University 8 May 1945 - GUELMA - October 2024

1^{st} year - Algebra 01

Series of exercises n° 02

* **Exercise 01.** Let $A = \{1, 2, 3\}$ and $B = \{0, 1, 2, 3\}$

- Find the following sets: $A \cap B$, $A \cup B$ and $A \times B$.
- Determine the cardinality: $n(A \times B)$ and $n(A) \times n(B)$.

* **Exercise 02** Let $A =]-\infty, 1[\cup]2, +\infty[$, $B =]-\infty, 1[$ and $C =]2, +\infty[$.

- Compare the following sets: $C_{\mathbb{R}}^{A}$, $C_{\mathbb{R}}^{B} \cap C_{\mathbb{R}}^{C}$ and $C_{\mathbb{R}}(B \cup C)$.

Exercise 03 Let A, B and C three subsets of a set E.

1) Prove that: $(A \cap B) \cup B^c = A \cup B^c$

$(A \setminus B) \setminus C = A \setminus (B \cup C)$

2) Simplify: $\overline{\overline{(A \cup B)} \cap \overline{(C \cup \overline{A})}}$, $\overline{\overline{(A \cap B)} \cup \overline{(C \cap \overline{A})}}$.

Exercise 04 Let A, B and C three subsets of a set E.

- Show that: 1) $C_E^A \setminus C_E^B = B \setminus A$

2) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

Exercise 05 Consider the function, $f : [0, 1] \longrightarrow [0, 2]$

$x \longmapsto f(x) = 2 - x$

1) Determine $f(\{\frac{1}{2}\})$, $f^{-1}(\{0\})$.
2) The function f is it bijective ?

Exercise 06 Let E a non-empty set and $A, B \in \mathcal{P}(E)$

- Using the definition of the symmetrical difference:

$A \Delta B = (A \setminus B) \cup (B \setminus A)$, show that:

1) $\forall A, B \in \mathcal{P}(E)$; $A \Delta B = (A \cup B) \setminus (A \cap B)$.

2) $\forall A, B \in \mathcal{P}(E)$, $A \Delta B = B \Delta A$.
3) $\forall A \in \mathcal{P}(E)$, $A \Delta \phi = A$ and $A \Delta A = \phi$

Exercise 07. Consider the function

$$g : \mathbb{R} \setminus \{1\} \longrightarrow \mathbb{R} \quad , \quad g(x) = \frac{1}{x-1}.$$

_The function is it injective, surjective and bijective.

Exercise 08. Consider the function, $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \frac{2x}{1+x^2}$$

1) Decide if f is one-to-one and onto function.
2) f is it invertible?

Exercise 09 Consider $f : \mathbb{R}_*^+ \to \mathbb{R}_*^+$ and $g : \mathbb{R}_*^+ \to \mathbb{R}$
defined by: $f(x) = \frac{1}{x}$ and $g(x) = \frac{x-1}{x+1}$
1) Prove that: $g \circ f = -g$.
2) Prove that: f is bijective ?

Exercise 10 Consider the bijection:

$$f : \mathbb{R} \setminus \{-2\} \longrightarrow \mathbb{R} \setminus \{1\}$$
$$x \longmapsto \frac{x+1}{x+2}.$$

_Find its inverse function ?

Exercise 11 E, F and G three finite sets, $f : E \to F$, $g : F \to G$
I) Prove that: 1) f and g are injective $\Rightarrow g \circ f$ injective ?
2) f and g surjective $\Rightarrow g \circ f$ surjective ?
II) Let $A_1, A_2 \subset E$ and $B_1, B_2 \subset F$. Prove that:
1) $f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)$, 2) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$
3) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$, 4) $f(f^{-1}(B)) \subset B$; $B \subset F$