University 8 May 1945 — **year 2024-2025**
first year Computer Engineering — **Analysis 1**

Set No.3 : Real sequences

Exercise 1 :
Calculate the limit of the following sequences with the general term

$$u_n = \frac{n}{n^2+1} + \frac{n}{n^2+2} + \frac{n}{n^2+3} + \cdots + \frac{n}{n^2+n}, \quad v_n = \frac{1}{n!}(1! + 2! + \cdots + n!),$$

$$w_n = \sqrt{n}(\sqrt{n-1} + \sqrt{n}.$$

Exercise 2 :

1. Let the sequence (u_n) define by the general term :

$$u_n = \frac{2^n + (-1)^n}{2^n}, n \in \mathbb{N}$$

   Show that $\lim u_n = 1$. For what values of n, $|u_n - 1|$ less then ε and less then 10^{-4}.
2. Using the definition of a sequence, show that :

$$\lim_{n \longrightarrow +\infty} \frac{2\ln(n+1)}{\ln n} = 0; \quad \lim_{n \longrightarrow +\infty} \frac{-5n^2 - 3}{4n} = -\infty, \quad \lim_{n \longrightarrow +\infty} \ln(\ln n) = +\infty$$

Exercise 3 :
Among the following sequences, show which ones are bounded

$$u_n = n^{(-1)^n}, \quad v_n = \Sigma_{k=1}^{n} \frac{1}{k+n}.$$

Study the monotony of the following sequences and deduce possibly their nature :

$$u_n = \Sigma_{k=1}^{n} \frac{k^2}{n^2}, \quad v_n = \Sigma_{k=1}^{n} \frac{1}{k+n}, \quad w_n = \frac{1 \times 3 \times 5 \times \cdots (2n-1)}{2 \times 4 \times 6 \times \cdots \times (2n)}.$$

Exercise 4 :
Let (u_n) and (v_n) be two sequences defined by $0 < u_0 < v_0$ and $u_{n+1} = \frac{2u_n v_n}{u_n + v_n}$ and $v_{n+1} = \frac{u_n + v_n}{2}$

1. Prove that $\forall n \in N, 0 < u_n < v_n$
2. Show that the two sequences u_n and v_n are convergent
3. Deduce that they converge towards the same limit. calculate this limit.

Exercise 5 :
Let the sequence (u_n) be defined by :

$$u_n = \begin{cases} u_1 = \frac{1}{2}, \\ u_{n+1} = u_n^2 + \frac{3}{16} \end{cases}.$$

1. Prove that $\forall n \geq 1$, $\frac{1}{4} < u_n < \frac{3}{4}$.
2. Study the nature of the sequence u_n and calculate its limit if it is convergent.
3. Let $E = \{u_n, n \geq 1\}$. Determine $\sup E$ and $\inf E$.

Exercise 6 : Find $\inf u_n$, $\sup u_n$, $\liminf u_n$ and $\limsup u_n$ if :

$$u_n = \frac{(-1)^n}{n} + \frac{1 + (-1)^n}{2}; \quad u_n = 1 + \frac{n}{n+1} \cos \frac{n\pi}{2}.$$