Task 3

$$\langle f_1, f_2 \rangle = \sum_{n \in \mathbb{Z}} f_1[n]\, f_2[n]$$
$$= \sum_{n \in \mathbb{Z}} \left(\delta[n] - \tfrac{1}{2}\delta[n-1] \right) f_2[n]$$
$$= f_2[0] - \tfrac{1}{2} f_2[1]$$
$$= 0 - \tfrac{1}{2}\left(\tfrac{1}{1^2}\right)$$
$$= -\tfrac{1}{2}$$

$$\langle f_1, f_3 \rangle = \sum_{n \in \mathbb{Z}} f_1[n]\, f_3[n]$$
$$= \sum_{n \in \mathbb{Z}} \left(\delta[n] - \tfrac{1}{2}\delta[n-1] \right) f_3[n]$$
$$= f_3[0] - \tfrac{1}{2} f_3[1]$$
$$= 1 - \tfrac{1}{2}\left(\tfrac{1}{|1|}\right)$$
$$= \tfrac{1}{2}$$

$$\langle f_2, f_3 \rangle = \sum_{n \in \mathbb{Z}} f_2[n]\, f_3[n$$
$$= \sum_{n=1}^{\infty} \left(\tfrac{1}{n^2}\right)\left(\tfrac{1}{|n|}\right)$$
$$= \sum_{n=1}^{\infty} \tfrac{1}{n^3}$$
$$= \zeta(3)$$
$$\approx 1.202$$

$$\langle f_4, f_5 \rangle = \sum_{n \in \mathbb{Z}} f_4[n]\, f_5[n]$$
$$= \sum_{n \in \mathbb{Z}} (1)\left(\tfrac{1}{2}\right)^n u[n]$$
$$= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n$$
$$= \frac{1 - (1/2)^{\infty+1}}{1 - (1/2)}$$
$$= \frac{1}{1/2}$$
$$= 2$$

$$\langle f_5, f_6 \rangle = \sum_{n \in \mathbb{Z}} f_5[n]\, f_6[n]$$
$$= \sum_{n \in \mathbb{Z}} \left(\tfrac{1}{2}\right)^n u[n] \left(\tfrac{3}{2}\right)^n u[n]$$
$$= \sum_{n=0}^{\infty} \left(\tfrac{3}{4}\right)^n$$
$$= \frac{1 - (3/4)^{\infty+1}}{1 - (3/4)}$$
$$= \frac{1}{1/4}$$
$$= 4$$