University 8 May 1945 year 2024-2025
first year Computer Engineering Analysis 1

Series of exercises No. 1 : Complex numbers

Exercise 1 :

1. Compute real and imaginary part of :

$$z_1 = \frac{i-4}{2i-3}, \quad z_2 = \left(\frac{1-2i}{2-i}\right)^2, \quad z_3 = (2-i)(3+8i)$$

2. Compute the modulus and the conjugate of

$$z = (1+i)^6, \quad w = i^{17}.$$

3. Write in the "algebraic" form the following complex numbers

$$z = i^5 + i + 1, \quad w = (3+3i)^8, \quad y = \frac{(1+i)^{2000}}{(i-\sqrt{3})^{1000}}$$

4. Write a given complex number in the algebraic form

$$2e^{i\frac{\pi}{4}}, \quad \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

5. Write in the "trigonometric" form the following complex numbers

$$z_1 = 6i, \quad z_2 = \left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^7, \quad z_3 = \frac{1-i\sqrt{3}}{1+i}.$$

6. Write in the "exponential" form the following complex numbers : $z_1, z_2, z_1.z_2$ such that $z_1 = 4e^{i\frac{\pi}{4}}, z_2 = 3ie^{i\frac{\pi}{6}}$.
7. Compute the cube roots of $z = -8$.

Exercise 2 :

1. Prove that $(1+i)^6 = -8i$
2. Deduce solution of equation (E) :$z^2 = -8i$
3. Write the two solutions of (E) in algebraic form, and in exponential form.
4. Deduce from the first question a solution to the equation $z^3 = -8i$

Exercise 3 :
Establish the following equalities :
– $(\cos(\frac{\pi}{7}) + i\sin(\frac{\pi}{7}))(\frac{1-i\sqrt{3}}{2})(1+i) = \sqrt{2}(\cos(\frac{5\pi}{84}) + i\sin(\frac{5\pi}{84}))$
– $(1-i)(\cos(\frac{\pi}{5}) + i\sin(\frac{\pi}{5}))(\sqrt{3}-i) = 2\sqrt{2}(\cos(\frac{13\pi}{60}) - i\sin(\frac{13\pi}{60}))$
– $\frac{\sqrt{2}(\cos(\frac{\pi}{12})+i\sin(\frac{\pi}{12}))}{1+i} = \frac{\sqrt{3}}{2} - \frac{1}{2}i.$

Exercise 4 :(Additional)
Write in algebraic form following complex numbers :

$$z_1 = 2e^{\frac{2i\pi}{3}}, \quad z_2 = \sqrt{2}e^{i\frac{\pi}{8}}, \quad z_3 = (2e^{\frac{i\pi}{4}})(e^{-\frac{3i\pi}{4}}), \quad z_4 = \frac{2e^{\frac{i\pi}{3}}}{3e^{-\frac{5i\pi}{6}}}$$