University 8 May 1945 **year 2024-2025**
first year Computer Engineering **Analysis 1**

Set of exercises No.4 : Real functions

Exercise 1 :
Determine the domain of definition of the following functions

$$f_1(x) = \frac{1}{\sqrt{\sin x}}, \quad f_2(x) = e^{\frac{1}{1-x}}\sqrt{x^2-1}, \quad f_3(x) = (1+\ln x)^{\frac{1}{x}},$$

$f_4(x) = \frac{1}{[x]}$, $\quad f_5(x) = \begin{cases} \sqrt{x-2}, & \text{if } x > 1 \\ \ln(x+2), & \text{if } x \leq 1. \end{cases}$

Exercise 2 :
Calculate the limit of the following functions

1. $\lim\limits_{x \longrightarrow +\infty} x \sin \dfrac{1}{x}$
2. $\lim\limits_{x \longrightarrow 0} \dfrac{\tan x}{x}$
3. $\lim\limits_{x \longrightarrow 0} \dfrac{x^2 + |x|}{x^2 - |x|}$
4. $\lim\limits_{x \longrightarrow 5} \dfrac{\sqrt{(x-5)^2}}{x-5}$
5. $\lim\limits_{x \longrightarrow \frac{\pi}{4}} \dfrac{\sin x - \cos x}{1 - \tan x}$
6. $\lim\limits_{x \longrightarrow \frac{\pi}{4}} \dfrac{\tan x - 1}{x - \frac{\pi}{4}}$
7. $\lim\limits_{x \longrightarrow 1} (1-x)\tan(\dfrac{x\pi}{2})$, $\quad$ such that $\tan(\alpha + \frac{\pi}{2}) = -\frac{1}{\tan(\alpha)}$.

Exercise 3 :
Using the definition of the limit of a function, show that

$$1)\ \lim_{x \longrightarrow 2} \frac{2x-1}{2x+1} = \frac{3}{2}, \quad 2)\ \lim_{x \longrightarrow 3} \sqrt{x+1} = 2, \quad 3)\ \lim_{x \longrightarrow -3^+} \frac{4}{x+3} = +\infty.$$

Exercise 4 :
Study the continuity of functions

$$f(x) = \begin{cases} x\sin(\frac{3}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \quad g(x) = \begin{cases} \frac{\sqrt{x+1}-1}{\tan x} & \text{if } \ x \neq 0 \\ \frac{1}{2} & \text{if } x = 0 \end{cases} \ ; \ h(x) = \begin{cases} \frac{\sin(x-2)}{x^2-2x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

Determine a and b so that the function f is continuous at $x_0 = 2$

$$f(x) = \begin{cases} \frac{x^2+x-a}{x-2}, & \text{if } x > 2 \\ \frac{2x+b}{3}, & \text{if } x \leq 2. \end{cases}$$

Exercise 5 :
Can we extend by continuity on $\mathbb{R}$ the functions :

$$f(x) = \frac{x}{|x|}, \quad g(x) = \frac{1-\cos(\sqrt{|x|})}{|x|}, \quad h(x) = 1 - x - \frac{2x \ln|x|}{x+1}.$$

Exercise 6 :
Let f be a function defined on $\mathbb{R}$ by

$$f(x) = x^5 - x^3 + x - 2$$

1. Show that $f(x) = 0$ admits a solution α with $1 < \alpha < 2$.
2. Determine the sign of the function $f(x)$, $\forall x \in \mathbb{R}$.