# CAPITAL THEORY: TECHNICAL PROGRESS AND CAPITAL STRUCTURE

## SOURCES OF MEASURED PRODUCTIVITY CHANGE: CAPITAL INPUT*

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### I. *Introduction*

In accord with the general theme, "Technical Progress and Capital Structure," the purpose of this paper is to study the relationship between the structure of capital and measured productivity change. Our first objective is to develop capital theoretic methods for the measurement of capital input. We apply these methods to the measurement of capital input in the U.S. private domestic economy, 1929–64. We then present evidence on the role of changes in the structure of capital in the explanation of measured changes in total factor productivity. In previous studies the effects of changes in the structure of capital have not been properly accounted for, giving rise to significant errors of measurement. Our conclusion is that such errors are an important source of measured productivity change. As a consequence the proportion of growth in output assigned to the residual, our ignorance, or advance of knowledge has been substantially overstated.

### II. *Capital Theory and Productivity Measurement*

If capital services were bought and sold by distinct economic units in the same way as labor services, there would be no conceptual or empirical difference between the construction of a quantity index of total capital input and the construction of the corresponding index of total labor input. Beginning with data on the value of transactions in each type of capital service, this value could be separated into a price of capital service or rental and a quantity of capital service in, say, machine-hours. A quantity index of total capital input would be constructed from the quantities of each type of capital service, using the relative shares of the rental value of each capital service in the rental value of all capital services as weights.

The measurement of capital services is less straightforward than the measurement of labor services because the consumer of a capital service

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is usually also the supplier of the service; the whole transaction is recorded only in the internal accounts of individual economic units. The obstacles to extracting this information for purposes of economic accounting are almost insuperable; the information must be obtained by a relatively lengthy chain of indirect inference. First, the calculation begins with data on the values of transactions in new investment goods. These values must be separated into prices and quantities of investment goods. Second, the quantity of new investment goods reduced by the quantity of old investment goods replaced must be added to accumulated stocks. Third, the quantity of capital services corresponding to each stock must be calculated.[1]

Paralleling the calculation of quantities of capital services, beginning with quantities of new investment goods, the prices of capital services must be calculated beginning with the prices of new investment goods. Finally, a quantity index of total capital input must be constructed from the quantities of each type of capital service, using the relative shares of the implicit rental value of each capital service in the implicit rental value of all capital services as weights. The implicit rental value of each capital service is obtained by simply multiplying the quantity of that service by the corresponding price. At this final stage the construction of a quantity index of capital input is formally identical to the construction of a quantity index of labor input. The chief difference between the construction of price and quantity indexes of capital input and any other aggregation problem is in the circuitous route by which the necessary data are obtained.

To represent the capital accounts that provide the basis for measuring capital input, we introduce the following notation:

$I_k$—quantity of output of the $k$th investment good,
$K_k$—quantity of input of the $k$th capital service,
$q_k$—price of the $k$th investment goods,
$p_k$—price of the $k$th capital service.

We assume that the proportion of an investment replaced in a given interval of time declines exponentially over time. A theoretical justification for this assumption is that replacement of investment goods is a recurrent event. Under this assumption the cumulated stock of past investments in the $k$th capital good, net of replacements, satisfies the well-known relationship:

$$I_k = K_k + \delta_k K_k, \tag{1}$$

[1] Here we assume that the "quantity" of a particular type of capital as an asset is proportional to its "quantity" as a service, whatever the age of the capital. If this condition is not satisfied, capital of each distinct age must be treated as a distinct asset and service. Output at each point of time consists of the usual output plus "aged" capital stock.

where $\delta_k$ is the instantaneous rate of replacement of the $k$th investment good. Similarly, in the absence of direct taxation the price of the $k$th capital service satisfies the relationship:

$$p_k = q_k\left[r + \delta_k - \frac{\dot{q}_k}{q_k}\right], \tag{2}$$

where $r$ is the rate of return on all capital, $\delta_k$ is the rate of replacement of the $k$th investment good, and $\dot{q}_k/q_k$ is the rate of capital gain on that good. Given these relationships between the price and quantity of investment goods and the price and quantity of the corresponding capital services, the only data beyond values of transactions in new investment goods required for the construction of price and quantity indexes of capital input are rates of replacement for each distinct investment good and the rate of return on all capital. We turn now to measurement of the rate of return.

First, to measure the values of output and input it is customary to exclude the value of capital gains from the value of input rather than to include the value of such gains in the value of output. This convention has the virtue that the value of output may be calculated directly from the value of transactions. Second, to measure total factor productivity, depreciation is frequently excluded from both input and output; this convention is adopted, for example, by Kendrick [12]. Exclusion of depreciation on capital introduces an entirely arbitrary distinction between labor input and capital input, since the corresponding exclusion of depreciation of the stock of labor services is not carried out.[2] To calculate the rate of return on all capital, our method is to subtract from the value of output plus capital gains the value of labor input and of replacement. This results in the rate of return multiplied by the value of accumulated stocks. The rate of return is calculated by dividing this quantity by the value of the stock.[3] The implicit rental value of the $k$th capital good is:

$$p_kK_k = q_k\left[r + \delta_k - \frac{\dot{q}_k}{q_k}\right]K_k. \tag{3}$$

To calculate price and quantity indexes for total capital input, the prices and quantities of each type of capital service are aggregated, using the relative shares of the implicit rental value of each capital service in the implicit rental value of all capital services as weights.

We have already noted that direct observations are usually available

[2] This point is made by Domar [6].

[3] Domar's procedure [6, p. 717, fn. 3] fails to correct for capital gains. Implicitly, Domar is assuming either no capital gains or that all capital gains are included in the value of output, whether realized or not.

only for values of transactions; the separation of these values into prices and quantities is based on much less complete information and usually involves indirect inferences. An error in the separation of the value of new investment goods into the price and quantity of investment goods will result in errors in measurement of the price and quantity of investment goods, of the price and quantity of capital services and of total factor productivity.[4] To measure the bias in the rate of growth of the quantity of investment goods, we let $Q^*$ be the relative error in the measurement of the price of investment goods, $I^*$ the "quantity" of investment goods output, calculated using the erroneous "price" of investment goods, and $I$ the actual quantity of investment goods output. The bias in the rate of growth of investment goods output is then:

$$\frac{\dot{I}^*}{I^*} - \frac{\dot{I}}{I} = -\frac{\dot{Q}^*}{Q^*}. \tag{4}$$

The rate of growth of this bias is negative if the rate of growth of the error in measurement of the price of investment goods is positive, and vice versa. If we let $K^*$ be the "quantity" of capital calculated using the erroneous "price" of investment goods and $K$ the actual quantity of capital:

$$K^* = \int_{-\infty}^{t} e^{-\delta(t-s)} I^*(s)\,ds = \int_{-\infty}^{t} e^{-\delta(t-s)} \frac{I(S)}{Q^*(s)}\,ds.$$

The bias in the rate of growth of the quantity of capital services is then:

$$\frac{\dot{K}^*}{K^*} - \frac{\dot{K}}{K} = \frac{I}{Q^*K^*} - \frac{I}{K}, \tag{5}$$

$$= \frac{I}{\displaystyle\int_{-\infty}^{t} e^{-\delta(t-s)} \frac{Q^*(t)}{Q^*(s)} I(s)\,ds} - \frac{I}{\displaystyle\int_{-\infty}^{t} e^{-\delta(t-s)} I(s)\,ds}$$

which is negative of the rate of growth of the error in measurement of the price of investment goods is positive, and vice versa.

To calculate the error of measurement in total factor productivity, we let $C$ represent the quantity of consumption goods and $L$ the quantity of labor input; second, we let $v_I$ represent the relative share of the value of investment goods in the value of total output and $v_C$ the relative share of consumption goods; finally, we let $w_K$ represent the relative share of the value of capital input in the value of total input and $w_L$ the relative share of labor. The rate of growth of total factor productivity

[4] The following analysis summarizes a more detailed argument in [12].

may be represented as:

$$\frac{\dot{P}}{P} = w_I \frac{\dot{I}}{I} + w_C \frac{\dot{C}}{C} - v_K \frac{\dot{K}}{K} - v_L \frac{\dot{L}}{L} \cdot$$

If we let $P^*$ represent the measured rate of growth of total factor productivity using the erroneous "price" of investment goods:

$$\frac{\dot{P}^*}{P^*} = w_I \frac{\dot{I}^*}{I^*} + w_C \frac{\dot{C}}{C} - v_K \frac{\dot{K}^*}{K^*} - v_L \frac{\dot{L}}{L} \cdot$$

Subtracting the first of these expressions from the second we obtain the bias in the rate of growth of total factor productivity:

$$\frac{\dot{P}^*}{P^*} - \frac{\dot{P}}{P} = w_I \left[\frac{\dot{I}^*}{I^*} - \frac{\dot{I}}{I}\right] - v_K \left[\frac{\dot{K}^*}{K^*} - \frac{\dot{K}}{K}\right] \cdot$$

Substituting expressions (5) and (4) for the biases in measured rates of growth of capital input and the output of investment goods, we have:

$$(6) \quad \frac{\dot{P}^*}{P^*} - \frac{\dot{P}}{P} = - w_I \frac{\dot{Q}^*}{Q^*}$$

$$- v_K \left[\frac{I}{\displaystyle\int_{-\infty}^{t} e^{-\delta(t-s)} \frac{Q^*(t)}{Q^*(s)} I(s)ds} - \frac{I}{\displaystyle\int_{-\infty}^{t} e^{-\delta(t-s)} I(s)ds}\right] \cdot$$

If investment and the error in measurement are growing at constant rates, the biases in the rates of growth of the quantity of investment goods produced and the quantity of capital services are equal, so that the net effect is equal to the rate of growth in the error in measurement of the price of investment goods multiplied by the difference between the capital share in total input and the investment share in total output.[5]

## III. *The Measurement of Capital Input*

We have demonstrated that an error in the measurement of investment goods prices results in errors in the measurement of investment goods output, capital input, and total factor productivity. Roughly speaking, a positive bias in the rate of growth of the investment goods prices index results in a positive bias in the rate of growth of total factor productivity, provided that the share of capital in the value of input exceeds the share of investment in the value of output. This condition is fulfilled for the U.S. private domestic sector throughout the period,

[5] Domar [7, p. 587, formula 51] considers a special case of this problem in which capital "is imported from the outside." This specialization is unnecessary, as suggested in the text.

1929–64. Hence we must examine the indexes of investment goods prices that underly our measurement for possible sources of bias.[6]

First, except for the price index for road construction the published price indexes for structures that underly the U.S. national accounts are indexes of the cost of input rather than the price of output. The use of input prices in place of output prices for structures results in an important error of measurement. To eliminate this error it is necessary to use an output price index in measuring prices of both investment goods output and capital services input. The Bureau of Public Roads prices index for a "standard mile" of highway construction is such an output price index. The corresponding input price index behaves in a manner very similar to that of the input price of all of new construction.[7] Second, the price indexes for equipment that underly the U.S. national accounts are based primarily on data from the wholesale price index of the Bureau of Labor Statistics. Since expenditures on the wholesale price index are less than those on the consumers' price index, adjustments for quality change are less frequent and less detailed. To eliminate this source of bias, we replace the implicit deflator for producers' durables by the corresponding deflator for consumers' durables. In view of the upward bias in the consumers' durables price index this adjustment is conservative.[8] Third, the price indexes for change in business inventories from the U.S. national accounts contain year-to-year fluctuations that appear to be highly implausible. To represent these movements more accurately, we replace the implicit deflator for change in inventories by the deflator for private domestic consumption. The level of this index generally coincides with that of the implicit deflator for change in business inventories; however, the fluctuations are much less than those for the inventory investment deflator.

To study the relationship between the structure of capital and measured productivity change we begin by constructing indexes of investment goods output and capital input without correcting for errors of measurement. As an initial index of investment goods output we take U.S. private domestic investment in constant prices as measured in the U.S. national product accounts. As an index of capital input we take the stock of capital in the private domestic sector of the U.S. economy. The stock of capital is the sum of land, plant, equipment, and inventories employed in this sector. To study the effects of errors in measurement on measured productivity change we weight the rate of growth of investment by its share in U.S. private domestic output and the rate of growth

[6] All data except those for the stock of land are taken from *U.S. Income and Output* [15] and subsequent revisions of the data given there. Data on the stock of land is from Goldsmith [9] and subsequent revisions.

[7] See Dacy [1] on this point and for discussion of the Bureau of Public Roads index.

[8] See Griliches [11].

TABLE 1

ERRORS OF MEASUREMENT IN THE PRICES OF INVESTMENT GOODS AND MEASURED PRODUCTIVITY CHANGE
(Annual Percentage Rates)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1930 | −47.93 | 2.10 | −6.49 | −49.77 | 2.50 | −6.84 | .36 |
| 1931 | −63.31 | .23 | −5.11 | −62.14 | .64 | −5.15 | .04 |
| 1932 | −264.28 | −1.22 | −4.42 | −290.46 | −.72 | −5.03 | .61 |
| 1933 | 14.92 | −2.85 | 1.26 | −2.15 | −2.30 | .63 | .64 |
| 1934 | 42.68 | −2.64 | 3.26 | 56.53 | −2.26 | 3.91 | −.66 |
| 1935 | 47.75 | −1.97 | 5.31 | 50.19 | −1.47 | 5.38 | −.07 |
| 1936 | 24.94 | −.69 | 3.07 | 23.72 | −.19 | 2.76 | .32 |
| 1937 | 20.07 | .33 | 2.73 | 26.90 | .73 | 3.56 | −.83 |
| 1938 | −76.58 | 1.04 | −6.82 | −72.69 | 1.73 | −6.73 | −.09 |
| 1939 | 31.18 | −.78 | 3.77 | 30.30 | −.13 | 3.44 | .33 |
| 1940 | 24.89 | .36 | 3.44 | 28.82 | .98 | 3.78 | −.34 |
| 1941 | 21.16 | 1.48 | 2.74 | 18.46 | 2.34 | 1.98 | .75 |
| 1942 | −94.83 | 2.55 | −7.50 | −124.59 | 3.21 | −9.81 | 2.30 |
| 1943 | −67.36 | −.25 | −2.24 | −92.99 | −.12 | −3.18 | .93 |
| 1944 | 8.61 | −1.51 | .92 | 17.23 | −1.52 | 1.27 | −.35 |
| 1945 | 29.63 | −1.26 | 2.26 | 31.74 | −1.14 | 2.34 | −.08 |
| 1946 | 61.96 | −.46 | 10.30 | 65.06 | −.38 | 10.77 | −.48 |
| 1947 | −1.73 | 4.07 | −1.68 | −.50 | 4.29 | −1.56 | −.12 |
| 1948 | 15.02 | 3.55 | 1.62 | 20.18 | 3.84 | 2.51 | −.89 |
| 1949 | −26.04 | 4.26 | −5.46 | −27.40 | 4.98 | −5.92 | .46 |
| 1950 | 30.74 | 2.27 | 5.49 | 36.26 | 2.84 | 6.42 | −.93 |
| 1951 | .99 | 4.53 | −1.50 | −4.64 | 5.74 | −3.07 | 1.57 |
| 1952 | −15.72 | 4.13 | −4.09 | −17.86 | 4.75 | −4.67 | .58 |
| 1953 | 1.17 | 2.79 | −.79 | 3.40 | 3.21 | −.59 | −.21 |
| 1954 | −3.07 | 2.61 | −1.40 | 2.41 | 3.13 | −.73 | −.67 |
| 1955 | 21.23 | 2.16 | 3.17 | 24.68 | 2.97 | 3.52 | −.35 |
| 1956 | −1.49 | 3.60 | −1.51 | −4.54 | 4.77 | −2.48 | .96 |
| 1957 | −8.11 | 3.18 | −2.48 | −7.44 | 3.96 | −2.63 | .16 |
| 1958 | −12.82 | 2.37 | −2.78 | −10.69 | 3.08 | −2.70 | −.07 |
| 1959 | 17.24 | 1.45 | 2.46 | 20.60 | 2.15 | 2.79 | −.33 |
| 1960 | −1.61 | 2.51 | −1.14 | .73 | 3.49 | −1.09 | −.05 |
| 1961 | −4.80 | 2.22 | −1.52 | −4.57 | 3.25 | −1.85 | .33 |
| 1962 | 12.99 | 1.76 | 1.51 | 11.43 | 2.66 | .93 | .58 |
| 1963 | 3.38 | 2.51 | −.36 | 3.26 | 3.28 | −.66 | .30 |
| 1964 | 4.86 | 2.55 | −.12 | 5.95 | 3.24 | −.19 | .07 |

1. Investment output.
2. Capital input.
3. Contribution of capital.
4. Investment output, corrected for errors in the measurement of prices.
5. Capital input, corrected.
6. Contribution of capital, corrected.
7. Contribution of errors in the measurement of prices.

of capital by its share in U.S. private domestic input. The difference is the contribution of capital to the measured rate of growth in total factor productivity.

Rates of growth of investment goods output, capital input, and the rate of growth of the contribution of capital to total factor productivity are given in columns 1–3 of Table 1. Since errors of measurement and cyclical and transitory phenomena induce large and negatively serially correlated fluctuations in measured annual percentage changes, we will analyze here only the average rates of changes for the whole 1929–64 period. During this period the average rate of growth of the contribution to changes in total factor productivity was −.12 percent per year. To eliminate errors of measurement in the prices of investment goods, we replace our initial index of investment goods output by a new index of U.S. gross private domestic investment in constant prices. This index is calculated with the Bureau of Public Roads price index for structures, the consumers' durables price index for equipment, and the private domestic consumption price index for change in inventories. We replace our initial index of capital input by the stock of capital calculated with these same price indexes. Columns 4–6 of Table 1 present rates of growth of the alternative indexes of investment goods output, capital input, and the rate of growth of their contribution to total factor productivity. The average rate of growth of their contribution to changes in total factor productivity is −.25 percent per year. Finally, column 7 gives the rate of growth of the contribution of errors in measurement of the prices of investment goods to measured productivity change. These errors account for .14 percent per year of the measured rate of growth of total factor productivity.

We have outlined a method for computing the price of capital services in the absence of direct taxation of business income. In the presence of direct taxes we may distinguish between the price of capital services before and after taxes. The expression (2) given above for the price of capital services is the price after taxes. The price of capital services before taxes is:

$$p_k = q_k \left[ \frac{1 - uv}{1 - u} r + \frac{1 - uw}{1 - u} \delta_k - \frac{1 - ux}{1 - u} \frac{\dot{q}_k}{q_k} \right] \tag{7}$$

where $u$ is the rate of direct taxation, $v$ the proportion of return to capital allowable as a charge against income for tax purposes, $w$ the proportion of replacement allowable for tax purposes and $x$ the proportion of capital gains included in income for tax purposes.

We estimate the variables describing the tax structure as follows: The rate of direct taxation is the ratio of profits tax liability to profits before taxes. The proportion of the return to capital allowable for tax purposes is the ratio of net interest to the total return to capital. Total return to capital is the after tax rate of return, $r$, multiplied by the current value of capital stock. The proportion of replacement allowable for tax purposes is the ratio of capital consumption allowances to the current value of replacement. The proportion of capital gains included in income is zero by the conventions of the U.S. national accounts. Given the value of direct taxes we estimate the after tax rate of return by subtracting from the value of output plus capital gains the value of labor input, replacement, and direct taxes. This results in the total return to capital. The rate of return is calculated by dividing this quantity by the current value of the stock of capital. Given data on the rate of return and the variables describing the tax structure, we calculate the price of capital services before taxes for each investment good. These prices of capital services are used in the calculation of indexes of capital input.

For the U.S. private domestic economy it is possible to distinguish four classes of investment goods—land, structures, equipment, and inventories. Although it is also possible to distinguish a number of subclasses within each of these groupings, we will employ only the four major groups in calculating an index of total capital input. For each group we first compute a before tax service price analogous to (7). We then compute an index of capital input as a Divisia index[9] of the services of land, structures, equipment, and inventories. This index may be compared with our initial index of capital input. In this index the relative share of the asset value of each investment good in the value of all assets is used as a weight. Our index eliminates the conceptual error arising from the implicit assumption that service prices are proportional to asset prices for different investment goods.[10] Similarly, we compute an index of investment goods output as a Divisia index of the output of structures, equipment, and inventories; we assume that the output of land as an investment good is zero for the U.S. private domestic economy.

Rates of growth of Divisia indexes of capital input, investment goods output, and the rate of growth of the contribution to total factor productivity with errors in the measurement of the prices of investment

[9] See Divisia [3–5] and Wold [16]. The basic formula for capital stock employed throughout is:

$$K_{t+1} = I_t + (1 - \delta)K_t,$$

where $I_t$ is the value of gross private domestic investment for each category in constant prices. The initial (1929) value of capital stock in constant prices of 1958 and the depreciation rates employed are as follows:

| | National Accounts Deflators | | Alternative Deflators | |
|---|---|---|---|---|
| | $K_{1929}$ | $\delta$ | $K_{1929}$ | $\delta$ |
| Land | 270,000 | 0 | 270,000 | 0 |
| Structures | 347,622 | 0.0437 | 214,287 | 0.0443 |
| Equipment | 69,587 | 0.1394 | 48,065 | 0.1290 |
| Inventories | 48,504 | 0 | 48,504 | 0 |

[10] For examples, see Kendrick [14, p. 106] and Denison [2, pp. 97–8]; see the comments by Griliches [10, p. 129].

TABLE 2

ERRORS OF AGGREGATION AND RATE OF UTILIZATION AND MEASURED PRODUCTIVITY CHANGE
(Annual Percentage Rates)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1930 | −50.54 | 2.90 | −7.07 | .23 | .16 | 2.45 | −6.92 | −.15 |
| 1931 | −63.13 | .88 | −5.30 | .16 | −7.52 | .43 | −5.16 | −.14 |
| 1932 | −377.24 | −.40 | −6.69 | 1.66 | −15.43 | −.85 | −6.56 | −.13 |
| 1933 | −5.99 | −5.39 | 1.44 | −.81 | 13.89 | −5.84 | 1.57 | −.13 |
| 1934 | 54.13 | −2.76 | 3.94 | −.03 | 11.44 | −3.21 | 4.09 | −.15 |
| 1935 | 50.21 | −2.15 | 5.61 | −.23 | 5.31 | −2.60 | 5.77 | −.15 |
| 1936 | 23.65 | −.09 | 2.71 | .04 | 11.99 | −.54 | 2.87 | −.16 |
| 1937 | 26.97 | .57 | 3.62 | −.07 | 9.57 | .12 | 3.78 | −.16 |
| 1938 | −76.72 | 2.25 | −7.25 | .52 | 5.02 | 1.80 | −7.10 | −.15 |
| 1939 | 30.68 | −.02 | 3.45 | −.01 | 9.99 | 2.13 | 2.71 | .74 |
| 1940 | 29.54 | 1.48 | 3.70 | .08 | 12.72 | 3.63 | 2.92 | .78 |
| 1941 | 18.85 | 3.17 | 1.73 | .26 | 21.31 | 5.32 | .90 | .82 |
| 1942 | −124.08 | 4.32 | −10.19 | .39 | 27.77 | 6.46 | −11.01 | .82 |
| 1943 | −89.79 | −.54 | −2.91 | −.27 | 21.08 | 1.61 | −3.71 | .80 |
| 1944 | 17.36 | −2.43 | 1.62 | −.35 | 12.78 | −.28 | .80 | .82 |
| 1945 | 31.21 | −1.62 | 2.49 | −.15 | 15.06 | .53 | 1.68 | .81 |
| 1946 | 64.04 | .17 | 10.41 | .36 | 22.19 | 2.32 | 9.66 | .75 |
| 1947 | −.03 | 5.84 | −2.03 | .46 | 23.44 | 7.99 | −2.77 | .74 |
| 1948 | 19.82 | 5.90 | 1.70 | .80 | 20.65 | 8.05 | .94 | .77 |
| 1949 | −26.80 | 6.63 | −6.42 | .50 | 9.31 | 8.78 | −7.19 | .77 |
| 1950 | 35.98 | 5.28 | 5.46 | .96 | 13.40 | 7.43 | 4.66 | .79 |
| 1951 | −5.19 | 6.32 | −3.39 | .32 | 23.07 | 8.46 | −4.20 | .81 |
| 1952 | −16.76 | 5.68 | −4.82 | .15 | 12.33 | 7.83 | −5.60 | .77 |
| 1953 | 3.64 | 4.18 | −.89 | .30 | 8.33 | 6.33 | −1.64 | .75 |
| 1954 | 2.24 | 4.71 | −1.31 | .58 | 8.63 | 6.86 | −2.06 | .76 |
| 1955 | 24.98 | 4.31 | 3.10 | .43 | 11.58 | 5.63 | 2.62 | .48 |
| 1956 | −4.62 | 5.11 | −2.61 | .13 | 16.90 | 6.43 | −3.06 | .45 |
| 1957 | −7.38 | 4.94 | −2.96 | .33 | 13.60 | 6.26 | −3.42 | .46 |
| 1958 | −10.69 | 4.18 | −3.09 | .39 | 9.67 | 5.50 | −3.56 | .47 |
| 1959 | 20.77 | 3.07 | 2.49 | .30 | 9.53 | 4.39 | 2.03 | .47 |
| 1960 | .75 | 4.22 | −1.34 | .25 | 9.67 | 5.54 | −1.80 | .46 |
| 1961 | −4.78 | 3.81 | −2.08 | .23 | 11.00 | 5.12 | −2.54 | .47 |
| 1962 | 11.69 | 2.81 | .91 | .01 | 12.88 | 4.13 | .44 | .48 |
| 1963 | 3.23 | 3.58 | −.77 | .11 | 12.08 | 4.90 | −1.25 | .48 |
| 1964 | 5.85 | 3.56 | −.32 | .14 | 11.52 | 4.88 | −.80 | .48 |

1. Investment output, Divisia index.
2. Capital input, Divisia index.
3. Contribution of capital, Divisia index.
4. Contribution of errors of aggregation and the conceptual error of assuming that service prices are proportional to asset prices.
5. Rate of return after taxes.
6. Capital input, corrected for variations in the rate of utilization.
7. Contribution of capital, corrected.
8. Contribution of the conceptual error of assuming that capital services are proportional to capital stock for each type of asset.

goods eliminated are presented in Table 2. These indexes eliminate errors of aggregation and the conceptual error of assuming that service prices are proportional to asset prices. With these errors removed the average rate of growth of the contribution of capital to changes in total factor productivity is $-.49$ percent per year. Thus such errors contribute .23 percent per year to the measured rate of growth of total factor productivity. A by-product of the computation of the Divisia index of capital input is an estimate of the rate of return after taxes in the U.S. private domestic economy. For completeness this rate of return is presented in column 5 of Table 2.

Up to this point we have assumed that capital services are proportional to capital stock for each type of asset. Adjustment of data on capital stock for variation in rates of utilization is essential if comparability of data on labor and capital services is to be preserved. Fortunately, relatively good data on the trend of hours per machine are available for electric motors in the manufacturing sector.[11] The relative utilization of electric motors provides an indicator of the relative utilization of capital in the manufacturing sector, since electric motors are the predominant source of direct power. We assume that the relative utilization of capital for the U.S. private domestic economy is the same as that for the manufacturing sector. When better data become available one should be able to improve upon this assumption.

The rate of growth of investment goods output is unchanged in correcting capital input for variations in the rate of utilization of capital stock. The rate of growth of capital input after this correction is presented in column 6 of Table 2. The rate of growth of the contribution of capital to changes in total factor productivity after correction is presented in column 7. The estimated average rate of growth of the contribution of capital is now $-.94$ percent per year, the conceptual error of assuming that capital service is proportional to capital stock accounting for .45 percent per year of the measured rate of growth of total factor productivity.

## IV. *Summary and Conclusion*

We have studied the relationship between the structure of capital and measured productivity change in the U.S. private domestic economy, 1929–64, and found that errors in measurement of the contribution of capital account for a substantial proportion of measured changes in total factor productivity. The total error in the measurement of total factor productivity from this source is .82 percent per year. Of this total .14 percent per year is due to errors in the measurement of prices of investment goods. Errors in aggregation and the conceptual error of assuming that service prices are proportional to asset prices account for .23 percent per year. The conceptual error of assuming that capital ser-

[11] See Foss [8]. Foss provides estimates on relative utilization only for 1929 and 1954. These have been updated to 1962 for this study. Thus, they catch only the trend in relative utilization and make no allowance for the shorter run cyclical fluctuations.

vices are proportional to capital stock for each type of asset accounts for .45 percent per year.[12]

The total error of measurement of the contribution of capital of .82 percent per year may be compared with Denison's average rate of growth of output per unit of input for the U.S. economy, 1929–57, of .93 percent per year. Denison's figure is corrected for errors in the measurement of labor services, but not for errors in the measurement of the contribution of capital. For the 1929–57 period, Denison's corrections for errors in the measurement of labor services amount to .77 per year. The effects of errors in the measurement of the contribution of capital we have studied are, if anything, slightly greater. Of course, our figures are not strictly comparable with those of Denison either in coverage or time span. Nevertheless, our results suggest that the residual, our ignorance, or advance of knowledge has been substantially overstated, even by Denison.

In explaining economic growth we suggest greater reliance than heretofore on the twin pillars of human and nonhuman capital, each supporting an important part of the capital structure. Perhaps the day is not far off when economists can remove the intellectual scaffolding of technical change altogether.

[12] Since some of these corrections are based on fragmentary evidence, all of these numbers should be taken only as indications of the probable orders of magnitude of the various errors. We need more research and better data for more precise estimates.

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