Delabastita and Rubens 2025

claude 4 sonnet, instructed by me

June 09, 2025, 14:06 +0900

# Equations and Variable Definitions from Coal Mining Study

## Main Equations

### Equation (1) - Cobb-Douglas Production Function

### Equation (2) - TFP Transition Process

### Labor Supply Function - Equation (3)

### Cost Minimization Objective - Equation (4)

### No Collusion Cost Minimization - Equation (5)

### First-Order Condition for Labor (No Collusion) - Equation (6)

### Markdown Under No Collusion - Equation (7)

### Perfect Collusion Cost Minimization - Equation (8)

### First-Order Condition for Labor (Perfect Collusion) - Equation (9)

### Markdown Under Perfect Collusion - Equation (10)

### General First-Order Condition - Equation (11)

### General Markdown Expression - Equation (12)

### Markup from Materials - Equation (13)

### Core Identification Equation - Equation (14)

### Conduct Parameter Definition - Equation (15)

### Moment Conditions for Production Function - Equation (16)

### GMM Moment Conditions - Equation (17)

## Additional Key Definitions

### Markdown Definition

### Percentage Markdown Definition

### Markup Definition

### Labor Market Share

### Revenue Share of Labor

### Revenue Share of Materials

### Inverse Labor Supply Elasticity (Firm-level)

### Inverse Materials Supply Elasticity (Firm-level)

## Variable Definitions

| Variable | Definition |
| --- | --- |
|  | Output (tonnage of coal) extracted by firm in year |
|  | Amount of effective labor throughout the year for firm in year |
|  | Amount of intermediate inputs purchased by firm in year |
|  | Capital stock (steam engines) used by firm in year |
|  | Logarithm of output |
|  | Logarithm of labor |
|  | Logarithm of materials |
|  | Logarithm of capital |
|  | Output elasticity of labor |
|  | Output elasticity of materials |
|  | Output elasticity of capital |
|  | Log total factor productivity |
|  | Unexpected productivity shock |
|  | Serial correlation parameter in productivity process |
|  | Wage in labor market in year |
|  | Wage for firm in year |
|  | Price of materials for firm in year |
|  | Market-level employment in market in year |
|  | Inverse market-level labor supply elasticity |
|  | Market-specific residual in labor supply |
|  | Collusion weight that firm puts on firm ’s costs |
|  | Conduct parameter (firm-level aggregate of bilateral conduct parameters) |
|  | Normalized conduct parameter ranging from 0 to 1 |
|  | Marginal cost for firm in year |
|  | Coal price for firm in year |
|  | Marginal revenue product of labor for firm in year |
|  | Wage markdown (ratio of MRPL to wage) |
|  | Wage markdown under no collusion |
|  | Wage markdown under perfect collusion |
|  | Percentage wage markdown |
|  | Price markup (ratio of price to marginal cost) |
|  | Labor market share of firm |
|  | Revenue share of labor for firm in year |
|  | Revenue share of materials for firm in year |
|  | Inverse firm-level labor supply elasticity |
|  | Inverse firm-level materials supply elasticity |
|  | Set of firms in market (where firm operates) in year |
|  | Agricultural wages in Belgium in year (instrument) |

## Tab-Separated Variable Definitions

Variable Definition  
Q\_ft Output (tonnage of coal) extracted by firm f in year t  
L\_ft Amount of effective labor throughout the year for firm f in year t  
M\_ft Amount of intermediate inputs purchased by firm f in year t  
K\_ft Capital stock (steam engines) used by firm f in year t  
q\_ft Logarithm of output Q\_ft  
l\_ft Logarithm of labor L\_ft  
m\_ft Logarithm of materials M\_ft  
k\_ft Logarithm of capital K\_ft  
beta\_l Output elasticity of labor  
beta\_m Output elasticity of materials  
beta\_k Output elasticity of capital  
omega\_ft Log total factor productivity  
u\_ft Unexpected productivity shock  
rho Serial correlation parameter in productivity process  
W\_l\_it Wage in labor market i in year t  
W\_l\_ft Wage for firm f in year t  
W\_m\_ft Price of materials for firm f in year t  
L\_it Market-level employment in market i in year t  
Psi\_l Inverse market-level labor supply elasticity  
eta\_it Market-specific residual in labor supply  
lambda\_fgt Collusion weight that firm f puts on firm g's costs  
lambda\_tilde\_ft Conduct parameter (firm-level aggregate of bilateral conduct parameters)  
lambda\_hat\_ft Normalized conduct parameter ranging from 0 to 1  
MC\_ft Marginal cost for firm f in year t  
P\_ft Coal price for firm f in year t  
MRPL\_ft Marginal revenue product of labor for firm f in year t  
m\_l\_ft Wage markdown (ratio of MRPL to wage)  
m\_l\_tilde\_ft Wage markdown under no collusion  
m\_l\_bar\_ft Wage markdown under perfect collusion  
d\_l\_ft Percentage wage markdown  
mu\_ft Price markup (ratio of price to marginal cost)  
s\_l\_ft Labor market share of firm f  
alpha\_l\_ft Revenue share of labor for firm f in year t  
alpha\_m\_ft Revenue share of materials for firm f in year t  
theta\_l\_ft Inverse firm-level labor supply elasticity  
theta\_m\_ft Inverse firm-level materials supply elasticity  
F\_i\_f\_t Set of firms in market i (where firm f operates) in year t  
w\_agri\_t\_minus\_1 Agricultural wages in Belgium in year t-1 (instrument)

# Table 1

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1: TABLE 1: Model Estimates   |  | log(Output) | log(Output) | log(Output) |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | A. Production Function, log(Output) | | | |  |  |  | | log(Labor) *bl* | .794 | .699 | .661 |  |  |  | |  | (.034) | (.327) | (.041) |  |  |  | | log(Materials) *bm* | .275 | .222 | .237 |  |  |  | |  | (.028) | (.138) | (.080) |  |  |  | | log(Capital) *bk* | 2.008 | .153 | .102 |  |  |  | |  | (.140) | (.075) | (.088) |  |  |  | | Serial correlation TFP *r* | .866 | .853 |  |  |  |  | |  | (.198) | (.157) |  |  |  |  | | Method | OLS | GMM | GMM |  |  |  | | RTS | Free | Free | Fixed at 1.05 |  |  |  | | *R*2 | .941 | .938 | .826 |  |  |  | | Hansen *J*-test |  | 2.34 | 2.72 |  |  |  | | Hansen *J*-test *p*-value |  | .126 | .255 |  |  |  | | Number of firms | 166 | 159 | 159 |  |  |  | | Observations | 4,480 | 4,005 | 4,005 |  |  |  | | B. Markdowns and Markups | | | |  |  |  | | Median markdown | 1.541 | 1.680 | 1.486 |  |  |  | |  | (.193) | (.450) | (.330) |  |  |  | | Average markdown | 1.676 | 1.828 | 1.616 |  |  |  | |  | (.224) | (.491) | (.361) |  |  |  | | Median markup | .884 | .714 | .763 |  |  |  | |  | (.112) | (.494) | (.287) |  |  |  | | Average markup | .946 | .764 | .816 |  |  |  | |  | (.120) | (.535) | (.315) |  |  |  | | Method | OLS | GMM | GMM |  |  |  | | RTS | Free | Free | Fixed at 1.05 |  |  |  | | C. Labor Supply | | | | | | | |  | log(Wage) |  | log(Wage) |  |  |  | |  | Est. | SE | Est. | SE |  |  | | log(Employment) | .066 | .006 | 1.009 | .265 |  |  | | Method | OLS |  | IV |  |  |  | | First-stage *F*-statistic |  |  | 462 |  |  |  | | Hansen *J*-test |  |  | 5.92 |  |  |  | | Hansen *J*-test *p*-value |  |  | .014 |  |  |  | | Observations | 1,990 |  | 1,990 |  |  |  | | Firm-level elasticity | 155.56 |  | 10.172 |  |  |  | |

## Derivation of (14)

Looking at equation (14) and the context provided, I can explain how the wage markdown is derived by combining the two markup estimates from the production approach.

### Cost minimization

From the cost minimization problem, the first-order condition for labor (equation 11) is:

Where the right side is the marginal revenue product of labor (MRPL).

Rearranging and using: - = output elasticity of labor - = revenue share of labor

The markup derived from the labor first-order condition becomes:

From (11)

From the Cobb-Douglas production function:

Taking the derivative with respect to labor:

Substitute this into the RHS of equation (11)

Multiply both sides by

From equation (3):

Taking the derivative:

Substitute this back to the above

Markup

Define the revenue share of labor: , therefore:

This gives us the markup expression derived from the labor first-order condition, which incorporates the wage markdown term in the denominator.

### Markup from materials (following De Loecker & Warzynski 2012)

For materials, since firms are price-takers (competitive input market), the standard markup formula applies:

Where is the revenue share of materials.

From the cost minimization problem, the first-order condition for materials is:

From the Cobb-Douglas production function:

Taking the derivative with respect to materials:

Substitute this into the RHS

Solve for the markup

Define the revenue share of materials:

Therefore:

This gives us the standard De Loecker & Warzynski (2012) markup formula:

### Equate the two markup expressions

Since both expressions equal :

Rearranging:

From equation (12), we know that:

Therefore:

This is equation (14) in the paper.

The wage markdown can be calculated in two equivalent ways:

1. **Labor supply approach**: (depends on conduct parameter and labor supply elasticity)
2. **Production approach**: (depends only on production function parameters and cost shares)