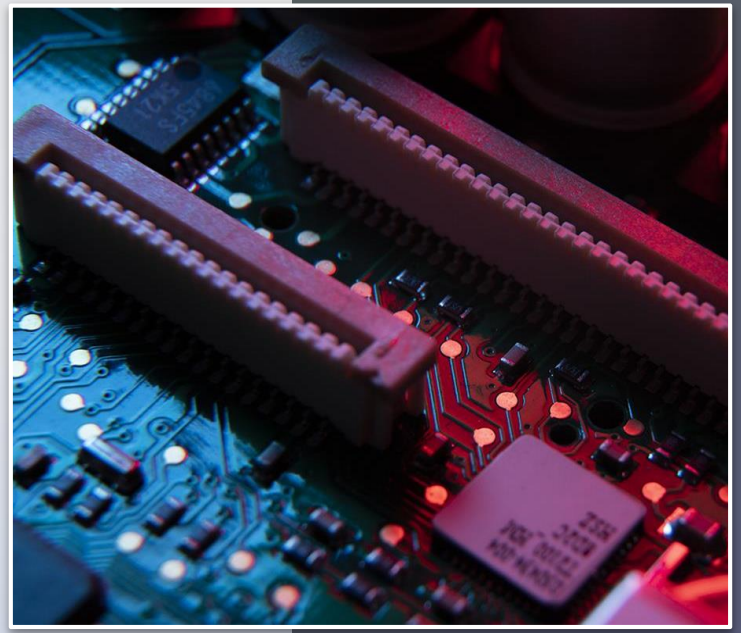
Continuous Regime Fundamental Theorems

DR. GUECHI Nassima



Course Plan

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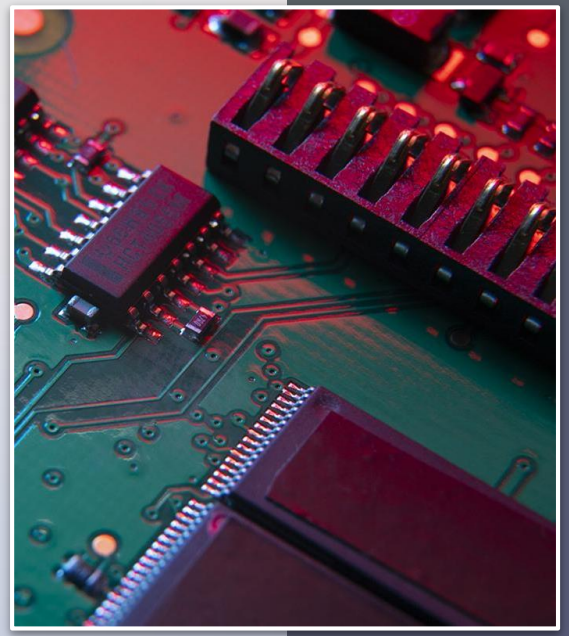
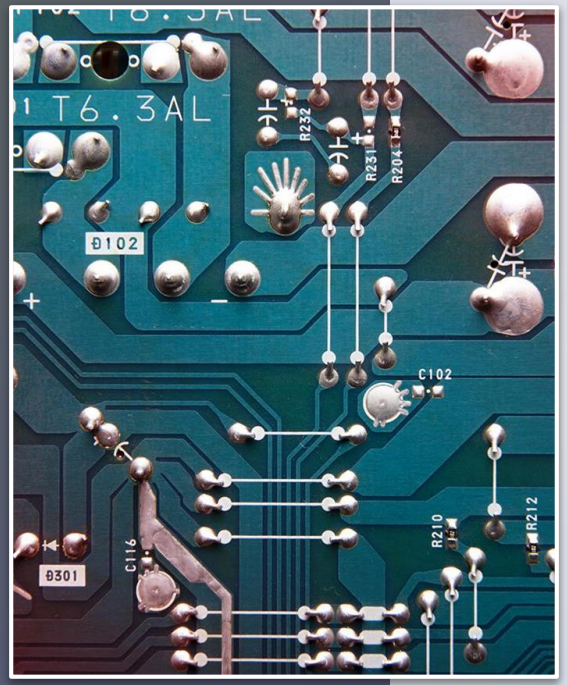




01

Introduction

What is an electrical circuit

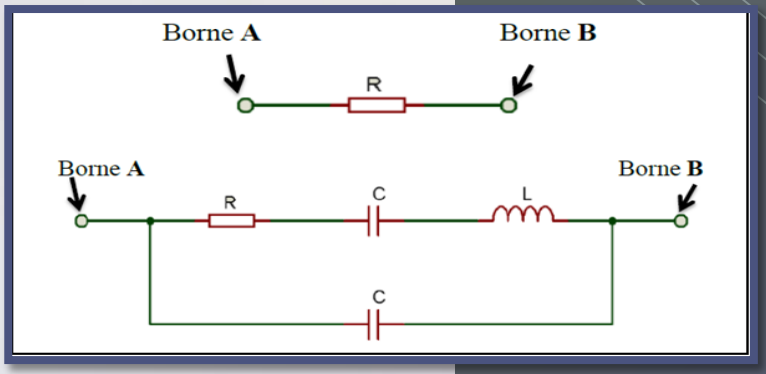
An electrical circuit consists of a set of interconnected components (elements or linear dipoles). Generally, the circuit includes at least one voltage or current source, passive components, and possibly one or more active components.

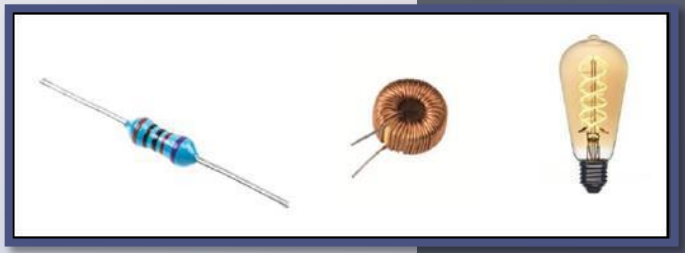
The components used in electronics have electrical terminals or poles that allow their connection in a network. They are distinguished as follows:

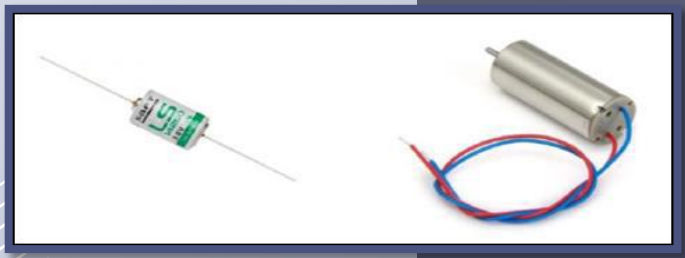
- Dipoles (2 poles) such as resistors, capacitors, inductors, ...
- Quadripoles (4 poles) such as filters, transformers, ..

Electrical Dipole

- An electrical dipole is a single component or a set of components connected to two terminals
- ***Passive Dipole:*** It is a dipole that consumes electrical energy and does not contain any energy source. Examples include resistors, inductors, and light bulbs.
- ***Active Dipole:*** It is a dipole that contains an energy source, such as a battery and a direct current (DC) motor.

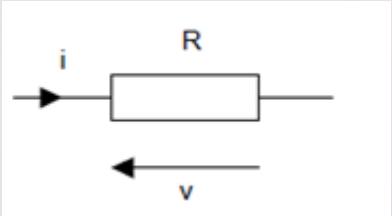


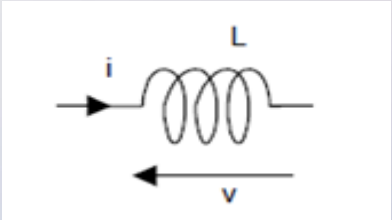


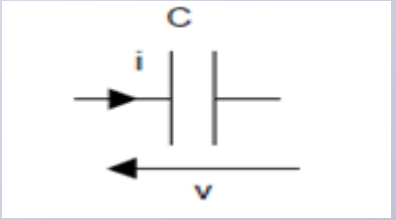


Elementary Dipoles R, L, and C

- ***Resistance***: A resistor is a dipole made of a conductive material and characterized by its resistance R expressed in Ohms (Ω)
- ***Coil (Inductance):*** It is a dipole made of a metallic conductor wound around a cylindrical support. When a current passes through it, it produces a magnetic field in the surrounding space. Inductance is expressed in henries (H).
- ***Capacitor:*** A capacitor is formed of two conductors, one completely surrounding the other (cylindrical capacitor), or two flat conductors separated by an insulator (flat capacitor). Capacitance is expressed in farads (F).

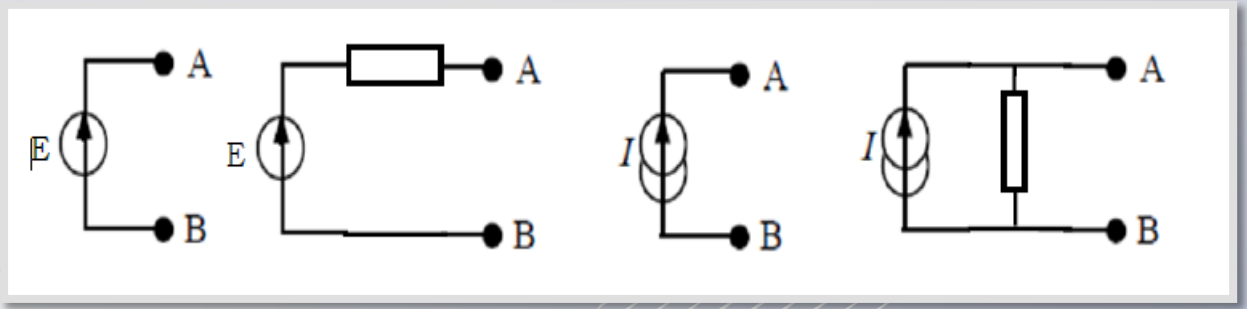


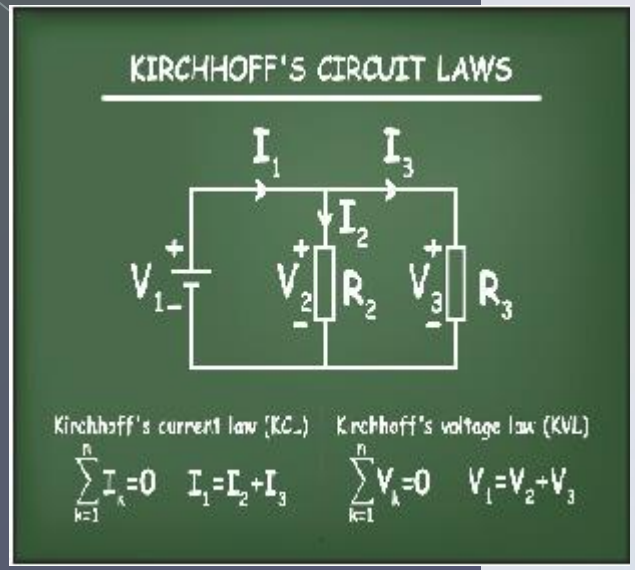




Voltage Source and Current Source

- ***A perfect voltage source*** is an active dipole that maintains a constant potential difference across its terminals regardless of the current it delivers. If the source has an internal resistance in series, it is referred to as a real voltage source
- ***An ideal current source*** provides a constant current regardless of the voltage appearing across its terminals. A real current source has a very high resistance placed in parallel.



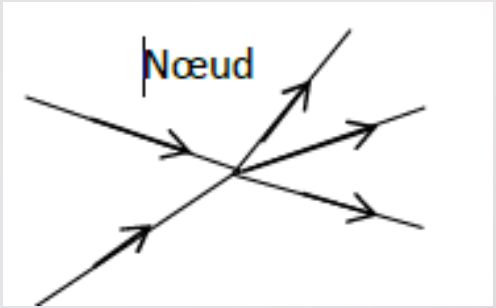


02

Kirchhoff's Laws

definitions

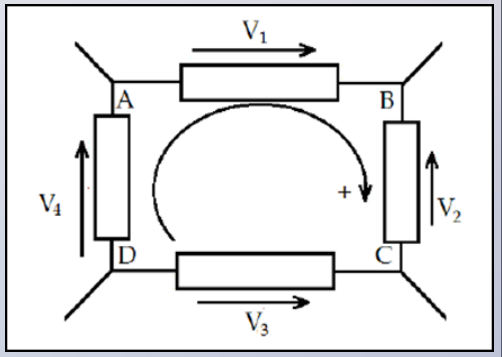
Node: A node is a point of connection between multiple dipoles (elements), at least three electrical wires that connect at the same location, and is often represented in a diagram by a dot.



Node Law: The algebraic sum of the currents entering a node is equal to the sum of the currents leaving the node.

$$I1 + I2 + I3 + I4 + I5 = 0$$

Loop: A loop is a closed path formed by a series of branches, but it never includes the same branch twice (it never passes through the same node more than once).



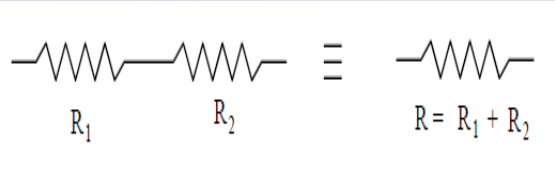
Loop Law: The sum of the voltages oriented along a loop of an electrical circuit is zero.

$$V1 - V2 - V3 + V4 = 0$$

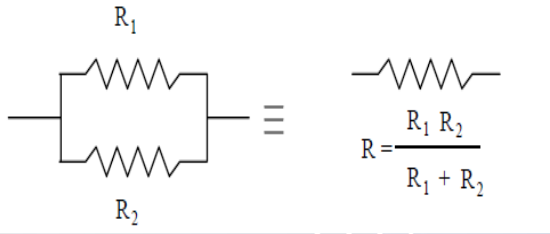
Association of Elementary Dipoles R and C

Association of Resistors

Series

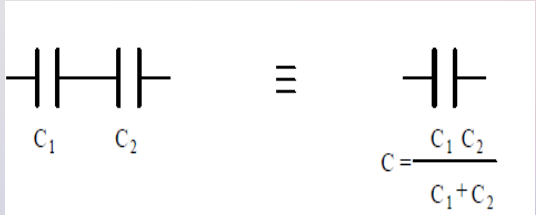


Parallel

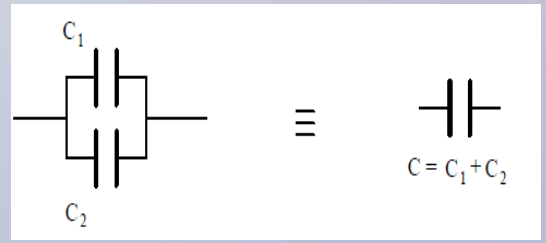


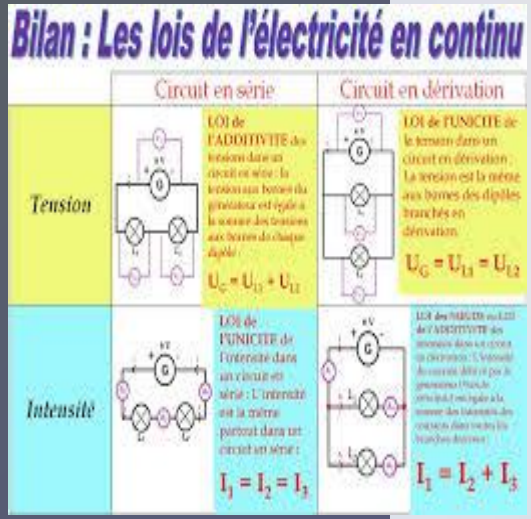
Association of Capacitors

Series



Parallel



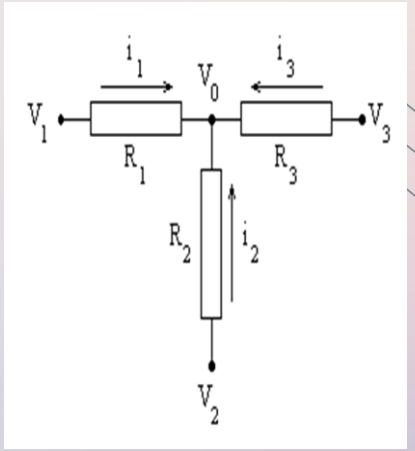


03

Fundamental Theorems

Millman's Theorem

- Millman's Theorem allows determining the potential at a node, given the potential at the end of each branch connected to that node.
- For example, we want to determine the potential V0 at the node connected to three branches. The potentials V_1, V_2 and V_3 as well as the resistances R_1, R_2 and R_3 are known. The currents I_1 , I_2 and I_3flowing towards the node are considered positive.
- Let's start by applying the known laws to the circuit.
- The node law allows us to write $I_1 + I_2 + I_3 = 0$.
- Ohm's law applied to the resistance R_1 gives $V_1 – V = R_1.I_1$.
- Ohm's law applied to the resistance R_2 gives $V_2 – V = R_2.I_2$.
- Ohm's law applied to the resistance R_3 gives $V_3 – V = R_3.I_3$.



The three Ohm's laws allow us to write the expressions for the currents. If we add these currents, we find

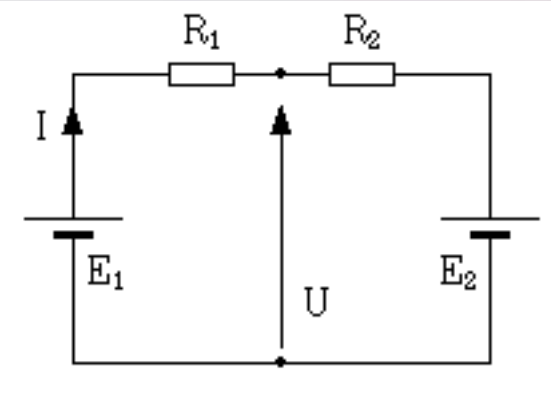
$$(V_1 – V)/R_1 + (V_2 – V)/R_2 + (V_3 – V)/R_3 = 0$$

A bit of manipulation on this expression allows us to isolate the potential V :

$$V = (V_1/R_1 + V_2/R_2 + V_3/R_3) / (1/R_1 + 1/R_2 + 1/R_3)$$

Superposition Theorem

In a circuit with multiple generators, the solution to the problem (the unknown voltages and currents) is the sum of the solutions found by considering only one generator at a time. To do this, each ideal voltage source is replaced by a short circuit, and each current source is replaced by an open circuit, except for the source whose influence we want to determine

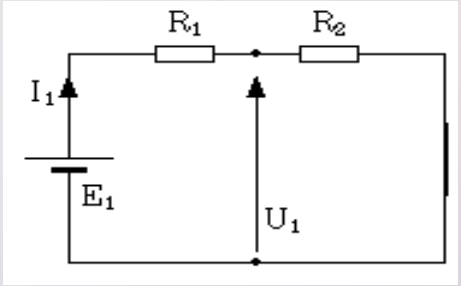


Superposition Theorem

Exemple

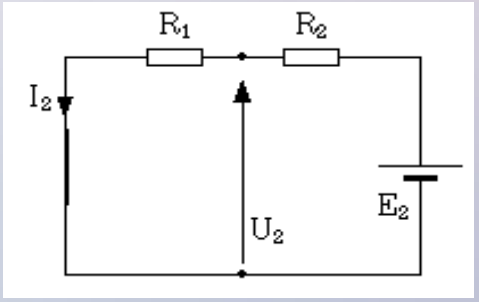
In the example above, we will start by removing E2 and calculate the voltage U with E1 alone. We then have a voltage divider:

$$U_1 = \frac{R_2}{R_1 + R_2} E_1$$



To obtain the contribution of E2, we then do the same thing by removing E1

$$U_2 = \frac{R_1}{R_1 + R_2} E_2$$



The total solution U is equal to the sum of the two previously found solutions:

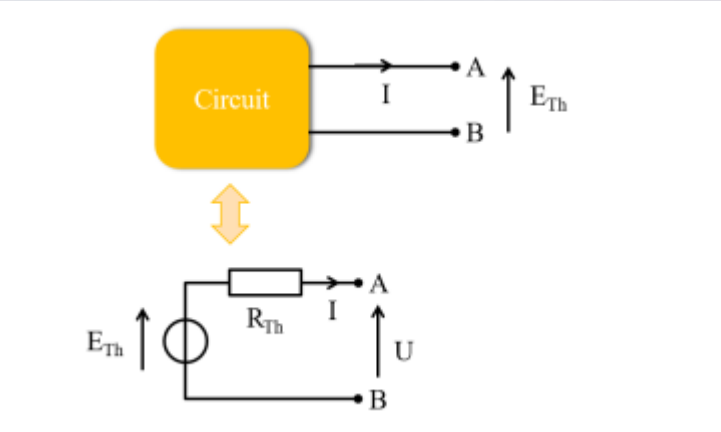
$$U = \frac{R_2}{R_1 + R_2} E_1 + \frac{R_1}{R_1 + R_2} E_2$$

Thevenin and Norton Theorems

The Thevenin and Norton theorems are the most important theorems in electronics. They allow for the resolution of complex problems in a minimal amount of time and by manipulating very few equations.

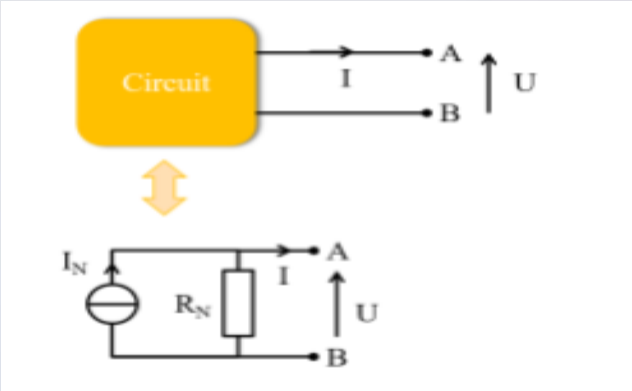
Thevenin theorem

The Thevenin theorem allows for the transformation of a circuit consisting of multiple loops with several voltage sources into a single-loop circuit composed of an equivalent resistance(R_{th} Thevenin resistance), and an equivalent voltage source(E_{th} Thevenin voltage).

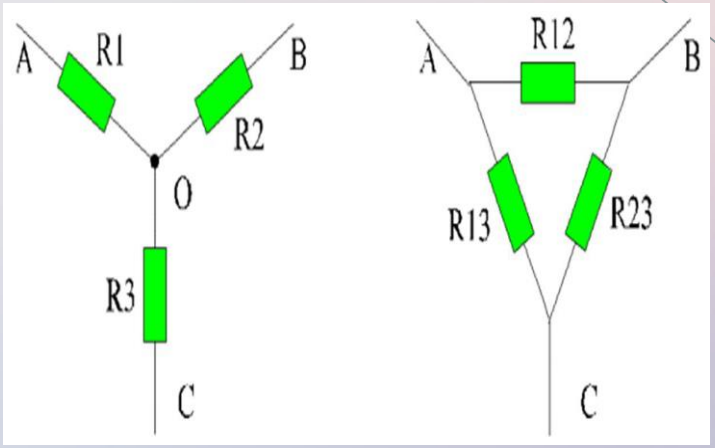


Norton Theorem

The Norton theorem is the dual of the Thevenin theorem: it allows for finding the equivalent real current source for a portion of a linear circuit containing sources and resistances



Kennelly Theorem

Presentation of the configurations in the form of a triangle (on the left) and a star (on the right). The Kennelly theorem, or triangle-star transformation, or Y-Δ transformation, or T-Π transformation, is a mathematical technique that simplifies the analysis of certain electrical networks.

This theorem, named in honor of Arthur Edwin Kennelly, allows for the conversion from a 'triangle' configuration (or Δ, or Π, depending on how the diagram is drawn) to a 'star' configuration (or, similarly, Y or T). The diagram on the side is drawn in the 'triangle-star' form; the diagrams below are in the T-Π form.