University 8 May 1945 **year 2024-2025**
first year Computer Engineering **Analysis 1**

Series of exercises No. 1 : Real numbers

Exercise 1 :

1. Prove that $\sqrt[3]{45+29\sqrt{2}}+\sqrt[3]{45-29\sqrt{2}}$ is integer number.
2. $\sqrt{4+2\sqrt{3}}+\sqrt{4-2\sqrt{3}}$ is not rational number
3. Let $(a,b)\in\mathbb{Q}^+\times\mathbb{Q}^+$ such that $\sqrt{ab}\notin\mathbb{Q}$. Prove that $\sqrt{a}+3\sqrt{b}\notin\mathbb{Q}$.
4. Knowing that if m is prime then $\sqrt{m}$ is irrational, show that $\sqrt{5}+\sqrt[3]{2}$ is irrational.

Exercise 2 :

1. $|x+y|=|x|+|y|\Longleftrightarrow x.y\geq 0$.
2. $||x|-|y||\leq|x-y|$
3. $|\sum_{k=1}^{n}x_k|\leq\sum_{k=1}^{n}|x_k|,\ \forall\{x_k\}_{k=1}^{n}\subset\mathbb{R}$
4. $|\sqrt{x}-\sqrt{y}|\leq\sqrt{|x-y|}$
5. $|x|+|y|\leq|x+y|+|x-y|$

Exercise 3 :
Let $[x]$ be the integer part of x; to show that $\forall x\in\mathbb{R}$

1. $[x+1]=[x]+1$
2. $[x]+[y]\leq[x+y]\leq[x]+[y]+1,\quad\forall x,y\in\mathbb{R}$
3. Calculate $\lim_{n\longrightarrow+\infty}\frac{1}{n^2}([x]+[2x]+[3x]+...+[nx])$

Exercise 4 :
Determine the sup, inf, max, and min of the following parts of $\mathbb{R}$.

$$A=[-1,1]\cup]2,4[,\quad B=[-1,\sqrt{2}]\cap\mathbb{Q},\quad C=\{\frac{5}{n};n\in\mathbb{N}^*\}$$

$$D=\{(-1)^n+\frac{3}{n^2},\ n\in\mathbb{N}^*\},E=\{\ \frac{2x-1}{x+4};\ |x-5|<2\},\quad F=\{\frac{m+n}{m.n},\ m,n\in\mathbb{N}^*\}$$

Exercise 5 :

1. Let A,B be a nonempty subsets of $\mathbb{R}$. define $A+B=\{x+y,\ x,y\in\mathbb{R}\}$.
   a) Prove that $\sup(A+B)=\sup(A)+\sup(B)$
2. Prove that if $B=\{\varepsilon x,x\in A\}$ then $\sup B=\varepsilon\sup A$.
3. Prove that $\sup(A\cup B)=\max(\sup A,\sup B)$.