2.2 Limitations of Memory System Performance*

The effective performance of a program on a computer relies not just on the speed of the processor but also on the ability of the memory system to feed data to the processor. At the logical level, a memory system, possibly consisting of multiple levels of caches, takes in a request for a memory word and returns a block of data of size *b* containing the requested word after *l* nanoseconds. Here, *l* is referred to as the *latency* of the memory. The rate at which data can be pumped from the memory to the processor determines the *bandwidth* of the memory system.

It is very important to understand the difference between latency and bandwidth since different, often competing, techniques are required for addressing these. As an analogy, if water comes out of the end of a fire hose 2 seconds after a hydrant is turned on, then the latency of the system is 2 seconds. Once the flow starts, if the hose pumps water at 1 gallon/second then the 'bandwidth' of the hose is 1 gallon/second. If we need to put out a fire immediately, we might desire a lower latency. This would typically require higher water pressure from the hydrant. On the other hand, if we wish to fight bigger fires, we might desire a higher flow rate, necessitating a wider hose and hydrant. As we shall see here, this analogy works well for memory systems as well. Latency and bandwidth both play critical roles in determining memory system performance. We examine these separately in greater detail using a few examples.

To study the effect of memory system latency, we assume in the following examples that a memory block consists of one word. We later relax this assumption while examining the role of memory bandwidth. Since we are primarily interested in maximum achievable performance, we also assume the best case cache-replacement policy. We refer the reader to the bibliography for a detailed discussion of memory system design.

Example 2.2 Effect of memory latency on performance

Consider a processor operating at 1 GHz (1 ns clock) connected to a DRAM with a latency of 100 ns (no caches). Assume that the processor has two multiply-add units and is capable of executing four instructions in each cycle of 1 ns. The peak processor rating is therefore 4 GFLOPS. Since the memory latency is equal to 100 cycles and block size is one word, every time a memory request is made, the processor must wait 100 cycles before it can process the data. Consider the problem of computing the dot-product of two vectors on such a platform. A dot-product computation performs one multiply-add on a single pair of vector elements, i.e., each floating point operation requires one data fetch. It is easy to see that the peak speed of this computation is limited to one floating point operation every 100 ns, or a speed of 10 MFLOPS, a very small fraction of the peak processor rating. This example highlights the need for effective memory system performance in achieving high computation rates. ■

2.2.1 Improving Effective Memory Latency Using Caches

Handling the mismatch in processor and DRAM speeds has motivated a number of architectural innovations in memory system design. One such innovation addresses the speed mismatch by placing a smaller and faster memory between the processor and the DRAM. This memory, referred to as the cache, acts as a low-latency high-bandwidth storage. The data needed by the processor is first fetched into the cache. All subsequent accesses to data items residing in the cache are serviced by the cache. Thus, in principle, if a piece of data is repeatedly used, the effective latency of this memory system can be reduced by the cache. The fraction of data references satisfied by the cache is called the cache *hit ratio* of the computation on the system. The effective computation rate of many applications is bounded not by the processing rate of the CPU, but by the rate at which data can be pumped into the CPU. Such computations are referred to as being *memory bound*. The performance of memory bound programs is critically impacted by the cache hit ratio.

Example 2.3 Impact of caches on memory system performance

As in the previous example, consider a 1 GHz processor with a 100 ns latency DRAM. In this case, we introduce a cache of size 32 KB with a latency of 1 ns or one cycle (typically on the processor itself). We use this setup to multiply two matrices A and B of dimensions 32 x 32. We have carefully chosen these numbers so that the cache is large enough to store matrices A and B, as well as the result matrix C. Once again, we assume an ideal cache placement strategy in which none of the data items are overwritten by others. Fetching the two matrices into the cache corresponds to fetching 2K words, which takes approximately 200 µs. We know from elementary algorithmics that multiplying two n x n matrices takes $2n^3$ operations. For our problem, this corresponds to 64K operations, which can be performed in 16K cycles (or 16 µs) at four instructions per cycle. The total time for the computation is therefore approximately the sum of time for load/store operations and the time for the computation itself, i.e., 200+16 µs. This corresponds to a peak computation rate of 64K/216 or 303 MFLOPS. Note that this is a thirty-fold improvement over the previous example, although it is still less than 10% of the peak processor performance. We see in this example that by placing a small cache memory, we are able to improve processor utilization considerably. ■

The improvement in performance resulting from the presence of the cache is based on the assumption that there is repeated reference to the same data item. This notion of repeated reference to a data item in a small time window is called *temporal locality* of reference. In our example, we had $O(n^2)$ data accesses and $O(n^3)$ computation. (See the Appendix for an explanation of the O notation.) Data reuse is critical for cache performance because if each data item is used only once, it would still have to be fetched once per use from the DRAM, and therefore the DRAM latency would be paid for each operation.

2.2.2 Impact of Memory Bandwidth

Memory bandwidth refers to the rate at which data can be moved between the processor and memory. It is determined by the bandwidth of the memory bus as well as the memory units. One commonly used technique to improve memory bandwidth is to increase the size of the memory blocks. For an illustration, let us relax our simplifying restriction on the size of the memory block and assume that a single memory request returns a contiguous block of four words. The single unit of four words in this case is also referred to as a *cache line*. Conventional computers typically fetch two to eight words together into the cache. We will see how this helps the performance of applications for which data reuse is limited.

Example 2.4 Effect of block size: dot-product of two vectors

Consider again a memory system with a single cycle cache and 100 cycle latency DRAM with the processor operating at 1 GHz. If the block size is one word, the processor takes 100 cycles to fetch each word. For each pair of words, the dot-product performs one multiply-add, i.e., two FLOPs. Therefore, the algorithm performs one FLOP every 100 cycles for a peak speed of 10 MFLOPS as illustrated in Example 2.2.

Now let us consider what happens if the block size is increased to four words, i.e., the processor can fetch a four-word cache line every 100 cycles. Assuming that the vectors are laid out linearly in memory, eight FLOPs (four multiply-adds) can be performed in 200 cycles. This is because a single memory access fetches four consecutive words in the vector. Therefore, two accesses can fetch four elements of each of the vectors. This corresponds to a FLOP every 25 ns, for a peak speed of 40 MFLOPS. Note that increasing the block size from one to four words did not change the latency of the memory system. However, it increased the bandwidth four-fold. In this case, the increased bandwidth of the memory system enabled us to accelerate the dot-product algorithm which has no data reuse at all.

Another way of quickly estimating performance bounds is to estimate the cache hit ratio, using it to compute mean access time per word, and relating this to the FLOP rate via the underlying algorithm. For example, in this example, there are two DRAM accesses (cache misses) for every eight data accesses required by the algorithm. This corresponds to a cache hit ratio of 75%. Assuming that the dominant overhead is posed by the cache misses, the average memory access time contributed by the misses is 25% at 100 ns (or 25 ns/word). Since the dot-product has one operation/word, this corresponds to a computation rate of 40 MFLOPS as before. A more accurate estimate of this rate would compute the average memory access time as 0.75 x 1 + 0.25 x 100 or 25.75 ns/word. The corresponding computation rate is 38.8 MFLOPS. ■

Physically, the scenario illustrated in Example 2.4 corresponds to a wide data bus (4 words or 128 bits) connected to multiple memory banks. In practice, such wide buses are expensive to construct. In a more practical system, consecutive words are sent on the memory bus on subsequent bus cycles after the first word is retrieved. For example, with a 32 bit data bus, the first word is put on the bus after 100 ns (the associated latency) and one word is put on each subsequent bus cycle. This changes our calculations above slightly since the entire cache line becomes available only after 100 + 3 x (memory bus cycle) ns. Assuming a data bus operating at 200 MHz, this adds 15 ns to the cache line access time. This does not change our bound on the execution rate significantly.

The above examples clearly illustrate how increased bandwidth results in higher peak computation rates. They also make certain assumptions that have significance for the programmer. The data layouts were assumed to be such that consecutive data words in memory were used by successive instructions. In other words, if we take a computation-centric view, there is a *spatial locality* of memory access. If we take a data-layout centric point of view, the computation is ordered so that successive computations require contiguous data. If the computation (or access pattern) does not have spatial locality, then effective bandwidth can be much smaller than the peak bandwidth.

An example of such an access pattern is in reading a dense matrix column-wise when the matrix has been stored in a row-major fashion in memory. Compilers can often be relied on to do a good job of restructuring computation to take advantage of spatial locality.

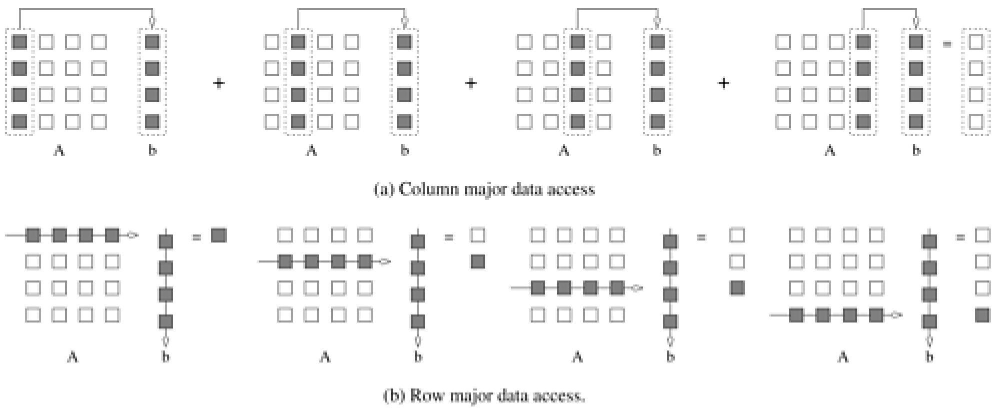
Example 2.5 Impact of strided access

Consider the following code fragment:

```
for (i = 0; i < 1000; i++)
        column_sum[i] = 0.0;
        for (j = 0; j < 1000; j++)
                column_sum[i] += b[j][i];
```

The code fragment sums columns of the matrix `b` into a vector `column_sum`. There are two observations that can be made: (i) the vector `column_sum` is small and easily fits into the cache; and (ii) the matrix `b` is accessed in a column order as illustrated in Figure 2.2(a). For a matrix of size 1000 x 1000, stored in a row-major order, this corresponds to accessing every 1000^{th} entry. Therefore, it is likely that only one word in each cache line fetched from memory will be used. Consequently, the code fragment as written above is likely to yield poor performance. ■

Figure 2.2. Multiplying a matrix with a vector: (a) multiplying column-by-column, keeping a running sum; (b) computing each element of the result as a dot product of a row of the matrix with the vector.



The above example illustrates problems with strided access (with strides greater than one). The lack of spatial locality in computation causes poor memory system performance. Often it is possible to restructure the computation to remove strided access. In the case of our example, a simple rewrite of the loops is possible as follows:

Example 2.6 Eliminating strided access

Consider the following restructuring of the column-sum fragment:

```
for (i = 0; i < 1000; i++)
        column_sum[i] = 0.0;
for (j = 0; j < 1000; j++)
        for (i = 0; i < 1000; i++)
                column_sum[i] += b[j][i];
```

In this case, the matrix is traversed in a row-order as illustrated in Figure 2.2(b). However, the reader will note that this code fragment relies on the fact that the vector `column_sum` can be retained in the cache through the loops. Indeed, for this particular example, our assumption is reasonable. If the vector is larger, we would have to break the iteration space into blocks and compute the product one block at a time. This concept is also called *tiling* an iteration space. The improved performance of this loop is left as an exercise for the reader.

So the next question is whether we have effectively solved the problems posed by memory latency and bandwidth. While peak processor rates have grown significantly over the past decades, memory latency and bandwidth have not kept pace with this increase. Consequently, for typical computers, the ratio of peak FLOPS rate to peak memory bandwidth is anywhere between 1 MFLOPS/MBs (the ratio signifies FLOPS per megabyte/second of bandwidth) to 100 MFLOPS/MBs. The lower figure typically corresponds to large scale vector supercomputers and the higher figure to fast microprocessor based computers. This figure is very revealing in that it tells us that on average, a word must be reused 100 times after being fetched into the full bandwidth storage (typically L1 cache) to be able to achieve full processor utilization. Here, we define full-bandwidth as the rate of data transfer required by a computation to make it processor bound.

The series of examples presented in this section illustrate the following concepts:

- Exploiting spatial and temporal locality in applications is critical for amortizing memory latency and increasing effective memory bandwidth.
- Certain applications have inherently greater temporal locality than others, and thus have greater tolerance to low memory bandwidth. The ratio of the number of operations to number of memory accesses is a good indicator of anticipated tolerance to memory bandwidth.
- Memory layouts and organizing computation appropriately can make a significant impact on the spatial and temporal locality.

2.2.3 Alternate Approaches for Hiding Memory Latency

Imagine sitting at your computer browsing the web during peak network traffic hours. The lack of response from your browser can be alleviated using one of three simple approaches:

(i) we anticipate which pages we are going to browse ahead of time and issue requests for them in advance; (ii) we open multiple browsers and access different pages in each browser, thus while we are waiting for one page to load, we could be reading others; or (iii) we access a whole bunch of pages in one go – amortizing the latency across various accesses. The first approach is called *prefetching*, the second *multithreading*, and the third one corresponds to