



**OXFORD JOURNALS**  
OXFORD UNIVERSITY PRESS

---

The Explanation of Productivity Change

Author(s): D. W. Jorgenson and Z. Griliches

Source: *The Review of Economic Studies*, Jul., 1967, Vol. 34, No. 3 (Jul., 1967), pp. 249-283

Published by: Oxford University Press

Stable URL: <https://www.jstor.org/stable/2296675>

---

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <https://about.jstor.org/terms>



Oxford University Press is collaborating with JSTOR to digitize, preserve and extend access to *The Review of Economic Studies*

# The Explanation of Productivity Change<sup>1</sup>

But part of the job of economics is weeding out errors.  
That is much harder than making them, but also  
more fun.—R. M. SOLOW

## 1. INTRODUCTION

Measurement of total factor productivity is based on the economic theory of production. For this purpose the theory consists of a production function with constant returns to scale together with the necessary conditions for producer equilibrium. Quantities of output and input entering the production function are identified with real product and real factor input as measured for social accounting purposes. Marginal rates of substitution are identified with the corresponding price ratios. Employing data on both quantities and prices, movements along the production function may be separated from shifts in the production function. Shifts in the production function are identified with changes in total factor productivity.

Our point of departure is that the economic theory underlying the measurement of real product and real factor input has not been fully exploited. As a result a number of significant errors of measurement have been made in compiling data on the growth of real product and the growth of real factor input. The result of these errors is to introduce serious biases in the measurement of total factor productivity. The allocation of changes in real product and real factor input between movements along a given production function and shifts of the production function must be corrected for bias due to errors of concept and measurement.

The purpose of this paper is to examine a hypothesis concerning the explanation of changes in total factor productivity. This hypothesis may be stated in two alternative and equivalent ways. In the terminology of the theory of production, if quantities of output and input are measured accurately, growth in total output is largely explained by growth in total input. Associated with the theory of production is a system of social accounts for real product and real factor input. The rate of growth of total factor productivity is the difference between the rate of growth of real product and the rate of growth of real factor input. Within the framework of social accounting the hypothesis is that if real product and real factor input are accurately accounted for, the observed growth in total factor productivity is negligible.

We must emphasize that our hypothesis concerning the explanation of real output is testable. By far the largest portion of the literature on total factor productivity is devoted to problems of measurement rather than to problems of explanation. In recognition of this fact changes in total factor productivity have been given such labels as The Residual or The Measure of Our Ignorance. Identification of measured growth in total factor productivity with embodied or disembodied technical change provides methods for measuring technical change, but provides no genuine explanation of the underlying changes in real output and input.<sup>2</sup> Simply relabelling these changes as Technical Progress or Advance of Knowledge leaves the problem of explaining growth in total output unsolved.

<sup>1</sup> The authors' work has been supported by grants from the National Science and Ford Foundations.

<sup>2</sup> See Jorgenson [35] for details.

The plan of this paper is as follows: We first discuss the definition of changes in total factor productivity from the point of view of the economic theory of production. Second, we provide operational definitions for the measurement of prices and quantities that enter into the economic theory of production. These definitions generate a system of social accounts for real product and real factor input and for the measurement of total factor productivity. Within this system we provide an operational definition of total factor productivity. This definition is fundamental to an empirical test of the hypothesis that if real product and real factor input are accurately accounted for, the observed rate of growth of total factor productivity is negligible.

Within our system of social accounts for real product and real factor input we can assess the consequences of errors of measurement that arise from conceptual errors in the separation of the value of transactions into price and quantity. Errors in making this separation may affect real product, real factor input, or both; for example, an error in the measurement of the price of investment goods results in a bias in total output and a bias in the capital accounts that underlie the measurement of total input. Within this system of social accounts we can suggest principles for correct aggregation of inputs and outputs and indicate the consequences of incorrect aggregation. Many of the most important errors of measurement in previous compilations of data on real product and real factor input arise from incorrect aggregation.

Given a system of social accounts for the measurement of total factor productivity we attempt to correct a number of common errors of measurement of real product and real factor input by introducing data that correspond more accurately to the concepts of output and input of the economic theory of production. After correcting for errors of measurement we examine the validity of our hypothesis concerning changes in total factor productivity. We conclude with an evaluation of past research and a discussion of implications of our findings for further research.

## 2. THEORY

Our definition of changes in total factor productivity is the conventional one. The rate of growth of total factor productivity is defined as the difference between the rate of growth of real product and the rate of growth of real factor input. The rates of growth of real product and real factor input are defined, in turn, as weighted averages of the rates of growth of individual products and factors. The weights are relative shares of each product in the value of total output and of each factor in the value of total input. If a production function has constant returns to scale and if all marginal rates of substitution are equal to the corresponding price ratios, a change in total factor productivity may be identified with a shift in the production function. Changes in real product and real factor input not accompanied by a change in total factor productivity may be identified with movements along a production function.

Our definition of change in total factor productivity is the same as that suggested by Abramovitz (1), namely, ". . . the effect of 'costless' advances in applied technology, managerial efficiency, and industrial organization (cost—the employment of scarce resources with alternative uses—is, after all, the touchstone of an 'input') . . ."<sup>1</sup> Of course, changes in total factor productivity or shifts in a given production function may be accompanied by movements along a production function. For example, changes in applied technology may be associated with the construction of new types of capital equipment. The alteration in patterns of productive activity must be separated into the part which is "costless", representing a shift in the production function, and the part which represents the employment of scarce resources with alternative uses, representing movements along the production function.

<sup>1</sup> Abramovitz [1, p. 764].

On the output side the quantities that enter into the economic theory of production correspond to real product as measured for the purposes of social accounting. Similarly, on the input side these quantities correspond to real factor input, also as measured for the purposes of social accounting. The prices that enter the economic theory of production are identified with the implicit deflators that underlie conversion of the value of total output and total input into real terms. The notion of real product is a familiar one to social accountants and has been adopted by most Western countries as the appropriate measure of the level of aggregate economic activity. The notion of real factor input is somewhat less familiar, since social accounting for factor input is usually carried out only in value terms or current prices. However, it is obvious that income streams recorded in value terms correspond to transactions in the services of productive factors. The value of these transactions may be separated into price and quantity and the resulting data may be employed to construct social accounts for factor input in constant prices. This type of social accounting is implicit in all attempts to measure total factor productivity.

The prices and quantities that enter into the economic theory of production will be given in terms of social accounts for total output and total input in current and constant prices. We observe that our measurement of total factor productivity is subject to all the well-known limitations of social accounting. Only the results of economic activities with some counterpart in market transactions are included in the accounts. No attempt is made to measure social benefits or social costs if these diverge from the corresponding private benefits or private costs. Throughout this study we adhere to the basic framework of social accounting. The measurement of both output and input is based entirely on market transactions; all prices reflect private benefits and private costs. That part of any alteration in the pattern of productive activity that is "costless" from the point of view of market transactions is attributed to change in total factor productivity. Thus the social accounting framework provides a definition of total factor productivity as the ratio of real product to real factor input.

To represent the system of social accounts that provides the basis for measuring total factor productivity, we introduce the following notation:

$Y_i$ —quantity of the  $i$ th output,

$X_j$ —quantity of the  $j$ th input,

$q_i$ —price of the  $i$ th output,

$p_j$ —price of the  $j$ th input.

Where there are  $m$  outputs and  $n$  inputs, the fundamental identity for each accounting period is that the value of output is equal to the value of input:

$$q_1 Y_1 + q_2 Y_2 + \dots + q_m Y_m = p_1 X_1 + p_2 X_2 + \dots + p_n X_n. \quad \dots(1)$$

This accounting identity is important in defining an appropriate method for measuring total factor productivity; it also provides a useful check on the consistency of any proposed definitions of total output and total input.

To define total factor productivity we first differentiate (1) totally with respect to time and divide both sides by the corresponding total value. The result is an identity between a weighted average of the sum of rates of growth of output prices and quantities and a weighted average of the sum of rates of growth of input prices and quantities:

$$\sum w_i \left[ \frac{\dot{q}_i}{q_i} + \frac{\dot{Y}_i}{Y_i} \right] = \sum v_j \left[ \frac{\dot{p}_j}{p_j} + \frac{\dot{X}_j}{X_j} \right], \quad \dots(2)$$

with weights  $\{w_i\}$  and  $\{v_j\}$  given by the relative shares of the value of the  $i$ th output in the value of total output and the value of  $j$ th input in the value of total input:

$$w_i = \frac{q_i Y_i}{\sum q_i Y_i}, \quad v_j = \frac{p_j X_j}{\sum p_j X_j}.$$

To verify that both sides of (2) are weighted averages, we observe that:

$$\begin{aligned} w_i &\geq 0, i = 1 \dots m; \\ v_j &\geq 0, j = 1 \dots n; \\ \sum w_i &= \sum v_j = 1. \end{aligned}$$

A useful index of the quantity of total output may be defined in terms of the weighted average of the rates of growth of the individual outputs from (2); denoting this index of output by  $Y$ , the rate of growth of this index is

$$\frac{\dot{Y}}{Y} = \sum w_i \frac{\dot{Y}_i}{Y_i},$$

an analogous index of the quantity of total input, say  $X$ , has rate of growth

$$\frac{\dot{X}}{X} = \sum v_j \frac{\dot{X}_j}{X_j}.$$

These quantity indexes are familiar as Divisia quantity indexes; the corresponding Divisia price indexes for total output and total input, say  $q$  and  $p$ , have rates of growth:

$$\frac{\dot{q}}{q} = \sum w_i \frac{\dot{q}_i}{q_i},$$

$$\frac{\dot{p}}{p} = \sum v_j \frac{\dot{p}_j}{p_j},$$

respectively.<sup>1</sup>

In terms of Divisia index numbers a natural definition of total factor productivity, say  $P$ , is the ratio of the quantity of total output to the quantity of total input:

$$P = \frac{Y}{X}. \quad \dots(3)$$

Using the definitions of Divisia quantity indexes,  $Y$  and  $X$ , the rate of growth of total factor productivity may be expressed as:

$$\frac{\dot{P}}{P} = \frac{\dot{Y}}{Y} - \frac{\dot{X}}{X} = \sum w_i \frac{\dot{Y}_i}{Y_i} - \sum v_j \frac{\dot{X}_j}{X_j}. \quad \dots(4)$$

or, alternatively, as:

$$\frac{\dot{P}}{P} = \frac{\dot{p}}{p} - \frac{\dot{q}}{q} = \sum v_j \frac{\dot{p}_j}{p_j} - \sum w_i \frac{\dot{q}_i}{q_i}.$$

These two definitions of total factor productivity are dual to each other and are equivalent by (2). In general, any index of total factor productivity can be computed either from indexes of the quantity of total output and total input or from the corresponding price indexes.<sup>2</sup>

Up to this point we have defined total factor productivity as the ratio of certain index numbers of total output and total input. An economic interpretation of this definition may be obtained from the theory of production. The theory includes a production function

<sup>1</sup> Divisia [17, 19]. Application of these indexes to the measurement of total factor productivity is suggested by Divisia in a later publication [18, pp. 53-54]. The economic interpretation of Divisia indexes of total factor productivity has been discussed by Solow [61] and Richter [52].

<sup>2</sup> The basic duality relationship for indexes of total factor productivity has been discussed by Siegel, 57, 58].

characterized by constant returns to scale; writing this function in implicit form, we have:

$$F(Y_1, Y_2, \dots, Y_m; X_1, X_2, \dots, X_n) = 0.$$

Shifts in the production function may be defined in terms of appropriate weighted average rates of growth of outputs and inputs,

$$G\dot{F} = \sum \left( \frac{F_i Y_i}{\sum F_i Y_i} \cdot \frac{\dot{Y}_i}{Y_i} \right) - \sum \left( \frac{F_j X_j}{\sum F_j X_j} \cdot \frac{\dot{X}_j}{X_j} \right), \quad \dots(5)$$

where  $F_i = \frac{\partial F}{\partial Y_i}$ ,  $F_j = \frac{\partial F}{\partial X_j}$  and:

$$\frac{1}{G} = \sum F_i Y_i = -\sum F_j X_j.$$

Changes in total factor productivity may be identified with shifts of the production function as opposed to movements along the production function by adding the necessary conditions for producer equilibrium—all marginal rates of transformation between pairs of inputs and outputs are equal to the corresponding price ratios—

$$\frac{\partial Y_i}{\partial X_j} = -\frac{F_j}{F_i} = \frac{p_j}{q_i}; \quad \frac{\partial Y_k}{\partial Y_l} = -\frac{F_k}{F_l} = \frac{q_l}{q_k}; \quad \frac{\partial X_j}{\partial X_l} = -\frac{F_l}{F_j} = \frac{p_l}{p_j}; \quad (i, k = 1 \dots m; \quad j, l = 1 \dots n).$$

Combining these conditions with the definition (5) of shifts in the production function, we obtain the definition (4) of total factor productivity:

$$G\dot{F} = \frac{\dot{P}}{P}.$$

The rate of growth of total factor productivity is zero if and only if the shift in the production function is zero.

The complete theory of production consists of a production function with constant returns to scale together with the necessary conditions for producer equilibrium. This theory of production implies the existence of a factor price frontier relating the prices of output to the prices of input. The dual to the definition (4) of total factor productivity may be identified with shifts in the factor price frontier.<sup>1</sup>

The economic interpretation of the index of total factor productivity is essential in measuring changes in total factor productivity by means of Divisia index numbers. As is well known,<sup>2</sup> the Divisia index of total factor productivity is a line integral so that its value normally depends on the path of integration; even if the path returns to its initial value the index of total factor productivity may increase or decrease. However, if price ratios are identified with marginal rates of transformation of a production function with constant returns to scale, the index will remain constant if the shift in the production function is zero.<sup>3</sup>

From either of the two definitions of the index of total factor productivity we have given it is obvious that the rate of growth of this index is not zero by definition. Even for a production function characterized by constant returns to scale with all factors paid the value of their marginal products, the rate of growth of real product may exceed or fall short of the rate of growth of real factor input; similarly, the rate of growth of the

<sup>1</sup> The notion of a factor price frontier has been discussed by Samuelson [54]; the factor price frontier is employed in defining changes in total factor productivity by Diamond [16] and by Phelps and Phelps [51].

<sup>2</sup> See, for example, Wold [64].

<sup>3</sup> See Richter [52]. We are indebted to W. M. Gorman for bringing this fact to our attention.

price of real factor input may exceed or fall short of the rate of growth of the price of real product.<sup>1</sup>

The economic theory of production on which our interpretation of changes in total factor productivity rests is not the only possible theory of production. From the definition of shifts in the production function (5) it is clear that the production function may be considered in isolation from the necessary conditions for producer equilibrium, provided that alternative operational definitions of the marginal rates of transformation are introduced. Such a production function may incorporate the effects of increasing returns to scale, externalities, and disequilibrium. Changes in total factor productivity in our sense could then be interpreted as movements along the production function in this more general sense.

To provide a basis for assessing the role of errors of measurement in explaining observed changes in total factor productivity, we first set out principles for measuring total output and total input. The measurement of flows of output and labour services is, at least conceptually, straightforward. Beginning with data on the value of transactions in each type of output and each type of labour service, this value is separated into a price and a quantity. A quantity index of total output is constructed from the quantities of each output, using the relative shares of the value of each output in the value of total output as weights. Similarly, a quantity index of total labour input is constructed from the quantities of each labour service, using the relative shares of the value of each labour service in the value of all labour services as weights.

If capital services were bought and sold by distinct economic units in the same way as labour services, there would be no conceptual or empirical difference between the construction of a quantity index of total capital input and the construction of the corresponding index of total labour input. Beginning with data on the value of transactions in each type of capital service, this value could be separated into a price of capital service or rental and a quantity of capital service in, say, machine hours. These data would correspond to the value of transactions in each type of labour service which could be separated into a price of labour service or wage and a quantity of labour service in, say, man hours. A quantity index of total capital input would be constructed from the quantities of each type of capital service, using the relative shares of the rental value of each capital service in the rental value of all capital services as weights.

The measurement of capital services is less straightforward than the measurement of labour services because the consumer of a capital service is usually also the supplier of the

<sup>1</sup> It is essential to distinguish our basic hypothesis from a misinterpretation of it recently advanced by Denison:

Since advances in knowledge cannot increase national product without raising the marginal product of one or more factors of production, they of course disappear as a source of growth if an increase in a factor's marginal product resulting from the advance of knowledge is counted as an increase in the quantity of factor input [14, p. 76].

In terms of our social accounting framework Denison suggests that we measure factor input as the sum of the increase in both prices and quantities; denoting the index of input implied by Denison's interpretation by  $X^D$ , gives:

$$\frac{\dot{X}^D}{X^D} = \sum v_j \frac{\dot{p}_j}{p_j} + \sum v_j \frac{\dot{X}_j}{X_j};$$

the corresponding index of output, say  $Y^D$ , would then be defined as:

$$\frac{\dot{Y}^D}{Y^D} = \sum w_i \frac{\dot{q}_i}{q_i} + \sum w_i \frac{\dot{Y}_i}{Y_i};$$

The resulting index of total factor productivity, say  $P^D$ , is constant by definition:

$$\frac{\dot{P}^D}{P^D} = \frac{\dot{Y}^D}{Y^D} - \frac{\dot{X}^D}{X^D} = 0.$$

By comparing this definition with our definition (4), the error in Denison's interpretation of our hypothesis is easily seen.

service; the whole transaction is recorded only in the internal accounts of individual economic units. The obstacles to extracting this information for purposes of social accounting are almost insuperable; the information must be obtained by a relatively lengthy chain of indirect inference. The data with which the calculation begins are the values of transactions in new investment goods. These values must be separated into a price and quantity of investment goods. Second, the quantity of new investment goods reduced by the quantity of old investment goods replaced must be added to accumulated stocks. Third, the quantity of capital services corresponding to each stock must be calculated.<sup>1</sup>

Paralleling the calculation of quantities of capital services beginning with the quantities of new investment goods, the prices of capital services must be calculated beginning with the prices of new investment goods. Finally, a quantity index of total capital input must be constructed from the quantities of each type of capital service, using the relative shares of the implicit rental value of each capital service in the implicit rental value of all capital services as weights. The implicit rental value of each capital service is obtained by simply multiplying the quantity of that service by the corresponding price. At this final stage the construction of a quantity index of total capital input is formally identical to the construction of a quantity index of total labour input or total output. The chief difference between the construction of price and quantity indexes of total capital input and any other aggregation problem is in the circuitous route by which the necessary data are obtained.

The details of the calculation of a price and quantity of capital services from data on the values of transactions in new investment goods depend on empirical hypotheses about the rate of replacement of old investment goods and the quantity of capital services corresponding to a given stock of capital. In studies of total factor productivity it is conventional to assume that capital services are proportional to capital stock. Where independent data on rates of utilization of capital are available, this assumption can be dispensed with. A number of hypotheses about the rate of replacement of old investment goods have been used in the literature: (1) Accounting depreciation measured by the straight-line method is set equal to replacement, possibly with a correction for changes in prices. (2) Gross investment in some earlier period is set equal to replacement. (3) A weighted average of past investment with weights derived from studies of the "survival curves" of individual pieces of equipment<sup>2</sup> is set equal to replacement. From a formal point of view, the last of these hypotheses includes the first two as special cases.

We assume that the proportion of an investment replaced in a given interval of time declines exponentially over time. A theoretical justification for this assumption is that replacement of investment goods is a recurrent event. An initial investment generates a series of replacement investments over time; each replacement generates a new series of replacements, and so on; this process repeats itself indefinitely. The appropriate model for replacement of investment goods is not the distribution over time of replacements for a given investment, but rather the distribution over time of the infinite stream of replacements generated by a given investment. The distribution of replacements for such an infinite stream approaches a constant fraction of the accumulated stock of investment goods for any "survival curve" of individual pieces of equipment and for any initial age distribution of the accumulated stock, whether the stock is constant or growing. But this is precisely the relationship between replacement and accumulated stock if an exponentially declining proportion of any given investment is replaced in a given interval of time.

The quantity of capital services corresponding to each stock could be measured directly, at least in principle. The stock of equipment would be measured in numbers of

<sup>1</sup> Here we assume that the "quantity" of a particular type of capital as an asset is proportional to its "quantity" as a service, whatever the age of the capital. If this condition is not satisfied, capital of each distinct age must be treated as a distinct asset and service. Output at each point of time consists of the usual output plus "aged" capital stock.

<sup>2</sup> Studies in which these three methods have been employed are (1) Jaszi, Wasson, and Grose [33], Goldsmith [25], and Kuznets [39]; (2) Meyer and Kuh [44] and Denison [15]; (3) Terborgh [63].

machines while the service flow would be measured in machine hours, just as the stock of labour is measured in numbers of men while the flow of labour services is measured in man hours. While the stock of equipment may be calculated by cumulating the net flow of investment goods, the relative utilization of this equipment must be estimated in order to convert stocks into flows of equipment services. For the purposes of this study we assume that the relative utilization of all capital goods is the same; we estimate the relative utilization of capital from the relative utilization of power sources. An adjustment for the relative utilization of equipment is essential in order to preserve comparability among our measurements of output, labour input, and capital input.

To represent the capital accounts which provide the basis for measuring total capital input, we introduce the following notation:

$I_k$ —quantity of output of the  $k$ th investment good,

$K_k$ —quantity of input of the  $k$ th capital service.

As before, we use the notation:

$q_k$ —price of the  $k$ th investment good,

$p_k$ —price of the  $k$ th capital service.

Under the assumption that the proportion of an investment replaced in a given interval of time declines exponentially, the cumulated stock of past investments in the  $k$ th capital good, net of replacements, satisfies the well-known relationship:

$$I_k = \dot{K}_k + \delta_k K_k, \quad \dots(6)$$

where  $\delta_k$  is the instantaneous rate of replacement of the  $k$ th investment good. Similarly, in the absence of direct taxation the price of the  $k$ th capital service satisfies the relationship:

$$p_k = q_k \left[ r + \delta_k - \frac{\dot{q}_k}{q_k} \right], \quad \dots(7)$$

where  $r$  is the rate of return on all capital,  $\delta_k$  is the rate of replacement of the  $k$ th investment good, and  $\dot{q}_k/q_k$  is the rate of capital gain on that good. Given these relationships between the price and quantity of investment goods and the price and quantity of the corresponding capital services, the only data beyond values of transactions in new investment goods required for the construction of price and quantity indexes of total capital input are rates of replacement for each distinct investment good and the rate of return on all capital. We turn now to the problem of measuring the rate of return.

First, to measure the values of output and input it is customary to exclude the value of capital gains from the value of input rather than to include the value of such gains in the value of output. This convention has the virtue that the value of output may be calculated directly from the values of transactions. Second, to measure total factor productivity, depreciation is frequently excluded from both input and output; this convention is adopted, for example, by Kendrick [37]. Exclusion of depreciation on capital introduces an entirely arbitrary distinction between labour input and capital input, since the corresponding exclusion of depreciation of the stock of labour services is not carried out.<sup>1</sup> To calculate the rate of return on all capital, our procedure is to subtract from the value of output plus capital gains the value of labour input and of replacement. This results in the rate of return multiplied by the value of accumulated stocks. The rate of return is calculated by dividing this quantity by the value of the stock.<sup>2</sup> The

<sup>1</sup> This point is made by Domar [21].

<sup>2</sup> Domar's procedure [21, p. 717, fn. 3] fails to correct for capital gains. Implicitly, Domar is assuming either no capital gains or that all capital gains are included in the value of output, whether realized or not.

implicit rental value of the  $k$ th capital good is:

$$p_k K_k = q_k \left[ r + \delta_k - \frac{\dot{q}_k}{q_k} \right] K_k.$$

To calculate price and quantity indexes for total capital input, the prices and quantities of each type of capital service are aggregated, using the relative shares of the implicit rental value of each capital service in the implicit rental value of all capital services as weights.

An almost universal conceptual error in the measurement of capital input is to confuse the aggregation of capital stock with the aggregation of capital service. This error may be exemplified by the following passage from a recent paper by Kendrick [38] devoted to theoretical aspects of capital measurement:

. . . the prices of the underlying capital goods, as established in markets or imputed by owners, can be appropriately combined (with variable quantity weights) to provide a deflator to convert capital values into physical volumes of the various types of underlying capital goods at base-period prices. Or, the result can be achieved directly by weighting quantities by constant prices.

As I view it, this is the most meaningful way to measure "real capital stock," since the weighted aggregate measures the physical complex of capital goods in terms of its estimated ability to contribute to production as of the base period.<sup>1</sup>

The "ability to contribute to production" is, of course, measured by the price of capital services, not the price of investment goods.<sup>2</sup>

We have already noted that direct observations are usually available only for values of transactions; the separation of these values into prices and quantities is based on much less complete information and usually involves indirect inferences; the presence of systematic errors in this separation is widely recognized. For output of consumption goods or input of labour services an error in separating the value of transactions into price and quantity results in an error in measurement of the price and quantity of total output or total labour input and in the measurement of total factor productivity. For example, suppose that the rate of growth of the price of a particular type of labour service is measured with an error; since all relative value shares remain the same, the resulting error in the price of total labour input has a rate of growth equal to the rate of growth of the error multiplied by the relative share of the labour service. The quantity of total labour input is measured with an error which is equal in magnitude but opposite in sign. The error in measurement of the rate of growth of total factor productivity is equal to the negative of the rate of growth of the error in the quantity of total labour input multiplied by the relative share of labour. The effects of an error in the rate of growth of the price of a particular type of consumption good are entirely analogous; of course, an upward bias in the rate of growth of output increases the measured rate of growth of total factor productivity, while an upward bias in the rate of growth of input decreases the measured rate of growth.

An error in the separation of the value of transactions in new investment goods into the price and quantity of investment goods will result in errors in measurement of the price and quantity of investment goods, of the price and quantity of capital services and of total

<sup>1</sup> Kendrick [38, p. 106]; see the comments by Griliches [27, p. 129]. Kendrick takes a similar position in a more recent paper [36]; see the comments by Jorgenson [35]. The treatment of capital input outlined above is based on our earlier paper [31]. The data have been revised to reflect recent revisions in the U.S. national accounts.

<sup>2</sup> The answer to Mrs. Robinson's [53] rhetorical question, "what units is capital measured in?" is dual to the measurement of the price of capital services. Given either an appropriate measure of the flow of capital services or a measure of its price, the other measure may be obtained from the value of income from capital. Since this procedure is valid only if the necessary conditions for producer equilibrium are satisfied, the resulting quantity of capital may not be employed to test the marginal productivity theory of distribution, as Mrs. Robinson and others have pointed out.

factor productivity. To measure the bias in the rate of growth of the quantity of investment goods, we let  $Q^*$  be the relative error in the measurement of the price of investment goods,  $I^*$  the "quantity" of investment goods output, calculated using the erroneous "price" of investment goods, and  $I$  the actual quantity of investment goods output. The bias in the rate of growth of investment goods output is then:

$$\frac{\dot{I}^*}{I^*} - \frac{\dot{I}}{I} = -\frac{\dot{Q}^*}{Q^*}. \quad \dots(8)$$

The rate of growth of this bias is negative if the rate of growth of the error in measurement of the price of investment goods is positive, and vice-versa. If we let  $K^*$  be the "quantity" of capital calculated using the erroneous "price" of investment goods and  $K$  the actual quantity of capital:

$$K^* = \int_{-\infty}^t e^{-\delta(t-s)} I^*(s) ds = \int_{-\infty}^t e^{-\delta(t-s)} \frac{I(s)}{Q^*(s)} ds.$$

The bias in the rate of growth of the quantity of capital services is then:

$$\frac{\dot{K}^*}{K^*} - \frac{\dot{K}}{K} = \frac{I}{Q^* K^*} - \frac{I}{K} = \frac{I}{\int_{-\infty}^t e^{-\delta(t-s)} \frac{Q^*(t)}{Q^*(s)} I(s) ds} - \frac{I}{\int_{-\infty}^t e^{-\delta(t-s)} I(s) ds}, \quad \dots(9)$$

which is negative if the rate of growth of the error in measurement of the price of investment goods is positive, and vice-versa.

To calculate the error of measurement in total factor productivity, we let  $C$  represent the quantity of consumption goods and  $L$  the quantity of labour input; second, we let  $w_I$  represent the relative share of the value of investment goods in the value of total output and  $w_C$  the relative share of consumption goods; finally, we let  $v_K$  represent the relative share of the value of capital input in the value of total input and  $v_L$  the relative share of labour. The rate of growth of total factor productivity may be represented as:

$$\frac{\dot{P}}{P} = w_I \frac{\dot{I}}{I} + w_C \frac{\dot{C}}{C} - v_K \frac{\dot{K}}{K} - v_L \frac{\dot{L}}{L}.$$

If we let  $P^*$  represent the measured index of total factor productivity using the erroneous "price" of investment goods:

$$\frac{\dot{P}^*}{P^*} = w_I \frac{\dot{I}^*}{I^*} + w_C \frac{\dot{C}}{C} - v_K \frac{\dot{K}^*}{K^*} - v_L \frac{\dot{L}}{L}.$$

Subtracting the first of these expressions from the second we obtain the bias in the rate of growth of total factor productivity:

$$\frac{\dot{P}^*}{P^*} - \frac{\dot{P}}{P} = w_I \left[ \frac{\dot{I}^*}{I^*} - \frac{\dot{I}}{I} \right] - v_K \left[ \frac{\dot{K}^*}{K^*} - \frac{\dot{K}}{K} \right].$$

Substituting expressions (9) and (8) for the biases in the measured rates of growth of capital input and the output of investment goods, we have:

$$\frac{\dot{P}^*}{P^*} - \frac{\dot{P}}{P} = -w_I \frac{\dot{Q}^*}{Q^*} - v_K \left( \frac{I}{\int_{-\infty}^t e^{-\delta(t-s)} \frac{Q^*(t)}{Q^*(s)} I(s) ds} - \frac{I}{\int_{-\infty}^t e^{-\delta(t-s)} I(s) ds} \right). \quad \dots(10)$$

If investment and the error in measurement are growing at constant rates, the biases in the rates of growth of the quantity of investment goods produced and the quantity of capital services are equal, so that the net effect is equal to the rate of growth in the error

in measurement of the price of investment goods multiplied by the difference between the capital share in total input and the investment share in total output.<sup>1</sup>

A second source of errors in measurement arises from limitations on the number of separate inputs that may be distinguished empirically. The choice of commodity groups to serve as distinct "inputs" and "outputs" involves aggregation within each group by simply adding together the quantities of all commodities within the group and aggregation among groups by computation of the usual Divisia quantity index. The resulting price and quantity indexes are Divisia price and quantity indexes of the individual commodities only if the rates of growth either of prices or of quantities within each group are identical.

Errors of aggregation in studies of total factor productivity have not gone unnoticed; however, these errors are frequently mislabelled as "quality change". Quality change in this sense occurs whenever the rates of growth of quantities within each separate group are not identical. For example, if high quality items grow faster than items of low quality, the rate of growth of the group is biased downward relative to an index treating high and low quality items as separate commodities. To eliminate this bias it is necessary to construct the index of input or output for the group as a Divisia index of the individual items within the group. Elimination of "quality change" in the sense of aggregation bias is essential to accurate social accounting and to measurement of changes in total factor productivity. Separate accounts should be maintained for as many product and factor input categories as possible. An attempt should be made to exploit available detail in any empirical measurement of real product, real factor input, and total factor productivity.

In some contexts the choice of an appropriate unit for the measurement of quantities of real product or real factor input is not obvious. For example, fuel may be measured in tons or in B.T.U. equivalents, tractor services may be measured in tractor hours or in horsepower hours, and so on. Measures of real product and real factor input may be adjusted for "quality change" by converting one unit of measurement to another. This procedure conforms to the principles of social accounting we have outlined and their interpretation in terms of the economic theory of production if the adjustment for quality change corrects errors of aggregation. In the examples we have given, if the marginal products of different types of fuel always move in proportion when fuel is measured in B.T.U. equivalents but fail to do so when fuel is measured in tons, the appropriate unit for the measurement of fuel is the B.T.U. Similarly, if the marginal products of tractor services measured in horsepower hours always move in proportion, but when measured in tractor hours fail to do so, tractor services should be measured in horsepower hours.

The appropriateness of any proposed adjustment for quality change may be confronted with empirical evidence on the marginal products of individual items within a commodity group. Under the assumption that these products are equal to the corresponding price ratios this evidence takes the form of data on relative price movements for the individual items. Under a more general set of assumptions the marginal products might be calculated from an econometric production function. The latter treatment would be especially useful for "linking in" new factors and products since the relevant prices cannot be observed until the new factors and products appear in the market. Any change in measured total factor productivity resulting from adjustments for quality change is explained by evidence on the movement of marginal products and is not the result of an arbitrary choice of definitions. The choice of appropriate units for measurement of

<sup>1</sup> Domar [22, p. 587, formula (5)] considers a special case of this problem in which capital "is imported from the outside". This specialization is unnecessary, as suggested in the text. A more detailed discussion of this issue is presented by Jorgenson [35].

For constant rates of growth of the relative error in the investment goods price index and the level of investment, formula (10) may be expressed in closed form:

$$\begin{aligned} \frac{\dot{P}^*}{P^*} - \frac{\dot{P}}{\bar{P}} &= -w_I \frac{\dot{Q}^*}{Q^*} + v_K \frac{\dot{Q}^*}{Q^*}, \\ &= (v_K - w_I) \frac{\dot{Q}^*}{Q^*}. \end{aligned}$$

real product and real factor input may go beyond selection among alternative scalar measured such as B.T.U. equivalents or tons; a commodity may be regarded as multi-dimensional and an appropriate unit of measurement may be defined implicitly by taking prices as given by so-called "hedonic" price indexes. The critical property of such price indexes is that when prices are given by a "hedonic" price index for the commodities within a group, all such commodities have marginal rates of transformation *vis-à-vis* commodities outside the group that move in proportion to each other. Insofar as this property is substantiated by empirical evidence, adjustment of the commodity group for "quality change" by means of such a price index is entirely legitimate and amounts to correcting an error of aggregation.<sup>1</sup> This is not to say that any proposed adjustment for quality change is legitimate. The appropriateness of each adjustment must be judged on the basis of the evidence. If no fresh evidence is employed, the choice of appropriate units is entirely arbitrary and any change in measured total factor productivity resulting from adjustment for "quality change" is simply definitional.

"Quality change" is sometimes used to describe a special type of aggregation error, namely, the error that arises in aggregating investment goods of different vintages by simply adding together quantities of investment goods of each vintage. If the quality of investment goods, as measured by the marginal productivity of capital, is not constant over all vintages, this procedure results in aggregation errors. An appropriate index of capital services may be constructed by treating each vintage of investment goods as a separate commodity. To construct such an index empirically, data on the marginal productivity of capital of each vintage at each point of time are required. If independent data on relative prices of capital services of different vintages are used in the construction of such a capital services index, any resulting reduction in measured productivity growth is not tautological. Only where the change in quality is measured indirectly from the resulting increase in total factor productivity, as suggested by Solow [60], does such a procedure result in the elimination of productivity change by definition.<sup>2</sup>

### 3. MEASUREMENT

#### 3.1. *Initial estimates*

We can now investigate the extent to which measured changes in total factor productivity are due to errors of measurement. We begin by constructing indexes of total output and total input for the United States for the twenty-year period following World War II, 1945-65, without correcting for errors of measurement. As an initial index of total output we take U.S. private domestic product in constant prices as measured in the U.S. national product accounts [48]. As an index of total input we take the sum of labour and capital services in constant prices. Labour and capital services are assumed to be proportional to stocks of labour and capital, respectively. The stock of labour is taken to be the number of persons engaged in the private domestic sector of the U.S. economy. The stock of capital is the sum of land, plant, equipment, and inventories employed in this sector.<sup>3</sup> The rate of growth of total factor productivity is equal to the difference in the rates of growth of total output and total input.

Indexes of total output, total input, and total factor productivity are given in Table I. The average annual rate of growth of total output over the period 1945-65 is 3.49 per cent. The average rate of growth of total input is 1.83 per cent. The average rate of growth of total factor productivity is 1.60 per cent. The rate of growth of total input explains 52.4

<sup>1</sup> See Griliches [28] and the references given there.

<sup>2</sup> Jorgenson [35].

<sup>3</sup> To make stocks of labour and capital precisely analogous, it would be necessary to go even further. Unemployed workers should be included in the stock of labour since unemployed machines are included in the stock of capital. Workers should be aggregated by means of discounted lifetime incomes since capital goods are aggregated by means of asset prices.

TABLE I

*Total output, input, and factor productivity, U.S. private domestic economy, 1945-65, initial estimates*

	1	2	3
1945	0·699	0·786	0·891
1946	0·680	0·817	0·836
1947	0·695	0·854	0·818
1948	0·729	0·876	0·836
1949	0·726	0·867	0·841
1950	0·801	0·891	0·901
1951	0·852	0·928	0·919
1952	0·873	0·947	0·924
1953	0·917	0·966	0·951
1954	0·904	0·954	0·949
1955	0·981	0·976	1·005
1956	0·999	1·001	0·998
1957	1·013	1·012	1·000
1958	1·000	1·000	1·000
1959	1·069	1·019	1·048
1960	1·096	1·036	1·057
1961	1·115	1·039	1·072
1962	1·189	1·057	1·123
1963	1·240	1·074	1·152
1964	1·307	1·097	1·188
1965	1·387	1·129	1·224

1. Output.    2. Input.    3. Productivity.

per cent of the growth in output; the remainder is explained by changes in total factor productivity.

### 3.2. Errors of aggregation

The first error of measurement to be eliminated is an error of aggregation. This error results from aggregating labour and capital services by summing quantities in constant prices. To eliminate the error, we replace our initial index of total input by a Divisia index of labour and capital input, as suggested by Solow [61]. A similar error results from aggregating consumption and investment goods output by adding together quantities in constant prices. This error may be eliminated by replacing our initial index of total output by a Divisia index of consumption and investment goods output. Indexes of total output, total input, and total factor productivity with these errors of aggregation eliminated are presented in Table II.

The average annual rate of growth of total output over the period 1945-65 with the error in aggregation of consumption and investment goods output eliminated is 3·39 per cent. The average rate of growth of total input with the error in aggregation of labour and capital services eliminated is 1·84 per cent. The resulting rate of growth of total factor productivity is 1·49 per cent. We conclude that these errors in aggregation result in an overstatement of the initial rate of growth of total factor productivity. With these errors eliminated total input explains 54·3 per cent of the growth in total output. This result may be compared with the 52·4 per cent of the growth in total output explained initially.

### 3.3. Investment goods prices

We have demonstrated that an error in the measurement of investment goods prices results in errors in the measurement of total output, total input, and total factor productivity.

Roughly speaking, a positive bias in the rate of growth of the investment goods price index results in a positive bias in the rate of growth of total factor productivity, provided that the share of capital in the value of input exceeds the share of investment in the value of output. This condition is fulfilled for the U.S. private domestic sector throughout the period, 1945-65. Hence, we must examine the indexes of investment goods prices that underlie our measurement for possible sources of bias.

Except for the price index for road construction the price indexes for structures that underlie the U.S. national accounts are indexes of the cost of input rather than the price of output. In the absence of changes in total factor productivity properly constructed

TABLE II  
*Total output, input, and factor productivity, U.S. private domestic economy, 1945-65, errors of aggregation eliminated*

	1	2	3
1945	0·713	0·783	0·912
1946	0·679	0·810	0·841
1947	0·694	0·847	0·824
1948	0·727	0·870	0·840
1949	0·727	0·864	0·845
1950	0·800	0·888	0·903
1951	0·851	0·925	0·921
1952	0·873	0·945	0·926
1953	0·918	0·964	0·953
1954	0·905	0·954	0·950
1955	0·981	0·976	1·005
1956	0·999	1·001	0·998
1957	1·013	1·012	1·000
1958	1·000	1·000	1·000
1959	1·070	1·019	1·049
1960	1·096	1·036	1·057
1961	1·115	1·038	1·073
1962	1·189	1·057	1·124
1963	1·240	1·073	1·153
1964	1·307	1·096	1·189
1965	1·387	1·128	1·225

1. Output.    2. Input.    3. Productivity.

price indexes for construction input would parallel the movements of price indexes for output. This is assured by the dual to the usual definition of total factor productivity (3). Dacy [12] has shown that the rate of growth of the price of inputs in highway construction is considerably greater than that of the price of construction output. Dacy's output price index grows from 0·805 to 0·982 from 1947 through 1959, while the input price index grows from 0·615 to 1·024 in the same period, both on a base 1·000 in 1958.<sup>1</sup> This empirical finding is simply another way of looking at the positive residual between rates of growth of total output and total input where total factor productivity is measured with error. Input price indexes are subject to the same errors of aggregation as the corresponding quantity indexes. Since input quantity indexes grow too slowly, input price indexes grow too rapidly.

<sup>1</sup> The growth of the output price index may be compared with that for personal consumption expenditures, which grows from 76·5 to 108·6 from 1947 through 1959. The close parallel between the output price index for construction and the price of consumption goods suggests an explanation for the difference in rates of growth of prices of consumption and investment goods described by Gordon [26]. This difference results from the error of measurement in using an input price index in place of an output price index for investment goods. If this error is corrected, the difference vanishes.

The use of input prices in place of output prices for structures results in an important error of measurement. To eliminate this error it is necessary to use an output price index in measuring prices of both investment goods output and capital services input. An index of this type has been constructed for the OBE 1966 Capital Stock Study [49]. Components of this index include the Bureau of Public Roads price index for highway structures, the Bell System price index for telephone buildings, and the Bureau of Reclamation price indexes for pumping plants and power plants. The resulting composite index may be compared with the implicit deflator for new construction from the U.S. national accounts [48]. The implicit deflator grows from 0·686 to 1·029 during the period 1947 through 1959 while the OBE Capital Goods Study price index for new construction output grows

TABLE III  
*Alternative investment deflators*

	1	2	3	4	5	6
1945	0·616	0·510	0·759	0·517	0·633	0·357
1946	0·672	0·570	0·768	0·575	0·705	0·638
1947	0·762	0·686	0·827	0·646	0·786	2·310
1948	0·854	0·770	0·863	0·703	0·827	1·023
1949	0·824	0·755	0·868	0·736	0·818	0·788
1950	0·778	0·791	0·878	0·752	0·823	0·818
1951	0·955	0·847	0·942	0·809	0·879	0·945
1952	0·983	0·876	0·954	0·822	0·896	0·949
1953	0·947	0·889	0·943	0·835	0·903	0·497
1954	0·894	0·886	0·929	0·840	0·914	0·772
1955	0·868	0·910	0·919	0·859	0·921	0·931
1956	0·982	0·956	0·949	0·918	0·945	0·978
1957	1·025	0·992	0·984	0·975	0·978	1·113
1958	1·000	1·000	1·000	1·000	1·000	0·994
1959	0·958	1·029	1·014	1·020	1·012	0·991
1960	0·936	1·042	1·009	1·022	1·026	1·020
1961	0·944	1·053	1·006	1·021	1·037	1·011
1962	0·980	1·069	1·008	1·023	1·048	1·001
1963	1·004	1·089	1·004	1·023	1·059	1·011
1964	1·014	1·119	1·004	1·031	1·071	1·014
1965	1·051	1·149	0·995	1·038	1·089	1·032

1. Structures II.  
2. Structures I.  
3. Equipment II.

4. Equipment I.  
5. Inventories II.  
6. Inventories I.

from 0·762 to 0·958 during the same period. Thus the relative bias in the input price index for all new construction as a measure of the price of construction output is roughly comparable to the relative bias in Dacy's input price index for highway construction as a measure of the price of highway construction output. The input price index, labelled Structures I, and the output price index, labelled Structures II, are given in Table III.

The price indexes for equipment that underlie the U.S. national accounts are based primarily on data from the wholesale price index of the Bureau of Labour Statistics [6]. Since expenditures on the wholesale price index are less than those on the consumers' price index [4], adjustments for quality change are less frequent and less detailed. A direct comparison of the durables components of the wholesale and consumers' price indexes gives some notion of the relative bias. The wholesale price index increases from 0·646 to 1·023 and the consumers' price index increases from 0·858 to 1·022 over the period 1947 to 1959, both on a base of 1·000 in 1958. A direct comparison of components common to both indexes reveals essentially the same relationship. To correct for bias

in the implicit deflator for producers' durables, we substitute for this deflator the implicit deflator for consumers' durables. The deflator for producers' durables increased from 0·646 in 1947 to 1·020 in 1959. Over this same period the deflator for consumers' durables increased from 0·827 to 1·014, both on a base of 1·000 in 1958. Thus the relative bias in the producers' durables price index as revealed by a comparison with components common to the wholesale and consumers' price indexes may be corrected by simply substituting the implicit deflator for consumers' durables for the producers' durables deflator. Both indexes are given in Table III; the producers' durables index is labelled Equipment I while the consumers' durables index is labelled Equipment II.

The durables component of the consumers' price index was itself subject to considerable upward bias in recent years. The consumers' price index for new automobiles increased 62 per cent from 1947 to 1959. It has been estimated that correcting this index for quality change would reduce this increase to only 31 per cent in the same period.<sup>1</sup> In view of the upward bias in the consumers' price index our adjustment for bias in the producers' durables price index is conservative. In order to reduce the error of measurement further, detailed research like that already carried out for automobiles is required for each class of producers' durable equipment.

The price indexes for change in business inventories from the U.S. national accounts contain year-to-year fluctuations that result from changes in the composition of investment in inventories; these changes are much more substantial than the corresponding changes in the composition of inventory stocks. The implicit deflator for change in inventories is not published; however, it may be computed from data on change in inventories in current and constant dollars. Changes that amount to nearly doubling or halving the index occur from 1946 to 1947, 1947 to 1948, and 1951 to 1952. The value of the index is 0·357 in 1945, 0·638 in 1946 and 2·310 in 1947, all on a base of 1·000 (or, to be exact, 0·994) in 1958. The index drops to 1·023 in 1948 and 0·788 in 1949. A less extreme but equally substantial movement in the index occurs from 1952 through 1957. Changes in the implicit deflator of this magnitude cannot represent movements in the price of all stocks of inventories considered as investment goods. To represent these movements more accurately, we replace the implicit deflator for change in inventories by the deflator for private domestic consumption expenditures. The level of this index generally coincides with that of the implicit deflator for change in business inventories; however, the fluctuations are much less. Both indexes are given in Table III; the implicit deflator for change in business inventories is labelled Inventories I while the implicit deflator for private domestic consumption expenditures is labelled Inventories II.

Indexes of total input, total output, and total factor productivity with errors in the measurement of prices of investment goods eliminated are presented in Table IV. The average rate of growth of total output over the period 1945-65 with these errors of measurement removed is 3·59 per cent. This rate of growth may be compared with the original rate of growth of total output of 3·49 per cent or with the rate of growth of 3·39 per cent for total output with errors of aggregation removed. The average rate of growth of total input over this period is 2·19 per cent. The original rate of growth of total input is 1·83 per cent; with errors of aggregation removed the rate of growth of total input is 1·84 per cent. The rate of growth of total factor productivity is 1·41 per cent. With errors in measurement of the prices of investment goods eliminated the rate of growth of total input explains 61·0 per cent of the rate of growth of total output.

### 3.4. Measurement of services

Up to this point we have assumed that labour and capital services are proportional to stocks of labour and capital. This assumption is obviously incorrect. In principle flows of capital and labour services could be measured directly. In fact it is necessary to

<sup>1</sup> Griliches [28, Table 8, last column, p. 397].

infer the relative utilization of stocks of capital and labour from somewhat fragmentary data. Okun [50] has attempted to circumvent the problem of direct observation of labour and capital services by assuming that the relative utilization of both labour and capital is a function of the unemployment rate for labour so that the gap between actual and "potential" output, that is, output at full utilization of both factors, may be expressed in terms of the unemployment rate. A similar notion has been used by Solow [62] to adjust stocks of labour and capital for relative utilization. Most of the available capacity utilization measures are based on the relationship of actual output to output at full utilization of both labour and capital, so that these measures also attempt to adjust both labour and capital simultaneously.

TABLE IV

*Total output, input, and factor productivity, U.S. private domestic economy, 1945-65,  
errors in investment goods prices eliminated*

	1	2	3
1945	0·692	0·759	0·913
1946	0·662	0·786	0·846
1947	0·679	0·822	0·829
1948	0·718	0·845	0·853
1949	0·717	0·842	0·854
1950	0·798	0·867	0·922
1951	0·839	0·908	0·925
1952	0·858	0·930	0·925
1953	0·905	0·950	0·954
1954	0·900	0·942	0·957
1955	0·982	0·966	1·016
1956	0·995	0·996	0·999
1957	1·009	1·010	1·000
1958	1·000	1·000	1·000
1959	1·076	1·022	1·052
1960	1·107	1·042	1·061
1961	1·127	1·049	1·073
1962	1·199	1·071	1·117
1963	1·249	1·091	1·142
1964	1·319	1·117	1·177
1965	1·400	1·153	1·209

1. Output.    2. Input.    3. Productivity.

Our approach to the problem of relative utilization is somewhat more direct in that we attempt to adjust capital and labour for relative utilization separately. Of course, this adjustment gives rise to a new concept of "potential" or capacity output, but we do not pursue this notion further in this paper. Our first assumption is that the relative utilization of capital is the same for all capital goods; while this is a very strong assumption it is weaker than the assumption underlying the Okun-Solow approach in which the relative utilization of capital and labour depends on that of labour. We estimate the relative utilization of capital from the relative utilization of power sources.<sup>1</sup> Data on the relative utilization of electric motors provides an indicator of the relative utilization of capital in manufacturing, since electric motors are the predominant source of power there. We assume that relative utilization of capital goods in the manufacturing and non-manufacturing sectors is the same. When more complete data become available, this assumption can be replaced by less restrictive assumptions. Unfortunately, this adjustment

<sup>1</sup> Foss [24]. See the Statistical Appendix for further details.

allows only for the trend in the relative utilization of capital; it does not adjust for short-term cyclical variations in capacity utilization. Thus we are unable to attain the objective of complete comparability between measures of labour and capital input.

The assumption that labour services are proportional to the stock of labour is obviously incorrect. On the other hand, the assumption that labour services can be measured directly from data on man-hours is equally incorrect, as Denison [14] has pointed out. The intensity of effort varies with the number of hours worked per week, so that labour input can be measured accurately only if data on man-hours are corrected for the effects of variations in the number of hours per man on labour intensity. Denison [15] suggests that the stock of labour provides an upper bound for labour services while the number of man-hours provides a lower bound. He estimates labour input by correcting man-hours for variations in labour intensity. We employ Denison's correction for intensity,

TABLE V  
*Total input and factor productivity, U.S. private domestic economy, 1945-65,  
errors in relative utilization eliminated*

	1	2
1945	0·716	0·968
1946	0·742	0·895
1947	0·777	0·877
1948	0·801	0·899
1949	0·802	0·897
1950	0·830	0·963
1951	0·873	0·963
1952	0·899	0·956
1953	0·924	0·980
1954	0·923	0·976
1955	0·959	1·023
1956	0·994	1·001
1957	1·009	1·000
1958	1·000	1·000
1959	1·035	1·038
1960	1·057	1·046
1961	1·067	1·054
1962	1·089	1·098
1963	1·114	1·118
1964	1·146	1·147
1965	1·189	1·172

1. Input.    2. Productivity.

but we apply this correction to actual hours per man rather than potential hours per man. Thus, our measure of labour input reflects short-run variations in labour intensity.

The assumption that labour and capital services are proportional to stocks of labour and capital results in an error in separating a given value of transactions into a price and a quantity. To correct this error we multiply the number of persons engaged by hours per man. The resulting index of man-hours is then corrected for variations in labour intensity. The corresponding error for capital is corrected by multiplying the stock of capital by the relative utilization of capital. Indexes of total input and total factor productivity after these errors have been eliminated are presented for the period 1945-65 in Table V. The average annual rate of growth of total output is the same as before these corrections, 3·59 per cent per year. The average rate of growth of total input is 2·57 per cent. The resulting average rate of growth of total factor productivity is 0·96 per cent. Total input now explains 71·6 per cent of the rate of growth in total output.

### 3.5. Capital services

In converting estimates of capital stock into estimates of capital services we have disregarded an important conceptual error in the aggregation of capital services. While investment goods output must be aggregated by means of investment goods or asset prices, capital services must be aggregated by means of service prices.

The prices of capital services are related to the prices of the corresponding investment goods; in fact, the asset price is simply the discounted value of all future capital services. Asset prices for different investment goods are not proportional to service prices because of differences in rates of replacement and rates of capital gain or loss among capital goods. Implicitly, we have assumed that these prices are proportional; to eliminate the resulting error in measurement, it is necessary to compute service prices and to use these prices in aggregating capital services.

We have already outlined a method for computing the price of capital services in the absence of direct taxation of business income. In the presence of direct taxes we may distinguish between the price of capital services before and after taxes. The expression (7) given above for the price of capital services is the price after taxes. The price of capital services before taxes is:

$$p_k = q_k \left[ \frac{1-u}{1-u} r + \frac{1-w}{1-u} \delta_k - \frac{1-ux}{1-u} \dot{q}_k \right] \quad \dots(11)$$

where  $u$  is the rate of direct taxation,  $v$  the proportion of return to capital allowable as a charge against income for tax purposes,  $w$  the proportion of replacement allowable for tax purposes, and  $x$  the proportion of capital gains included in income for tax purposes

We estimate the variables describing the tax structure as follows: The rate of direct taxation is the ratio of profits tax liability to profits before taxes. The proportion of the return to capital allowable for tax purposes is the ratio of net interest to the total return to capital. Total return to capital is the after tax rate of return,  $r$ , multiplied by the current value of capital stock. The proportion of replacement allowable for tax purposes is the ratio of capital consumption allowances to the current value of replacement. The proportion of capital gains included in income is zero by the conventions of the U.S. national accounts. Given the value of direct taxes we estimate the after tax rate of return by subtracting from the value of output plus capital gains the value of labour input, replacement, and direct taxes. This results in the total return to capital. The rate of return is calculated by dividing this quantity by the current value of the stock of capital. Given data on the rate of return and the variables describing the tax structure, we calculate the price of capital services before taxes for each investment good.<sup>1</sup> These prices of capital services are used in the calculation of indexes of capital input, total input, and total factor productivity.

For the U.S. private domestic economy it is possible to distinguish five classes of investment goods—land, residential and non-residential structures, equipment, and inventories. Although it is also possible to distinguish a number of sub-classes within each of these groupings, we will employ only the five major groups in calculating an index of total capital input. For each group we first compute a before tax service price analogous to (11). We then compute an index of capital input as a Divisia index of the services of land, structures, equipment and inventories. In constructing this index we eliminate the conceptual error that arises from the implicit assumption that service prices are proportional to asset prices for different investment goods. In eliminating this conceptual error we also eliminate the error of aggregation that results from adding together capital services in constant prices to obtain an index of total capital input. To eliminate the corresponding error in our index of investment goods output we replace our initial index by a Divisia index of investment in structures, equipment, and inventories. Indexes of total output, total input and total factor productivity resulting from the elimination of these errors are

<sup>1</sup> Further details are given in the Statistical Appendix.

presented in Table VI. The after tax rate of return implicit in the new index of capital input is also given in Table VI.

The average rate of growth of total output over the period 1945-65 with the error in aggregation of investment goods eliminated is 3·59. This rate of growth is essentially the same as for total output with errors in the aggregation of consumption and investment goods and errors in the measurement of investment goods prices eliminated. The average rate of growth of total input with errors in aggregation of capital services eliminated is 2·97 per cent. This rate of growth may be compared with the initial rate of growth of 1·83 per cent.

TABLE VI

*Total input and factor productivity, U.S. private domestic economy, 1945-65,  
errors in aggregation of capital input eliminated; implicit rate of return after taxes*

	1	2	3	4
1945	0·692	0·671	1·030	0·158
1946	0·661	0·698	0·950	0·198
1947	0·678	0·735	0·926	0·237
1948	0·717	0·765	0·940	0·223
1949	0·716	0·773	0·930	0·126
1950	0·797	0·804	0·992	0·095
1951	0·837	0·850	0·986	0·242
1952	0·857	0·880	0·976	0·143
1953	0·905	0·908	0·997	0·091
1954	0·900	0·911	0·988	0·078
1955	0·982	0·951	1·032	0·113
1956	0·995	0·987	1·008	0·175
1957	1·009	1·005	1·004	0·138
1958	1·000	1·000	1·000	0·107
1959	1·077	1·039	1·035	0·097
1960	1·107	1·063	1·040	0·105
1961	1·127	1·076	1·046	0·118
1962	1·199	1·099	1·089	0·138
1963	1·250	1·126	1·107	0·131
1964	1·320	1·160	1·134	0·127
1965	1·401	1·206	1·157	0·141

1. Output. 2. Input. 3. Productivity. 4. Rate of return.

The resulting rate of growth of total factor productivity is 0·58 per cent. The index of total factor productivity with these errors eliminated is presented in Table VI. With these errors eliminated total input explains 82·7 per cent of the growth in total output. The original index of total input explains 52·4 per cent of this growth.

### 3.6. Labour services

We have eliminated errors of aggregation that arise in combining capital services into an index of total capital input. Similar errors arise in combining different categories of labour services into an index of total labour input. Implicitly, we have assumed that the price per man-hour for each category of labour services is the same; to eliminate the resulting error of measurement it is necessary to use prices per man-hour for each category in computing an index of total labour input. Second, to eliminate the error of aggregation that results from adding together labour services in constant prices, we replace our initial index of labour input by a Divisia index of the individual categories of labour services.

The Divisia index of total labour input is based on a weighted average of the rates

of growth of different categories of labour, using the relative shares in total labour compensation as weights. To represent our index of total labour input, we let  $L_l$  represent the quantity of input of the  $l$ th labour service, measured in man-hours. The rate of growth of the index of total labour input, say  $\dot{L}$ , is:

$$\frac{\dot{L}}{L} = \sum v_l \frac{\dot{L}_l}{L_l}$$

where  $v_l$  is the relative share of the  $l$ th category of labour in the total value of labour input. The number of man-hours for each labour service is the product of the number of men, say  $n_l$ , and hours per man, say  $h_l$ ; using this notation the index of total labour input may be rewritten:

$$\frac{\dot{L}}{L} = \sum v_l \frac{\dot{n}_l}{n_l} + \sum v_l \frac{\dot{h}_l}{h_l}.$$

For comparison with our initial indexes of labour input we separate the rate of growth of the index of labour input into three components—change in the total number of men, change in hours per man, and change in labour input per man-hour. We have assumed that the number of hours per man is the same for all categories of labour services, say  $H$ . Letting  $N$  represent the total number of men and  $e_l$  the proportion of the workers in the  $l$ th category of labour services, we may write the index of total labour input in the form:

$$\frac{\dot{L}}{L} = \frac{\dot{H}}{H} + \frac{\dot{N}}{N} + \sum v_l \frac{\dot{e}_l}{e_l}. \quad \dots(12)$$

Our initial index of labour input was simply  $N$ , the number of persons engaged; we corrected this index by taking into account the number of hours per man,  $H$ . To eliminate the remaining errors of aggregation we must correct the rate of growth of man-hours by adding to it an index of labour input per man-hour. The third term in the expression (12) for total labour input given above provides such an index. We will let  $E$  represent this index, so that:

$$\frac{\dot{E}}{E} = \sum v_l \frac{\dot{e}_l}{e_l}. \quad \dots(13)$$

For computational purposes it is convenient to note that the index may be rewritten in the form:

$$\frac{\dot{E}}{E} = \sum \frac{p_l}{\sum p_l e_l} \dot{e}_l = \sum p'_l \dot{e}_l,$$

where  $p_l$  is the price of the  $l$ th category of labour services and  $p'_l$  is the relative price. The relative price is the ratio of the price of the  $l$ th category of labour services to the average price of labour services,  $\sum p_l e_l$ .

In principle it would be desirable to distinguish among categories of labour services classified by age, sex, occupation, number of years schooling completed, industry of employment, and so on. An index of labour input per man-hour based on such a breakdown requires detailed research far beyond the scope of this study. We will compute such an index only for males and only for categories of labour broken down by the number of school years completed. The basic computation is presented in Table VII. Data on relative prices for labour services are available for the years 1939, 1949, 1956, 1958, 1959 and 1963.<sup>1</sup> Combining these prices with changes in the distribution of the labour force provides a measure of the change in labour input per man-hour.<sup>2</sup>

<sup>1</sup> Additional details on relative prices for labour services are presented in the Statistical Appendix, Table XII.

<sup>2</sup> Additional details on the distribution of the labour force are presented in the Statistical Appendix, Table XI.

TABLE VII  
*Relative prices,\* changes in distribution of the labour force, and indexes of labour-input per man-hour,  
 U.S. males, the civilian labour force, 1940-64*

School year completed	$p'_t$	$\Delta e_t$	$p_t$	$\Delta e_t$	$p'_t$	$\Delta e_t$						
1939	1940-48	1949	1948-52	1956	1952-57	1958	1957-59	1959	1959-62	1963	1962-65	
Elementary 0-4	0.497	-2.3	0.521	-0.3	0.452	-1.3	0.409	-0.8	0.498	-0.8	0.407	-0.8
5-6 or 5-7	0.672	-3.1	0.685	-0.5	0.624	-0.2	0.565	-1.0	0.688	-0.9	0.562	-1.5
7-8 or 8	0.887	-6.8	0.813	-1.8	0.796	-3.3	0.753	-1.2	0.801	-1.9	0.731	-1.2
High School 1-3	1.030	2.4	0.974	-1.3	0.955	0.7	0.923	0.6	0.912	-0.6	0.886	-0.3
4	1.241	7.0	1.143	1.0	1.159	2.6	1.113	0.9	1.039	1.6	1.087	3.2
College 1-3	1.442	1.4	1.336	1.2	1.356	0.2	1.392	0.7	1.255	1.3	1.269	0.0
4+ or 4	1.947	1.3	1.866	1.6	1.810	1.3	1.840	0.9	1.569	1.0	1.571	0.2
5+	...	...	...	...	...	...	...	...	1.888	0.3	1.730	0.4
Percentage change in labour input per man-hour	6.45		2.50		2.97		2.39		2.36		2.13	
Annual percentage change	0.78		0.62		0.59		1.20		0.79		0.72	

SOURCE: Derived from Tables 11 and 12, Statistical Appendix.

\* The relative prices are computed using the appropriate beginning period distribution of the labour force as weights.

Indexes of total input and total factor productivity with errors in the aggregation of labour services eliminated are presented in Table VIII. The average rate of growth of total input over the period 1945-65 with the error in aggregation of labour services eliminated is 3·47. This rate of growth may be compared with the initial rate of growth of total input of 1·83 per cent. The resulting rate of growth of total factor productivity is 0·10 per cent. With these errors eliminated total input explains 96·7 per cent of the growth in total output.

TABLE VIII  
*Total input and factor productivity, U.S. private domestic economy 1945-65,  
errors in aggregation of labour input eliminated*

	1	2
1945	0·634	1·090
1946	0·661	1·001
1947	0·700	0·971
1948	0·732	0·981
1949	0·743	0·966
1950	0·776	1·026
1951	0·823	1·017
1952	0·857	1·002
1953	0·887	1·020
1954	0·894	1·007
1955	0·936	1·048
1956	0·976	1·019
1957	0·997	1·012
1958	1·000	1·000
1959	1·047	1·027
1960	1·077	1·027
1961	1·096	1·027
1962	1·125	1·064
1963	1·158	1·076
1964	1·200	1·096
1965	1·255	1·112

1. Input.      2. Productivity.

#### 4. SUMMARY AND CONCLUSION

##### 4.1. Summary

The purpose of this paper has been to examine the hypothesis that if quantities of output and input are measured accurately, growth in total output may be largely explained by growth in total input. The results are given in Table IX and Charts 1, 2 and 3. We first present our initial estimates of rates of growth of output, input, and total factor productivity. These estimates include many of the errors made in attempts to measure total factor productivity without fully exploiting the economic theory underlying the social accounting concepts of real product and real factor input. We begin by eliminating errors of aggregation in combining investment and consumption goods and labour and capital services. We then eliminate errors of measurement in the prices of investment goods arising from the use of prices for inputs into the investment goods sector rather than outputs from this sector. We remove errors arising from the assumption that the flow of services is proportional to stocks of labour and capital by introducing direct observations on the rates of utilization of labour and capital stock. We present rates of growth that result from correct aggregation of investment goods and capital services. Finally, we give rates of growth that result from correcting the aggregation of labour services.

The rate of growth of input initially explains 52·4 per cent of the rate of growth of output. After elimination of aggregation errors and correction for changes in rates of utilization of labour and capital stock the rate of growth of input explains 96·7 per cent of the rate of growth of output; change in total factor productivity explains the rest. In the terminology of the theory of production, movements along a given production function explain 96·7 per cent of the observed changes in the pattern of productivity activity; shifts in the production function explain what remains.

This computation is based on the 1945-65 period, measuring total factor productivity peak to peak. If one were to choose a different set of years, the numerical results would be slightly different, but their main thrust would be the same. For example, starting with the Post-Korean peak year of 1953, the rate of growth of input initially explains only 37·3 per cent of the rate of growth of output. After all the corrections the rate of growth of input explains 79·2 per cent of the growth in output between 1953 and 1965, reducing the estimated rate of change in total factor productivity from 2·12 per cent per year to

TABLE IX

*Total output, input, and factor productivity, U.S. private domestic economy, 1945-65,  
average annual rates of growth*

	Output	Input	Productivity
1. Initial estimates	3·49	1·83	1·60
Estimates after correction for:			
2. Errors of aggregation	3·39	1·84	1·49
3. Errors in investment goods prices	3·59	2·19	1·41
4. Errors in relative utilization	3·59	2·57	0·96
5. Errors in aggregation of capital services	3·59	2·97	0·58
6. Errors in aggregation of labour services	3·59	3·47	0·10

0·72. We conclude that our hypothesis is consistent with the facts. If the economic theory underlying the measurement of real product and real factor input is properly exploited, the role to be assigned to growth in total factor productivity is small.

#### 4.2. Evaluation of past research

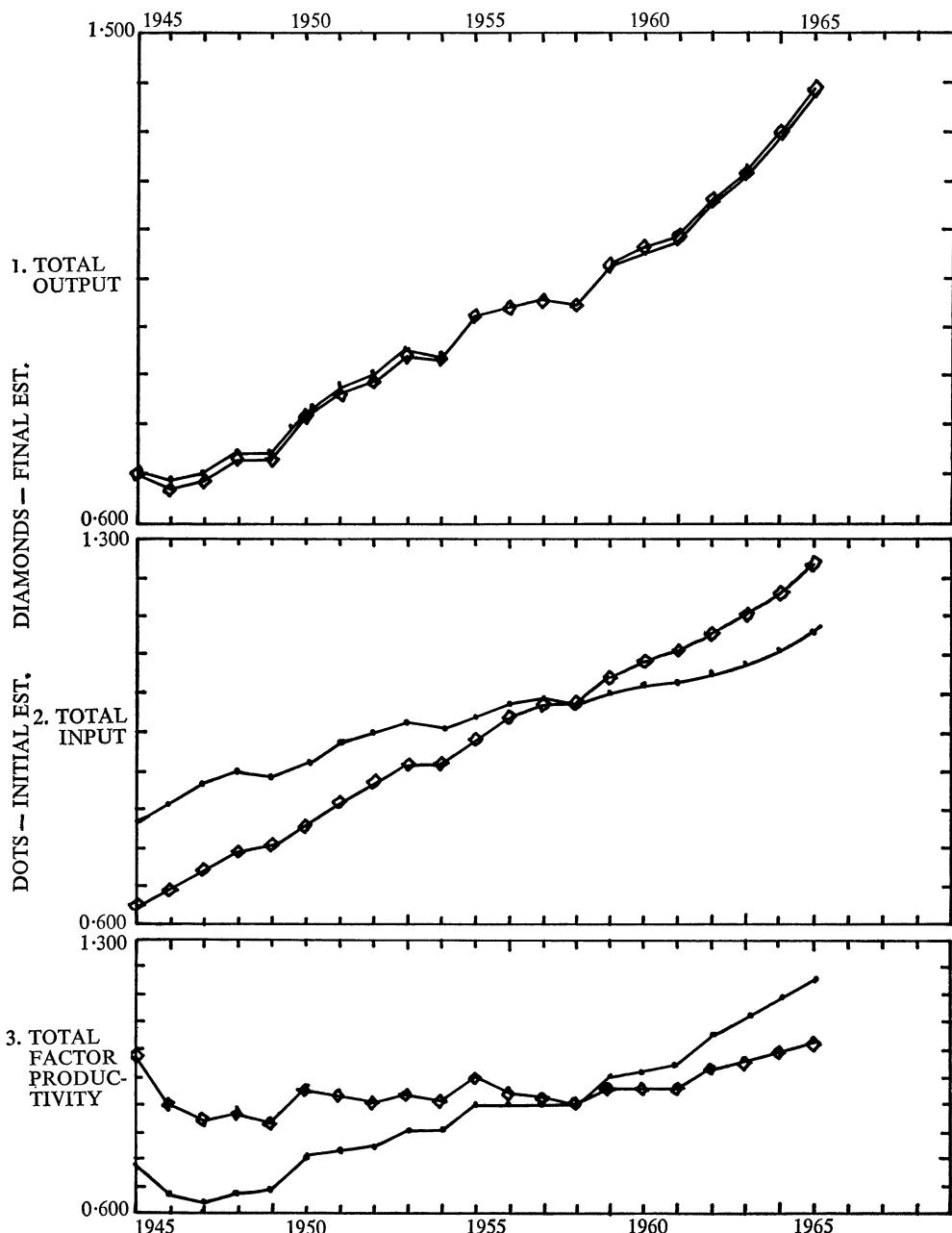
Our conclusion that most of the growth in total output may be explained by growth in total input is just the reverse of the conclusion drawn from the great body of past research on total factor productivity, the research of Schmookler [55], Mills [46], Fabricant [23], Abramovitz [2], Solow [61], and Kendrick [37]. These conclusions, stated by Abramovitz, are ". . . that to explain a very large part of the growth of total output and the great bulk of output *per capita*, we must explain the increase in output per unit of conventionally measured inputs. . ." <sup>1</sup>. This conclusion results from inadequacies in the basic economic theory underlying the social accounts employed in productivity measurements. The increase in output per unit of conventionally measured inputs is characterized by very substantial errors of measurement, equal in magnitude to the alleged increase in productivity. We have given a concrete and detailed list of errors of this type.

Our results differ from those of Denison [15] in that we correct changes in total factor productivity for errors in the measurement of output, capital services, and labour services, while Denison corrects only for errors in the measurement of labour services.

<sup>1</sup> Abramovitz [1, p. 776].

To get some idea of the relative importance of errors in the measurement of labour and errors in the measurement of output and capital, we may observe that the rate of growth of total factor productivity is reduced from 1·60 per cent per year to 0·10 per cent per year. Of the total reduction of 1·50 per cent per year errors in the measurement of output and capital account for 1·17 per cent per year while errors in the measurement of labour

INDEXES OF TOTAL OUTPUT, TOTAL INPUT AND TOTAL FACTOR PRODUCTIVITY (1958 = 1·0), U.S. PRIVATE DOMESTIC ECONOMY, 1945-1965



account for 0·33 per cent per year. We conclude that errors of measurement of the type left uncorrected by Denison are far more important than the type of errors he corrects.<sup>1</sup>

Our results suggest that the residual change in total factor productivity, which Denison attributes to Advance in Knowledge, is small. Our conclusion is not that advances in knowledge are negligible, but that the accumulation of knowledge is governed by the same economic laws as any other process of capital accumulation. Costs must be incurred if benefits are to be achieved. Although we have made no attempt to isolate the effects of expenditures on research and development from expenditures on other types of current inputs or investment goods, our results suggest that social rates of return to this type of investment are comparable to rates of return on other types of investment. Of course, our inference is indirect and a better test of this proposition could be provided by direct observation of private and social rates of return to investment in scientific research and development activities. Unfortunately, many of the direct observations on these rates of return available in the literature attribute all or part of the measured increase in total factor productivity to investment in research and development;<sup>2</sup> since these measured increases are subject to all the errors of measurement we have enumerated, satisfactory direct tests of the hypothesis that private and social rates of return to research and development investment are equal to private rates of return to other types of investment are not yet available.

Another implication of our results is that discrepancies between private and social returns to investment in physical capital may play a relatively minor role in explaining economic growth. Under the operational definitions of total factor productivity we have adopted, a positive discrepancy between social and private rates of return would appear as a downward bias in the rate of growth of input, hence an upward bias in the rate of growth of total factor productivity. The effects of such discrepancies are lumped together with the effects of other sources of growth in total factor productivity we have measured. The fact that the growth of the resulting index is small indicates that the contribution of investment to economic growth is largely compensated by the private returns to investment. This implication of our findings is inconsistent with explanations of economic growth such as Arrow's model of learning by doing [3], which are based on a higher social than private rate of return to physical capital.<sup>3</sup>

Of course, ours is not the first explanation of productivity change that does not rely primarily on discrepancies between private and social rates of return. An explanation of this type has been proposed by Solow [60], namely, embodied technical change. As Solow [59] points out, explanation of measured changes in total factor productivity as embodied technical change does not require discrepancies between private and social rates of return: ". . . the fact of expectable obsolescence reduces the private rate of return on saving below the marginal product of capital as one might ordinarily calculate it. But this discrepancy is fully reflected in a parallel difference between the marginal product of

<sup>1</sup> Errors in the aggregation of labour services account for 0·48 per cent per year, but this is offset by errors of measurement in the relative utilization of labour of -0·15 per cent per year so that the net correction for errors of measurement of labour is 0·33 per cent per year.

An alternative interpretation of our results may be provided by analogy with the conceptual framework for technical change discussed by Diamond [16]. Errors of measurement in the growth of labour services may be denoted labour-diminishing errors of measurement; capital-diminishing errors of measurement may be separated into embodied and disembodied errors. Errors in capital due to errors in the measurement of prices of investment goods are analogous to embodied technical change. Finally, some of the errors in measurement affect levels of output; these errors may be denoted output-diminishing errors of measurement.

A decomposition of total errors of measurement into labour-diminishing, capital-diminishing, embodied and disembodied, and output-diminishing is as follows: Labour-diminishing errors of measurement contribute 0·33 per cent per year to the initial measured rate of growth of total factor productivity. Embodied capital-diminishing errors contribute 0·28 per cent per year and disembodied capital-diminishing errors contribute 0·99 per cent per year. Finally, output-diminishing errors of measurement of 0·10 per cent per year must be set off against the input-diminishing errors totalling 1·60 per cent per year.

<sup>2</sup> See, for example, the studies of Minasian [47] and Mansfield [42].

<sup>3</sup> See Levhari [40, 41] for an elaboration of this point.

capital and the social rate of return on saving. So . . . the private and social rates of return coincide"<sup>1</sup>. In referring to "capital as one might ordinarily calculate it", Solow explicitly does not identify quality-corrected or "surrogate" capital with capital input and "surrogate" investment with investment goods output. In Solow's framework the marginal product of "surrogate" capital is precisely equal to the private and social rate of return on saving. The difference between Solow's point of view and ours is that the private and social rates of return are equal by definition in his framework, where the equality between private and social rates of return is a testable hypothesis within our framework.<sup>2</sup>

#### 4.3. *Implications for future research*

The problem of measuring total factor productivity is, at bottom, the same as the estimation of national product and national factor input in constant prices. The implication of our findings is that the predominant part of economic growth may be explained within a conventional social accounting framework. Of course, precise measurement of productivity change requires attention to reliability as well as accuracy. Our catalogue of errors of measurement could serve as an agenda for correction of errors in the measurement of output and for incorporation of the measurement of input into a unified social accounting framework. Given time and resources we could attempt to raise all of our measurements to the high standards of the U.S. National Product Accounts in current prices. This could be done with some difficulty for rates of relative utilization of labour and capital stock and the prices of investment goods, which require the introduction of new data into the social accounts. The elimination of aggregation errors in measuring capital services and investment goods requires a conceptual change to bring these concepts into closer correspondence with the economic theory of production. The measurement of appropriate indexes of labour input, corrected for errors of aggregation, necessitates fuller exploitation of existing data on wage differentials by education, occupation, sex, and so on.

The most serious weakness of the present study is in the use of long-term trends in the relative utilization of capital and labour to adjust capital input and labour input to concepts appropriate to the underlying theory of production. As a result of discrepancies between these trends and year-to-year variations in relative utilization of capital and labour, substantial errors of measurement have remained in the resulting index of total factor productivity. Examination of any of the alternative indexes we have presented reveals substantial unexplained cyclical variation in total factor productivity. An item of highest priority in future research is to incorporate more accurate data on annual variations in relative utilization. Hopefully, elimination of these remaining errors will make it possible to explain cyclical changes in total factor productivity along the same lines as our present explanation of secular changes. Cyclical changes are very substantial so that even our secular measurements could be improved with better data. For example, the use of the period 1945-58, a peak in total factor productivity to a trough, reveals a drop in total factor productivity of nine per cent; the use of the period 1949-65, a trough to a peak, yields an increase in total factor productivity of eleven and a half per cent.

In compiling data on labour input we have relied upon observed prices of different types of labour services. Given a broader accounting framework it would be possible to treat human capital in a manner that is symmetric with our measurement of physical capital. Investment in human capital could be cumulated into stocks along the lines suggested by Schultz [56]. The flow of investment could be treated as part of total output. The rate of return to this investment could then be measured and compared with the rate of return to physical capital. Similarly, investment in scientific research and development could be separated from expenditures on current account and cumulated into stocks.

<sup>1</sup> Solow [59, p. 58-59].

<sup>2</sup> For further discussion of this point, see Jorgenson [35].

The rate of return to research activity could then be computed. In both of these calculations it would be important not to rely on erroneously measured residual growth in total output for measurement of the social return to investment.

It is obvious that further disaggregation of our measurements would be valuable in order to provide a more stringent test of the basic hypothesis that growth in output may be explained by growth in input. The most important disaggregation of this type is to estimate levels of output and input by individual industries. The statistical raw material for disaggregation by industry is already available for stocks of labour and capital and levels of output. However, data for relative utilization of labour and capital and for disaggregation of different types of labour and capital within industry groups would have to be developed. Once these data are available, it will be possible to estimate rates of return to capital for individual industries and to study the effects of the distribution of productive factors among industries along the lines suggested by Massell [43]. The fact that past observations do not reveal significant changes in productivity does not imply that the existing allocation of productive resources is efficient relative to allocations that could be brought about by policy changes. In such a study it might be useful to extend the scope of productivity measurements to include the government sector. This would be particularly desirable if educational investment, which is largely produced in that sector, is to be incorporated into total output.

Finally, our results suggest a new point of departure for econometric studies of production function at every level of aggregation. While some existing studies [29, 30] employ data on output, labour, and capital corrected for errors of measurement along the lines we have suggested, most estimates of production functions are based on substantial errors of measurement. Econometric production functions are not an alternative to our methods for measuring total factor productivity, but rather supplement these methods in a number of important respects. Such production functions provide one means of testing the assumptions of constant returns to scale and equality between price ratios and marginal rates of transformation that underlie our measurement. A complete test of the hypothesis that growth in total output may be explained by growth in total input requires the measurement of input within a unified social accounting framework, the measurement of rates of return to both human and physical capital, further disaggregation, and new econometric studies of production functions. A start has been made on this task, but much interesting and potentially fruitful research remains to be done.

*University of California, Berkeley  
University of Chicago*

D. W. JORGENSON  
Z. GRILICHES.

#### STATISTICAL APPENDIX

1. As our initial estimate of output we employ gross private domestic product which is defined as gross national product less gross product, general government, and gross product, rest of the world, all in constant prices of 1958. These data are obtained from the U.S. national accounts. Our second estimate of output requires data on gross private domestic investment and gross private domestic consumption, defined as gross private domestic product less gross private domestic investment, in both current and constant prices of 1958. These data are also obtained from the U.S. national accounts.

As our initial estimate of labour input we employ private domestic persons engaged, defined as persons engaged for the national economy less persons engaged, general government, and persons engaged, rest of the world. These data are obtained from the U.S. national accounts [48]. Our initial estimate of capital input is obtained by the perpetual inventory method based on double declining balance estimates of replacement. For structures and equipment the lifetimes of individual assets are based on the "Bulletin F lives" employed by Jaszi, Wasson and Grose [33]. Data for gross private domestic

investment prior to 1929 are unpublished estimates that underlie the capital stock estimates of Jaszi, Wasson and Grose [33]. For inventories and land, the initial values of capital stock in constant prices of 1958 are derived from Goldsmith [25]. The stock of land in constant prices is assumed to be unchanged throughout the period we consider. Estimates of the value of land in current prices are obtained from Goldsmith [25].

The estimates of gross private domestic investment are subsequently revised by introducing alternative deflators to those employed in the U.S. national accounts. These deflators are given in Table III of the text. Gross private domestic consumption is left unchanged in this calculation. We compute stocks of land, structures, residential and non-residential, equipment, and inventories separately for each set of deflators. The basic formula is:

$$K_{t+1} = I_t + (1 - \delta)K_t, \quad \dots(14)$$

where  $I_t$  is the value of gross private domestic investment for each category in constant prices. The initial (1929) value of capital stock in constant prices of 1958 and the depreciation rates are as follows:

	National accounts deflators		Alternative deflators	
	$K_{1929}$	$\delta$	$K_{1929}$	$\delta$
Land	254,700	0	254,700	0
Structures	183,234	0·0386	162,708	0·0384
	163,205	0·0513	142,670	0·0509
Equipment	74,851	0·1325	51,701	0·1226
Inventories	48,504	0	48,504	0

2. In dropping the assumption that services are proportional to stock for both labour and capital, we require data on hours/man and hours/machine. The data on hours/man are derived from Kendrick's data on man-hours in the U.S. private domestic economy, extended through 1965.

To estimate hours/machine we first estimate the relative utilization of electric motors in manufacturing. Estimates have been given by Foss [24] for 1929, 1939 and 1954. We have updated these estimates to 1962. The basic computation is given in Table X. The 1954 data and the basic method of computation are taken from Foss [24, Table II, p. 11]. The 1954 data differ from the figures given by Foss due to a revision of the 1954 horsepower data by the Bureau of the Census and omission of the "fractional horsepower motors" adjustment. The latter, applied to both 1954 and 1962, would not have affected the estimated change in relative utilization. The horsepower data for 1962 and 1954 are from the 1963 *Census of Manufactures* [7], "Power Equipment in Manufacturing Industries," MC63(1)-6. Consumption of electric energy is taken from the 1962 *Survey of Manufactures* [11], Chapter 6. The 1962 total (388·2) is reduced by the consumption of electric power for nuclear energy (51·5) as shown in Series S81-93 of Bureau of the Census, *Continuation to 1962 of Historical Statistics of the U.S.* [9].

3. To estimate service prices for capital from the formula (11) given in the text we require data on the tax structure and on the rate of return. The variable  $u$ , the rate of direct taxation, is the ratio of profits tax liability to profits before taxes for the corporate sector. These data are from the U.S. national accounts. The variable  $v$ , the proportion of return to capital allowable as a charge against income for tax purposes, is the ratio of

private domestic net interest to the after tax rate of return,  $r$ , multiplied by the current value of capital stock. Private domestic net interest is net interest less net interest for the rest of the world sector. These data are taken from the U.S. national accounts. We discuss estimation of the after tax rate of return below. The current value of capital stock is the sum of stock in land, structures, equipment, and inventories. Each of the four components is the product of the corresponding stock in constant prices of 1958, multiplied by the investment deflator for the component. Finally, the variable  $w$ , the proportion of replacement allowable for tax purposes, is the ratio of capital consumption allowances to the current value of replacement. Capital consumption allowances are taken from the U.S. national accounts. The current value of replacement is the sum of replacement in

TABLE X  
*Relative utilization of electric motors, manufacturing, 1954 and 1962*

	Unit	1954	1962
1. Horsepower of electric motors, total	Thousand horsepower	91,505	126,783
2. Available kilowatt-hours of motors (line 1 $\times$ 7261)	Billions of kilowatt-hours	664·4	920·6
3. Electric power actually consumed, all purposes	Billions of kilowatt-hours	222·1	336·7
4. Per cent power used for electric motors	...	64·6	65·6
5. Power consumed by motors (line 3 $\times$ line 4)	Billions of kilowatt-hours	143·5	220·9
6. Per cent utilization (line 5/line 2 $\times$ 100)	...	21·6	24·0
7. Number of equivalent 40 hour weeks (line 6 $\times$ 4·2/100)	...	0·907	1·008
8. Index	1954 = 100	100·0	111·1

Line 2: The adjustment is derived as follows: It is assumed "that each electric motor could work continuously throughout the year . . . , 8760 . . . Horsepower hours are converted to kilowatt-hours; . . . 1 horsepower-hour = 0·746 kilowatt hours. The result [is] . . . adjusted upward by dividing through 0·9, since modern electric motors have an efficiency of approximately 90 per cent. . . ." Foss [23, p. 11].  $8760 \times 0·746/0·9 = 7261$ .

Line 4: Per cent power used for electric motors in 1962 computed using the industry distribution in 1945 given by Foss [24] in his Table I, and the 1962 consumption of total electric power by industries from the 1962 *Survey of Manufacturers* [11, Chapter 6].

Line 7: There are 4·2 forty-hour shifts in a full week of 168 hours.

current prices for structures and equipment. Replacement in current prices is the product of replacement in constant prices of 1958 and the investment deflator for the corresponding component. Replacement in constant prices is a by-product of the calculation of capital stock by formula (14) given above. Replacement is simply  $\delta K_t$ , where  $K_t$  is capital stock in constant prices.

To estimate the rate of return we define the value of capital services for land, structures, equipment and inventories as the product of the service price (11) and the corresponding stock in constant prices. Setting this equal to total income from property, we solve for the rate of return. Total income from property is gross private domestic product in constant prices less private domestic labour income. Private domestic labour income is private domestic compensation of employees from the U.S. national accounts multiplied by the ratio of private domestic persons engaged in production to private domestic full-time equivalent employees, both from *The National Income and Product Accounts of the United States, 1929-1965* [49]. This amounts to assuming that self-employed individuals have the same average labour income as employees.

The final formula for the rate of return is then the ratio of total income from property less profits tax liability less the current value of replacement plus the current value of capital gain to the current value of capital stock. The current value of capital gain is the

sum of capital gains for all assets; the capital gain for each asset is the product of the rate of growth of the corresponding investment deflator and the value of the asset in constant prices of 1958.

4. The basic sources of data underlying Table VII of the text are summarized in Tables XI and XII. Table XI presents estimates of the distribution of the male labour force by school years completed for 1940, 1948, 1952, 1957, 1959, 1962 and 1964. These data are taken from various issues of the *Special Labor Force Reports* [5] and *Current*

TABLE XI

*Civilian labour force, males 18 to 64 years old, by educational attainment  
per cent distribution by years of school completed*

School year completed	1940	1948	1952	1957	1959	1959†	1962†	1965†
Elementary 0-4	10·2	7·9	7·6	6·3	5·5	5·9	5·1	4·3
5-6 or 5-7*	10·2	7·1	6·6 11·6	11·4	10·4	10·7	9·8	8·3
7-8 or 8*	33·7	26·9	25·1 20·1	16·8	15·6	15·8	13·9	12·7
High School 1-3	18·3	20·7	19·4	20·1	20·7	19·8	19·2	18·9
4	16·6	23·6	24·6	27·2	28·1	27·5	29·1	32·3
College 1-3	5·7	7·1	8·3	8·5	9·2	9·4	10·6	10·6
4+ or 4	5·4	6·7	8·3	9·6	10·5	6·3	7·3	7·5
5+	...	...	...	...	...	4·7	5·0	5·4

SOURCE: The basic data for columns 1, 3, 4, 5 and 6 are taken from U.S. Department of Labor, *Special Labor Force Report* [5], No. 1, "Educational Attainment of Workers, 1959". The 5-8 years class is broken down into the 5-7 and 8 (5-6 and 7-8 for 1940, 1948, and 1952) on the basis of data provided in *Current Population Report* [10], Series P-50, Nos. 14, 49 and 78. The 1940 data were broken down using the 1940 *Census of Population* [8], Vol. III, Part 1, Table 13. The 1952 breakdown for translating the 5-7 class into 5-6 and 7-8 was done using the information on the educational attainment of all males by single years of school completed from the 1950 *Census of Population* [8], Detailed Characteristics, U.S. Summary. The 1962 data are from *Special Labor Force Report* [11], No. 30, and the 1965 figures are from *Special Labor Force Report* [11], No. 65, "Educational Attainment of Workers, March 1965".

\* 5-6 and 7-8 for 1940, 1948 and the first part of 1952, 5-7 and 8 thereafter.

† Employed, 18 years and over.

TABLE XII

*Mean annual earnings of males, 25 years and over by school years completed,  
selected years*

School year completed	1939	1949	1956	1958	1959	1963
Elementary 0-4	665	1724	2127	2046	2935	2465
5-6 or 5-7	900	2268	2927	2829	4058	3409
7-8 or 8	1188	2693 2829	3732	3769	4725	4432
High School 1-3	1379	3226	4480	4618	5379	5370
4	1661	3784	5439	5567	6132	6588
College 1-3	1931	4423	6363	6966	7401	7693
4+ or 4	2607	6179	3490	9206	9255	9523
5+	...	...	...	...	11,136	10,487

SOURCE: Columns 1, 2, 3, 4, H. P. Miller [45, Table 1, p. 966]. Column 5 from 1960 *Census of Population* [8], PC(2)-7B, "Occupation by Earnings and Education". Column 6 computed from *Current Population Reports* [10], Series P-60, No. 43, Table 22, using midpoints of class intervals and \$44,000 for the over \$25,000 class. The total elementary figure in 1940 broken down on the basis of data from the 1940 *Census of Population* [8]. The "less than 8 years" figure in 1949 split on the basis of data given in H. S. Houthakker [32]. In 1956, 1958, 1959 and 1963, split on the basis of data on earnings of males 25-64 from the 1959 1-in-a-1000 Census sample. We are indebted to G. Hanoch for providing us with this tabulation.

Earnings in 1939 and 1959; total income in 1949, 1958 and 1963.

*Population Reports* [10], with some additional data from the 1940, 1950 and 1960 *Census of Population* [8] used to break down several classes into sub-classes. We could have used data from the 1950 and 1960 Censuses on educational attainment. The increase in the number of links did not seem to offset the decrease in comparability that would be introduced by the use of different sources of data. Table II presents estimates of the mean incomes of males (25 years and over) for these classes. These data are largely taken from Miller [45], supplemented by Census and *Current Population Reports* [10] data. Table VII of the text presents the relative incomes, the first differences of the educational distribution, and the computation of an appropriate index of the change in the average education per man.

## REFERENCES

- [1] Abramovitz, Moses, "Economic Growth in the United States", *American Economic Review*, **52**, No. 4 (September 1962), pp. 762-782.
- [2] Abramovitz, Moses, *Resource and Output Trends in the United States since 1870*, Occasional Paper 63, New York, National Bureau of Economic Research, 1950.
- [3] Arrow, K. J. "The Economic Implications of Learning by Doing", *Review of Economic Studies*, **29** (3) No. 80 (June 1962), 155-173.
- [4] Bureau of Labor Statistics, *Consumers' Price Index*, Washington, U.S. Department of Labor, various monthly issues.
- [5] Bureau of Labor Statistics, *Special Labor Force Reports*, U.S. Government Printing Office, Washington, D.C.
- [6] Bureau of Labor Statistics, *Wholesale Prices and Price Indexes*, Washington, U.S. Department of Labor, various monthly issues.
- [7] Bureau of the Census, *Census of Manufactures*, U.S. Government Printing Office, Washington, D.C.
- [8] Bureau of the Census, *Census of Population*, U.S. Government Printing Office, Washington, D.C.
- [9] Bureau of the Census, *Continuation to 1962 of Historical Statistics of the U.S.*, U.S. Government Printing Office, Washington, D.C.
- [10] Bureau of the Census, *Current Population Reports*, U.S. Government Printing Office, Washington, D.C.
- [11] Bureau of the Census, *Survey of Manufactures*, U.S. Government Printing Office, Washington, D.C.
- [12] Dacy, D., "A Price and Productivity Index for a Nonhomogeneous Product", *Journal of the American Statistical Association*, **59**, No. 306 (June 1964), 469-480.
- [13] Denison, E. F., "Discussion", *American Economic Review*, **66**, No. 2 (May 1966), 76-78.
- [14] Denison, E. F., "Measurement of Labor Input: Some Questions of Definition and the Adequacy of Data", in Conference on Research in Income and Wealth, *Output, Input, and Productivity Measurement*, Studies in Income and Wealth, Vol. 25, Princeton, Princeton University Press, 1961, pp. 347-372.
- [15] Denison, E. F., *The Sources of Economic Growth in the United States and the Alternatives Before Us*, Supplementary Paper No. 13, New York, Committee for Economic Development, 1962.

- [16] Diamond, P. A., "Technical Change and the Measurement of Capital and Output", *Review of Economic Studies*, **32** (4), No. 92 (October 1965), 289-298.
- [17] Divisia, F., *Économique Rationnelle*, Paris, Gaston Doin et Cie, 1928.
- [18] Divisia, F., *Exposés d'économique*, Vol. I, Paris, Dunod, 1952.
- [19] Divisia, F., "L'indice monétaire et la théorie de la monnaie", *Revue d'Économie Politique*, 39<sup>e</sup> Année, N° 4, 5, 6; Juillet-Août, Septembre-Octobre, Novembre-Décembre, 1925, pp. 842-861, 980-1008, 1121-1151.
- [20] *Ibid.*, 40<sup>e</sup> Année, N° 1, Janvier-Février, pp. 49-81.
- [21] Domar, E. D., "On the Measurement of Technological Change", *Economic Journal*, **71**, No. 284 (December 1961), 709-729.
- [22] Domar, E. D., "Total Productivity and the Quality of Capital", *Journal of Political Economy*, **71**, No. 6 (December 1963), 586-588.
- [23] Fabricant, S., *Basic Facts on Productivity Change*, Occasional Paper 63, New York, National Bureau of Economic Research, 1959.
- [24] Foss, M., "The Utilization of Capital Equipment", *Survey of Current Business*, **43**, No. 6 (June 1963), 8-16.
- [25] Goldsmith, R., *A Study of Saving in the United States*, Princeton, Princeton University Press, 1955.
- [26] Gordon, R. A., "Price Changes: Consumers' and Capital Goods", *American Economic Review*, **51**, No. 5 (December 1961), 937-957.
- [27] Griliches, Z., "Comment", *American Economic Review*, **51**, No. 2 (May 1961), 127-130.
- [28] Griliches, Z., "Notes on the Measurement of Price and Quality Changes", in Conference on Research in Income and Wealth, *Models of Income Determination*, Princeton, Princeton University Press, 1964, pp. 381-404.
- [29] Griliches, Z., "Production Functions in Manufacturing: Some Preliminary Results", in Conference on Research in Income and Wealth, *Production Relations*, New York, Columbia University Press, forthcoming.
- [30] Griliches, Z., "The Sources of Measured Productivity Growth: United States Agriculture, 1940-60", *Journal of Political Economy*, **71**, No. 4 (August 1963), 331-346.
- [31] Griliches, Z., and Jorgenson, D., "Sources of Measured Productivity Change: Capital Input", *American Economic Review*, **56**, No. 2 (May 1966), 50-61.
- [32] Houthakker, H. S., "Education and Income", *Review of Economics and Statistics*, **41**, No. 1 (February 1959), 24-28.
- [33] Jaszi, G., Wasson, R., and Grose, L., "Expansion of Business Fixed Capital in the United States", *Survey of Current Business*, **42** (November 1962), 9-18.
- [34] Jorgenson, D., "Alternative Methods for Measuring Capital Input: Comment", in Conference on Research in Income and Wealth, *The Industrial Composition of Income and Product*, New York, Columbia University Press, forthcoming.
- [35] Jorgenson, D., "The Embodiment Hypothesis", *Journal of Political Economy*, **74**, No. 1 (February 1966), 1-17.
- [36] Kendrick, J. W., "Industry Changes in Non-Labor Costs", in Conference on Research in Income and Wealth, *The Industrial Composition of Income and Product*, New York, Columbia University Press, forthcoming.

T

- [37] Kendrick, J. W., *Productivity Trends in the United States*, Princeton, Princeton University Press, 1961.
- [38] Kendrick, J. W., "Some Theoretical Aspects of Capital Measurement", *American Economic Review*, **51**, No. 2 (May 1961), 102-111.
- [39] Kuznets, S., *Capital in the American Economy*, Princeton, Princeton University Press, 1962.
- [40] Levhari, D., "Extensions of Arrow's 'Learning by Doing'", *Review of Economic Studies*, **33** (2), No. 94 (April 1966), 117-132.
- [41] Levhari, D., "Further Implications of Learning by Doing", *Review of Economic Studies*, **33** (1), No. 93 (January 1966), 31-38.
- [42] Mansfield, E., "Rates of Return from Industrial Research and Development", *American Economic Review*, **55**, No. 2 (May 1965), 310-322.
- [43] Massell, B. F., "A Disaggregated View of Technical Change", *Journal of Political Economy*, **69**, No. 6 (December 1961), 547-557.
- [44] Meyer, J., and Kuh, E., *The Investment Decision*, Cambridge, Harvard University Press, 1957.
- [45] Miller, H. P., "Annual and Lifetime Income in Relation to Education", *American Economic Review*, **50**, No. 5 (December 1960), 962-986.
- [46] Mills, F. C., *Productivity and Economic Progress*, Occasional Paper 38, New York, National Bureau of Economic Research, 1952.
- [47] Minasian, J., "The Economics of Research and Development", in Universities-National Bureau Committee for Economic Research, *The Rate and Direction of Inventive Activity*, Princeton, Princeton University Press, 1962, pp. 93-142.
- [48] Office of Business Economics, 1966 *Capital Stock Study*, Washington, D.C., U.S. Department of Commerce, no date.
- [49] Office of Business Economics, *The National Income and Product Accounts of the United States, 1929-1965, A Supplement to the Survey of Current Business*, Washington, D.C., U.S. Department of Commerce, 1966.
- [50] Okun, A. M., "Potential GNP: Its Measurement and Significance", *Proceedings of the Business and Economic Statistics Section of the American Statistical Association*, 1962, pp. 98-104.
- [51] Phelps, E. S., and Phelps, C., "Factor-Price-Frontier Estimation of a 'Vintage' Production Model of the Postwar U.S. Nonfarm Business Sector", *Review of Economics and Statistics*, **48**, No. 3 (August 1966), 251-265.
- [52] Richter, M. K., "Invariance Axioms and Economic Indexes", *Econometrica*, forthcoming.
- [53] Robinson, J., "The Production Function and the Theory of Capital", *Review of Economic Studies*, **21** (2), No. 55 (1953-1954), 81-106.
- [54] Samuelson, P. A., "Parable and Realism in Capital Theory: The Surrogate Production Function", *Review of Economic Studies*, **29** (3), No. 80 (June 1962), 193-206.
- [55] Schmookler, J., "The Changing Efficiency of the American Economy, 1869-1938", *Review of Economics and Statistics*, **34**, No. 3 (August 1952), 214-231.
- [56] Schultz, T. W., "Education and Economic Growth", in N. B. Henry (ed.), *Social Forces Influencing American Education*, Chicago, University of Chicago Press, 1961.

- [57] Siegel, I. H., *Concepts and Measurement of Production and Productivity*, U.S. Bureau of Labor Statistics, March 1952.
- [58] Siegel, I. H., "On the Design of Consistent Output and Input Indexes for Productivity Measurement", in Conference on Research in Income and Wealth, *Output, Input and Productivity Measurement*, Studies in Income and Wealth, Vol. 25, Princeton, Princeton University Press, 1961, pp. 23-41.
- [59] Solow, R. M., *Capital Theory and the Rate of Return*, Chicago, Rand-McNally, 1964.
- [60] Solow, R. M., "Investment and Technical Progress", in K. J. Arrow, S. Karlin and P. Suppes (eds.), *Mathematical Methods in the Social Sciences*, 1959, Stanford, Stanford University Press, 1960, pp. 89-104.
- [61] Solow, R. M., "Technical Change and the Aggregate Production Function", *Review of Economics and Statistics*, 39, No. 3 (August 1957), 312-320.
- [62] Solow, R. M., "Technical Progress, Capital Formation, and Economic Growth", *American Economic Review*, 52, No. 3 (May 1962), 76-86.
- [63] Terborgh, G., *Sixty Years of Business Capital Formation*, Washington, Machinery and Allied Products Institute, 1960.
- [64] Wold, H., *Demand Analysis*, New York, Wiley and Sons, 1953.