Exercise 4 : Task 4

$f_o[n] = e^{-n}$ $\qquad n = 0, 1, \ldots N-1$

$f_1[n] = \begin{cases} e^{-n} & n = 0, 1, \ldots N-1 \\ 0 & n = N, N+1, \ldots 2N-1 \end{cases}$

$f[n] = \begin{cases} e^{-n} & n = 0, 1, \ldots N-1 \\ 0 & \text{for all other } n \in \mathbb{Z} \end{cases}$

1) $F_o[k] = \mathcal{F}\{f_o[n]\}$

$F_o[k] = \sum_{n=0}^{N-1} f_o[n] \, e^{-j2\pi n \frac{k}{N\Delta T}}$ $\qquad , \quad \Delta T = 1$

$F_o[k] = \sum_{n=0}^{N-1} e^{-n} \, e^{-j2\pi n \frac{k}{N}}$

$F_o[k] = \dfrac{1 - e^{-N}e^{-j2\pi k}}{1 - e^{-1}e^{-j2\pi k/N}}$ $\qquad , \quad e^{-j2\pi m} = 1$

$F_o[k] = \dfrac{1 - e^{-N}}{1 - e^{-1}e^{-j2\pi k/N}}$

2) $F_1[k] = \mathcal{F}\{f_1[n]\}$

$F_1[k] = \sum_{n=0}^{2N-1} f_1[n] \, e^{-j2\pi n \frac{k}{2N\Delta T}}$ $\qquad , \quad \Delta T = 1$

$F_1[k] = \sum_{n=0}^{N-1} e^{-n} \, e^{-j\pi n \, k/N}$

$F_1[k] = \dfrac{1 - e^{-N}e^{-j\pi k}}{1 - e^{-1}e^{-j\pi k/N}}$ $\qquad e^{-j\pi k} = (-1)^k$

$F_1[k] = \dfrac{1 - (-1)^k \, e^{-N}}{1 - e^{-1}e^{-j\pi k/N}}$

3) for $n = 0, 1, \ldots N-1$, $\qquad F_o[n] = F_1[2n]$

4) $F(e^{j\omega}) = \sum_{n \in \mathbb{Z}} f[n] \, e^{-j\omega n}$

$F(e^{j\omega}) = \sum_{n=0}^{N-1} e^{-n} \, e^{-j\omega n}$

$F(e^{j\omega}) = \dfrac{1 - e^{-N}e^{-j\omega N}}{1 - e^{-1}e^{-j\omega}}$

5)
$F_o[k]$ and, $F_1[k]$ represent the frequency components of the same function, but $F_1[k]$ is sampled twice as densely in the frequency space. Both $F_o[k]$ and $F_1[k]$ are discretized versions of $F(e^{j\omega})$ within the frequency domain.

6)
Zero padding in the time domain increases the frequency domain resolution of the DFT.