

Chapter 6: Inner product spaces

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A: Inner products and Norms

Problem: 1

Show that the function that takes $((x_1, x_2), (y_1, y_2)) \in \mathbb{R}^2 \times \mathbb{R}^2$ to $|x_1 y_1| + |x_2 y_2|$ is not an inner product on \mathbb{R}^2

Proof. For $((1, 1), (1, 1)), ((-1, -1), (1, 1)) \in \mathbb{R}^2 \times \mathbb{R}^2$, we have

$$|1 \cdot 1| + |1 \cdot 1| = 2$$

for both two vectors. But on the same hand, we also have

$$\begin{aligned} ((1, 1), (1, 1)) + ((-1, -1), (1, 1)) &= ((0, 0), (1, 1)) \\ &= |0 \cdot 1| + |0 \cdot 1| = 0 \end{aligned}$$

□

This could not be an inner product since it violates the additivity property of inner products.

Problem: 2

Show that the function that takes $((x_1, x_2, x_3), (y_1, y_2, y_3)) \in \mathbb{R}^3 \times \mathbb{R}^3$ to $x_1 y_1 + x_3 y_3$ is not an inner product on \mathbb{R}^3

Proof. It take $(0, 1, 0)$ to zero while $(0, 1, 0) \neq 0$

□

Problem: 3

Suppose $\mathbb{F} = \mathbb{R}$ and $V \neq \{0\}$. Replace the positivity condition (which states that $\langle v, v \rangle \geq 0$ for all $v \in V$) in the definition of an inner product with the condition that $\langle v, v \rangle > 0$ for some $v \in V$. Show that this new definition of an inner product does not change the set of functions from $V \times V$ to \mathbb{R} that are inner products on V .

Proof. We show that the two condition are equivalent on the given space. Suppose the positivity condition is satisfied, then the new condition is obviously true for some $v \in V$.

Now we suppose the new condition is satisfied, then for all $v \in V$, we have

$$\langle v, v \rangle =$$

□

Problem: 8