Appendix A: Two polar models of labor market power

This section derives two examples on how labor market imperfections translate into labor market power that can be measured by wedges between wages and marginal revenue products of labor. I first discuss a simple efficient bargaining model in which employees possess labor market power. Following this, I present a model of monopsonistic labor markets. In both models, labor market power materializes in wedges between wages and marginal revenue products of labor. The framework of the main text nests both models who are polar labor market power models, where either only the firm or only the workforce has labor market power. For a theoretical combination of both models, I refer the interested reader to Falch & Strøm (2007). Notably, I do not intend to describe strategic interactions between product and labor market power with these models as I am only interested in understanding how output elasticities and labor and product market power relate to labor shares without drawing any inferences on the causal relation between labor and product market power. Therefore, the purpose of this section is simply to illustrate how labor market power translates into wedges between wages and marginal revenue products, which is the measure of labor market power in the main text. Throughout this section, I heavily draw on Dobbelaere & Mairesse (2013).

Case 1: Employee-side labor market power – efficient bargaining model

Firms compete in imperfect product markets. As in Dobbelaere & Mairesse (2013), risk-neutral workers collectively bargain with the firm over wages (w_{it}) and employment (L_{it}). Ultimately, this coordination of labor supply, i.e. the absence of a competitive pool of workers that compete over firms' labor demand, will lead to employee-side labor market power.

Employees maximize their utility function, given by:

(A.1)
$$U\left(w_{it},\ L_{it}\right) = w_{it} L_{it} + \left(\overline{L}_{it} - L_{it}\right)\overline{w}_{it},$$

where $\overline{w}_{it} \leq w_{it}$ is the reservation wage and $\overline{L}_{it}$ is the competitive employment level.

As in the main text, firms produce output using the production function:

(A.2)
$$Q_{it} = Q_{it}(.) = Q_{it}\left(L_{it}, K_{it}, M_{it}, e^{\omega_{it}}\right).$$

Capital is a fixed production input. For mathematical convenience, I assume that labor and intermediate inputs are both flexible. This limits the source of labor market power to pure bargaining power within the Nash-bargaining process between firms and employees (e.g. due to the presence of unions). However, generally, one can additionally allow for inflexible contracts to create employee-side labor market power by defining that a part of the wage bill cannot be adjusted in the short-run.[1] With $R_{it} = P_{it}Q_{it}$ denoting revenue, this implies that firms maximize the following short-run objective function:

(A.3)
$$\Pi_{it} = R_{it} - w_{it}L_{it} - z_{it}M_{it},$$

where z_{it} denotes the unit costs for intermediate inputs. Intermediate input markets are perfectly competitive. Thus, firms can unilaterally set M_{it} given z_{it} (this is not necessary but eases computation). Since employees collectively bargain with firms, wage and employment levels are decided from a bargaining game in which employees have some degree of bargaining power, denoted by $\phi_{it} \in [0, 1]$. As shown in Dobbelaere & Mairesse (2013), the outcome of this bargaining is the generalized Nash-solution:

(A.4)
$$\left(w_{it}L_{it} + \left(\overline{L}_{it} - L_{it}\right)\overline{w}_{it}\right)^{\phi_{it}}\left(R_{it} - w_{it}L_{it} - z_{it}M_{it}\right)^{1-\phi_{it}}.$$

Maximization with respect to w_{it} and L_{it} gives:

[1] In such a framework, employee-side labor market power can for instance result from employees exploiting long contract durations or institutional dismissal protections to spend below efficient effort levels.

(A.5) $$\chi_{it}\left[\frac{R_{it}-w_{it}L_{it}-z_{it}M_{it}}{L_{it}}\right]$$

and

(A.6) $$\phi_{it}\left[\frac{R_{it}-MRPL_{it}L_{it}-z_{it}M_{it}}{L_{it}}\right],$$

where $\chi_{it} = \frac{\phi_{it}}{1-\phi_{it}}$ denotes the relative extent of rent sharing. In this simple framework all the labor market power of the workforce is collected in ϕ_{it}. As equations (A.5) and (A.6) show, when employees possess positive bargaining power ($\phi_{it} > 0$), wages are above the marginal revenue product of labor. Note that equations (A.5) and (A.6) also nicely show that if firms can hire from a competitive pool of workers that do not coordinate their actions (i.e. a case where firms and workers do not bargain with each other), wages and marginal revenue products of labor equalize. In that sense, the source of labor market power in the efficient bargaining model is the fact that firms are bound to hire workers from an organized community. This essentially constitutes a hiring friction (more details can be found in McDonald & Solow (1981)).

Case 2: Employer-side labor market power – monopsonistic labor market

On a monopsonistic labor market, firms set wages such that wages are below the marginal revenue product of labor. To do so, firms need to face a labor supply curve that is imperfectly elastic (Dobbelaere & Mairesse (2013)). Imperfectly elastic labor supply curves are typically motivated by labor market frictions that prevent workers from a costless switching between many firms. Among others, such frictions include imperfect information, local preferences, or moving costs (Boal & Ransom (1997); Burdett & Mortensen (1998); Bhaskar and To (1999); Dobbelaere & Mairesse (2013)). In the following, I derive an expression showing how imperfectly elastic labor supply curves translate into labor market power that allows firms to pay wages below marginal revenue products of labor.

Firms produce output using the production function (A.2). Now, firms do not bargain with a community of workers. Instead, firms unilaterally set wages. Consequently, the firm's objective is to maximize the following version of equation (A.3):

(A.7) $$\Pi_{it}(w_{it}, z_{it}, L_{it}, M_{it}) = R_{it}(L_{it}, M_{it}) - w_{it}(L_{it})L_{it} - z_{it}M_{it}.$$

Maximization with respect to labor gives:

(A.8) $$MRPL_{it} = w_{it} + \frac{\partial w_{it}}{\partial L_{it}}L_{it} = w_{it}(1 + \frac{1}{\varepsilon^{L}_{it}}).$$

where $\varepsilon^{L}_{it} \geq 0$ denotes the labor supply elasticity. After reformulating equation (A.8), one receives:

(A.9) $$\frac{\varepsilon^{L}_{it}}{1+\varepsilon^{L}_{it}}MRPL_{it} = w_{it}.$$

Equation (A.9) shows that only if firms face an imperfectly elastic labor supply curve, unilateral wage setting of a firm leads to wages that are below the marginal revenue product of labor. In absence of employee-side adjustment frictions that give firms' labor market power, we have $\varepsilon^{L}_{it} = \infty$ and $w_{it} = MRPL_{it}$.

A simple combination between the two models discussed above can be formalized by defining that the outside option or the reservation wage in the efficient bargaining model equals the wage in a monopsonistic labor market. For a model applying this idea, I refer the interested reader to Falch & Strøm (2007). In an alternative approach, the online Appendix B.3 shows how one can extend a standard firm-level cost-minimization framework to allow for arbitrary firm- and employee-side labor market power.

Appendix B: Details on deriving firm labor market power

Appendix B.1: Deriving a parameter for labor market power

First, I derive equation (3) from the main text, which measures the degree of firms' output market power. This is done from a cost-minimization framework. The key assumption to derive (3) as a measure of output market power is that intermediate input markets are *flexible and that intermediate input prices are exogenous to firms*, i.e. that unit costs for intermediates equal marginal revenue products of intermediate inputs (De Loecker & Warzynski (2012)). Using firms' production function (1) and the periodic cost function, $C(.) = r_{it}K_{it} + w_{it}L_{it} + z_{it}M_{it}$, where r_{it}, w_{it}, and z_{it} respectively denote the unit costs for capital (K_{it}), labor (L_{it}), and intermediates (M_{it}), we can formulate the following Lagrangian:

(B.1) $$L_{it} = r_{it}K_{it} + w_{it}L_{it} + z_{it}M_{it} + \lambda_{it}\left(Q_{it} - Q_{it}(.)\right),$$

Given the above assumptions, the following first order condition holds:

(B.2) $$z_{it} = \lambda_{it}\frac{\partial Q_{it}}{\partial M_{it}}.$$

where $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$, with P_{it} and μ_{it} being the firm's output price and the firm's price setting output market power (De Loecker & Warzynski (2012)). Expanding (B.2) with $\frac{M_{it}}{Q_{it}}$ and reformulating leads to equation (3) of the main text:

(3) $$\mu_{it} = \theta^{M}_{it} * \frac{P_{it}Q_{it}}{z_{it}M_{it}},$$

where $\theta^{X}_{it} = \frac{\partial Q_{it}}{\partial X_{it}}\frac{X_{it}}{Q_{it}}$ denotes the output elasticity of input $X = \{L, M, K\}$.

From equation (2) of the main text, i.e. from $\left(1 + \tau^{L}_{it}\right) = \frac{w_{it}}{MRPL_{it}}$, one can derive a similar expression. To see this, first use the assumptions that intermediates are a flexible input and that input prices are exogenous to firms, which implies that marginal revenue products of intermediate inputs equal intermediate input unit costs (we applied these assumptions already to derive (3) above, i.e. we could also reformulate (3) and substitute it in). From that, we can expand (2) in the following way:

(B.3) $$\left(1 + \tau_{it}^{L}\right) = \frac{w_{it}}{MRPL_{it}} \frac{MRPM_{it}}{z_{it}} = \frac{w_{it}}{MPL_{it} * MR_{it}} \frac{MPM_{it} * MR_{it}}{z_{it}},$$

where $MRPM_{it}$, MR_{it}, MPL_{it}, and MPM_{it} respectively denote the marginal revenue product of intermediates, the marginal revenue, the marginal product of labor and the marginal product of intermediates. Rewriting (B.3) and expanding with $\left(\frac{\frac{M_{it}}{Q_{it} M_{it}}}{\frac{L_{it}}{Q_{it} L_{it}}} = 1\right)$ gives:

(B.4) $$\left(1 + \tau_{it}^{L}\right) = \frac{w_{it}}{z_{it}} \frac{\frac{\partial Q_{it}}{\partial M_{it}} \frac{M_{it}}{Q_{it}}}{\frac{\partial Q_{it}}{\partial L_{it}} \frac{L_{it}}{Q_{it}}} * \frac{L_{it}}{M_{it}} = \frac{w_{it}}{z_{it}} \frac{\theta_{it}^{M}}{\theta_{it}^{L}} \frac{L_{it}}{M_{it}}.$$

Expanding with $\frac{P_{it} Q_{it}}{P_{it} Q_{it}}$, substituting (3) into (B.4), and rearranging gives equation (4) of the main text:

(4) $$\mu_{it} = \theta_{it}^{L} \frac{P_{it} Q_{it}}{w_{it} L_{it}} \left(1 + \tau_{it}^{L}\right),$$

Which, after using (3), is equivalent to:

(B.5) $$\theta_{it}^{M} \frac{P_{it} Q_{it}}{z_{it} M_{it}} = \theta_{it}^{L} \frac{P_{it} Q_{it}}{w_{it} L_{it}} \left(1 + \tau_{it}^{L}\right).$$

Finally, rearranging yields equation (5) of the main text:

(5) $$\gamma_{it} \equiv \frac{1}{\left(1 + \tau_{it}^{L}\right)} = \frac{\theta_{it}^{L}}{\theta_{it}^{M}} * \frac{z_{it} M_{it}}{w_{it} L_{it}},$$

where γ_{it} denotes a measure of the firm's labor market power.

Appendix B.2: Cost-minimization with labor market power

In this section, I extent the cost-minimization framework of De Loecker & Warzynski (2012) to allow for imperfections on labor markets that create labor market power. For convenience, I start by only allowing for monopsonistic firm labor market power and subsequently discuss how one can further extent the described framework to additionally allow for employee-side labor market power.[2] The latter is important for explaining why some firms in the data pay wages above marginal revenue products of labor.

As in the main text, firms produce output using the production function:

$$Q_{it} = Q_{it}(.) = Q_{it}\left(L_{it}, K_{it}, M_{it}, e^{\omega_{it}}\right).$$

Firms take intermediate input prices as given and have some wage setting market power in the labor market. To keep the derivations simple, I abstract from capital market imperfections. Together, this motivates the periodic cost function $C_{it} = w_{it}(L_{it})L_{it} + z_{it}M_{it} + r_{it}K_{it}$, where w_{it}, z_{it}, and r_{it} are unit input costs for labor, intermediate inputs, and capital. Note that wages are a function of the firms' amount of labor. As in the online Appendix B.1, firms minimize costs, which allows me to consider the following Lagrangian:

(B.6) $$L_{it} = r_{it}K_{it} + w_{it}\left(L_{it}\right)L_{it} + z_{it}M_{it} + \lambda_{it}\left(Q_{it} - Q_{it}(.)\right),$$

The first order conditions for M_{it} and L_{it} write:

(B.7) $$z_{it} = \lambda_{it}\frac{\partial Q_{it}(.)}{\partial M_{it}}$$

and

(B.8) $$w_{it}\left(1 + \frac{\partial w_{it}}{\partial L_{it}}\frac{L_{it}}{w_{it}}\right) = w_{it}\left(1 + \frac{1}{\varepsilon^{L}_{it}}\right) = \lambda_{it}\frac{\partial Q_{it}(.)}{\partial L_{it}},$$

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[2] A cost-minimization framework that allows for monopsonistic input market power, while abstracting from market power on the input supply side, can also be found in Morlacco (2019).

where $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$ (as in the online Appendix B.1). ε^{L}_{it} denotes the labor supply elasticity and $\lambda_{it}\frac{\partial Q_{it}(.)}{\partial X_{it}}$ is the marginal revenue product of production input $X = \{M, L\}$. Using this latter definition, we can show that (B.8) equals equation (2) of the main text:

$$\frac{\varepsilon^{L}_{it}}{1+\varepsilon^{L}_{it}} = \frac{w_{it}}{MRPL_{it}} = \left(1 + \tau^{L}_{it}\right).$$

Reformulating (B.7) and (B.8) respectively gives:

(B.9)
$$\mu_{it} = \theta^{M}_{it} * \frac{P_{it}Q_{it}}{z_{it}M_{it}}$$

and

(B.10)
$$\mu_{it} = \theta^{L}_{it} * \frac{P_{it}Q_{it}}{w_{it}L_{it}}\frac{\varepsilon^{L}_{it}}{1+\varepsilon^{L}_{it}}.$$

(B.9) and (B.10) are equivalent to equations (3) and (4) of the main text. Combining (B.9) and (B.10) gives equation (5) of the main text, which I call (B.11) here:

(B.11)
$$\gamma_{it} \equiv \left(1 + \frac{1}{\varepsilon^{L}_{it}}\right) = \frac{\theta^{L}_{it}}{\theta^{M}_{it}} * \frac{z_{it}M_{it}}{w_{it}L_{it}}.$$

Until now, we abstracted from worker-side labor market power, which, as can be seen from (B.11), would imply that $\gamma_{it}\geq 1$. Allowing for worker-side labor market power introduces an additional term in the Lagrangian (B.6) which allows for the outcome $\gamma_{it} < 1$. Depending on the nature of this worker-side labor market power, one might even want to consider a dynamic optimization problem, as adjustment costs and worker-side labor market power are closely related.

In any case (be it a static or dynamic model), under the presence of worker-side labor market power, the first order condition for labor changes to:

(B.12)
$$w_{it}\left(1 + \frac{1}{\varepsilon^{L}_{it}} + F_{it}(.)\right) = \lambda_{it}\frac{\partial Q_{it}(.)}{\partial L_{it}},$$

where $-1\leq F_{it}(.)\leq 0$ and the arguments of $F_{it}(.)$ depend on the specific source of workers-side labor market power and might potentially include dynamic factors (e.g.

bargaining power, hiring/firing costs, etc.). Total firm labor market power (equation (5) of the main text) is then be given by:

$$\gamma_{it} \equiv \left(1 + \frac{1}{\varepsilon^{L}_{it}} + F_{it}(.)\right) = \frac{\theta^{L}_{it}}{\theta^{M}_{it}} * \frac{z_{it}M_{it}}{w_{it}L_{it}}, \tag{B.13}$$

which, again, is equivalent to the expression used to measure labor market power in the main text. Finally, note that the index i in ε^{L}_{it} and $F_{it}(.)$ highlights that this framework is entirely consistent with simultaneously observing some firms in which the monopsonistic labor market power term (i.e. the inverse of the labor supply elasticity) dominates, while having other firms in the data in which the rent-sharing term $F_{it}(.)$ is in absolute terms larger than $\frac{1}{\varepsilon^{L}_{it}}$. The degree of each firm's total firm labor market power, which is an average parameter across the firm's entire workforce (see Mertens (2021)), depends on the net effect of rent-sharing and monopsonistic exploitation. This in turn depends on the firm's workforce characteristics and labor market setting that are both unrestricted by the above production side framework as this framework is consistent with any labor market model and does not depend on the presence of specific workforce characteristics.

Appendix B.3: Comparison with De Loecker, Eeckhout, & Unger (2020)

In this section, I show how one can transfer my estimates on product and labor market power to the markup estimates of De Loecker, Eeckhout, & Unger (2020). In particular, I show that their markup estimate, denoted by μ_{it}^{DLEU} is a function of my product (μ_{it}) and labor market power parameter (γ_{it}).

De Loecker et al. (2020) define a production model consisting of two production factors. One combines intermediate and labor inputs into a joint production factor, $V_{it} = M_{it} + L_{it}$, and the other is capital, K_{it}. Using a cost-minimization framework, as detailed above, they derive their markup from the wedge between the output elasticity of V_{it} and its inverse expenditure share in revenues:

(B.14)
$$\mu_{it}^{DLEU} = \theta_{it}^{V} * \frac{P_{it}Q_{it}}{P_{it}^{V}V_{it}} = \theta_{it}^{V}\frac{P_{it}Q_{it}}{z_{it}M_{it}+w_{it}L_{it}},$$

Where P_{it}^{V} is the (average) unit input cost of V_{it} and $P_{it}^{V}V_{it} = z_{it}M_{it} + w_{it}L_{it}$. Note that $\theta_{it}^{V} = \theta_{it}^{M} + \theta_{it}^{L}$ because bundling M_{it} and L_{it} into one production factor imposes a joint output elasticity for these two inputs. In absence of product and factor market distortions, we have: $\theta_{it}^{V} = \frac{P_{it}^{V}V_{it}}{P_{it}Q_{it}} = \frac{z_{it}M_{it}}{P_{it}Q_{it}} + \frac{w_{it}L_{it}}{P_{it}Q_{it}}$.

Now consider that labor markets are characterized by some degree of labor market power, either hold by firms or workers. Intermediate input markets are competitive. As firms make separate decision on M_{it} and L_{it} we can consider separate first-order conditions for these two inputs. As can be seen from equations (B.7), (B.12), and (B.13), the first order conditions for M_{it} and L_{it} are:

(B.15)
$$z_{it} = \lambda_{it}\frac{\partial Q_{it}(.)}{\partial M_{it}} \quad \Rightarrow \quad \mu_{it} = \theta_{it}^{M} * \frac{P_{it}Q_{it}}{z_{it}M_{it}}$$

and

(B.16)
$$w_{it}\gamma_{it} = \lambda_{it}\frac{\partial Q_{it}(.)}{\partial L_{it}} \quad \Rightarrow \quad \mu_{it}\gamma_{it} = \theta^{L}_{it} * \frac{P_{it}Q_{it}}{w_{it}L_{it}}.$$

Substituting (B.15) and (B.16) into (B.14) gives:

(B.17)
$$\mu^{DLEU}_{it} = \theta^{V}_{it}\frac{P_{it}Q_{it}}{\theta^{M}_{it}\frac{P_{it}Q_{it}}{\mu_{it}} + \theta^{L}_{it}\frac{P_{it}Q_{it}}{\mu_{it}\gamma_{it}}}.$$

Reformulating and using $\theta^{V}_{it} = \theta^{M}_{it} + \theta^{L}_{it}$ finally gives:

(B.18)
$$\mu^{DLEU}_{it} = \frac{\theta^{M}_{it}+\theta^{L}_{it}}{\theta^{M}_{it}\gamma_{it}+\theta^{L}_{it}}\mu_{it}\gamma_{it}.$$

Note that if labor markets are competitive ($\gamma_{it} = 1$), equation (B.18) reduces to $\mu^{DLEU}_{it} = \mu_{it}$. With labor market power, the scaling factor $\frac{\theta^{M}_{it}+\theta^{L}_{it}}{\theta^{M}_{it}\gamma_{it}+\theta^{L}_{it}}$ adjusts for differences in firms' input market power in input markets for intermediates and labor.

As can further be seen from equation (B.16), estimating the input wedge between labor's output elasticity and expenditure share, combines product and labor market power and only measures true product market power if labor markets are competitive.