

Exercise 3 : Task 3

$$\begin{aligned}\langle f_1, f_2 \rangle &= \sum_{n \in \mathbb{Z}} f_1[n] f_2[n] \\&= \sum_{n \in \mathbb{Z}} (\delta[n] - \tfrac{1}{2} \delta[n-1]) f_2[n] \\&= f_2[0] - \tfrac{1}{2} f_2[1] \\&= 0 - \tfrac{1}{2} \left(\tfrac{1}{\sqrt{2}}\right) \\&= -\tfrac{1}{2}\end{aligned}$$

$$\begin{aligned}\langle f_1, f_3 \rangle &= \sum_{n \in \mathbb{Z}} f_1[n] f_3[n] \\&= \sum_{n \in \mathbb{Z}} (\delta[n] - \tfrac{1}{2} \delta[n-1]) f_3[n] \\&= f_3[0] - \tfrac{1}{2} f_3[1] \\&= 1 - \tfrac{1}{2} \left(\tfrac{1}{i\sqrt{1}}\right) \\&= \tfrac{1}{2}\end{aligned}$$

$$\begin{aligned}\langle f_2, f_3 \rangle &= \sum_{n \in \mathbb{Z}} f_2[n] f_3[n] \\&= \sum_{n=1}^{\infty} \left(\tfrac{1}{n^2}\right) \left(\tfrac{1}{i|n|}\right) \\&= \sum_{n=1}^{\infty} \tfrac{1}{n^3} \\&= \zeta(3) \\&\approx 1.202\end{aligned}$$

$$\begin{aligned}\langle f_4, f_5 \rangle &= \sum_{n \in \mathbb{Z}} f_4[n] f_5[n] \\&= \sum_{n \in \mathbb{Z}} (1) \left(\tfrac{1}{2}\right)^n u[n] \\&= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n \\&= \frac{1 - (\frac{1}{2})^{\infty+1}}{1 - (\frac{1}{2})} \\&= \frac{1}{\frac{1}{2}} \\&= 2\end{aligned}$$

$$\begin{aligned}\langle f_5, f_6 \rangle &= \sum_{n \in \mathbb{Z}} f_5[n] f_6[n] \\&= \sum_{n \in \mathbb{Z}} \left(\tfrac{1}{2}\right)^n u[n] \left(\tfrac{3}{2}\right)^n u[n] \\&= \sum_{n=0}^{\infty} \left(\tfrac{3}{4}\right)^n \\&= \frac{1 - (\frac{3}{4})^{\infty+1}}{1 - (\frac{3}{4})} \\&= \frac{1}{\frac{1}{4}} \\&= 4\end{aligned}$$