Production Approach Workshop@IDE-JETRO
Session 2

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March 26, 2024

Session 2: Production Approach with Revenue Data (1)
Identification

- With output quantity data, the production approach can identify firm-level price markup under weak assumptions on the market structure and the demand function.
- However, typical firm-level production datasets include only revenue, not output quantity.

Ad hoc use of revenue as quantity

- Many studies use log revenue r_i as log output y_i, assuming perfect competition:

$$r_i = y_i + \underbrace{p}_{\text{constant}} \Rightarrow dr_i = dy_i$$

- Long criticized since Marschak and Andrews (1944): they are different under imperfect competition:

$$r_i = y_i + p(y_i, \underbrace{\epsilon_i}_{\text{shock}}) \Rightarrow dr_i = \frac{MC_i}{P_i} dy_i$$

Using an Industry Price Index as a Price

- Many studies divide revenue by an industry price index to mimic output quantity:

$$r_{it} - p_t$$

- Industry price index $\neq$ price
  - No cross-sectional variation
  - A measure of price changes, not price levels
  - $p_t = \ln 1 = 0$ when t is the base year

Conventional Production Approach to Markups

- Estimate a revenue production function

$$r_{it} = \phi_t \left(m_{it}, k_{it}, l_{it}, z_{it}^d, \omega_{it} \right)$$

- Calculate a markup by

$$\mu_{it}^{Conv} = \frac{\partial r_{it} / \partial m_{it}}{\alpha_{it}^M}$$

- μ_{it}^{Conv} should underestimate μ_{it} because $dr_{it} = dy_{it} + p'(y_{it})dy_{it} < dy_{it}$

Bias in Conventional Markup Estimates

- Bond et al. (2021) showed

$$\frac{\partial r_{it}/\partial m_{it}}{\alpha_{it}^M} = \frac{\partial r_{it}}{\partial y_{it}} \frac{\partial r_{it}/\partial m_{it}}{\alpha_{it}^M}$$
$$= \frac{1}{\mu_{it}} \mu_{it}$$
$$= 1$$

- μ_{it}^{Conv} contains no information of markup if ϕ_t is precisely estimated.
  - Note: when ϕ_t is misspecified, μ_{it}^{Conv} could contain information of markup.

Bias in Production Function Estimation: Simulation

- Cobb-Douglas production function with AR1 TFP shock:

$$
\begin{gathered}
y_{it} = 0.4m_{it} + 0.3k_{it} + 0.3\ell_{it} + 0.3import_{it} + \omega_{it}, \\
\omega_{it} = 0.8\omega_{it-1} + \eta_{it},\ \eta_{it} \sim N(0, (0.13)^2)
\end{gathered}
$$

- HSA demand system (Matsuyama and Ushchev, 2017) with MA1 demand shock

$$
\begin{gathered}
r_{it} = -2 + 8.3 \ln\{\exp(0.12(y_{it} - a(\mathrm{y}_t))) + 0.5\epsilon_{it}\}, \\
\epsilon_{it} = 0.8\xi_{it-1} + \xi_{it}, \quad \xi_{it} \perp\!\!\!\perp \xi_{it-1} \sim U[0,1],
\end{gathered}
$$

$a(\mathrm{y}_t)$: aggregate quantity index (endogenously determined)

Simulation: Data Generating Process

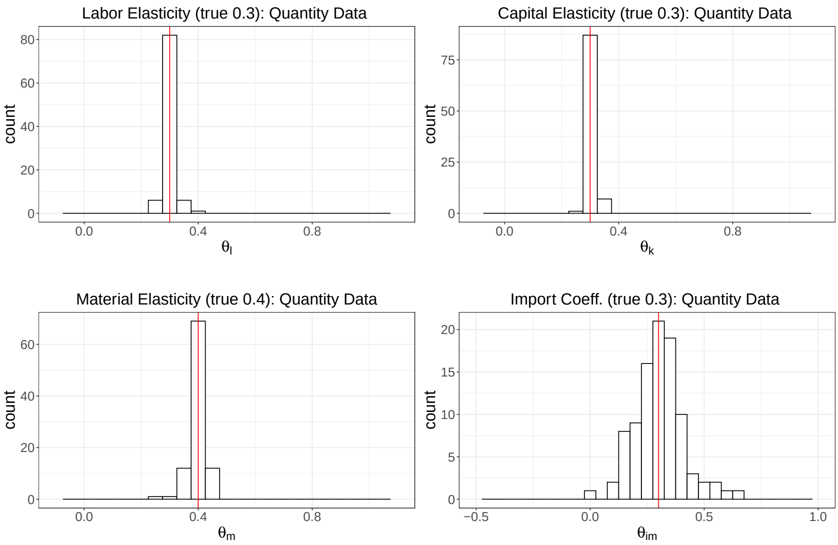
- Monopolistic competition without entry
  - m_{it} maximizes profit and correlates with ω_{it} and ϵ_{it}
  - $(k_{it}, \ell_{it}, import_{it})$: exogenous DGP
- Moderate markups

	Min	Median	Max
Markup	1.01	1.27	1.78

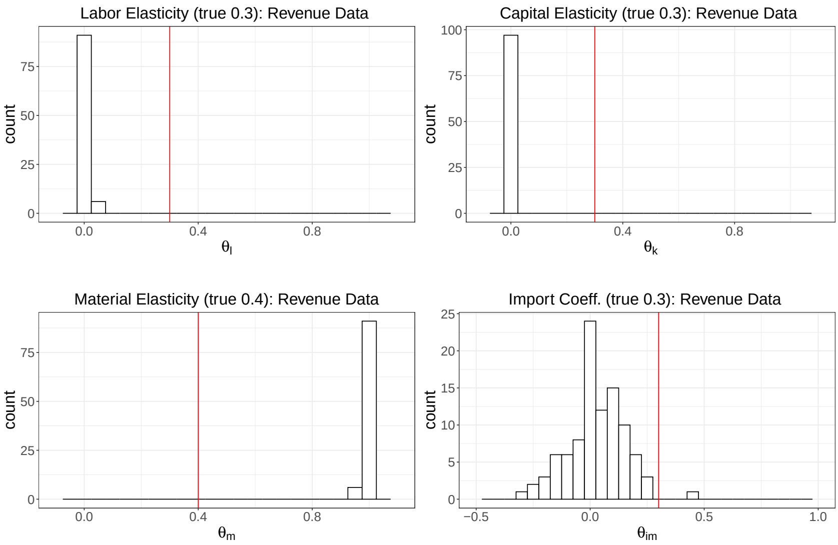
Simulation: Estimation Method

- $N = 300$ firms, $T = 5$ periods
- 100 replications
- ACF method with constant returns to scale restriction ($\theta_m + \theta_k + \theta_l = 1$) (Ackerberg, Caves and Frazer, 2015; Flynn, Gandhi and Traina, 2019)
  - Positive marginal products restriction: $\theta_l, \theta_m, \theta_k \in [0, 1]$

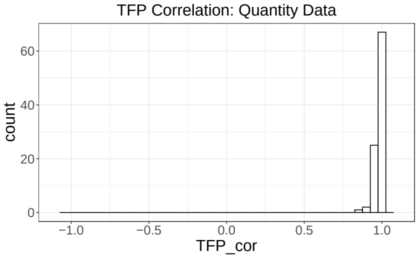
ACF with Quantity Data

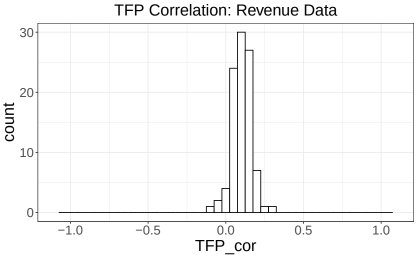


ACF with Revenue Data

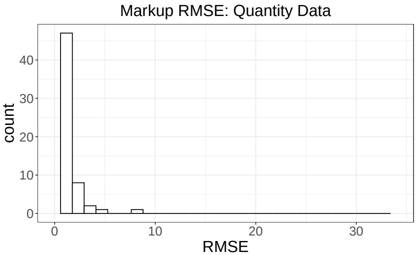


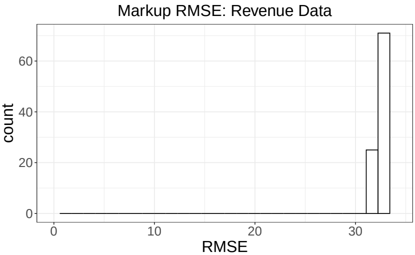
Correlations between True and Estimated TFPs





RMSE of Markup Estimates





RMSE (Root Mean Squared Error)

Production Approach with Revenue Data

- Assume monopolistic competition and estimate a demand function
- Klette and Griliches (1996);De Loecker (2011)

$$p_{it} = a - (b+1)\, y_{it} \Rightarrow r_{it} = a + by_{it}$$

  - Common and constant markups $\mu_{it} = 1/b$.
- Kasahara and Sugita (2020; 2023)

$$p_{it} = \tilde{\psi}_t \left(y_{it}, z_{it}^d, \epsilon_{it} \right)$$

  - ψ_t: nonparametric; ϵ_{it}: unobserved demand shock (e.g., elasticity shock)
  - "Standard" assumptions and data found in typical applications (e.g., Levinsohn and Petrin, 2003; Ackerberg et al., 2015)

Setup

- Inverse demand with MA1 shock $\epsilon_{it} = \theta\xi_{it-1} + \xi_{it}$ normalized by the CDF:

$$p_{it} = \tilde{\psi}_t\left(y_{it}, z_{it}^d, \epsilon_{it}\right) = \psi_t\left(y_{it}, z_{it}^d, u_{it}\right), \;\; u_{it} := \underset{\text{CDF}}{F_\epsilon(\epsilon_{it})} \overset{iid}{\sim} U[0,1]$$

  where $u_{it} \perp (m_{it-2}, k_{it-1}, l_{it-1}, z_{it-1}^d)$ (temporary demand shock)

- Revenue

$$r_{it} = \psi_t\left(y_{it}, z_{it}^d, u_{it}\right) + y_{it} = \varphi_t\left(y_{it}, z_{it}^d, u_{it}\right),$$

$$\text{Markup} = \frac{1}{\partial\varphi_t\left(y_{it}, z_{it}^d, u_{it}\right)/\partial y_{it}}$$

Setup

- Production function with AR1 TFP process:

$$y_{it} = f_t(x_{it}) + \omega_{it}, \text{ with } \quad x_{it} = (m_{it}, k_{it}, \ell_{it})'$$

$$\omega_{it} = \bar{h}(\omega_{it-1}) + \eta_{it}, \quad \eta_{it} \overset{iid}{\sim} G_\eta$$

- Independence ($\perp$)
  - $\eta_{it} \perp k_{it}, l_{it}, m_{it-1}, z^d_{it}$
  - $u_{it} \perp m_{it-2}, k_{it-1}, l_{it-1}, z^d_{it-1}$
  - $\eta_{it} \perp u_{it}$
  - m_{it} is correlated with η_{it} and u_{it}

Control function

- Inverse of demand function for $m_{it} = \mathbb{M}_t(\overset{+}{\omega}_{it}, k_{it}, l_{it}, u_{it})$ (Levinsohn and Petrin 2003; Ackerberg et al. 2015)

$$\omega_{it} = \mathbb{M}_t^{-1}(x_{it}, z_{it}^d, u_{it}) \text{ where } x_{it} \equiv (m_{it}, k_{it}, \ell_{it})'$$

- Model
    - Revenue

$$\begin{aligned} r_{it} &= \varphi_t\left(f_t(x_{it}) + \mathbb{M}_t^{-1}(x_{it}, z_{it}^d, u_{it}), z_{it}^d, u_{it}\right) \\ &\equiv \phi_t\left(x_{it}, z_{it}^d, u_{it}\right), \ \ u_{it} \overset{iid}{\sim} U(0,1) \end{aligned}$$

    - TFP process

$$\begin{aligned} \mathbb{M}_t^{-1}\left(x_{it}, z_{it}^d, u_{it}\right) &= \bar{h}\left(\mathbb{M}_t^{-1}(x_{it-1}, z_{it-1}^d, u_{it-1})\right) + \eta_{it} \\ &\equiv h_t(x_{it-1}, z_{it-1}^d, u_{it-1}) + \eta_{it} \end{aligned}$$

Normalization

- Constraints

$$
\begin{aligned}
\varphi_t^{-1}(r_{it}, z_{it}^d, u_{it}) &= f_t(x_{it}) + \mathbb{M}_t^{-1}(x_{it}, z_{it}^d, u_{it}) \\
\mathbb{M}_t^{-1}\left(x_{it}, z_{it}^d, u_{it}\right) &= h_t(x_{it-1}, z_{it-1}^d, u_{it-1}) + \eta_{it}
\end{aligned}
$$

- For any constant $(a_1, a_2, b) \in \mathbb{R}^2 \times \mathbb{R}_{++}$,

$$
\begin{aligned}
\tilde{\varphi}_t^{-1} &= a_1 + a_2 + b\varphi_t^{-1};\ \tilde{f}_t = a_1 + bf_t; \\
\tilde{\mathbb{M}}_t^{-1} &= a_2 + b\mathbb{M}_t^{-1}; \tilde{h}_t = a_2 + bh_t
\end{aligned}
$$

  also satisfy the constraints.
- Need to fix location (a_1, a_2) and scale b.

Normalization of $\mathbb{M}_t^{-1}$

- We first identify $\{\varphi_t^{-1}(\cdot), f_t(\cdot), \mathbb{M}_t^{-1}(\cdot)\}$ up to scale and location by fixing (a_1, a_2, b), equivalently,

$$f_t(m_0^*, k^*, l^*) = 0$$
$$\mathbb{M}_t^{-1}\left(m_0^*, k^*, l^*, z^{d*}, u^*\right) = 0 \text{ and } \mathbb{M}_t^{-1}\left(m_1^*, k^*, l^*, z^{d*}, u^*\right) = 1$$

  for some $(m_0^*, m_1^*, k^*, l^*, z^{d*}, u^*)$.
- Later, we identify (a_1, a_2, b) from additional assumptions or data.

Step 1: Identification of demand shock

- Conditional quantile function with endogenous variables

$$r_{it} = \phi_t \left(x_{it}, z_{it}^d, u_{it} \right), \ u_{it} \overset{iid}{\sim} U(0,1)$$

- IV quantile regression (Chernozhukov and Hansen, 2005) using $u_{it} \perp (m_{it-2}, k_{it-1}, l_{it-1})$:

$$\Pr \left[r_{it} \leq \phi_t \left(x_{it}, z_{it}^d, u \right) | m_{it-2}, k_{it-1}, l_{it-1}, z_{it-1}^d \right] = u \text{ for all } u \in [0,1]$$

Proof

$$
\begin{aligned}
&\Pr\left[r_{it} \leq \phi_t\left(x_{it}, z^d_{it}, u\right) | m_{it-2}, k_{it-1}, l_{it-1}, z^d_{it-1}\right] \\
&= \Pr\left[\phi_t\left(x_{it}, z^d_{it}, u_{it}\right) \leq \phi_t\left(x_{it}, z^d_{it}, u\right) | m_{it-2}, k_{it-1}, l_{it-1}, z^d_{it-1}\right] \\
&= \Pr\left[u_{it} \leq u | m_{it-2}, k_{it-1}, l_{it-1}, z^d_{it-1}\right] \\
&= \Pr\left[u_{it} \leq u\right] \\
&= u.
\end{aligned}
$$

Step 1: Identification of demand shock

- The moment condition

$$E\left[1\left\{r_{it} \leq \phi_t\left(x_{it}, z_{it}^d, u\right)\right\} - u \,\middle|\, m_{it-2}, k_{it-1}, l_{it-1}, z_{it-1}^d\right] = 0, \forall u \in [0, 1]$$

$\Longrightarrow \phi_t(\cdot)$ and $u_{it} = \phi_t^{-1}(r_{it}, x_{it}, z_{it}^d)$ are identified

Step 2: Identification of Control Function $\mathbb{M}_t^{-1}$

- Transformation model (Horowitz, 1996)

$$\mathbb{M}_t^{-1}\left(\overset{+}{m}_{it}, k_{it}, l_{it}, z_{it}^d, u_{it}\right) = h(x_{it-1}, z_{it-1}^d, u_{it-1}) + \eta_{it}$$

- Chiappori, Komunjer and Kristensen (2015): identification from conditional CDF $G_{m_t|v_t}(m_{it}|v_{it})$,$v_{it} := (k_{it}, l_{it}, z_{it}^d, u_{it}, x_{it-1}, z_{it-1}^d, u_{it-1})$, up to scale and location normalization.

Proof

- m_{it} and η_{it} are in one-to-one when v_{it} is conditioned:

$$\underbrace{G_{m_t|v_t}\left(m_{it}|v_{it}\right)}_{\text{Data}} = G_{\eta_t|v_t}\left(\eta_{it}|v_{it}\right) = G_{\eta_t}\left(\eta_{it}\right) \text{ from } \eta_{it} \perp v_{it}.$$

$$= G_{\eta_t}\left(\mathbb{M}_t^{-1}\left(m_{it}, k_{it}, l_{it}, z_{it}^d, u_{it}\right) - h(x_{it-1}, z_{it-1}^d, u_{it-1})\right)$$

- Differentiate by $q_t \in \{x_t, u_t\}$ and $q_{t-1} \in \{x_{t-1}, u_{t-1}\}$ at some point $(\tilde{x}_{t-1}, \tilde{z}_{t-1}^d, \tilde{u}_{t-1})$:

$$\frac{\partial G_{m_t|v_t}\left(m_t|\tilde{v}_t\right)}{\partial q_t} = \frac{\partial \mathbb{M}_t^{-1}\left(x_t, z_t^d, u_t\right)}{\partial q_t} g_{\eta_t}\left(\tilde{\eta}_t\right)$$

$$\frac{\partial G_{m_t|v_t}\left(m_t|\tilde{v}_t\right)}{\partial q_{t-1}} = \frac{\partial \bar{h}_t(\tilde{x}_{t-1}, \tilde{z}_{t-1}^d, \tilde{u}_{t-1})}{\partial q_{t-1}} g_{\eta_t}\left(\tilde{\eta}_t\right)$$

where $\tilde{v}_t := (k_t, l_t, z_t^d, u_t, \tilde{x}_{t-1}, \tilde{z}_{t-1}^d, \tilde{u}_{t-1})$

Proof

$$\Rightarrow \frac{\partial \mathbb{M}_t^{-1}(x_t, z_t, u_t)}{\partial q_t} = -\underbrace{\frac{\partial G_{m_t|v_t}(m|\tilde{v}_t)/\partial q_t}{\partial G_{m_t|v_t}(m|\tilde{v}_t)/\partial q_{t-1}}}_{\text{Data}} \frac{\partial \overline{h}_t(\tilde{x}_{t-1}, \tilde{z}_{t-1}^d, \tilde{u}_{t-1})}{\partial q_{t-1}}$$

From the location and scale normalization,

$$\begin{aligned} 1 &= \mathbb{M}_t^{-1}\left(m_1^*, k^*, l^*, z^{d*}, u^*\right) \\ &= \underbrace{\mathbb{M}_t^{-1}\left(m_0^*, k^*, l^*, z^{d*}, u^*\right)}_{=0} + \int_{m_0^*}^{m_1^*} \frac{\partial \mathbb{M}_t^{-1}\left(m, k^*, l^*, z^{d*}, u^*\right)}{\partial m_t} dm \\ &= -\frac{\partial \overline{h}_t(\tilde{x}_{t-1}, \tilde{z}_{t-1}^d, \tilde{u}_{t-1})}{\partial q_{t-1}} \underbrace{\int_{m_0^*}^{m_1^*} \frac{\partial G_{m|v}(m|v_t^*)/\partial m_t}{\partial G_{m|v}(m|v_t^*)/\partial q_{t-1}} dm}_{\equiv S_t} \end{aligned}$$

where $v_t^* \equiv \left(k^*, l^*, z^{d*}, u^*, \tilde{x}_{t-1}, \tilde{z}_{t-1}^d, \tilde{u}_{t-1}\right)$

Proof

The derivatives of $\mathbb{M}_t^{-1}\left(x_t, z_t^d, u_t\right)$ are identified for $q_t \in \{x_t, z_t^d, u_t\}$as

$$\frac{\partial \mathbb{M}_t^{-1}\left(x_t, z_t^d, u_t\right)}{\partial q_t} = \frac{1}{S_t}\frac{\partial G_{m|v}\left(m_t|\tilde{v}_t\right)/\partial q_t}{\partial G_{m|v}\left(m_t|\tilde{v}_t\right)/\partial q_{t-1}}.$$

Then, we identify

$$\begin{aligned}
&\mathbb{M}_t^{-1}\left(x_t, z_t^d, u_t\right)\\
=&\underbrace{\mathbb{M}_t^{-1}\left(m_0^*, k^*, l^*, z^{d*}, u^*\right)}_{=0} + \int_{m_0^*}^{m_t} \frac{\partial \mathbb{M}_t^{-1}(s, k_t, l_t, z_t^d, u_t)}{\partial m_t} ds\\
+&\int_{k^*}^{k_t} \frac{\partial \mathbb{M}_t^{-1}(m_{t0}^*, s, l_t, z_t^d, u_t)}{\partial k_t} ds + \int_{l^*}^{l_t} \frac{\partial \mathbb{M}_t^{-1}(m_{t0}^*, k_t^*, s, z_t^d, u_t)}{\partial l_t} ds\\
+&\int_{z^{d*}}^{z_t^d} \frac{\partial \mathbb{M}_t^{-1}(m_{t0}^*, k_t^*, l_t^*, s, u)}{\partial u_t} ds + \int_{u^*}^{u_t} \frac{\partial \mathbb{M}_t^{-1}(m_{t0}^*, k_t^*, l_t^*, z^{d*}, u)}{\partial u_t} ds.
\end{aligned}$$

Step 3: Identification of Production Function, Price and Quantity

- Differentiating $\varphi^{-1}\left(\phi_t\left(x_{it}, z_{it}^d, u_{it}\right), z_{it}^d, u_{it}\right) = f_t(x_{it}) + \mathbb{M}_t^{-1}\left(x_{it}, z_{it}^d, u_{it}\right)$ by $q_{it} \in \{m_{it}, k_{it}, l_{it}\}$, z_{it}^d and u_{it}

$$\frac{\partial \varphi_t^{-1}}{\partial r_{it}} \underbrace{\frac{\partial \phi_t}{\partial q_{it}}}_{\text{known}} = \frac{\partial f_t}{\partial q_{it}} + \underbrace{\frac{\partial \mathbb{M}_t^{-1}}{\partial q_{it}}}_{\text{known}}$$

$$\frac{\partial \varphi_t^{-1}}{\partial r_{it}} \underbrace{\frac{\partial \phi_t}{\partial z_{it}^d}}_{\text{known}} + \frac{\partial \varphi_t^{-1}}{\partial z_{it}^d} = \underbrace{\frac{\partial \mathbb{M}_t^{-1}}{\partial z_{it}^d}}_{\text{known}}$$

$$\frac{\partial \varphi_t^{-1}}{\partial r_{it}} \underbrace{\frac{\partial \phi_t}{\partial u_t}}_{\text{known}} + \frac{\partial \varphi_t^{-1}}{\partial u_t} = \underbrace{\frac{\partial \mathbb{M}_t^{-1}}{\partial u_t}}_{\text{known}}.$$

- 5 equations for 6 unknown derivatives of $f_t(\cdot)$ and $\varphi_t(\cdot)$

Step 3: Identification of Production Function, Price and Quantity

- First order condition for m_{it}

$$\left(\frac{\partial\varphi_t^{-1}\left(r_{it}, z_{it}^d, u_{it}\right)}{\partial r_{it}}\right)^{-1}\frac{\partial f_t\left(x_{it}\right)}{\partial m_{it}} = \underbrace{\alpha_{it}^M}_{\text{Data}}$$

- Markup

$$\frac{\partial\varphi_t^{-1}}{\partial r_{it}} = \left(\frac{\partial\phi_t}{\partial m_{it}} - \alpha_{it}^M\right)^{-1}\frac{\partial\mathbb{M}_t^{-1}}{\partial m_{it}}$$

- Elasticities

$$\frac{\partial f_t}{\partial q_{it}} = \left(\frac{\partial\phi_t}{\partial m_{it}} - \alpha_{it}^M\right)^{-1}\frac{\partial\mathbb{M}_t^{-1}}{\partial m_{it}}\frac{\partial\phi_t}{\partial m_{it}} - \frac{\partial\mathbb{M}_t^{-1}}{\partial q_{it}}$$

Step 3: Identification of Production Function, Price and Quantity

- Production function

$$f_t(m_{it}, k_{it}, l_{it}) = \underbrace{f_t(m_0^*, k^*, l^*)}_{=0} + \int_{m^*}^{m_{it}} f_t\left(s, k_{it}, l_{it}\right) ds + \int_{k^*}^{k_{it}} f_t\left(m^*, s, l_{it}\right) ds + \int_{l^*}^{l_{it}} f_t\left(m^*, k^*, s\right) ds$$

- Output and price

$$y_{it} = f_t(m_{it}, k_{it}, l_{it}) + \mathbb{M}_t^{-1}\left(x_t, z_t^d, u_t\right)$$

$$p_{it} = r_{it} - y_{it}.$$

Alternative Identification Settings

- First order Markov process with observable shifters:

$$\omega_{it} = h(\omega_{it-1}, z^h_{it-1}) + \eta_{it}$$

- Endogenous labor (l_{it-1} is used as an IV)
- Observable production function shifters (endogenous/exogenous; continuous/discrete)

$$\varphi_t(y_{it}, z^d_{it}, u_{it}) \text{ and } f_t(x_{it}, z^s_{it})$$

Location and scale normalization across different periods

- $\{\varphi_t^{*-1}, f_t^*, \mathbb{M}_t^{*-1}\}$: true structure
- $\{\varphi_t^{-1}, f_t, \mathbb{M}_t^{-1}\}$: identified structure.
- Location and scale normalization, (a_{1t}, a_{2t}, b_t):

$$\varphi_t^{-1} = a_{1t} + b_t \varphi_t^{*-1},\ f_t = a_{2t} + b_t f_t^*,\ \mathbb{M}_t^{-1} = a_t + b_t \mathbb{M}_t^{*-1}.$$

- Location and scale normalization differs period-by-period:

$$(a_{1t}, a_{2t}, b_t) \neq (a_{1,t+1}, a_{2,t+1}, b_{t+1}).$$

Example of identifiable objects

- Markup ratio of two firms

$$\frac{\mu_{it}}{\mu_{jt}} = \frac{\mu^*_{it}}{\mu^*_{jt}}$$

- TFP deviation from mean ratio of two firms

$$\frac{\omega_{it} - E[\omega_t]}{\omega_{jt} - E[\omega_t]} = \frac{\omega^*_{it} - E[\omega^*_t]}{\omega^*_{jt} - E[\omega^*_t]}$$

- Elasticity ratio of two factors

$$\frac{\partial f_t/\partial k}{\partial f_t/\partial l} = \frac{\partial f^*_t/\partial k}{\partial f^*_t/\partial l}$$

Fix Location and Scale Parameters

- Additional assumptions can fix location and scale normalizations across periods
  - Constancy of some object over periods.
  - Local constant returns to scale: for some x_t,

$$\frac{\partial f_t(x_t)}{\partial m_t} + \frac{\partial f_t(x_t)}{\partial l_t} + \frac{\partial f_t(x_t)}{\partial k_t} = 1 \qquad \text{for} \;\; t \;\; \text{and} \;\; t+1.$$

  - Output or price information for some firms

Compare scale normalization across different periods

- b_t/b_{t-1} can be identified by assuming the stability of some function, e.g.,
  - Variance $Var(\eta_t) = Var(\eta_{t-1})$
  - Elasticity $\partial f_t^*(\hat{x})/\partial v = \partial f_{t-1}^*(\hat{x})/\partial v$ for some $\hat{x}$ and some factor v
  - Local returns to scale: for some $\hat{x}$,

$$\frac{\partial f_t^*(\hat{x})}{\partial m_t} + \frac{\partial f_t^*(\hat{x})}{\partial l_t} + \frac{\partial f_t^*(\hat{x})}{\partial k_t} = \frac{\partial f_{t-1}^*(\hat{x})}{\partial m_{t-1}} + \frac{\partial f_{t-1}^*(\hat{x})}{\partial l_{t-1}} + \frac{\partial f_{t-1}^*(\hat{x})}{\partial k_{t-1}}$$

Example of identifiable objects

- Markup change

$$\frac{\mu_{it}}{\mu_{it-1}} = \frac{b_t}{b_{t-1}} \frac{\mu^*_{it}}{\mu^*_{it-1}}$$

- Elasticity change

$$\frac{\partial f_t/\partial k}{\partial f_{t-1}/\partial k} = \frac{b_t}{b_{t-1}} \frac{\partial f^*_t/\partial k}{\partial f^*_{t-1}/\partial k}$$

- TFP deviation from mean change

$$\frac{\omega_{it} - E[\omega_t]}{\omega_{jt} - E[\omega_t]} = \frac{b_t}{b_{t-1}} \frac{\omega^*_{it} - E[\omega^*_t]}{\omega^*_{jt} - E[\omega^*_t]}$$

Local Constant Returns to Scale

- The local constant returns to scale for some $\tilde{x}$,

$$\frac{\partial f_t^*(\tilde{x})}{\partial k_t} + \frac{\partial f_t^*(\tilde{x})}{\partial l_t} + \frac{\partial f_t^*(\tilde{x})}{\partial m_t} = 1,$$

can identify the scale parameter b_t:

$$\frac{\partial f_t(\tilde{x})}{\partial k_t} + \frac{\partial f_t(\tilde{x})}{\partial l_t} + \frac{\partial f_t(\tilde{x})}{\partial m_t} = b_t \left(\frac{\partial f_t^*(\tilde{x})}{\partial k_t} + \frac{\partial f_t^*(\tilde{x})}{\partial l_t} + \frac{\partial f_t^*(\tilde{x})}{\partial m_t} \right) = b_t.$$

- Example of identifiable objects
  - Markup μ_{it}
  - Elasticity $\partial f_t / \partial k_t$, $\partial f_t / \partial l_t$, $\partial f_t / \partial m_t$
  - TFP deviation from mean $\omega_{it} - E[\omega_t]$

Identification of Demand System

- Homothetic utility with a single index (HSA)(Matsuyama and Ushchev, 2017)
$$\frac{P_{it} Y_{it}}{H_t} = S\left(\frac{Y_{it}}{A(\mathrm{Y}, \mathrm{u})}, u_{it}\right)$$
$H_t \equiv \sum_{i \in I} P_{it} Y_{it}$ is the total industry expenditure; $A(\mathrm{Y}, \mathrm{u})$ an index of Y_{it}, u_{it}.
- HSA nests the CES demand, the translog demand and the constant response demand

Identification of Demand System

Proposition

(Matsuyama and Ushchev, 2017, Lemma1). Suppose $\varphi_t(y, u)$ is identified up to location (i.e., scale parameter b_t is identified). Define

$$S(Y_{it}, u_{it}) \equiv \frac{\exp\left(\varphi_t\left(\ln Y_{it}, u_{it}\right)\right)}{H_t}$$

and identify $A\left(\mathrm{Y}', \mathrm{u}'\right)$ by solving

$$\sum_{i \in I_t} S\left(\frac{\mathrm{Y}'}{\mathrm{A}(\mathrm{Y}', \mathrm{u}')}, \mathrm{u}'\right) = 1$$

for given $(\mathrm{Y}', \mathrm{u}')$. Then, $\{S(\cdot), A(\cdot)\}$ is a HSA demand system.

Identification of Utility Function

Theorem

(Matsuyama and Ushchev, 2017, Lemma1).There exists a unique monotone, convex, continuous and homothetic rational preference that generates the HSA demand system. The associated utility function U is obtained as

$$\ln U(\mathrm{Y}', \mathrm{u}) = \ln A(Y, u_t) + \sum_{i \in I_t} \int_{c(u_t)}^{Y_i/A(Y, u_t)} \frac{S(\xi, u_{it})}{\xi} d\xi \qquad (1)$$

where $c(u_t)$ is a constant for given u_t.

Ackerberg, Daniel A, Kevin Caves, and Garth Frazer, "Identification Properties of Recent Production Function Estimators," *Econometrica*, 2015, *83* (6), 2411–2451.

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch, "Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data," *Journal of Monetary Economics*, 2021, *121*, 1–14.

Chernozhukov, Victor and Christian Hansen, "An IV model of quantile treatment effects," *Econometrica*, 2005, *73* (1), 245–261.

Chiappori, Pierre-André, Ivana Komunjer, and Dennis Kristensen, "Nonparametric Identification and Estimation of Transformation Models," *Journal of Econometrics*, 2015, *188* (1), 22–39.

Flynn, Zach, Amit Gandhi, and James Traina, "Measuring Markups with Production Data," Unpublished 2019.

Horowitz, Joel L, "Semiparametric Estimation of a Regression Model with an Unknown Transformation of the Dependent Variable," *Econometrica*, 1996, pp. 103–137.

Kasahara, Hiroyuki and Yoichi Sugita, "Nonparametric identification of production function, total factor productivity, and markup from revenue data," *arXiv preprint arXiv:2011.00143*, 2020.

— **and** — , "Nonparametric identification of production function, total factor productivity, and markup from revenue data," 2023.

Klette, Tor Jakob and Zvi Griliches, "The Inconsistency of Common Scale Estimators When Output Prices Are Unobserved and Endogenous," *Journal of Applied Econometrics*, 1996, *11* (4), 343–361.

Levinsohn, James and Amil Petrin, "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 2003, pp. 317–341.

Loecker, Jan De, "Product Differentiation, Multiproduct Firms, and Estimating the Impact of Trade Liberalization on Productivity," *Econometrica*, 2011, *79* (5), 1407–1451.

Marschak, J. and W.H. Andrews, "Random Simultaneous Equations and the Theory of Production," *Econometrica*, 1944, *12*, 143–205.

Matsuyama, Kiminori and Philip Ushchev, "Beyond CES: Three Alternative Cases of Flexible Homothetic Demand Systems," Buffett Institute Global Poverty Research Lab Working Paper 17-109 2017.