# A summary on empirical approaches to labour market power

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(労働市場力実証研究に関する要約)

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**Abstract** I summarize the various proposed methods of production approach to markups/markdowns/TFP estimation. I make distinctions between formulae and production parameter estimation.

The original method of Hall (1988) only uses cost minimization, so do De Loecker and Warzynski (2012), and derive a formula for markup. Yeh, Macaluso, and Hershbein (2022) give a formula of markdown and markup using both cost minimization and monopolist profit maximization FOCs. Rubens (2023) also uses cost minimization and monopolist profit maximization FOCs and shows a markdown/markup formula under nonsubstitutability of materials.

Gandhi, Navarro, and Rivers (2020) proves the nonidentification of gross production function when one inverts a flexible input demand function and in dynamic panel estimator. In their way of identification, they consider the output price markups yet use price-taking profit maximization FOC, which can be their limitation.

# Contents

## Summary

- Generally speaking, concentration may be a misleading measure of market power.

- Monopolistic competition of heterogenous firms can result in a negative relationship between market power and concentration.

- A more solid definition of market power is given by output price markups (downstream) or input price markdowns (upstream).

- But using concentration can be OK in some markets: Studies on US/online labour markets show concentration is positively related to markdowns/supression of wages.

- So one can start with concentration, then proceed to wage markdowns.

- Many studies estimate markups/markdowns at the establishment level with financial statement data using the "production approach".

- It only assumes cost minimization of firms.

- Production approach looked very useful, but the criticism of nonidentification results by Gandhi, Navarro, and Rivers (2020).

- Literature has not come up with a method that responds to the criticism.

- For the time being, one can start with the Rubens' or DJ's method to estimate wage markdowns.

## Introduction

Following Hall (1988)'s industry level estimation, De Loecker and Warzynski (2012) developed the "production approach" to TFPR/markup estimation at the plant/firm level, by using only financial statement of firms.

Their algorithm follows the tradition of inverting a factor demand function to obtain (to "proxy") productivity $\omega_{jt}$ (Olley and Pakes 1996; Levinsohn and Petrin 2003; synthesized by Ackerberg, Caves, and Frazer 2015).

There are debates over ACF-DLW method:

- Invertibility of factor demand, aka scalar unobservable ass. and monotinicity ass. (Bond et al. 2021; but see Appendix O.6.3 of Yeh, Macaluso, and Hershbein 2022; while Demirer 2022 allows labour augmenting shocks in addition to Hicks neutral shocks under the homothetic production function)

- Nonidentification results when one uses profit maximization FOC on a flexible input (materials, energey) in a gross output production function (Gandhi, Navarro, and Rivers 2020).

- Nonidentification results when using deflated revenues in place of physcal quantity and profit maximation FOC (Bond et al. 2021; see also a reply by De Loecker 2021 of practices of controlling output prices and also input prices).

- Endogeneity of inputs (or what the valid IVs are, Doraszelski and Jaumandreu 2021)

- Hicks neutral (factor non-augmenting) tech assumption (Doraszelski and Jaumandreu 2013; Demirer 2022; Raval 2023)

## Market power and concentration measures

Originally, below is the problem.

Output market power := Markup, or the difference between price and marginal cost.

Input market power := Markdown, or the difference between factor price and marginal revenue product.

Problem: We do not observe marginal X.

### Old method: Structure-conduct-performance literature

Herfindahl-Hirschman index (HHI, a summary measure of market shares) was used as a proxy in the IO literature up to 1980's. The practice is used outside the IO even now.

Problem: Monopolistically competitive models with heterogenous firms can predict: concentration↑ ∼ markup↓

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┌─────────────────────┐                          ┌─────────────────────┐
│  concentration ↑    │- - - - - - - - - - ->│    markups ↑        │
└─────────────────────┘       Cournot           └─────────────────────┘
          ↑
          ┊
   ╭──────────────────────────╮
  ( productivity dispersion ↑ )
   ╰──────────────────────────╯
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concentration ↑  - - →  markups ↓

monopolistically competitive

firm exits ↑

range of products to compete with ↑

substitutability of goods ↑
=
price senstivity of consumers ↑
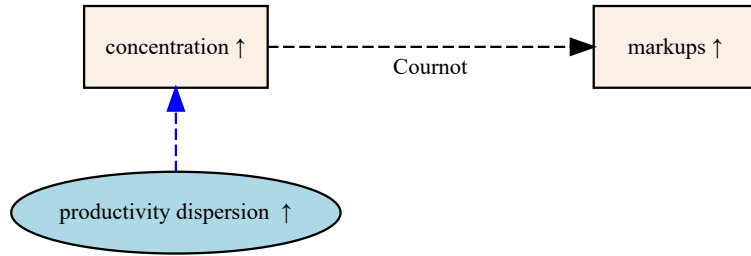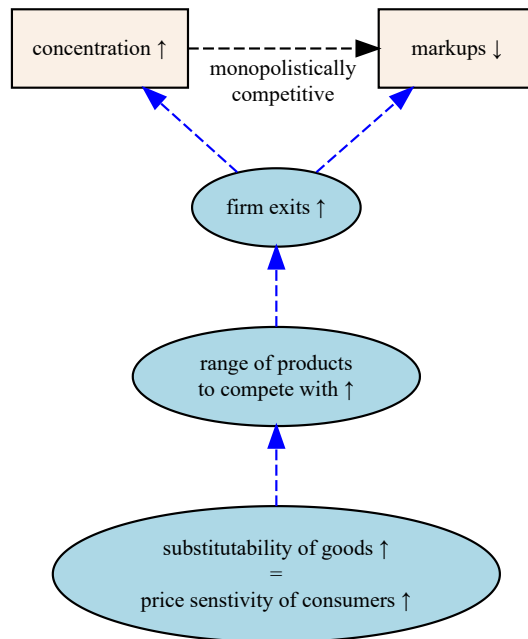
## Current method

More recently, one uses markups (and markdowns) by obtaining marginal "X" in some way.

Despite concerns, the old method may be valid in some markets. A positive relationship between market power and concentration:

- Employment share and markdowns (Yeh, Macaluso, and Hershbein 2022)

- Employment effects of minimum wages (if small, originally large markdown) and HHI (Azar et al. 2019)

- Job application elasticity (if small, large power) and HHI (Azar, Marinescu, and Steinbaum 2019)

- Earnings or wages (low) and HHI (Azar et al. 2020; Benmelech, Bergman, and Kim 2022)

- Income inequality and HHI (Rinz 2022)

- Slower wage growth and HHI (by hospital mergers) (Prager and Schmitt 2021)

# DLW method on markup

Marginal cost can be obtained from a cost minimization problem.

Conditional factor demand function (of factor price-taking cost minimization) and factor demand function (of price-taking profit maximization) need not to coincide (equality needs markup in cost minimization = inverse demand elasticity plus one in profit maximization).

## Markup formula

Production function with a measurement error $\epsilon$ (subscripts $i, t$ are suppressed)

$$Y = Q^* e^\epsilon, \quad Q^* \leqslant F(L, K, M) e^\omega.$$

Cost minimization on variable input $V = L, M$:

$$\mathcal{L} = P^V V + rK + \lambda[Q^* - F(L, K, M)e^\omega].$$

FOC:

$$P^V - \lambda F_V e^\omega = 0 \quad \lambda = \frac{P^V}{F_V e^\omega}.$$

Note that $\lambda$ is a measure of marginal cost. Markup $\mu = \frac{P}{MC}$ is[1]

$$
\begin{aligned}
\mu &= \frac{P}{\lambda} = \frac{P F_V e^\omega}{P^V}, \\
&= \frac{P F_V e^\omega Y V}{P^V V F e^{\epsilon+\omega}}, \\
&= \frac{PY}{P^V V} \frac{F_V V}{F} e^{-\epsilon}, \\
&= \frac{\theta^V}{S^V} e^{-\epsilon}, \quad S^V \equiv \frac{P^V V}{PY}, \ \theta^V \equiv \frac{F_V V}{F} = \frac{\partial F}{\partial V} \frac{V}{F}.
\end{aligned}
$$

Once we back out $\epsilon$, we get $\mu$. With subscripts, firm specific markup $\mu_{it} = \frac{\theta^V_{it}}{S^V_{it}} e^{\epsilon_{it}}$ is affected by $\hat{\epsilon}_{it}$. Need to get rid of it.

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[1]Following the notation convention of the literature, I use $S^V$, not $\alpha^V$ as in this paper, for factor revenure share of $V$.

## Factor demand inversion

DLW suggests to invert the log factor demand $m = m(k, l, \mathbf{p}, \omega)$ to substitute with $\omega = h(k, l, m, \mathbf{p})$,[2] so log production is

$$y = f(k, l, m, \mathbf{p}) + h(k, l, m, \mathbf{p}) + \epsilon = \phi(k, l, m, \mathbf{p}) + \epsilon,$$

where

$$\phi(k, l, m, \mathbf{p}) = f(k, l, m, \mathbf{p}) + \omega.$$

Then DLW suggests to regress $y$ on polynomials on $k, l, m, \mathbf{p}$ to get $\hat{\phi}$, and compute

$$\hat{\epsilon} = y - \hat{\phi}.$$

In inverting the factor demand, one needs:

1. Scalar unobservable assumption: There is only one latent factor $\omega$ in demand equation $m$.

2. Monotonicity assumption: The demand function is monotonic in $\omega$.

Monotonicity ass. is not too strong. But scalar unobservable ass. is.

Other implicit assumptions:

3. There are no firm fixed effects $\alpha_i$ (or $\alpha_i = 0$).[3]
4. Output demand conditions which affect input demand differently by firms are all controlled by the observables.

DLW acknowledges the strong imposition of 4. Covariates $\mathbf{z}_{it}$ added to control for output market conditions are:

- DLW: Export status, lagged inputs in $h(k, l, m, \mathbf{p})$.[4]

- De Loecker et al. (2016): Location dummies, output prices, product dummies, market shares, input prices, export status, input tariffs, output tariffs (p.466).

- De Loecker (2011): Product information and import quota protection (assumed to be exogenous to individual firms).

Some can be endogenous to production but no discussions in DLW on IVs on firm state variables $\mathbf{z}_{it}$.

**Algorithm (DLW 2012)** on $y_{it} = f(\mathbf{x}_{it}, \beta) + \epsilon_{it} + \omega_{it}$ with a choice of $f$.

1. Get $\hat{\phi}_{it}$ by regressing $y_{it}$ on polynomials in $k_{it}, l_{it}, m_{it}, \mathbf{p}'_{it} = \mathbf{x}'_{it}, \mathbf{p}'_{it}$.

2. Impute $\hat{\omega}_{it}(\beta) = \hat{\phi}_{it} - f(\mathbf{x}_{it}, \beta)$ and $\hat{\omega}_{it-1}(\beta) = \hat{\phi}_{it-1} - f(\mathbf{x}_{it-1}, \beta)$.

3. Assume $\omega_{it} = g(\omega_{it-1}) + \xi_{it}$ where $g$ is a polynomial of choice in $\omega_{it-1}$. Get $\hat{\xi}_{it}(\beta) = \hat{\omega}_{it}(\beta) - g\{\hat{\omega}_{it-1}(\beta)\}$.

4. Estimate $\beta$ with GMM using $\varepsilon[\hat{\xi}_{it}(\beta)v_{it}] = 0$ where $v_{it}$ is a flexible input chosen by firm $i$ before observing $\xi_{it}$. Use additional moment conditions (e.g., capital as a predetermined input $v_{it} = k_{it}$) to gain efficiency.

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[2]If, for example,

$$m = a_1 k + a_2 l + a_3' \mathbf{p} + \omega,$$

then:

$$\omega = \underbrace{m - (a_1 k + a_2 l + \mathbf{a}_3' \mathbf{p})}_{\equiv h(k, l, m, \mathbf{p})}.$$

[3]But time-variant, partly random heterogenous productivity $\omega_{it}$ is incorporated. Not sure if this is a major drawback…
[4]DLW 2012 mention export status (p.2446), lagged variable inputs as IVs for current inputs (Appendix p.5), and is not explicit on what variables are used in $\mathbf{z}_{it}$.

5. Bootstrap to estimate covariance matrix: Set $B$ as number of bootstrapping. Randomly sample firms with replacement for its entire lifespan. Do 1.-4. for $B$ runs.

Comments on DLW algorithm:

- It takes at least 2 period panel data (vs. dynamic panel takes 3 periods).

- Scalar unobservable (= only 1 latent factor) is crucial.

- Materials costs: Cost of Goods and Services (COGS) in US manufacturing data. Some procured goods may be used for fixed investments or non-production purposes, so it is not a perfect measure.

- Formula is not new and inversion is done by OP. DLW (p.2444) notes their contribution is being able to recover firm-specific estimates of markup while providing consistent estimates of output elasiticities and allowing some inputs (capital, in some cases labour) to face adjustment costs.

- One of OP's motivation for factor demand inversion is "preserving the typically enormous cross-sectional variation for the sake of identification" (De Loecker and Syverson 2021).

TABLE 2—MODEL ESTIMATES

| | Ordinary least squares | | Dynamic panel | |
|---|---|---|---|---|
| | Est. | SE | Est. | SE |
| *Panel A. Production function* | | | | |
| Output elasticity of labor | 0.563 | 0.082 | 0.532 | 0.147 |
| Output elasticity of capital | 0.569 | 0.066 | 0.630 | 0.105 |
| Scale parameter | 1.132 | 0.044 | 1.162 | 0.060 |
| $R^2$ | 0.91 | | 0.92 | |
| Observations | 1,130 | | 849 | |
| | | | | |
| *Panel B. Leaf price markdown* | | | | |
| Average | 2.934 | 0.414 | 2.904 | 0.442 |
| Median | 2.134 | 0.066 | 2.126 | 0.079 |

*Notes:* Panel A reports the estimated output elasticities using both OLS and the dynamic panel estimator. Panel B contains the leaf markdown moment estimates. Standard errors are block-bootstrapped with 200 iterations.

Comparision with the dynamic panel (Blundell and Bond) estimator.

- BB double-differences out the confounds as it poses weaker assumptions on the correlation between unobservables and covariates $\varepsilon[\xi_{it} - \xi_{it-1} + (\epsilon_{it} - \rho\epsilon_{it-1}) - (\epsilon_{it} - \rho\epsilon_{it-1})|k_{it-1}, l_{it-1}] = 0$ (see 2.3.1 of Ackerberg 2023). This eliminates the need to invert a demand to net out $\epsilon_{it}$, or of the scalar unobservable ass.

- By not inverting $\omega$, BB cannot separately identify $\omega + \epsilon$.

- BB estimates tend to be noisier due to weaker statistical power. See Table 2 from Rubens (2023).

- Weaker statistical power leads to the use of additional moment conditions ("system GMM" estimator).[5]

- BB's double-differencing eliminates the firm fixed effects $\alpha_i$ while DLW assumes $\alpha_i = 0$.

- BB assumes AR(1) in $\omega_{it}$, not polynomials (as under first-order Markov).

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[5]But the economic validity of additional moment conditions can be ambiguous.
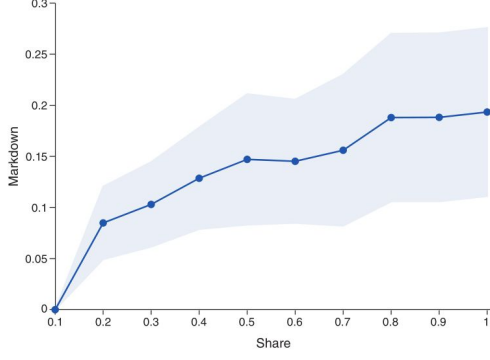
FIGURE 1. MARKDOWN-SIZE RELATIONSHIP

*Notes:* The figure shows point estimates and 95-percent confidence intervals of plant-specific markdowns on size (as measured by employment share) indicators, controlling for indicators for plant age and industry, as well as state and year fixed effects. The omitted group is the smallest size indicator, so coefficients reflect deviations relative to this baseline. The indicator labeled "0.1" is equal to unity for those plants with employment shares $s \in (0, 0.1]$. Other indicators are defined similarly. Standard errors are clustered at the industry level.

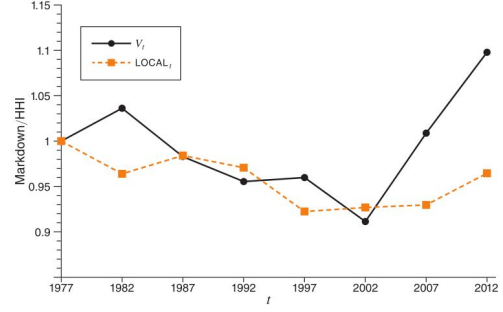*Source:* Authors' own calculations from ASM/CM data in 1976–2014.

FIGURE 6. AGGREGATE MARKDOWN AND LOCAL CONCENTRATION, 1977–2012

*Notes:* The solid black line shows the time series for the aggregate markdown as in (14), and the dashed orange line shows the time series of local employment concentration as in (18). Both are normalized to their initial respective values in 1977.

*Source:* Authors' own calculations from quinquennial CM data from 1977–2012.

Figure 1: US wage markdown in Yeh et al. (2022)

# YMH method on markdown

## Formula

Yeh, Macaluso, and Hershbein (2022) showed markdown can be estimated with a formula similar to DLW.

- It uses a monopolist profit maximization FOC.

Monopolist profit maximization (with a single input)

$$\max_{L} \quad R(L) - w(L)L$$

FOC

$$R'(L) = w'(L)L + w(L) = \left( \frac{w'(L)L}{w(L)} + 1 \right) w(L) = \left( \varepsilon_s^{-1} + 1 \right) w(L)$$

where $\varepsilon_s^{-1}$ is an inverse elasticity of input $L$ supply (or input price elasticity of input supply function). This gives the markdown[6] $\psi^L$ for a monopolist:

$$\psi^L \equiv \frac{R'(L)}{w(L)} = \varepsilon_s^{-1} + 1.$$

(Variable) Cost minimization (with a single input)

$$\mathcal{L} = w(L)L + \lambda[Q^* - F(L; \mathbf{V}, \omega)].$$

FOC

$$w'(L)L + w(L) = \lambda F_L \quad \Leftrightarrow \quad \underbrace{\frac{w'(L)L}{w(L)} + 1}_{=\varepsilon_s^{-1}+1} = \frac{\lambda F_L}{w(L)}.$$

Then

$$\psi^L = \varepsilon_s^{-1} + 1 = \frac{\lambda}{P} \frac{F_L L}{Y} \frac{PY}{w(L)L} = \mu^{-1} \frac{\theta^L}{S^L}.$$

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[6]How much more revenue is generated with an additional input cost. $> 1$ under monopsony/oligoposny.

With any other flexible input $V = Energy, M$ and markup $\mu^V$ derived from $V$, input price markdown for $L$ is given by:[7]

$$\psi^L = \frac{\left(\frac{\theta^L}{S^L}\right)}{\left(\frac{\theta^V}{S^V}\right)}.$$

For implementation, Yeh et al. (2022) use the DLW algorithm.

Wage markdown is defined as the above. Taking $V = M$, we have $\psi^L = \left(\frac{\theta^L}{S^L}\right) / \left(\frac{\theta^M}{S^M}\right)$. It is important to note that, for markdown to be unaffected by market power in other factor markets $\frac{\theta^V}{S^V}$, the labour monopsony firm needs to be a price taker in the factor market for $V$. This is the assumption which must hold when using the above formula.[8]

Is this assumption realistic? The answer depends on the context. However, an oligopsonistic market power in materials and energy markets is less likely to hold than oligopsonistic market power in a labour market. Labour market transactions are inherently tied to location which limits the competition among the buyers. In the case of materials, they are more mobile hence there will be more firms that demand the same materials. In the case of energy, its price is often regulated by the government.

## Data

### Census of Manufactures

Establishment level data of all manufactures in 1976-2014, collected in years ending with 2 and 7. Revenues and inputs (capital, labor, materials, energy) in monetary units.

*Census Bureau*

### NBER-CES Manufacturing Database

Annual industry-level data from 1958-2018 on output, employment, payroll and other input costs, investment, capital stocks, TFP, and various industry-specific price indexes.

*National Bureau of Economic Research*

### Annual Survey of Manufactures

Establishment level data of all manufactures in 1976-2014, rotating panel collected in every year. Large plants are sampled with near certainty, small plants are sampled less frequently.

*Census Bureau*

US all industry aggregate markdown decreased (1977-2002) but rose after 2002. Markdown is increasing in the local concentration measure (HHI).

# Rubens' method on markup and markdown

## Formula under nonsubstitutability of materials

### Prevalence of nonsubstitutability in production processes

In modern manufacturing, there are materials and energy that are difficult to substitute with other inputs. In agriculture, seeds/seedlings and water cannot be substituted with any other inputs. Chemicals (fertilizer,

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[7]This formula has an extra benefit that it cancels out bias terms of using deflated revenues in place of quantity. See Proposition 2 of Appendix.

[8]Because market for $V = M$ is assumed to be competitive, one cannot consider material input price markdown defined as $\psi^M = \frac{\left(\frac{\theta^M}{S^M}\right)}{\left(\frac{\theta^L}{S^L}\right)}$ which implies $\psi^L = \left(\psi^M\right)^{-1}$.

pesticide, fungicide, etc.) and facilities (nets, fences) are more substitutable. Nonsubstitutability of materials implies production with Leontief in (raw) materials (tobacco leaves)[9] [10]

**Effects of nonsubstitutability**

Yeh et al. (2022) estimated markups and markdowns simultaneously. Rubens (2023) showed this is no longer possible (without further assumptions) once material is nonsubstitutable.

Consider a production technology with Leontief in materials:

$$Q^* \leqslant \min \left\{ \beta^M M, H(L,K) e^\omega \right\}$$

$M$ is more like a constraint that firm has to keep up with when changing other inputs... Cost minimization of planned production (free of $\epsilon$):

$$\mathcal{L} = W^M(M)M + W^L(L)L + \lambda \left[ Q^* - \min \left\{ \beta^M M, H(L,K) e^\omega \right\} \right].$$

FOC

$$W^{M'}(M)M \frac{dM}{dL} + W^M(M) \frac{dM}{dL} + W^{L'}(L)L + W^L(L) = \lambda H_L e^\omega.$$

Rewriting

$$\underbrace{\left[ \frac{W^{M'}(M)}{W^M} M + 1 \right]}_{\equiv \psi^M} W^M \frac{dM}{dL} + \underbrace{\left[ \frac{W^{L'}(L)}{W^L} L + 1 \right]}_{\equiv \psi^L} W^L = \lambda \underbrace{H_L e^\omega}_{\frac{\partial Q^*}{\partial L}}$$

$\psi^M$ is inverse material supply elasticity $+ 1$ and $\psi^L$ is inverse labor supply elasticity $+ 1$. So

$$\psi^M W^M \frac{\frac{dM}{dL}}{\frac{\partial Q^*}{\partial L}} + \psi^L W^L \frac{1}{\frac{\partial Q^*}{\partial L}} = \lambda.$$

In Leontief, any increase in $M$ increases $Q^*$ at the same rate, or at $\frac{Q^*}{M}$:

$$\frac{\partial Q^*}{\partial L} \frac{dL}{dM} = \frac{Q^*}{M} = \frac{Y e^{-\epsilon}}{M}.$$

This gives

$$\psi^M W^M \frac{1}{\frac{Y e^{-\epsilon}}{M}} + \psi^L W^L \frac{1}{\frac{\partial Y e^{-\epsilon}}{\partial L}} = \lambda.$$

Usual substitution gives

$$\lambda = P\psi^M \frac{W^M M}{PY e^{-\epsilon}} + P\psi^L \frac{W^L L}{PY} \frac{1}{\frac{\partial Y e^{-\epsilon}}{\partial L}} \frac{1}{\frac{L}{Y}},$$

$$= P\psi^M S^M e^\epsilon + P\psi^L S^L \frac{1}{\theta^L} e^\epsilon,$$

or[11]

$$\mu = \left[ S^M \psi^M + S^L \psi^L \frac{1}{\theta^L} \right]^{-1} e^{-\epsilon}. \tag{4a}$$

Note: There is no input demand inversion involved up to here.

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[9]$\epsilon$ is missing, but it will not be a problem, because, in the end the paper does not invert a factor demand. But for comparability, I add it in below. To be exact with the paper, assume $\epsilon = 0$.

[10]DLW also consider Leontief in materials to which they call the "value added production function" as opposed to "gross output production function" in which they treat materials as substitutable. However, even with their version of "VA production function," they subtract the materials as if it is a preprocess of cost minimization and do not consider how materials affect FOCs.

[11](4a) nests DLW (substitutable $S^M = 0$ and input market competition $\psi^V = 1$), Morlacco (2017, substitutable $S^M = 0$ and input market non-competition $\psi^V > 1$), and De Loecker and Scott (2022, nonsubstitutable $S^M > 0$ and input market competition $\psi^V = 1$) as special cases.

- To be concrete, DLW's formula does not necessarily imply its implementation (algorithm with inversion). Neither does YMH's or Rubens'.

- For markup/markdown estimation, one can either invert a factor demand and/or use a dynamic panel estimator.

But we need to deal with $\epsilon$.

## Identification strategy "choices"

### Design choice

In (4a):

- 1 equation, 6 variables

- 3 unknowns: $\mu, \psi^L, \psi^M$

- 3 knowns (1 estimable: $\theta^L$, 2 observables: $S^L, S^M$)

Need to come up with a structure that gives values to 2 unknowns.

- $\psi^L = 1$, $\mu_i = \mu = 1$ and exogenously set by wholesalers, estimate $\psi^M$ (main text).
  - Other values of $\mu$ in Appendix.

- $\psi^L = 1$, $\psi^M$ is given by a nested logit (over farmer occupations) framework (with many accompanying assumptions) as a function of estimated parameters of occupation share equation, estimate $\mu$ (Appendix A, C.1).
  - Noisy estimates.

- Others are possible.

Why are $\psi^L = 1$, $\mu_i = \mu$ plausible?

- Wage rates did not change before and after consolidation.

- Wholesaler (state monopoly) unilaterally sets $P$.

Once we assume uniform exogenous markup and $\psi^L = 1$, we have:

$$
\begin{aligned}
\mu &= \frac{\beta^L}{S_{it}^L + \beta^L S_{it}^M \psi_{it}^M}, \\
(S_{it}^L + \beta^L S_{it}^M \psi_{it}^M)\mu &= \beta^L, \\
\psi_{it}^M &= \frac{\beta^L - \mu S_{it}^L}{\mu \beta^L S_{it}^M}, \\
&= \frac{1}{S_{it}^M}\left(\frac{1}{\mu} - \frac{S_{it}^L}{\beta^L}\right).
\end{aligned}
\tag{8}
$$

### Moment condition choice

How do we deal with $\epsilon$?

- Proxy variable approach eliminates $\epsilon$ in the moment condition by inverting a factor demand to substitute for $\omega$.

- Dynamic panel estimator leaves $\epsilon$ in the moment condition.

Rubens chooses the latter (proxy variable approach is placed in Appendix), because:

- Input demand (and its inversion) must control for (markups and) markdowns. This requires an additional structure or additional restrictions on their distributions. This limits the data from expressing heterogenous markups/markdowns.
- With additional parametric restrictions on productivity dynamics in dynamic panel estimator, heterogenous markdowns are obtained. Since markdown estimation is the main objective, this is better suited to the purpose.[12]

## Estimation steps

Main results set $\epsilon = 0$ and use dynamic panel estimator.

Robustness check results invert a factor demand to purge $\epsilon$.

One could have retained $\epsilon$ and estimated dynamic panel, too.

### Estimation without $\epsilon$

1. Assume a production function with quality-unadjusted inputs $\tilde{l}, \tilde{k}$ and quality-unadjusted revenue with a quality adjustment function $a\left(p, w^L\right)$:

$$q = \delta\left(\tilde{l}, \tilde{k}, \beta\right) + a\left(p, w^L\right) + \omega$$

2. Incorporate a consolidation dummy in productivity dynamics and assume AR(1) in redisdual productivity:

$$\omega_{it} = \beta^{z'} \mathbf{z}_{it} + \tilde{\omega}_{it}, \quad \tilde{\omega}_{it} = \rho \tilde{\omega}_{it-1} + \nu_{it}$$

3. By $\rho$-differencing, form moment conditions $\varepsilon[\hat{\nu}_{it}(\tilde{l}_{it-1}, \tilde{k}_{it}, \tilde{k}_{it-1})] = 0$, etc. and estimate $\beta$.

### Estimation with $\epsilon$

Appendix C.1. deals with $\epsilon \neq 0$. It assumes leaf as flexible but nonsubstitutable inputs and inverts its demand (termed as "control function approach").[13]

- The "CF Approach" uses Leontief: leaf price per case = leaf costs/output in cases. Use leaf price per case $w^M$ in the leaf demand inversion to control for differences in input price (input quality).
- Input demand is given by $m(p, w^L, w^M, \tilde{\mathbf{x}}, \beta, \mu, \psi^M, \omega)$ in Appendix E[14] and substitute $\mu = \mu(p, \gamma^P, \mathbf{s})$ with $\mathbf{s}$ is a vector of various market shares and $\gamma^P$ is price elasticity of output demand. $\gamma^P$ is assumed to be unique and to be subsumed in the intercept term (a strong ass.). $\psi^M$ is derived as $\psi^M(w^M, \mathbf{s})$. Everything is sort of first-order Taylor approximation.

$$\begin{aligned} q &= \delta\left(\tilde{\mathbf{x}}, \beta\right) + a\left(p, w^L\right) + \omega + \epsilon, \\ &= \delta\left(\tilde{\mathbf{x}}, \beta\right) + a\left(p, w^L\right) + h\left(p, w^L, w^M, \tilde{\mathbf{x}}, \beta, \gamma^P, \mathbf{s}\right) + \epsilon, \\ &= \phi\left(\tilde{\mathbf{x}}, \beta, p, w^L, w^M, \mathbf{s}\right) + \epsilon. \end{aligned}$$

- Productivity is backed out with the use of quality adjustment function:

$$\omega = \hat{\phi} - \delta\left(\tilde{\mathbf{x}}, \beta\right) - a\left(p, w^L\right).$$

- Results do not change much (Table A4).

"(T)he production approach to markup measurement does not hinge on a particular approach to estimate the production function. Rather it is an approach that delivers markups (and potentially marginal costs) for each individual producer (and time period), by exploiting standard cost minimization of a variable input in production."

— De Loecker, 2020

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[12]I do not fully understand this argument.

[13]Control function approach, proxy variable approach, production approach...all of them seem to refer to the same thing.

[14]Just derive $Q^*(\cdot)$ from cost minimzation FOCs and form $\frac{Q^*}{\beta^M}$ where $\beta^M$ is the quantity per case.

# GNR criticism and the proposed solution

Gandhi, Navarro, and Rivers (2020) showed that, for gross production functions, in the absence of material demand variations (say, by input prices which may or maynot be observed), parameters are not indentified when using profit maximization FOC in material demand inversion and using IVs to estimate other parameters.

There are four points in relationship with DLW:

1. GNR criticism is on gross production function estimation.
   - A *gross production function* describes production technology with material inputs.

   - A *value added production function* describes production technology only with non-materials, i.e., labour, and various forms of capital.

2. DLW formula and implementation (obtaining production function parameters) are separate objects. DLW formula is unrelated to GNR criticism.

3. One needs a source of variations across firms in material input demand. If material input prices do not vary through time, material input demand, conditional on lagged inputs, varies across firms only by shocks $\xi_{it}$ to TFP, which are by design orthogonal to IVs, so IVs cannot identify parameters.

4. GNR shows the criticism also applies to dynamic panel estimators.

Is this a fatal blow to the "production approach"?

## Criticism: Nonidentification of elasticity

In the absence of time-series variations in relative prices $\frac{P_t}{P_t^V}$, IV estimation (GMM) using lagged flexible inputs as instruments for current flexible inputs does not give unique values for output elasticity $\theta^K, \theta^L$ on $k, l$.

Even with time-series variations in relative prices, IV estimates suffer from weak identification. This is also shown in Monte Carlo by Kasahara and Sugita (2023).

## Why

$$y_{it} = f^0(\mathbf{x}_{it}) + \omega_{it} + \epsilon_{it},$$
$$= f^0(\mathbf{x}_{it}) - f_M^0(\mathbf{x}_{it}) - \varepsilon[\epsilon_{it}] + d_t + \epsilon_{it}.$$

where we used price-taking expected profit maximization FOC

$$\max_{M_{it}} \quad \varepsilon\left[P_{it}F^0(K_{it}, L_{it}, M_{it})e^{\omega_{it}+\epsilon_{it}} - \rho_{it}M_{it}\right]$$

$$\frac{\partial F^0(\mathbf{x}_{it}) e^{\omega_{it}}\varepsilon[e^{\epsilon_{it}}]}{\partial M_{it}} = \frac{\rho_{it}}{P_{it}}.$$

$$f_M^0(\mathbf{x}_{it}) + \omega_{it} + \varepsilon[\epsilon_{it}] = \underbrace{\ln\rho_t - p_t}_{\equiv d_t}, \quad f_M^0(\mathbf{x}_{it}) = \ln\frac{\partial F^0(\mathbf{x}_{it})}{\partial M_{it}},$$

so one can invert

$$\omega_{it} = \underbrace{-f_M^0(\mathbf{x}_{it}) - \varepsilon[\epsilon_{jt}]}_{\equiv M^{-1}(\mathbf{x}_{it})} + d_t,$$

$$y_{it} = f^0(\mathbf{x}_{it}) + \omega_{it} + \epsilon_{it},$$
$$= \underbrace{f^0(\mathbf{x}_{it}) + M^{-1}(\mathbf{x}_{it})}_{\equiv \phi_G(\mathbf{x}_{it})} + d_t + \epsilon_{it}. \quad (\phi_G + d_t = \phi \text{ of DLW})$$

Using $\phi_G$

$$\omega_{it} = f^0(\mathbf{x}_{it}) + M^{-1}(\mathbf{x}_{it}) + d_t - f^0(\mathbf{x}_{it}),$$
$$= \phi_G(\mathbf{x}_{it}) + d_t - f^0(\mathbf{x}_{it}),$$
$$\omega_{it-1} = \phi_G(\mathbf{x}_{it-1}) + d_{t-1} - f^0(\mathbf{x}_{it-1}).$$

First-order Markov ass. gives

$$\omega_{it} = h^0(\omega_{it-1}) + \xi_{it}, \quad (h^0 = g \text{ of DLW})$$
$$= h^0\{\phi_G(\mathbf{x}_{it-1}) + d_{t-1} - f^0(\mathbf{x}_{it-1})\} + \xi_{it}.$$

So, with price-taking profit max FOC and FO Markov, we get

$$y_{it} = f^0(\mathbf{x}_{it}) + h^0\{\phi_G(\mathbf{x}_{it-1}) + d_{t-1} - f^0(\mathbf{x}_{it-1})\} + \xi_{it} + \epsilon_{it}.$$

The statement: If $d_t = d$ for all $t$,

- Conditional expectation (conditioned on IVs $\Gamma_{it}$) of $y_{it} = f^0(\mathbf{x}_{it}) + h^0(z) + \xi_{it} + \epsilon_{it}$ can be expressed as $E\left[f^0(\mathbf{x}_{it}) \mid \Gamma_{it}\right] + E\left[h^0(z) \mid \Gamma_{it}\right]$.

- But, when we define new functions $\tilde{f} \neq f^0, \tilde{h} \neq h^0$ by mixing a nuisance function at a constant proportion $a$ for any $a \in (0,1)$ as in the below, we can show (in Theorem 1) that their conditional (on IVs) expectations are same $E\left[f^0(\mathbf{x}_{it}) \mid \Gamma_{it}\right] + E\left[h^0(z) \mid \Gamma_{it}\right] = E\left[\tilde{f}(\mathbf{x}_{it}) \mid \Gamma_{it}\right] + E\left[\tilde{h}(z) \mid \Gamma_{it}\right]$.

$$\tilde{f}(\mathbf{x}_{it}) \equiv (1-a)f^0(\mathbf{x}_{it}) + a\phi_G(\mathbf{x}_{it}),$$
$$\tilde{h}(z) \equiv ad + (1-a)h^0\left(\frac{z - ad}{1 - a}\right).$$

- For true functions $f^0, h^0$, there are infinitely many *wrong* functions $\tilde{f}, \tilde{h}$, or different "parameters", whose conditional expectations (of functional values) are identical (*observationally equivalent*).[15] Different functions give different parameter values. Conditional expectations using IVs map to multiple (this case infiintely many) different parameter values. So conditional expectations of true functions cannot be identified uniquely by using IVs, or we say, functions $f^0, h^0$ are *not point identified*.[16]

- Note: When one regresses $y_{it}$ on $\Gamma_{it}$, [given the same "what can be known from data" ($\phi_G$ of Lewbel 2019), which is usually the distributions of data under certain i.i.d. sampling] **the same data/distributions of data/model value $m$ map to different parameter values** $f_v^0 \neq \tilde{f}_v = f_v^0 + a\frac{\partial M^{-1}}{\partial v}$ for any $a \in (0,1)$ if $\frac{\partial M^{-1}}{\partial v} \neq 0$. So (true) parameters are not point identified.

- GNR Appendix O1 shows that nonidentification results also applies to dynamic panel estimators.

## GNR solution: Use price-taking profit maximization FOC to pin down material demand partial derivative

Their Theorem 2 and 3 show that using the *share regression* recovers all parameters of production function nonparametrically:

$$s_{it} = \ln D^\epsilon(\mathbf{x}_{it}) + u_{it}^\epsilon. \tag{s}$$

**Algorithm (in theory)**

1. Estimate share regression: log shares on $\mathbf{x}_{it}$.

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[15] When two different distributions give the same moments/likelihood, the parameters are not uniquely identified.

[16] When any two different model values $m_1, m_2 \in \mathcal{M}$ are not observationally equivalent (i.e., parameters $\theta_1 \neq \theta_2$ are different), the parameters $\theta$ of the model are said to be *point identified*. See Lewbel (2019).

2. Get $\hat{\epsilon}_{it} = -\left(s_{it} - \ln \hat{D}^\epsilon(\mathbf{x}_{it})\right)$ and $\hat{\varepsilon}$ by the mean of $e^{\hat{\epsilon}_{it}}$, $\hat{f}_m = \frac{\hat{D}^\epsilon(\mathbf{x}_{it})}{\hat{\varepsilon}}$.

3. Integrate $\hat{f} = \int \hat{f}_m dm_{it}$ and compute $\hat{\mathcal{Y}}_{it} = y_{it} - \hat{f} - \hat{\epsilon}_{it}$.

4. Regress $\hat{\mathcal{Y}}_{it}$ on $k_{it}, l_{it}, k_{it-1}, l_{it-1}, \mathcal{Y}_{it-1}$, possible interactions of these and recover derivatives $\mathcal{C}_v$ of $v = k_{it}, l_{it}$.

5. Integrate $\int \hat{\mathcal{C}}_v dv = \hat{\mathcal{C}} + c$ which gives $\hat{\mathcal{C}}$ up to a constant.

6. Compute $\hat{f}(\mathbf{x}_{it}) + c = \hat{f}(\mathbf{x}_{it}) - \hat{\mathcal{C}}$, $\hat{\omega}_{it} + c = y_{it} - \hat{f}(\mathbf{x}_{it}) - \hat{\epsilon}_{it}$.

**Algorithm (for implementation using Siev estimator)**

**Deriving the share regression**

Price-taking profit maximization gives

$$P_t \frac{\partial F^0(\mathbf{x}_{it})}{\partial M_{it}} e^{\omega_{it}} \varepsilon[e^{\epsilon_{it}}] = \rho_t.$$

Noting

$$\frac{\partial \ln F^0(\mathbf{x}_{it})}{\partial m_{it}} = \frac{\partial \ln F^0(\mathbf{x}_{it})}{\partial F^0} \frac{\partial F^0}{\partial M_{it}} \frac{\partial M_{it}}{\partial m_{it}} = \frac{1}{F^0} F^0_M M_{it},$$

where the last derivative is given by:

$$1 = \frac{\partial M}{\partial M} = \frac{\partial M}{\partial m} \frac{\partial m}{\partial M} = \frac{\partial M}{\partial m} \frac{1}{M} \quad \Rightarrow \quad \frac{\partial M}{\partial m} = M,$$

so,

$$F^0_M = \frac{\partial \ln F^0(\mathbf{x}_{it})}{\partial m_{it}} \frac{F^0}{M_{it}} = f^0_m(\mathbf{x}_{it}) \frac{F^0}{M_{it}}, \quad f^0_m(\mathbf{x}_{it}) \equiv \frac{\partial \ln F^0(\mathbf{x}_{it})}{\partial m_{it}}.$$

Plugging this in gives:

$$P_t f^0_m F^0(\mathbf{x}_{it}) e^{\omega_{it}} \varepsilon[e^{\epsilon_{it}}] = \rho_t M_{it},$$

$$P_t f^0_m \underbrace{F^0(\mathbf{x}_{it}) e^{\omega_{it}} e^{\epsilon_{it}}}_{\equiv Y_{it}} e^{-\epsilon_{it}} \varepsilon[e^{\epsilon_{it}}] = \rho_t M_{it},$$

$$f^0_m e^{-\epsilon_{it}} \varepsilon[e^{\epsilon_{it}}] = \frac{\rho_t M_{it}}{P_{it} Y_{it}} \equiv S_{it}.$$

Taking logs and denoting $\ln E[e^{\epsilon_{it}}] = \ln \varepsilon$ gives (s):

$$s_{it} = \underbrace{\ln f^0_m + \ln \varepsilon}_{\equiv \ln D^\epsilon(\mathbf{x}_{it})} - \epsilon_{it}, \quad D^\epsilon(\mathbf{x}_{it}) = f^0_m(\mathbf{x}_{it}) \varepsilon[e^{\epsilon_{it}}].$$

**Thereom 2 states that one can recover $\hat{f}_m$ by using a share regression:**

Given $\omega_{it}, \mathbf{x}_{it} \in \mathcal{J}_{it}$ we have $E[\epsilon_{it}|\mathcal{J}_{it}] = 0$, so regressing $s_{it}$ on a flexible function of $\mathbf{x}_{it}$ identifies $f^0_m$ as a set of coefficients on the part of $m_{it}$.

$$E[s_{it}|\mathbf{x}_{it}] = \ln D^\epsilon(\mathbf{x}_{it}).$$

This identifies $D^\epsilon$. This gives $\epsilon_{it}$ and $\varepsilon = E[e^{\epsilon_{it}}]$:

$$\hat{\epsilon}_{it} = \ln \hat{D}^\epsilon - s_{jt}, \quad \hat{\varepsilon} = E\left[e^{\ln \hat{D}^\epsilon - s_{jt}}\right].$$

This gives:

$$\ln \hat{f}_m = \ln \hat{D}^\epsilon - \ln \hat{\varepsilon},$$

$$\hat{f}_m = \frac{\hat{D}^\epsilon}{\hat{\varepsilon}}.$$

16

**Thereom 3 states that one can recover the rest of $f$ up to a constant if $\hat{f}_m$ is known:**

By the fundamental theorem:

$$\int f_m^0 \left( \mathbf{x}_{it} \right) dm_{it} = f^0 \left( \mathbf{x}_{it} \right) + \mathcal{C}(k_{it}, l_{it}), \quad \text{or} \quad f^0 \left( \mathbf{x}_{it} \right) = \int f_m^0 \left( \mathbf{x}_{it} \right) dm_{it} - \mathcal{C}(k_{it}, l_{it}). \tag{f}$$

where the constant of integration $\mathcal{C}$ is allowed to depend on $k_{it}, l_{it}$, or $\mathcal{C} = c_0 + c_1(k_{it}, l_{it})$ without loss of generality. So if we can compute $\mathcal{C}$, then we can identify all of $f^0 \left( \mathbf{x}_{it} \right)$. GNR says we can identify $f^0 \left( \mathbf{x}_{it} \right)$ up to a constant, so we can can still discuss partials $f_v$ and do analysis on productivity $\omega_{it}$ by taking a ratio to the most/least productive firm $i = 0$ with $\hat{\omega}_{it} - \omega^{0t}$.

Start with the production function in logs:

$$\omega_{it} = y_{it} - f^0 - \epsilon_{it},$$
$$= y_{it} - \left\{ \int f_m^0 \left( \mathbf{x}_{it} \right) dm_{it} - \mathcal{C}(k_{it}, l_{it}) \right\} - \epsilon_{it}.$$

Define

$$\mathcal{Y}_{it} \equiv y_{it} - \int f_m^0 \left( \mathbf{x}_{it} \right) dm_{it} - \epsilon_{it},$$
$$= \omega_{it} - \mathcal{C}(k_{it}, l_{it}).$$

Note that given we have $\hat{\epsilon}_{it}$ and $\hat{f}_m^0$, we can get $\hat{\mathcal{Y}}_{it}$. Using first-order Markov on $\omega_{it}$

$$\mathcal{Y}_{it} = h^0 \left\{ \omega_{it-1} \right\} + \xi_{it} - \mathcal{C}(k_{it}, l_{it}),$$
$$= h^0 \left\{ \mathcal{Y}_{it-1} + \mathcal{C}(k_{it-1}, l_{it-1}) \right\} + \xi_{it} - \mathcal{C}(k_{it}, l_{it}).$$

Denote $\Gamma_y = (k_{it}, l_{it}, k_{it-1}, l_{it-1}, \mathcal{Y}_{it-1})$. Then

$$E\left[ \mathcal{Y}_{it} | \Gamma_y \right] = -\mathcal{C}(k_{it}, l_{it}) + h^0 \left\{ \mathcal{Y}_{it-1} + \mathcal{C}(k_{it-1}, l_{it-1}) \right\}.$$

Then we get the partial derivatives of $c_1(k_{it}, l_{it})$ of integration $\mathcal{C}$ as (negative) coefficients on $k_{it}, l_{it}$ conditional on $\mathcal{Y}_{it-1} k_{it-1}, l_{it-1}$. As we cannot identify $c_0$ part, the constant of integration $\mathcal{C}$ is identified up to a constant. Using this in (f) gives $\hat{f}^0$ up to a constant.

## Limitations of GNR solution

### Under monopoly, share regression does not give $f_m^0$ nonparametrically

We are trying to estimate markdowns and markups. Why a price taker?

Monopolist profit maximization gives

$$\rho_t = \left( P_t' F^0 \left( \mathbf{x}_{it} \right) + P_t \frac{\partial F^0 \left( \mathbf{x}_{it} \right)}{\partial M_{it}} \right) e^{\omega_{it}} \varepsilon[e^{\epsilon_{it}}],$$
$$= \left( \frac{\partial P}{\partial Y_{it}} \frac{F^0 \left( \mathbf{x}_{it} \right)}{P_t} + f_m^0 \left( \mathbf{x}_{it} \right) \frac{F^0}{M_{it}} \right) P_t e^{\omega_{it}} \varepsilon[e^{\epsilon_{it}}],$$
$$= \left( -\eta^{-1}(\mathbf{x}_{it}, \omega_{it}, \epsilon_{it}) + f_m^0 \left( \mathbf{x}_{it} \right) \frac{F^0 e^{\omega_{it}} \varepsilon[e^{\epsilon_{it}}]}{M_{it}} \right) P_t,$$
$$S_{it} = -\eta^{-1}(\mathbf{x}_{it}, \omega_{it}, \epsilon_{it}) \frac{M_{it}}{Y_{it}} + f_m^0 \left( \mathbf{x}_{it} \right),$$
$$= -\frac{\eta^{-1}(\mathbf{x}_{it}, \omega_{it}, \epsilon_{it})}{\beta^M} + f_m^0 \left( \mathbf{x}_{it} \right), \quad \beta^M \equiv \frac{M_{it}}{Y_{it}}.$$

$\mathbf{x}_{it}$ appears both in $\eta$ and $f_m^0$ and the two functions cannot be separately identified.

**GNR shows share regression gives $f_m^0$ if output demand is iso-elastic:**

Iso-elastic demand function:

$$\frac{P_{it}}{\Pi_t} = \left(\frac{Y_{it}}{Y_t}\right)^{\frac{1}{\sigma_t}} e^{\chi_{it}}.$$

Price elasticity does not depend on the scale of $Y_{it}$ and is a constant $\sigma_t$. Of course, this is equivalent to assuming away the problem: If $\eta(\mathbf{x}_{it}, \omega_{it}, \epsilon_{it}) = \eta$, then, no problem.

They also show that one can only recover a linear combination of productivity and demand shock $\frac{1+\sigma_t}{\sigma_t}\omega_{it} + \chi_{it}$.

GNR Appendix O6-1 shows how one can use share regression (under price taker FOC) and achieve identification in dynamic panel estimators with firm fixed effects under an AR(1) productivity dynamics.

## What shall we do?

- Some papers (e.g., Kasahara and Sugita 2023) are working on the solution.

- A parametric solution under a Cobb-Douglas production had been proposed (Doraszelski and Jaumandreu 2013).

- We can first do concentration, then proceed to Rubens' or DJ's method, and wait to see if any non-parametric (non-functional form dependent) alternative comes up.

## Appendix: Markdown in profit maximization FOC and cost minimization FOC

A monopsonist cost minimization problem

$$\min_V \quad P^V V + \lambda[Q^* - F(K, L, M)e^\omega \varepsilon[e^\epsilon]]$$

FOC

$$P^V + P^{V\prime}V - \lambda F_V e^\omega = 0.$$

This gives

$$\underbrace{\left(1 + P^{V\prime}\frac{V}{P^V}\right)}_{\equiv \psi^V} P^V = \lambda F_V e^\omega,$$

$$\psi^V \frac{P^V V}{PF e^\omega} = \frac{\lambda}{P} F_V \frac{V}{F e^\omega} e^\omega,$$

$$\psi^V S^V = \frac{1}{\mu}\theta^V e^\omega,$$

$$\psi^{V|min} = \frac{e^\omega}{\mu}\frac{\theta^V}{S^V}. \tag{1}$$

A profit maximization problem

$$\max_V \quad PF(K, L, M)e^\omega \varepsilon[e^\epsilon] - P^V V$$

FOC

$$PF_V e^\omega + P'Fe^\omega \frac{dQ}{dV} - P^V - P^{V\prime}V = 0.$$

This gives, noting $\frac{dQ}{dV} = F_V e^\omega$,

$$\psi^V P^V = PF_V e^\omega + P'Fe^\omega F_V e^\omega,$$

$$= PF_V\left(e^\omega + \frac{\partial P}{\partial Q}\frac{Fe^\omega}{P}e^\omega\right),$$

$$\psi^V \frac{P^V V}{PF e^\omega} = F_V \frac{V}{F e^\omega}\left(1 - \eta^{-1}\right)e^\omega,$$

$$\psi^V S^V = \theta^V\left(1 - \eta^{-1}\right)e^\omega,$$

$$\psi^{V|max} = (1 - \eta^{-1})e^\omega \frac{\theta^V}{S^V}. \tag{2}$$

A monopolist-monopsonist profit maximization problem can be written as an optimal production scale problem with a cost function $C(\cdot)$:

$$\max_Q \quad PQe^\omega \varepsilon[e^\epsilon] - C(Qe^\omega)$$

FOC

$$Pe^\omega + P'Qe^\omega - C'(Q)e^\omega = 0,$$

$$C'(Q) = P\left(1 + \frac{\partial P}{\partial\left(Qe^\omega\right)}\frac{Qe^\omega}{P}\right),$$

$$\frac{1}{\mu} = \frac{C'(Q)}{P} = 1 - \eta^{-1}.$$

Plugging this into (1), we have (2):

$$\psi^{V|min}\Big|_{\mu^{-1}=1-\eta^{-1}} = \left(1 - \eta^{-1}\right)e^\omega\frac{\theta^V}{S^V} = \psi^{V|max}.$$

We also note that $\psi^{V|min}$ or $\psi^{V|max}$ is equal to factor input price markdown. This is seen by solving the profit maximization problem formulated as:

$$\max_V \quad R(V)\varepsilon[e^\epsilon] - P^V V$$

with FOC

$$R'(V) - P^{V'}V - P^V = 0,$$

so

$$\text{markdown} \equiv \frac{R'(V)}{P^V} = 1 + \frac{\partial P^V}{\partial V}\frac{V}{P^V} > 1,$$

where $\frac{\partial P^V}{\partial V}\frac{V}{P^V}$ is an inverse factor supply elasticicty (or a price elasticity of factor supply) which is assumed to be upwardly sloped.

Given that cost minimization problem does not consider the output price $P$, the appearance of $\mu = \frac{P}{\lambda}$ is superfluous and is a clever accident. So (1) should read as, given markup $\mu$, markdown $\psi^V$ is determined such that (1) holds.

In contrast, profit maximization takes the output prices explicitly. So (2) reads as, given the demand elasticity of output price $\eta$, markdown $\psi^V$ is determined such that (2) holds.

This suggests that there is no reason to expect $\psi^{V|min}$ to be the same as $\psi^{V|max}$ given that only the latter specifies what sort of output price optimization is conducted. This is also seen that only after an explicit output price optimization $\psi^{V|min}$ changes to $\psi^{V|max}$.

We also note the markup of monopolist-monopsonist is given by

$$\mu = \frac{\eta}{\eta - 1} > 1.$$

**Rererences**

Ackerberg, Daniel A., Kevin Caves, and Garth Frazer. 2015. "Identification Properties of Recent Production Function Estimators." *Econometrica* 83 (6): 2411–51. https://doi.org/10.3982/ECTA13408.

Azar, José, Emiliano Huet-Vaughn, Ioana Marinescu, Bledi Taska, and Till Von Wachter. 2019. "Minimum Wage Employment Effects and Labor Market Concentration." *NBER Working Paper Series*. Vol. 26101. National Bureau of Economic Research. https://www.nber.org/system/files/working_papers/w26101/w26101.pdf.

Azar, José, Ioana Marinescu, and Marshall Steinbaum. 2019. "Measuring Labor Market Power Two Ways." *AEA Papers and Proceedings* 109 (May): 317–21. https://doi.org/10.1257/pandp.20191068.

Azar, José, Ioana Marinescu, Marshall Steinbaum, and Bledi Taska. 2020. "Concentration in US Labor Markets: Evidence from Online Vacancy Data." *Labour Economics* 66: 101886. https://doi.org/10.1016/j.labeco.2020.101886.

Benmelech, Efraim, Nittai K. Bergman, and Hyunseob Kim. 2022. "Strong Employers and Weak Employees." *Journal of Human Resources* 57 (S): S200–250. https://doi.org/10.3368/jhr.monopsony.0119-10007R1.

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch. 2021. "Some Unpleasant Markup Arithmetic: Production Function Elasticities and Their Estimation from Production Data." *Journal of Monetary Economics* 121: 1–14. https://doi.org/10.1016/j.jmoneco.2021.05.004.

De Loecker, Jan. 2011. "Product Differentiation, Multiproduct Firms, and Estimating the Impact of Trade Liberalization on Productivity." *Econometrica* 79 (5): 1407–51. https://doi.org/10.3982/ECTA7617.

———. 2021. "Comment on (Un)pleasant ... By Bond Et Al (2020)." *Journal of Monetary Economics* 121: 15–18. https://doi.org/10.1016/j.jmoneco.2021.04.009.

De Loecker, Jan, Pinelopi K. Goldberg, Amit K. Khandelwal, and Nina Pavcnik. 2016. "Prices, Markups, and Trade Reform." *Econometrica* 84 (2): 445–510. https://doi.org/10.3982/ECTA11042.

De Loecker, Jan, and Chad Syverson. 2021. "An Industrial Organization Perspective on Productivity." In *Handbook of Industrial Organization*, 4:141–223. 1. Elsevier.

De Loecker, Jan, and Frederic Warzynski. 2012. "Markups and Firm-Level Export Status." *American Economic Review* 102 (6): 2437–71. https://doi.org/10.1257/aer.102.6.2437.

Demirer, Mert. 2022. "Production Function Estimation with Factor-Augmenting Technology: An Application to Markups." https://demirermert.github.io/Papers/Demirer_production_function%202.pdf.

Doraszelski, Ulrich, and Jordi Jaumandreu. 2013. "R& D and productivity: Estimating endogenous productivity." *The Review of Economic Studies* 80 (4): 1338–83. https://doi.org/10.1093/restud/rdt011.

———. 2021. "Reexamining the de Loecker & Warzynski (2012) Method for Estimating Markups."

Gandhi, Amit, Salvador Navarro, and David A. Rivers. 2020. "On the Identification of Gross Output Production Functions." *Journal of Political Economy* 128 (8): 2973–3016. https://doi.org/10.1086/7077 36.

Hall, Robert E. 1988. "The Relation Between Price and Marginal Cost in US Industry." *Journal of Political Economy* 96 (5): 921–47.

Levinsohn, James, and Amil Petrin. 2003. "Estimating Production Functions Using Inputs to Control for Unobservables." *Review of Economic Studies* 70 (2): 317–41.

Lewbel, Arthur. 2019. "The Identification Zoo: Meanings of Identification in Econometrics." *Journal of Economic Literature* 57 (4): 835–903. https://doi.org/10.1257/jel.20181361.

Olley, G. Steven, and Ariel Pakes. 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica* 64 (6): 1263–97.

Prager, Elena, and Matt Schmitt. 2021. "Employer Consolidation and Wages: Evidence from Hospitals." *American Economic Review* 111 (2): 397–427. https://doi.org/10.1257/aer.20190690.

Raval, Devesh. 2023. "Testing the production approach to markup estimation." *The Review of Economic Studies* 90 (5): 2592–2611. https://doi.org/10.1093/restud/rdad002.

Rinz, Kevin. 2022. "Labor Market Concentration, Earnings, and Inequality." *Journal of Human Resources* 57 (S): S251–83. https://doi.org/10.3368/jhr.monopsony.0219-10025R1.

Rubens, Michael. 2023. "Market Structure, Oligopsony Power, and Productivity." *American Economic Review* 113 (9): 2382–2410. https://doi.org/10.1257/aer.20210383.

Yeh, Chen, Claudia Macaluso, and Brad Hershbein. 2022. "Monopsony in the US Labor Market." *American Economic Review* 112 (7): 2099–2138. https://doi.org/10.1257/aer.20200025.