Chapter 2: Finite dimensional vector space

Linear Algebra Done Right, by Sheldon Axler

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A: The vector space of linear maps

Problem S

uppose $b, c \in \mathbb{R}$. Define $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T(x, y, z) = (2x - 4y + 3z + b, 6x + cxyz).$$

Show that T is a linear map if and only if $b = 0$ and $c = 0$.

Proof. Consider $T(1, 1, 1) = T(1, 0, 0) + T(0, 1, 1)$

$$T(1, 1, 1) = (1 + b, 6 + c) \tag{1}$$
$$T(1, 0, 0) = (2 + b, 6) \tag{2}$$
$$T(0, 1, 1) = (-1 + b, 0) \tag{3}$$

Therefore, $b = 0, c = 0$ □