards, three cards are drawn at random. Find the probability that each card is from different suit.

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- From a pack of cards, three cards are drawn at random. Find the probability that each card is from different suit.
- Total number of cases = $^{52}C_3$
  One card each should be selected from a different suit. The three suits can be chosen in $^{4}C_3$ was
  The cards can be selected in a total of $(^{4}C_3)$ x $(^{13}C_1)$ x $(^{13}C_1)$ x $(^{13}C_1)$
  Probability = $^{4}C_3 \times (^{13}C_1)^3 / {}^{52}C_3$
  = $4 \times (13)^3 / {}^{52}C_3$

Question 7

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- Find the probability that a leap year has 52 Sundays.
- A leap year can have 52 Sundays or 53 Sundays. In a leap year, there are 366 days out of which there are 52 complete weeks & remaining 2 days. Now, these two days can be (Sat, Sun) (Sun, Mon) (Mon, Tue) (Tue, Wed) (Wed, Thur) (Thur, Friday) (Friday, Sat).
  So there are total 7 cases out of which (Sat, Sun) (Sun, Mon) are two favorable cases. So, P (53 Sundays) = 2 / 7
  Now, P(52 Sundays) + P(53 Sundays) = 1
  So, P (52 Sundays) = 1 - P(53 Sundays) = 1 – (2/7) = (5/7)

Question 8

A certain candy similar to Skittles is manufactured with the following properties: 30% of sweet ones and 70% of pieces are sour. Each candy piece is colored with red or blue (but not both). If a candy piece is sweet, then it is colored blue in 80%, and red in 20%. If it is sour, 80% is in red and 20% is in blue.

The candy pieces are mixed together and randomly before they are sold.

(a) If you choose a piece at random from the jar, what is the probability that you choose a blue piece?

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P = 0.3*0.8+0.7*0.2 = 0.38

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(b) Given that the piece you chose is blue, what is probability that the piece is sour?

We want $p(sour|blue) = p(sour \cap blue)/p(blue)$, but we cannot find $p(sour \cap blue)/p(blue)$ in a direct way.

But we can expand it more, $p(sour \cap blue) = p(blue|sour)*p(sour)$.

So, $p(sour|blue) = p(blue|sour)*p(sour)/p(blue)$.

$=0.2*0.7/0.38 = 0.3684\ldots$

Question 9

- Suppose we have a deck of four cards: A-spade, 2-spade, A-heart, 2-heart. After being dealt two random cards, facing down, the dealer tells us that we have at least one ace in our hand.

(a) What is the probability that our hand has both aces? In other words, what is P(Two aces| At least one ace)?

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$$P(2\,A \mid A) = P(A \mid 2A)\,P(2A) / P(A) = 1*(1/6) / (5/6) = 1/5$$

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2. What if the dealer tells us we have the A-spade in our hand, what is P(Two aces| A-spade)?

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P(2A | A-spade) = P(A-spade | 2A) P(2A) / P(A-spade) = 1*(1/6) / (3/6) = 1/3

Question 10

A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighborhood have the flu, while the other 10% are sick with measles. Let F stand for an event of a child being sick with flu and M stand for an event of a child being sick with measles. Assume for simplicity that $F \cup M = S$, i.e., that there no other maladies in that neighborhood.

A well-known symptom of measles is a rash (the event of having which we denote R). Assume that the probability of having a rash if one has measles is $P(R \mid M) = 0.95$. However, occasionally children with flu also develop rash, and the probability of having a rash if one has flu is $P(R \mid F) = 0.08$.

Upon examining the child, the doctor finds a rash. What is the probability that the child has measles?

$$P(M|R) = P(R|M)\, P(M) \,/\, P(R) = P(R|M)\, P(M) \,/\, (P(R,M) + P(R,F))$$
$$= P(R|M)\, P(M) \,/\, (P(R|M)\, P(M) + P(R|F)\, P(F))$$

Question 11

- Three bags contain 3 red, 7 black; 8 red, 2 black, and 4 red & 6 black balls respectively. 1 of the bags is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the third bag.

Question 11

- Let E1, E2, E3 and A are the events defined as follows.
  E1 = First bag is chosen
  E2 = Second bag is chosen
  E3 = Third bag is chosen
  A = Ball drawn is red
- Since there are three bags and one of the bags is chosen at random, so P (E1) = P(E2) = P(E3) = 1 / 3
  If E1 has already occurred, then first bag has been chosen which contains 3 red and 7 black balls. The probability of drawing 1 red ball from it is 3/10. So, P (A/E_1) = 3/10, similarly $P(A/E_2)$ = 8/10, and $P(A/E_3)$ = 4/10. We are required to find $P(E_3/A)$ i.e. given that the ball drawn is red, what is the probability that the ball is drawn from the third bag by Baye's rule.

Question 11

$$= \frac{\frac{1}{3}\times\frac{4}{10}}{\frac{1}{3}\times\frac{3}{10}+\frac{1}{3}\times\frac{8}{10}+\frac{1}{3}\times\frac{4}{10}} = \frac{4}{15}.$$