
Set of exercises No.5 : Differentiable function

Exercise 1 :
Study the differentiability at x_0 of functions :

$$f(x) = (x-1)|x-1|;\ x_0 = 1, \quad g(x) = |x-1| + |x+1|, \quad x_0 = -1$$

h(x)=x+(x-1)arcsin $\sqrt{\frac{x}{x+1}}$; $x_0 = 1$.

Exercise 2 :
Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$.

1. Is f differentiable at $x = 0$?
2. Is f' continuous at $x = 0$?

Under what condition does the function

$$g(x) = \begin{cases} x^n \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

admits a continuous derivative at the point $x_0 = 0$?

Exercise 3 :
Determine the values of α and β for which the function

$$f(x) = \begin{cases} \alpha + \beta x^2, & \text{if } |x| < 1 \\ \frac{1}{|x|}, & \text{if } |x| \geq 1, \end{cases}$$

is continuous and differentiable on $\mathbb{R}$.

Exercise 4 :
Calculate the derivatives of the following functions

$$f(x) = \ln(1 + \ln(x)), \quad g(x) = e^{\ln(x)+2}, \quad h(x) = \begin{cases} x(1 + e^{\frac{1}{x}})^{-1}, & \text{if}; x < 0 \\ \ln(3 + \sqrt[3]{x^5}), & \text{if } \ x \geq 0. \end{cases}$$

L(x)=$\tan^2(1-2x)$

Exercise 5 :
Let the function $y(x) = \frac{2-x^2}{x^2}$.

1. Show that $y'(x) \neq 0$ on $[-1, 1]$
2. Is there a contradiction with the theorem of Rolle ?