Exercise 4 : Task 2

(4.2) a)

$$\delta(t-t_0) = \begin{cases} \infty & , \quad t = t_0 \\ 0 & , \quad t \neq t_0 \end{cases}$$

(b)

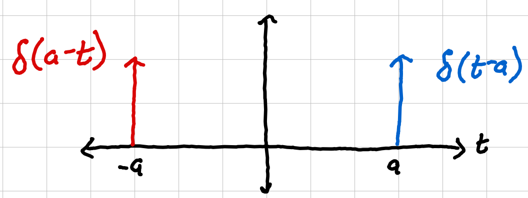
$$\delta(x-x_0) = \begin{cases} 1 & , \quad x = x_0 \\ 0 & , \quad x \neq x_0 \end{cases}$$

(c)

$$\delta(a-t) =$$
$$\delta(-(t-a))$$

$$\delta(t-a) \neq \delta(-(t-a))$$

These two functions will always be mirrored across $x=0$. $\delta(t-a)$ has an impulse at $t=a$, $\delta(a-t)$ has an impulse at $t=-a$



(4.3) → Repeat with $f(t) = A$ for $0 \leq t \leq T$

(a)

$$f(t-t_0) = \begin{cases} A & , \quad t_0 \leq t < T+t_0 \\ 0 & , \quad \text{else} \end{cases}$$

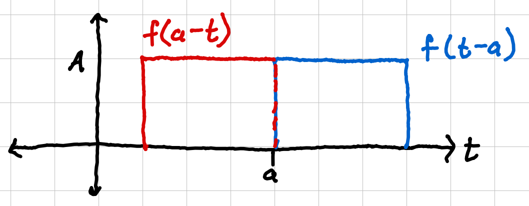
(b)

$$f(x-x_0) = \begin{cases} A & , \quad x_0 \leq x < T+x_0 \\ 0 & , \quad \text{else} \end{cases}$$

(c)

$$f(a-t) = f(-(t-a)) = \begin{cases} A & a \geq t > a-T \\ 0 & \text{else} \end{cases}$$

$$f(t-a) = \begin{cases} A & a \leq t < T+a \\ 0 & \text{else} \end{cases}$$



$a \geq t > a-T$
$a-T \leq t \leq a$

These two functions are mirrored across the line $x=a$.

$$0 \leq a-t < T$$
$$-a \leq -t < T-a$$
$$a \geq t > a-T$$
$$a-T < t \leq a$$

(4.11)

$$\mathcal{F}\{(f*h)(t)\} =$$

$$\int_{-\infty}^{\infty} (f*h)(t)\, e^{-j2\pi\mu t}\, dt =$$

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau\right] e^{-j2\pi\mu t}\, dt =$$

$$\int_{-\infty}^{\infty} f(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau)\, e^{-j2\pi\mu t}\, dt\right] d\tau =$$

$$\int_{-\infty}^{\infty} f(\tau)\, H(\mu)\, e^{-j2\pi\mu\tau}\, d\tau =$$

$$H(\mu) \int_{-\infty}^{\infty} f(\tau)\, e^{-j2\pi\mu\tau}\, d\tau =$$

$$H(\mu)\, F(\mu) =$$

$$(F \cdot H)(\mu)$$

$$\mathcal{F}^{-1}\{(F*H)(\mu)\} =$$

$$\int_{-\infty}^{\infty} (F*H)(\mu)\, e^{j2\pi\mu t}\, d\mu =$$

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} F(\nu)\, H(\mu-\nu)\, d\nu\right] e^{j2\pi\mu t}\, d\mu =$$

$$\int_{-\infty}^{\infty} F(\nu) \left[\int_{-\infty}^{\infty} H(\mu-\nu)\, e^{j2\pi\mu t}\, d\nu\right] d\mu =$$

$$\int_{-\infty}^{\infty} F(\nu)\, h(t)\, e^{j2\pi\nu t}\, d\mu =$$

$$h(t) \int_{-\infty}^{\infty} F(\nu)\, e^{j2\pi\nu t}\, d\mu =$$

$$h(t)\, f(t) =$$

$$(f \cdot h)(t)$$