1^{st} year , Algebra 1 October 2024.

Series of exercises n = 03

Exercise 01. We define on $\mathbb{R}^*$ the relation $\mathcal{R}$ as:

$$\forall x, y \in \mathbb{R}^*, \ x \mathcal{R} y \Leftrightarrow x.y > 0.$$

1) Show that $\mathcal{R}$ is an equivalence relation.
2) Determine the equivalence class of $x \in \mathbb{R}^*$.
3) Determine the set of all equivalence classes.

Exercise 2 In $\mathbb{N}^*$, we define the relation $\mathcal{S}$ by:

$$\forall n, m \in \mathbb{N}^*, \ n \mathcal{S} m \Leftrightarrow \exists a \in \mathbb{N}^* : n = am.$$

1) Prove that $\mathcal{S}$ is a partial order relation (partial ordering)
2) The partial ordering is it a total ordering.

Exercise 3. We define in $\mathbb{Z}$ the relation $\mathcal{R}$ as:

$$a \mathcal{R} b \Leftrightarrow \frac{a+2b}{3} \in \mathbb{Z}$$

1) Show that $\mathcal{R}$ is an equivalence relation.
2) Determine the equivalence class of -1, 0 and 1.

Exercise 4. We define on $\mathbb{R}^2$ the relation $\mathcal{R}$ as:

$$(a,b) \mathcal{R} (c,d) \Leftrightarrow a^2 + b^2 = c^2 + d^2.$$

1) Show that $\mathcal{R}$ is an equivalence relation
2) Give the equivalence class of (a,b) in $\mathbb{R}^2$.

Exercise 5. We define the relation $\leq$ on $\mathbb{N} \times \mathbb{N}$ by:

$$\forall (a,b), (c,d) \in \mathbb{N} \times \mathbb{N} : (a,b) \leq (c,d) \Leftrightarrow a \leq c \text{ and } b \leq d$$

1) Show that $\leq$ is a partial order relation
2) Prove that the partial order is not a total order relation.