Exercise Set 3

Fys-2010

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Exercise 1

From the book (3rd edition): 3.14 (a)

(4th edition): 3.26 (a)

Exercise 2

Let h be the discrete signal $h[n] = \frac{1}{2}\delta[n] + \frac{1}{3}\delta[n-1]$, with δ being the Kronecker delta function.

1. A function f which satisfies $f[n] = 0$ for all $n < 0$ is called a causal function. We want to find a **causal** solution $g[n]$ of the equation $(h * g)[n] = \delta[n]$.

   (a) Write explicitly the finite difference equation $(h * g)[n] = \delta[n]$.

   (b) What is the value of $g[0]$?

   (c) Give the relationship between $g[n]$ and $g[n-1]$ for all $n > 0$.

   (d) Deduce from the previous results the expression of $g[n]$.

2. Does the solution belong to the space of absolutely convergent sequences $\ell_1()$?

Exercise 3

Reminder : the partial sum of a geometric series is $\sum_{n=0}^{N} q^n = \frac{1-q^{N+1}}{1-q}$ for $q \neq 1$. For the calculations, you may need the Riemann Zêta function:

$$\zeta(s) = \sum_{n=1}^{+\infty} \frac{1}{n^s}.$$

The series converges for all $s > 1$ (with $\zeta(1) = +\infty$) ; some particular values $\zeta(2) = \frac{\pi^2}{6}$ and $\zeta(3) \approx 1.202$ (Apéry constant).

We introduce the following functions:

$$\begin{aligned}
f_1[n] &= \delta[n] - \frac{1}{2}\delta[n-1], \\
f_2[n] &= \begin{cases} \frac{1}{n^2} & n \geq 1 \\ 0 & n \leq 0 \end{cases}, \\
f_3[n] &= \begin{cases} \frac{1}{|n|} & n \neq 0 \\ 1 & n = 0 \end{cases}, \\
f_4[n] &= 1, \\
f_5[n] &= (\frac{1}{2})^n u[n], \\
f_6[n] &= (\frac{3}{2})^n u[n].
\end{aligned}$$

Compute the following scalar products: $\langle f_1, f_2 \rangle$, $\langle f_1, f_3 \rangle$, $\langle f_2, f_3 \rangle$, $\langle f_4, f_5 \rangle$, $\langle f_5, f_6 \rangle$. The scalar products are defined as $\langle f_i, f_j \rangle = \sum_{n\in} f_i[n] f_j[n]$.

Exercise 4

Matlab/Python exercise about histogram and histogram equalization.

(a) Show the histograms of the images corresponding to Fig. 3.16 in the book.

(b) Implement a (MATLAB/Python) program that performs histogram equalization on these images, and display the histograms before and after the equalization.

Exercise 5

Implement a smoothing filter using convolution. The smoothing filter may perform a regular average of the pixels encompassed by the filter. Predefined convolution function in 2D can be used. For example, conv2 in matlab, and ”from scipy.signal import convolve2d” in python. Try to re-create Fig. 3.33 in the book.

Exercise 6

Implement image sharpening using a Laplacian mask. Again, conv2 may be used. Try to re-create Fig. 3.46 (3.38) (images of the North Pole of the moon) in the book.