

Series of exercises n° 04

Exercise 01. Let  $G = \mathbb{R}^* \times \mathbb{R}$  and  $*$  is a composition law in  $G$  defined by :

$$\forall (x,y), (x',y') \in G : (x,y) * (x',y') = (xx', xy' + y)$$

- 1) Prove that  $*$  is an internal composition law.
- 2) Show that  $*$  is associative, has a neutral element and inverse element.
- 3) What can we deduce?
- 4) Prove that  $*$  is not commutative.

Exercise 02. In  $\mathbb{R}_+^*$  we define an internal composition law by :

$$\forall x, y \in \mathbb{R}_+^*, x * y = \sqrt{x^2 + y^2}$$

- a) Prove that  $*$  is commutative, associative and 0 is the neutral element
- b) Show that all elements of  $\mathbb{R}_+^*$  have no inverse for  $*$

Exercise 03. Let  $*$  an internal composition law defined on a set  $A = ]-1, 1[$  by :

$$\forall x, y \in A, x * y = \frac{x+y}{1+xy}$$

- 1) Prove that  $(A, *)$  is an abelian group
- 2) Let  $B = ]0, 1[$ , the set  $(B, *)$  is it a subgroup of  $(A, *)$ .

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Exercise 04. Let  $A = [1, +\infty]$  and  $\Delta$  is a composition law in  $A$  defined by:

$$\forall x, y \in A, x \Delta y = (x-1)(y-1) + 1$$

1) Show that  $\Delta$  is an internal composition law.

2) Prove that  $(A, \Delta)$  is a commutative group.

3) The set  $(\{\frac{4}{3}, \frac{3}{2}, 2, 3, 4\}, \Delta)$  is it a subgroup of a group  $(A, \Delta)$

Exercise 05. Let  $G = \mathbb{R} \times \mathbb{R}$  be a set, we define two internal composition laws in  $G$  by:

$$\forall (x, y), (x', y') \in G,$$

$$(x, y) + (x', y') = (x+x', y+y')$$

$$(x, y) * (x', y') = (xx', xy' + x'y)$$

1) Prove that  $(G, +)$  is an abelian group.

2) a) Show that the law  $*$  is commutative

b) Show that the law  $*$  is associative

c) Give the neutral element of  $*$

d) Prove that  $(G, +, *)$  a commutative ring

$(G, +, *)$  is it a ring with 1, (with unity)?

Exercise 06

① Prove that  $(\mathbb{Z}^2, +, \cdot)$  is a ring

② Prove that  $(\mathbb{Z}/n\mathbb{Z}, +, \cdot)$  is a commutative group

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EXERCISE 07. Let  $(E, \Delta)$  be an abelian group,  $E = \mathbb{R} \setminus \{3\}$

and,  $\forall a, b \in E$ :  $a \Delta b = ab + 3(a+b) + 6$

Let,  $f: (E, \Delta) \rightarrow (\mathbb{R}^*, \cdot)$ , defined by:  $f(a) = 2a + 3$

- Determine  $\lambda$  so that  $f$  is a group homomorphism

Exercise 08. Let,  $g: (\mathbb{R}^*, \cdot) \rightarrow (\mathbb{R}, +)$  be a bijective function, such that:  $g(x) = \ln x$

- Prove that  $g$  is an isomorphism.

Permutations.

Exercise 09. List all the elements of  $S_3$ .

Exercise 10. Consider  $S_4$ , let:

$$\varphi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}, \quad \varphi_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 1 \end{pmatrix}$$

① Is  $\varphi_3$  an element of  $S_4$ ? Explain?

② Determine the following:  $\varphi_1^2, \varphi_1 \varphi_2, \varphi_2 \varphi_1$ .

③ Find  $\varphi_1^{-1}, \varphi_2^{-1}, \varphi_2 \varphi_1^{-1}, (\varphi_1 \varphi_2)^{-1}$ .

Exercise 11. Consider the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 5 & 4 & 8 & 7 & 6 & 2 & 1 \end{pmatrix}$$

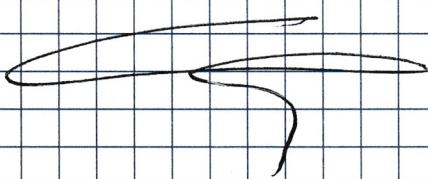
1) Compute  $\sigma^{-1}$ .

2) Write  $\sigma$  as a product of cycles, with disjoint supports

3) Give sign  $\sigma$ .

4) Write  $\sigma^2$  in table notation and in cycles notation

5) Give sign  $\sigma^2$ .



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