Electron Diffraction

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January 17, 2026

1 Brief Introduction

This experiment employs electron diffraction to ascertain the lattice parameters of a polycrystalline graphite sample and to validate the de Broglie hypothesis. De Broglie proposed that matter possesses wave-like properties, with a wavelength defined by:

$$\lambda = \frac{h}{mv}$$

When an electron is accelerated across a potential difference V, its wavelength is described as:

$$\lambda = \frac{h}{\sqrt{2mVe}}$$

To test this hypothesis, we apply Bragg's law of diffraction:

$$2d \sin\theta = n\lambda$$

The diffraction angle θ is derived by measuring the diameter of the observed concentric rings. Based on the geometric arrangement, this is calculated as:

$$\theta = \frac{1}{2} \tan^{-1}\left(\frac{D}{2L}\right)$$

2 Prelab

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Prelab

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1) a) $p_p = p_e$

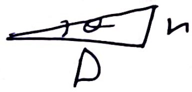
$\Rightarrow \lambda = h/p$

$\Rightarrow \frac{h}{p_p} = \frac{h}{p_e}$

$\Rightarrow \lambda_p = \lambda_e$

Also, $n\lambda = 2d\sin\theta$

$\Rightarrow \theta_p = \theta_e$



& $r_p = r_e$

$\Rightarrow$ Same diameter for equal momenta.

b) $\frac{p_p^2}{2m_p} = \frac{p_e^2}{2m_e} \Rightarrow \frac{p_p}{p_e} = \sqrt{\frac{m_p}{m_e}}$

$\Rightarrow \frac{\lambda_p}{\lambda_e} = \sqrt{\frac{m_e}{m_p}} \Rightarrow \frac{\sin\theta_p}{\sin\theta_e} = \sqrt{\frac{m_e}{m_p}}$

Taking small angle approximation

$\frac{r_p}{r_e} \approx \sqrt{\frac{m_e}{m_p}}$

Since $m_p >> m_e \Rightarrow r_p << r_e$

$\Rightarrow$ proton has a much smaller diameter.

2.) • The cathode releases electrons via thermal emission.
- The collimator acts as a filter. Negative biasing restricts the energy of the electrons.
- The accelerator contains a constant voltage which accelerates the electrons to a fixed energy.

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2.1 Data

Electron Diffraction

Data Sheet

V = 2.0 kV

V = 4.0 kV

V = 2.5 kV

V = 4.5 kV

V = 3.0 kV

V = 4.8 kV

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V = 3.5 kV

$L = 14.0 \pm 0.1$ cm $\quad \Delta V = 0.1$ V

V(kV)	r_1 (mm)	r_2 (mm)
2.0	$\frac{15.09+20.00}{2}$ = 17.54	$\frac{29.07+24.6}{2}$ = 58.46/2 = 29.23
2.5	$\frac{14.52+17.10}{2}$ = 15.81	$\frac{26.76+29.39}{2}$ = 28.08
3.0	$\frac{12.45+15.56}{2}$ = 14.00	$\frac{23.6+26.59}{2}$ = 25.00
3.5	$\frac{11.59+14.45}{2}$ = 13.02	$\frac{22.49+25.82}{2}$ ($\cancel{62.96+66.06}$) = 24.16
4.0	$\frac{11.47+13.07}{2}$ ($\cancel{52.65+54.08}$) = 12.27	$\frac{19.86+22.90}{2}$ ($\cancel{61.09+63.60}$) = 21.38
4.5	$\frac{10.99+12.62}{2}$ = 11.80	$\frac{20.05+21.90}{2}$ = 20.98
5.0	$\frac{10.63+12.08}{2}$ = 11.36	$\frac{18.40+21.41}{2}$ = 19.90

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3 Graphs

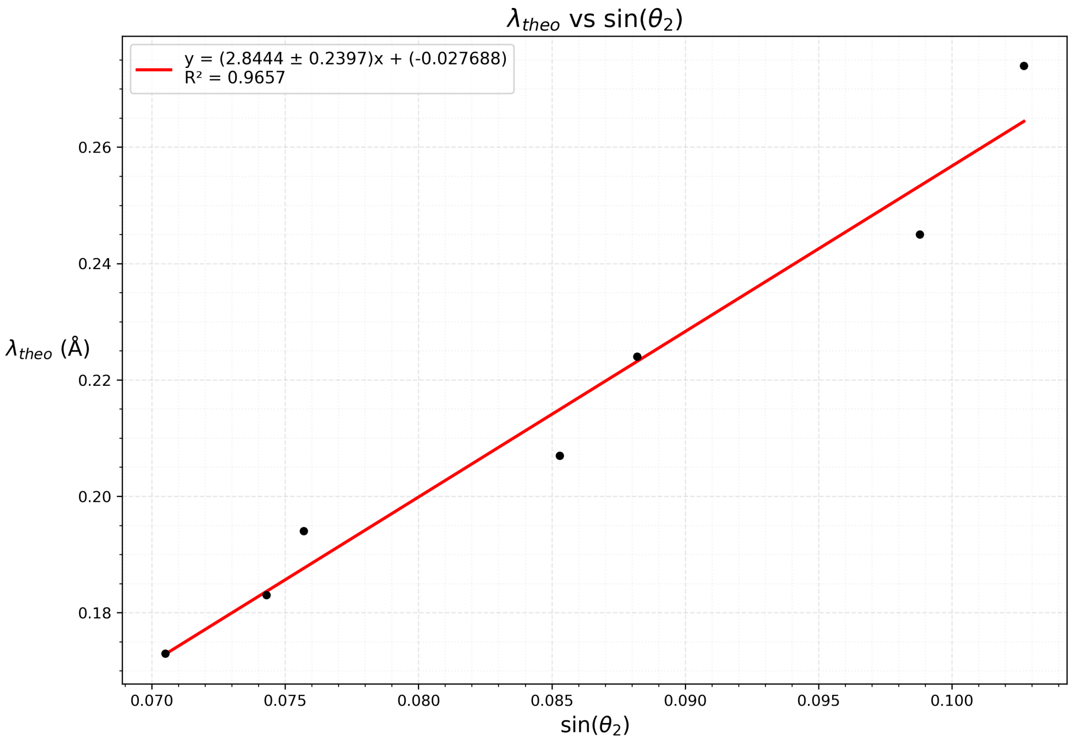

Figure 1: Plot of λ_{theo} vs $\sin(\theta_2)$

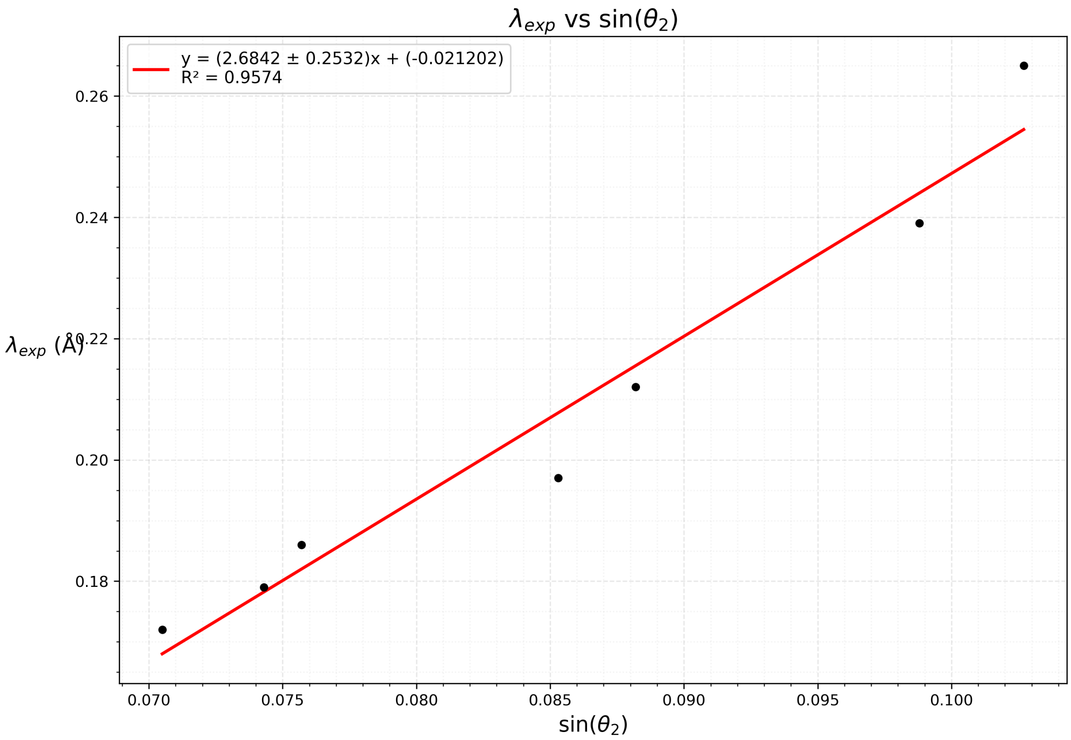

Figure 2: Plot of λ_{exp} vs $\sin(\theta_2)$

4 Analysis

4.1 Verification of de Broglie Hypothesis

To verify the hypothesis, we compare the theoretical de Broglie wavelength (λ_{theo}) with the experimental wavelength (λ_{exp}) derived from the diffraction pattern of the first ring (D_1).

The theoretical wavelength is calculated using the accelerating voltage V:

$$\lambda_{theo} = \frac{h}{\sqrt{2m_e eV}} \approx \frac{12.27}{\sqrt{V_k}}\,\text{Å} \tag{1}$$

The experimental wavelength is determined using Bragg's Law:

$$\lambda_{exp} = 2d_1 \sin(\theta_1) \tag{2}$$

where $d_1 = 2.13\,\text{Å}$ (inter-planar distance for graphite) and the diffraction angle θ_1 is derived from the ring radius r_1 and the distance $L = 14.0\,\text{cm}$:

$$\theta_1 = \frac{1}{2}\tan^{-1}\left(\frac{r_1}{L}\right) \tag{3}$$

The results are summarized in Table 1.

Table 1: Experimental Verification of de Broglie Hypothesis

Voltage (kV)	r_1 (mm)	λ_{theo} (Å)	θ_1 (rad)	$\sin(\theta_1)$	λ_{exp} (Å)	% Error
2.0	17.54	0.274	0.0623	0.0623	0.265	3.28%
2.5	15.81	0.245	0.0562	0.0562	0.239	2.45%
3.0	14.00	0.224	0.0498	0.0498	0.212	5.35%
3.5	13.02	0.207	0.0463	0.0463	0.197	4.83%
4.0	12.27	0.194	0.0437	0.0437	0.186	4.12%
4.5	11.80	0.183	0.0420	0.0420	0.179	2.19%
5.0	11.36	0.173	0.0405	0.0405	0.172	0.58%

Clearly, the experimental wavelengths closely match the theoretical predictions, with percentage errors below 6%, thus confirming the de Broglie hypothesis.

4.2 Determination of Inter-planar Distance d_2

The second diffraction ring corresponds to an unknown inter-planar distance d_2. According to Bragg's law ($n\lambda = 2d_2 \sin\theta_2$), a plot of λ versus $\sin(\theta_2)$ yields a straight line with a slope equal to $2d_2$.

The data for the second ring is compiled in Table 2, with λ_{theo} and λ_{exp} as before.

Table 2: Data for Determination of d_2

Voltage (kV)	r_2 (mm)	θ_2 (rad)	$\sin(\theta_2)$	λ_{theo} (Å)	λ_{exp} (Å)
2.0	29.23	0.1029	0.1027	0.274	0.265
2.5	28.08	0.0989	0.0988	0.245	0.239
3.0	25.00	0.0883	0.0882	0.224	0.212
3.5	24.16	0.0854	0.0853	0.207	0.197
4.0	21.38	0.0758	0.0757	0.194	0.186
4.5	20.98	0.0744	0.0743	0.183	0.179
5.0	19.90	0.0706	0.0705	0.173	0.172

The plots of λ versus $\sin(\theta_2)$ are shown in the Graphs section. According to Bragg's law, the relationship $\lambda = 2d_2 \sin(\theta_2)$ implies that a linear fit should yield a slope equal to $2d_2$.

4.2.1 Analysis Using Theoretical Wavelength

From the linear regression of λ_{theo} versus $\sin(\theta_2)$, we obtain:

- Slope: (2.8444 ± 0.2397) Å
- Intercept: -0.0277 Å
- $R^2 = 0.9657$

The inter-planar distance is determined from the slope:

$$d_2 = \frac{\text{slope}}{2} = \frac{2.8444}{2} = (1.422 \pm 0.120)\,\text{Å}$$

4.2.2 Analysis Using Experimental Wavelength

From the linear regression of λ_{exp} versus $\sin(\theta_2)$, we obtain:

- Slope: (2.6842 ± 0.2532) Å
- Intercept: -0.0212 Å
- $R^2 = 0.9574$

The inter-planar distance is:

$$d_2 = \frac{\text{slope}}{2} = \frac{2.6842}{2} = (1.342 \pm 0.127)\,\text{Å}$$

Both analyses yield high R^2 values (> 0.95), confirming excellent linear fits and validating Bragg's law.

The average inter-planar distance from both methods is:

$$d_2 = \frac{1.422 + 1.342}{2} = 1.382\,\text{Å}$$

The uncertainty is calculated as:

$$\Delta d_2 = \frac{0.120 + 0.127}{2} = 0.124\,\text{Å}$$

Thus, we have:

$$d_2 = (1.382 \pm 0.124)\,\text{Å}$$

The known inter-planar spacing for the (1120) plane of graphite is approximately 1.23 Å, which is reasonably close to our experimental results considering the uncertainty.

4.3 Error Analysis

The uncertainty in the experimental wavelength, $\Delta\lambda_{exp}$, is calculated using the propagation of errors from the measurement of the diffraction ring radius r.

The relationship for wavelength is given by Bragg's Law:

$$\lambda = 2d_1 \sin\theta \tag{4}$$

$$\Rightarrow \Delta\lambda = 2d_1 \cos\theta \cdot \Delta\theta \tag{5}$$

The diffraction angle θ is determined by the geometry of the setup:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{r}{L}\right) \tag{6}$$

The uncertainty in θ, denoted as $\Delta\theta$, depends on the uncertainty in the radius measurement (Δr) and the distance L. Assuming L is constant relative to the variation in r:

$$\Delta\theta \approx \frac{1}{2}\frac{1}{1+(r/L)^2}\frac{\Delta r}{L} \tag{7}$$

Here, Δr is estimated as half the width of the diffraction ring (calculated from the spread in the raw inner/outer measurements).

Table 3 summarizes the uncertainty calculations. The theoretical wavelength λ_{theo} generally falls within the range $\lambda_{exp} \pm \Delta\lambda$, indicating the deviations are largely accounted for by measurement limitations.

Table 3: Uncertainty Analysis for λ_{exp}

Voltage (kV)	Δr (est) (mm)	$\Delta\theta$ (rad)	λ_{exp} (Å)	$\Delta\lambda$ (Å)	λ_{theo} (Å)
2.0	2.45	0.0087	0.265	0.037	0.274
2.5	1.29	0.0046	0.239	0.020	0.245
3.0	1.55	0.0055	0.212	0.024	0.224
3.5	1.43	0.0051	0.197	0.022	0.207
4.0	0.80	0.0029	0.186	0.012	0.194
4.5	0.81	0.0029	0.179	0.012	0.183
5.0	0.72	0.0026	0.172	0.011	0.173

5 Conclusion

- The experiment successfully verified the de Broglie hypothesis, with experimental wavelengths closely matching theoretical predictions within a 6% error margin.
- The inter-planar distance d_2 for the second diffraction ring was determined to be approximately 1.382 ± 0.124 Å, quite close to the known value of 1.23 Å for graphite.
- Uncertainties in the experimental values of wavelengths were determined, with the theoretical values falling within the calculated error bounds.
- Measurement uncertainties, particularly in ring diameter and path length, were the primary sources of error.