A Theory of Persistent Income Inequality

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This paper explores the dynamics of income inequality by studying the evolution of human capital investment and neighborhood choice for a population of families. Parents affect the conditional probability distribution of their children's income through the choice of a neighborhood in which to live. Neighborhood location affects children both through local public finance of education as well as through sociological effects. These forces combine to create incentives for wealthier families to segregate themselves into economically homogeneous neighborhoods. Economic stratification combines with strong neighborhoodwide feedback effects to transmit economic status across generations, leading to persistent income inequality.

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1. Introduction

Starting with Becker and Tomes (1979) and Loury (1981), many researchers have examined models explaining a nondegenerate cross-section income distribution (see Galor and Zeira, 1993, and Bénabou, 1993, 1994, for some important recent contributions). In much of this literature, differences in human capital investment by parents in children play a major role in generating cross-section inequality. Generally, human capital markets are taken to be incomplete in the sense that human capital formation cannot be financed by issuing claims against a child's future earnings due to the lack of enforceability of such contracts. As a result, high-income families are better able than poor families to invest in human capital, and income disparities are passed on across generations. Consequently, imperfect human capital markets can induce substantial serial correlation in the time-series profile of income distribution as relative income rankings change slowly over time.

Despite the ability of these models to explain some stickiness in relative income rankings, this work has centered on models with a striking implication for the average behavior of families over time. With the exception of Galor and Zeira (1993), these models generally predict that average incomes are equal for all families, when computed over sufficiently long time horizons. Further, the models imply that there is no asymptotic tendency for one family to rank above another in income. Becker and Tomes (1986), in fact, argue that this feature is empirically accurate, as a number of studies comparing parent and child income have found small intergenerational correlation coefficients, .3 or below, suggesting that family incomes converge rapidly.¹

Although there exists evidence that the overall cross-section income distribution exhibits mean reversion, there also exists substantial evidence of persistence in the distribution's

tails. For example, Brittian (1977) has found substantial correlation in relative economic status between fathers and sons in the United States. Among fathers whose relative status ranking was in the top 10 percent of the sample, the average son's percentile ranking was 13 percent, whereas for fathers whose percentile ranking was 90 percent or below, the ranking was 71.8 percent. Similarly, Cooper, Durlauf, and Johnson (1994) have found evidence that the intergenerational correlation coefficient varies widely across families depending on characteristics of the county they resided in, ranging from .02 to over .4, with higher values associated with relatively affluent or poor countries. In a related literature, many scholars have argued in favor of the existence of a class of chronically poor people who are trapped in ghettos. Wilson (1987) has documented the growth and persistence of the chronically poor in a number of studies. Wilson's work has emphasized the idea that as middleand upper-class blacks have moved outside of historically segregated neighborhoods, the remaining residents have found themselves confronted by a breakdown of social and economic institutions that has rendered poverty in these neighborhoods self-perpetuating. This breakdown has been attributed to economic factors such as the lack of an adequate tax base to support schools, as well as to sociological factors such as the lack of successful role models to motivate children to try to leave the ghetto (formally modeled in Streufert, 1991) or peer group effects that imply that the lack of educational attainment on the part of some students will hurt the ability of others to learn (formally modeled in deBartolome, 1991, and Bénabou, 1993). These ideas are consistent with the empirical findings of Datcher (1982) and Corcoran, Gordon, Laren, and Solon (1989) that neighborhood characteristics are an important determinant of individual income levels. One important implication of the empirical work is that a family's income is not a sufficient statistic for determining whether poverty persists across generations.

This paper attempts to understand persistent income inequality by constructing a dynamic model of income distribution. Our model contains two key features. First, we explicitly model communitywide influences on individual occupational attainment. Education is locally financed; intercommunity borrowing is ruled out.² In addition, the distribution of productivity shocks among offspring is allowed to depend on neighborhood composition in order to capture various sociological influences. These factors create a feedback from the community income distribution to the realized income of offspring. Second, families choose which neighborhoods in which to live, subject to minimum and maximum income requirements, which proxy for zoning restrictions. Homogeneous neighborhoods benefit the wealthy due to the positive spillover effects induced by high per capita incomes whereas larger, heterogeneous neighborhoods provide the advantage of lower average costs to education. Together, these features induce a complex pattern of intergenerational neighborhood formation and income dynamics. We describe conditions under which uniformly poor and prosperous communities can emerge among a population of initially nonpoor families. Further, the economic stratification of neighborhoods creates a link between cross-sectional and intertemporal inequality. The basic model is also able to describe the process by which heterogeneous urban communities can be transformed into ghettos as wealthier families move to suburbs. Together, these results indicate how community factors strongly influence whether a family is trapped in poverty.

By modeling individual education levels and productivity as functions of neighborhood

behavior, we introduce a mechanism by which each family's opportunity set is affected by the choices of others. This idea has been the basis for much recent work on theories of multiple equilibria and coordination failure (see Cooper and John, 1988). One important distinction between our model and previous work is that we do not rely on uniform positive feedbacks between all agents to generate multiplicity in long-run behavior. Instead, our analysis relies on "endogenous stratification" of the economy—that is, the tendency for agents with similar characteristics to interact only with one another. Endogenous stratification, which in our model means that the rich and poor live in separate communities, produces multiplicity in long-run behavior by allowing different agents to experience different interaction environments.³

Our analysis also provides a way of understanding how agents evolve toward different long-run equilibria. The coordination failure/multiple equilibrium literature has generally concentrated on demonstrating the existence of multiple steady states in an economy, without explaining how different equilibria actually come about. Further, this literature generally assumes that all agents end up at the same equilibrium. Our results illustrate how distinct long-run equilibria can emerge among groups of agents as a consequence of the particular sample path realization of the economy.

Section 2 of the paper describes a baseline model of family income. Section 3 characterizes the equilibrium income distribution when all human capital investment is private. Section 4 analyzes the aggregate equilibrium when human capital investment is determined at an economy-wide level. Section 5 characterizes the behavior of the economy with endogenous neighborhood formation and local public finance. Some sufficient conditions are provided for poverty, prosperity, and persistent inequality to emerge in an economy. Section 6 considers the breakup of urban centers and the emergence of inner-city poverty. Section 7 contains a summary and conclusions. Proofs for all theorems are found in a technical appendix that is available from the author upon request.

2. A Model of Evolving Families

2.1. Population Structure

The population consists of a finite set I of families, indexed by i. Family i, t is composed of the agent i, t-1, who is born at t-1 and his offspring. The vector of family incomes at t, $\{Y_{1,t}, \ldots Y_{I,t}\}$, is denoted as Y_t and is the main object of our study.

Agents live two periods. Each agent receives education when young and has one child and works when old. Families live in neighborhoods indexed by d. The set of families occupying neighborhood d at t is $N_{d,t}$; # $(N_{d,t})$ denotes the number of families in the neighborhood. The number of neighborhoods is at least as large as the number of families.

2.2. Preferences

Agent i, t-1's total utility $U_{i,t-1}$ is determined by consumption when old, $C_{i,t}$, and the expected income of his offspring as an adult, $Y_{i,t+1}$.⁵ Expectations are based on \mathfrak{I}_t , the

history of the economy up to t:

$$U_{i,t-1} = E(u(C_{i,t}) + v(Y_{i,t+1}) \mid \Im_t). \tag{1}$$

The functions $u(\cdot)$ and $v(\cdot)$ are continuous, increasing, and concave and obey

$$u(\infty) = v(\infty) = \infty; \quad u'(\infty) = v'(\infty) = 0.$$
 (2)

2.3. Production Technology

Each old agent has a fixed labor endowment \bar{L} , which is applied to one of a set of occupations indexed by k; $O_{i,t}$ denotes the occupation of agent i, t-1. Aggregate output Y_t is determined by a linear function of the amount of labor devoted to each occupation, $L_{k,t}$,

$$Y_t = \sum_{k=1}^{\infty} w_k L_{k,t}. \tag{3}$$

All workers are paid their marginal product, so that if $O_{i,t}=k$, then $Y_{i,t}=w_k\bar{L}$. The sequence $\overline{Y}^1=w_1\bar{L}$, $\overline{Y}^2=w_2\bar{L}$, ... thus defines the state space for family i,t's income. A family is defined as experiencing poverty if a parent's income is less than or equal to \overline{Y}^p .

2.4. Education Constraints on Occupation

Agent i, t-1's occupation is determined by two factors: the amount of human capital invested in him during youth, $H_{i,t-1}$, and a human capital productivity shock $\zeta_{i,t}$. Formally,

$$\exists$$
 numbers $e_1 \dots e_{\infty}$ such that if $e_r \leq H_{i,t-1} < e_{r+1}$ then $O_{i,t} = O(r, \zeta_{i,t})$. (4)

The e_r 's represent minimum threshold requirements for different levels of human capital investment. These thresholds are treated as discrete since human capital investment helps determine an individual's choice among a discrete class of occupations; it is possible to normalize $O_{i,t}$ so that O(r,0) = r, which would directly relate the thresholds to occupational training.

We place several restrictions on $O(\cdot, \cdot)$. First, the function is nondecreasing in both arguments. Second, for fixed $\zeta_{i,t}$,

$$\frac{O(r+1,\zeta_{i,t}) - O(r,\zeta_{i,t})}{e_{r+1} - e_r}$$
 (5)

is nonincreasing in r, so the transformation of human capital into output exhibits nonincreasing returns to scale; this restriction is used in proving the existence of an equilibrium neighborhood education level as it ensures that preferences over tax rates within a neighborhood are single-peaked. Third, $O(r, \zeta_{i,t}) - E(O(r, \zeta_{i,t}) \mid \Im_{t-1})$ has uniformly bounded support (independent of r), which allows sufficient education to shield a person from poverty. Fourth, for fixed r, (5) is nondecreasing in $\zeta_{i,t}$, so favorable productivity shocks will not reduce the rate of return to education, which helps ensure that rich neighborhoods spend more on education than poor ones.

2.5. Properties of Productivity Shocks

The conditional probability of a productivity shock depends on $\hat{F}_{Y,d,t}$, the empirical probability distribution over incomes for families in neighborhood d at t. This dependence is designed to capture peer group and role model effects that create a feedback from a neighborhood's composition to the economic outcomes of its children:

$$Prob(\zeta_{i,t+1} \mid \Im_t) = Prob(\zeta_{i,t+1} \mid \hat{F}_{Y,d,t}) \text{ if } i \in N_{d,t}.$$
(6)

The probability of favorable productivity shocks is increased by rightward shifts in a neighborhood's empirical income distribution. Letting \hat{F}_1 and \hat{F}_2 denote two realizations of $\hat{F}_{Y.d.t}$,

If
$$\hat{F}_1(a) \le \hat{F}_2(a) \,\forall a$$
,
then $\int_{-\infty}^b Prob(\zeta_{i,t+1} \mid \hat{F}_1) \,d\zeta \le \int_{-\infty}^b Prob(\zeta_{i,t+1} \mid \hat{F}_2) \,d\zeta \,\forall b$. (7)

No mechanism exists whereby one group of families can compensate another group for its effect on a neighborhood's productivity shocks, introducing a first form of market incompleteness.

Finally, we assume that innovations to productivity shocks consist of neighborhood-specific and family-specific components:

$$\zeta_{i,t+1} - E(\zeta_{i,t+1} \mid \hat{F}_{Y,d,t}) = \gamma_{d,t+1} + \psi_{i,t+1}. \tag{8}$$

Innovations across neighborhoods are independent of one another, whereas innovations within a neighborhood may positively correlate, which can capture the effects of a range of common within-neighborhood influences. This assumption is made for simplicity as it allows us to study the one-generation ahead transition dynamics of each neighborhood in isolation.

2.6. Budget Constraint

Agent i, t-1 divides his income between consumption $C_{i,t}$ and taxes $T_{i,t}$:

$$Y_{i,t} = C_{i,t} + T_{i,t} \tag{9}$$

2.7. Human Capital Formation

All children in a neighborhood receive the same human capital investment.⁷ The cost function $NH_{d,t}$ measures the required within-neighborhood per capita expenditures to achieve a given level of per capita human capital in d at t:

$$NH_{d,t} = NH(H_{i,t}, \#(N_{d,t}), \hat{F}_{Y,d,t}). \tag{10}$$

The third argument in $NH(\cdot, \cdot, \cdot)$ reflects the effect of neighborhood composition on the education process; rightward shifts of $\hat{F}_{Y,d,t}$ are assumed to reduce the total and marginal

costs of education.⁸ While NH is increasing and convex with respect to $H_{i,t}$, the function is decreasing in neighborhood size, creating an incentive for large neighborhoods. These decreasing average costs are assumed to be bounded, which will turn out to ensure that conditions exist under which more than one neighborhood will emerge given some realizations of the empirical income distribution. For all $H_{i,t}$ and $\hat{F}_{y,d,t}$, there exists a level $M < \infty$ such that

$$0 < NH(H_{i,t}, 1, \hat{F}_{Y,d,t}) - NH(H_{i,t}, \#(I), \hat{F}_{Y,d,t}) \le M. \tag{11}$$

2.8. Structure of Taxes

Given realized family incomes at t, each neighborhood chooses a proportional tax rate $\tau_{d,t} \in (0,1)$ whose proceeds are used to finance education. Communities cannot borrow in order to finance education, which is a second form of market incompleteness:

$$T_{i,t} = \tau_{d,t} Y_{i,t} \text{ where } i \in N_{d,t}.$$

A given $\tau_{d,t}$ is an equilibrium rate for $N_{d,t}$ if at least one half of the members of $N_{d,t}$ prefer the rate to any fixed alternative. The highest such rate is always chosen. (We also assume that families resolve indifference between neighborhoods in favor of the one with the highest expected human capital formation.) Without loss of generality, each community is required to choose a high enough tax rate so that $H_{i,t} \geq e_1$.

The model will be complete once the distribution of families by neighborhood is determined. We now consider the implications of different assumptions on neighborhood structure for the dynamics of income inequality.

3. Equilibrium with Family-Specific Feedback Effects

This section describes a model of private human capital formation. The model, essentially equivalent to Loury (1981), explores the dynamics of income when there are no cross-family effects.

Assumption 1. Human capital formation is a private good: *Each family is a member of a separate neighborhood.*

Given this assumption, it is straightforward to verify that there exist equilibrium human capital and consumption sequences for each family.

Theorem 1. Existence and probability structure of equilibrium in economy with family-specific feedback effects: For all families i, there exists a sequence of income, consumption and human capital levels such that each old agent maximizes expected utility in each period. Each family's income obeys

$$Prob(Y_{i,t+1} \mid \mathcal{I}_t) = Prob(Y_{i,t+1} \mid Y_{i,t}). \tag{13}$$

The behavior of relative income in this model will depend on the properties of the implied Markov chain that characterizes the evolution of family income. In particular, Theorem 2 illustrates the conditions under which long-run inequality can emerge in this economy.

Theorem 2. Conditions for presence of permanent income inequality: For the process characterizing individual family income, if \overline{Y}_m communicates with \overline{Y}_n for any n greater that any m, then either income inequality is not permanent, i.e.

$$Prob(Y_{i,t+s} - Y_{j,t+s} > 0 \,\forall \, s > 0 \mid Y_{i,t} - Y_{j,t} > 0) = 0 \tag{14}$$

or for any finite f,

The set
$$\{\overline{Y}_1, \dots \overline{Y}_f\}$$
 is transient.⁹ (15)

The conditions of the theorem permit one family to always have greater income than another, but only if both family incomes become infinite with probability 1. Permanent inequality due to poverty can therefore occur only if there are low income-absorbing states for the Markov process describing family income. The low income-absorbing states are those in which families cannot invest sufficient human capital in their offspring to allow their descendants to ever attain high-income occupations.

4. Equilibrium with Economywide Feedback Effects

We next consider the evolution of the economy when all families occupy a common neighborhood. The model represents an extreme version of the public education economy studied by Glomm and Ravikumar (1992). The key feature of this section is that all children experience the same intertemporal feedback effects.

Assumption 2. Human capital formation is an economywide good: All families are members of a common neighborhood N at all times t.

The existence of an equilibrium in this economy is equivalent to the existence of a sequence of equilibrium tax rates. As is well known, an equilibrium tax rate may not exist due to nontransitive voting behavior. It is easy to establish that preferences are single-peaked with respect to tax rates in this economy, which rules out nonexistence problems. To see this, consider the choice of the tax rate that agent i, t-1 would make if he could dictate to the entire community. Our restrictions on the utility and production functions jointly imply that individual utility must be monotonically increasing over τ values below an individual's most preferred tax rate and decreasing over τ values above that level. Therefore, there must exist at least one tax rate in each period such that at least half of the population would be opposed to any change in the rate, which means that there exists at least one equilibrium tax rate. The existence of an equilibrium sequence of tax rates in turn implies the existence of an equilibrium stochastic process for family income, consumption, and human capital.

Theorem 3. Existence and probability structure of equilibrium with economywide feedback effects: There exists a joint stochastic process over income, consumption, and

human capital levels for all families at all dates with taxes determined by majority voting each period. Each family's income obeys

$$Prob(Y_{i,t+1} \mid \Im_t) = Prob(Y_{i,t+1} \mid \hat{F}_{Y,N,t}, \#(I)).^{10}$$
(16)

This economy cannot generate any intertemporal income inequality, unlike the economy in Section 3. The difference follows immediately from the elimination of any differences in the human capital formation and productivity shock probability distributions across the young. The models in Sections 3 and 4 thus represent polar cases illustrating how forces in the economy that affect the degree of income stratification of neighborhoods can affect intertemporal income equality. Homogeneous neighborhoods allow different income classes to evolve separately from one another. Heterogeneous neighborhoods promote intertemporal equality through the contemporaneous equalization of human capital as well as through common peer group and role model effects across children. The way in which neighborhoods are determined therefore becomes a key feature in understanding the dynamics of the income distribution and the persistence of poverty and inequality.

5. Equilibrium with Endogenous Neighborhood Formation and Local Feedback Effects

We now consider an economy where families organize themselves into neighborhoods. The dynamics of this economy depend critically on the evolution of the distribution of families by neighborhood. This evolution in turn depends on the interaction of several factors. Decreasing per capita costs in the production of human capital promote income heterogeneity within neighborhoods. On the other hand, the proportional tax assumption means that heterogeneity also leads to redistribution from rich to poor, creating incentives for the wealthy to form isolated communities. The rich may also choose to isolate themselves from the poor in order to induce a more favorable conditional probability distribution for their children's productivity shocks. Alternatively, a poor minority may wish to avoid high tax rates preferred by more affluent families. Inequality emerges in response to forces that act to promote the stratification of communities by income.

5.1. Characterization of Equilibrium

We first define a set of rules for the formation of neighborhoods. Communities are allowed to erect income requirements for entry. Minimum income requirements act as a type of zoning restriction that permits wealthy neighborhoods to keep out poor families; maximum income requirements allow the poor to live in isolation if they choose. Ideally, one would want a model in which house price differences could act to produce a configuration of families into different neighborhoods defined by income classes such that no family wishes to move to a different neighborhood. However, there does not necessarily exist an equilibrium with these characteristics. Nonexistence may occur for two reasons. First, there may not exist a set of house prices that support desired segregation by the rich. 11 In this case, the rich will

keep forming new neighborhoods in response to the entry of the poor, precluding any stable neighborhood configuration at a point in time. Second, house price differences do not, in isolation, provide a mechanism by which poor families can avoid living with rich families. In the absence of such a barrier, no stable neighborhood configuration will exist if the poor wish to be isolated. This can occur, for example, when the fixed-cost component of some educational threshold e_r is high so that the rich want to live with the poor in order to expand the tax base and reduce per capita costs, whereas the poor want low taxes due to the high marginal utility of consumption. 12

In order to ensure the existence of a stable configuration of neighborhoods, families are therefore assumed to sequentially choose neighborhoods during a time period. Once a family enters a neighborhood, the family may not move again. ¹³ This sequential structure is only one of many possible ways to ensure existence; all subsequent analysis holds for any neighborhood formation rule that allows wealthier families to isolate themselves from the rest of the population.

Assumption 3. Rules for neighborhood formation: (A) At the beginning of time t all productivity shocks $\zeta_{i,t}$ are realized. All agents know the realizations of these shocks as well as $\hat{F}_{Y,d,t-1}$ and $\#(N_{d,t-1})$ for all neighborhoods d. (B) Families move sequentially according to some known order, each either entering a neighborhood in which it is eligible to move or entering an empty neighborhood. No family moves more than once. (C) A majority of families who have entered any neighborhood during time t can vote to erect a minimum and/or maximum income requirement for joining the neighborhood. The minimum barrier cannot be set above the lowest income among families that have already entered the neighborhood and the maximum barrier cannot be set below the highest income among families currently in the neighborhood. 14

Observe that the sequential structure of moves by families resolves the chasing problem: the poor and rich can each isolate themselves when so desired. Different orderings will produce different configurations of families by neighborhoods; however, all subsequent theorems hold for any ordering of families. Assumption 3 leads to Theorem 4.

Theorem 4. Existence and probability structure of equilibrium in economy with endogenous neighborhood formation and local feedback effects: There exists a joint stochastic process over income, consumption, human capital, and neighborhood membership for all families at all dates such that each old agent maximizes expected utility. Each family's income obeys

$$Prob(Y_{i,t+1} \mid \Im_t) = Prob(Y_{i,t+1} \mid \hat{F}_{Y,d,t}, \#(N_{d,t})). \tag{17}$$

This economy can exhibit complex dynamics over time. Neighborhoods emerge and disappear as families trade off incentives for heterogeneous neighborhoods versus homogeneous neighborhoods to determine the level of economic stratification. Alternative specifications of the functional forms and parameters of our model can produce extremely different cross-section and intertemporal behavior across family incomes. We now describe some sufficient conditions under which poverty, prosperity, and persistent inequality can emerge

along different sample path realizations of the economy, when all families are initially nonpoor. Since neighborhood effects are so fundamental to the model, these conditions are expressed as conditional probability statements relating offspring outcomes to neighborhood characteristics. These restrictions can always be replaced with restrictions on the primitives of the model; Cooper (1992) and Durlauf (1994) provide examples of specific parameterizations of economies whose equilibria fulfill these sufficient conditions.¹⁵

5.2. The Emergence of Persistent Poverty

In order to understand how persistent poverty can emerge in this model, it is necessary to bound the conditional probability characterizing the occupation choices of children born in uniformly poor neighborhoods. This bound may be written as

$$Prob(Y_{i,t+1} \le \overline{Y}^p \,\forall \, i \in N_{d,t} \mid Y_{i,t} \le \overline{Y}^p \,\forall \, i \in N_{d,t}, \#(N_{d,t})) = 1 - \epsilon. \tag{18}$$

When ϵ is small, uniform neighborhood poverty is persistent; when $\epsilon = 0$, the poverty is permanent. In terms of the underlying stochastic process characterizing family income, this means that low income levels do not communicate with high income levels. The case where $\epsilon = 0$ is the stochastic generalization of the poverty trap derived by Galor and Zeira (1993), Bénabou (1993), and others.

The endogenous neighborhoods model can capture the main features of the previous work on poverty traps through the feedback of the empirical neighborhood income distribution to the income of offspring. Equation (18) may be used to identify what joint restrictions must exist on the level of human capital investment and the distribution of productivity shocks in poor neighborhoods. First, human capital investment in poor children is low—that is, there exists a *b* (which may depend on the neighborhood income distribution) such that

If
$$Y_{i,t} \leq \overline{Y}^p \, \forall i \in N_{d,t}$$
, then neighborhood d chooses $H_{i,t} \leq e_b$. (19)

Given the borrowing constraints in the model, which require that all educational expenditures are financed by within-neighborhood taxes, this condition will hold, for example, if the feedback from the neighborhood income distribution to the human capital production function (equation (10)) and the occupation function (equation (4)) is such that human capital investment by poor neighborhoods is sufficiently unproductive, or if the costs of education are sufficiently high and $u'(\cdot)$ is sufficiently large at low consumption levels that high human capital investment is not utility maximizing for poor communities. Second, the distibution of productivity shocks must be such that if the human capital received when young is less than or equal to e_b , this barrier (almost always) leads to adult poverty:

$$Prob(\zeta_{i,t+1} \text{ such that } O(b, \zeta_{i,t+1}) \le p \,\forall i \in N_{d,t} \mid Y_{i,t} \le \overline{Y}^p \,\forall i \in N_{d,t}) = 1 - \epsilon.$$
 (20)

This equation will hold for ϵ small or zero if the effect of uniform poverty on the distribution of productivity shocks as captured in equations (6) and (7) is strong enough.

The existence of low-income near-absorbing or absorbing states does not, however, mean that any families are actually trapped in poverty for substantial lengths of time, when

the economy starts off with all nonpoor families. The emergence of poverty requires the possibility of some type of downward mobility. One type of downward mobility is embodied in Assumption 4, which states that the maximum income among families within a neighborhood can decrease over time, for incomes below some threshold. This assumption is extremely weak as it is equivalent to requiring only that the offspring of the wealthiest family in a community can have a lower income than his parent. The alternative of requiring that all families can experience downward mobility every period regardless of neighborhood and income is, on the other hand, a very strong assumption, since the poorest family in a neighborhood will benefit from the positive spillover effects induced by its more affluent neighbors. For any specification of preferences there will exist a wide range of specifications of $O(\cdot, \cdot)$ and probability distributions of $S_{i,t}$ which fulfill the assumption by inducing a large enough support for the probability distribution of occupations obtainable with a given education level.

Assumption 4. Conditions for downward income movements: There exists an income level $\overline{Y}^{thresh1} > \overline{Y}^{p}$ such that for any neighborhood $N_{d,t}$,

$$Prob(\max_{i \in N_{d,t}} Y_{i,t+1} < \max_{i \in N_{d,t}} Y_{i,t} \mid \overline{Y}^{thresh1} > \max_{i \in N_{d,t}} Y_{i,t} > \overline{Y}^{p}) > 0.$$
 (21)

Assumption 4 implies Theorem 5.

Theorem 5. Persistent poverty: Under Assumptions 3 and 4, for any family i, there exists a set A_1 of realizations of Y_t with all families nonpoor and value of ϵ in equation (18) such that, with positive probability, the family becomes trapped in poverty over an arbitrarily long time interval. For some t' > t, there exists a set B_1 with $Prob(Y_{t'} \in B_1 \mid Y_t \in A_1) > 0$ such that for any fixed S and $\xi > 0$,

$$Prob(Y_{i,t'+s} \leq \overline{Y}^p \,\forall \, s \in (1,\ldots,S) \mid \underset{\sim}{Y_{t'}} \in B_1) > 1 - \xi. \tag{22}$$

Theorem 5 allows for temporary human capital growth among all families; the key to our results on the possibility of persistent poverty as well as those on persistent inequality is that a family's human capital accumulation can slow down and reverse itself along some sample path realizations of the economy.

5.3. The Emergence of Prosperity

We next consider how some families always avoid poverty. If $\epsilon=0$ in equation (18), it is important that some families can remain nonpoor, otherwise the only long-run equilibrium in the model is permanent poverty among all families. One possibility is that there exists an income threshold such that if neighborhood income exceeds this level, family income can never subsequently move down. Such an assumption would require either that negative productivity shocks disappear among the nonpoor or that nonpoor families invest in children

at such a rate that negative shocks never cause downward mobility. The elimination of any downward mobility seems unnatural even if poverty were a persistent state, since arguments that imply that positive shocks among poor children are unlikely do not imply that wealthy children will always do as well as their parents. Further, an assumption that sufficiently wealthy families will invest enough human capital to eliminate downward mobility would strongly restrict preferences.

An alternative approach to explaining how some families escape poverty may be based on the idea that nonpoor families partially insure their children against negative shocks by choosing an increasing sequence of human capital investments over time, generating growth in family income. Growth in family income, in turn, attenuates the probability of a family ever becoming poor by rendering the sequence of shocks necessary to drive a family into poverty less and less likely. Assumption 5 provides a means of formalizing the link between growth and long-run prosperity by stating that sufficiently wealthy neighborhoods exhibit a weak form of expected income growth among offspring, as the lowest income among offspring is expected to exceed the lowest income among parents. ¹⁶

Assumption 5. Conditions for evolution of family incomes outside of poverty: (A) For uniformly nonpoor neighborhoods, the lowest income among offspring exceeds the lowest income among parents with positive probability:

$$Prob(\min_{i \in N_{d,t}} Y_{i,t+1} > \min_{i \in N_{d,t}} Y_{i,t} \mid Y_{i,t} > \overline{Y}^p \, \forall \, i \in N_{d,t}, \#(N_{d,t})) > 0.$$
 (23)

(B) For sufficiently wealthy neighborhoods, the lowest income among offspring exceeds the lowest income among parents in expected value. There exists a pair of numbers $\overline{Y}^{thresh2}$ and v, with $\overline{Y}^{thresh2} > \overline{Y}^p$ and v > 0, such that

$$E(\min_{i \in N_{d,t}} Y_{i,t+1} - \min_{i \in N_{d,t}} Y_{i,t} \mid Y_{i,t} > \overline{Y}^{thresh2} \, \forall \, i \in N_{d,t}, \#(N_{d,t})) \ge \nu. \tag{24}$$

Assumption 5 is sufficient to prove that some families can escape poverty.

Theorem 6. Permanent prosperity: Let A_2 denote the set of all possible realizations of Y_t such that all families are nonpoor. Under Assumptions 3 through 5, for any family i, if $Y_t \in A_2$, then with positive probability the family remains out of poverty in all periods,

$$Prob(Y_{i,t+s} > \overline{Y}^p \,\forall \, s > 0 \mid Y_t \in A_2) > 0. \tag{25}$$

The conditional probability that the family remains out of poverty for all future periods can become arbitrarily close to one. For some t' > t, there exists a set B_2 with $Prob(Y_{t'} \in B_2 \mid Y_t \in A_2) > 0$ such that for any fixed $\xi > 0$,

$$Prob(Y_{i,t'+s} > \overline{Y}^p \,\forall \, s > 0 \mid Y_{t'} \in B_2) > 1 - \xi. \tag{26}$$

5.4. Endogenous Stratification and the Emergence of Persistent Inequality

Theorems 5 and 6 provide general conditions for very different types of family income behavior to develop along different sample path realizations of the economy. Neither theorem requires multiple neighborhoods, although as discussed below, the probability of observing poverty or prosperity is strongly affected by the presence or absence of economic stratification of neighborhoods. Multiple neighborhoods are necessary, however, for the emergence of inequality. Since families within a neighborhood experience common influences that eliminate expected intertemporal income differences, persistent inequality requires that distinct neighborhoods emerge over time in response to cross-section income inequality. The emergence of multiple neighborhoods, in turn, will depend on the way in which decreasing average costs of education are traded off against the implicit redistribution that occurs in heterogeneous neighborhoods. In fact, multiple neighborhoods will always form when there is a large enough gap between relatively rich and poor families, as stated in Theorem 7.

Theorem 7. Endogenous stratification: For any income level \overline{Y}^{low} there exists an income level \overline{Y}^{high} such that if all families either have incomes $Y_{i,t} \geq \overline{Y}^{high}$ or $Y_{i,t} \leq \overline{Y}^{low}$, no neighborhood will form that contains both a family with income $Y_{i,t} \geq \overline{Y}^{high}$ and a family with income $Y_{i,t} \leq \overline{Y}^{low}$.

This result implies that there exist configurations of family incomes such that the economy exhibits both persistently poor and nonpoor families, as stated in Theorem 8.

Theorem 8. Persistent income inequality: Under Assumptions 3 through 5, for any pair of families i and j, there exists a set A_3 of realizations of Y_t with all families nonpoor and a value of ϵ in equation (18) such that, with positive probability, the economy evolves so that one family exhibits persistent poverty whereas the other family exhibits persistent prosperity over an arbitrarily long time interval. For some t' > t, there exists a set B_3 with $Prob(Y_{t'} \in B_3 \mid Y_t \in A_3) > 0$ such that for any fixed S and $S \in S$ and $S \in S$ on

$$Prob(Y_{i,t'+s} > \overline{Y}^p \ge Y_{j,t'+s} \,\forall \, s \in (1,\ldots,S) \mid Y_{t'} \in B_3) > 1 - \xi.$$
 (27)

Theorems 7 and 8 provide a direct relationship between an economy's cross-section income distribution and the dynamics of inequality. By inducing economic stratification, cross-section inequality can alter the interaction environments for offspring and thereby place families on very different income trajectories. As a result, a sufficiently large degree of inequality at a point in time can cause inequality to persist over many generations. Since one can always parameterize the function $O(r, \zeta_{i,t})$ so that the set A_3 can be obtained with positive probability even if all families start with identical incomes above the poverty line, endogenous stratification means that long-run inequality can emerge as a consequence of the sample path realization of the economy given a wide range of initial conditions. Finally, observe that since the emergence of persistent poverty for individual families occurs through

the emergence of uniformly poor communities, the model also illustrates how ghettos can endogenously develop from nonpoor initial conditions.

The private human capital formation economy of Section 3 and the single neighborhood model of Section 4 are special cases of the endogenous neighborhoods model. When the effects of increased neighborhood size on required per capita expenditure for a fixed human capital level are small enough, there is no incentive for any family to share a community with a poorer family, which means each family must occupy a distinct neighborhood. The general endogenous neighborhoods model, however, possesses far richer dynamics than the private education special case. For example, unlike the private education model, the general endogenous neighborhoods model predicts that the persistence of poverty will differ according to whether a family is located in a uniformly poor neighborhood. This difference is an implication of both the empirical work on neighborhood effects as well as of the evidence of Card and Krueger (1992) that once school quality is controlled for, there is little relation between parental education and income and the rate of return to years of schooling. In addition, the emergence of uniformly poor neighborhoods itself depends on the cross-section income distribution for the entire economy, inducing a complex sequence of interactions between families over time.

Alternatively, when sufficiently high fixed human capital production costs and mobility costs are present, one can similarly show that the endogenous neighborhoods model will collapse to the single neighborhood model. The ability of the rich to isolate themselves from the poor is essential not only in allowing inequality, but in increasing the probability of observing some families experiencing poverty. To see this, suppose that all productivity innovations are idiosyncratic, so that when the number of families is large, it is likely at each point in time that a nonnegligible set of families exists who have experienced large negative innovations. The children of these families will experience lower income feedback effects only if they are isolated from other children. This allows a situation to arise, for some specifications of the model, where the law of large numbers implies that when all families live in the same neighborhood, the feedback economywide neighborhood income distribution makes poverty among offspring unlikely, whereas when the rich isolate themselves from the poor, the feedback from the lower tail of the income distribution makes poverty likely for the offspring of the less affluent. Put differently, endogenous stratification, by destroying the implicit insurance that exists between families located in heterogeneous communities, can increase the probability that an individual experiences poverty both as a child and as an adult.

The analysis of this section has shown how endogenous stratification can lead to a very strong form of inequality in the sense that poor and wealthy communities can coexist for long periods of time. The model is also capable of producing long periods of inequality between nonpoor neighborhoods. To see this, observe that rightward shifts in a neighborhood's empirical income distribution, through the effects embodied in equations (5) and (10), will increase the marginal product of each level of per capita educational expenditure. Unless this increase is offset by enough concavity in the function that converts education into occupations, wealthy communities will, for a wide range of preferences, choose higher average income growth rates than less wealthy communities, causing relative inequality to grow over time.

6. Urban and Suburban Interactions

In this section, we examine a variant of the endogenous neighborhoods model to see how income inequality can emerge in the context of the breakup of urban communities. Much of the literature on the urban poor (see Wilson, 1987) has emphasized that this group is, to a large extent, composed of the residual members of heterogeneous urban neighborhoods where the wealthier members have departed for suburbs. Further, a number of authors have argued that the historical experience of many American cities is well described as a precipitous withdrawal of wealthy whites from urban centers in response to small changes in the economic and racial composition of neighborhoods, a phenomenon known as tipping (see Schelling, 1971, for a discussion and references). These features can be captured by studying the decision of families to leave a heterogeneous city community in favor of homogeneous neighborhoods.

We alter the model in Section 5 in order to characterize conditions under which wealthier members of a heterogeneous urban population will disperse to suburbs. Assumption 6 introduces an original common neighborhood for all families, which we treat as an urban center. The urban center augments the endogenous neighborhoods model as it is open to all families regardless of income.

Assumption 6. Properties of urban center: (A) All families are members of neighborhood 1 at time 0. (B) Neighborhoods are formed according to the rules of Assumption 3. However, no family may be excluded from neighborhood 1 through minimum or maximum income requirements.

As before, when proportional taxes cause the implicit redistribution from rich to poor in the urban center to be large enough, wealthy agents have an incentive to leave and form their own neighborhoods. If the tax base of the initially heterogeneous neighborhood deteriorates sufficiently, the urban center can become a poverty trap. Theorem 9 illustrates how the realized cross-section income distribution can cause the urban center to break up.

Theorem 9. Properties of urban and suburban communities model: Suppose that all families i, t-1 inhabit neighborhood 1. Given Assumptions 3 through 6, there exist nonempty sets A_4 and A_5 such that (A) if $Y_t \in A_4$, then the urban center will be preserved, and (B) if $Y_t \in A_5$, then the urban center will break up at t. Theorems 4 through 8 all hold.

The emergence of a residual poor neighborhood will depend on the sample path realization of the income distribution; given the zero/one nature of the decision by each family to remain in the urban center, small changes in the cross-section income distribution can lead to large changes in the composition of $N_{1,t}$. This is an immediate implication of the fact that some elements of A_4 and A_5 must be "near" each other since the union of these sets is the space of all possible incomes. Unlike most tipping models, Theorem 9 does not exclusively rely on the percentage of less affluent families to drive the wealthy out of an urban center, although this percentage strongly affects the willingness of the affluent to remain in the urban center.

7. Summary and Conclusions

This paper describes the evolution of the distribution of income and the possible emergence of poverty in an economy in which education is locally financed and in which the empirical income distribution in a community affects the eventual occupational status of offspring. Wealthy families have an incentive to isolate themselves from the rest of the economy in order to provide the highest level of education for their children at the lowest cost. Decreasing average costs in human capital formation, on the other hand, create incentives for communities to emerge with heterogeneity in income across agents. When the forces leading to homogeneity are strong enough, endogenous stratification of the economy will occur, causing poor families to be isolated from the rest of the population. This isolation can induce persistent or permanent poverty among some families as they are unable to jointly generate sufficient human capital investment in their children to escape from low-paying occupations.

Several interesting extensions exist to the current paper. First, the critical role of the cross-section income distribution in inducing persistent inequality makes it important to better understand how this distribution is determined. Specifically, it would be valuable to identify when $O(r, \zeta_{i,t})$ is likely to produce substantial cross-section inequality for different initial conditions. Explicit analysis of this function will also permit one to compute aspects of the transition dynamics of the model such as the mean first passage time out of poverty for individual families. Second, the model may be used to study how a society should structure income redistribution policies in order to maximize some social welfare criterion. One can think of redistribution schemes as mechanisms that act to complete those missing markets whose effects manifest themselves through inequality. The dynamic structure of the model, however, suggests that the equity and efficiency tradeoffs embedded in different policies will be very complex. For example, the efficacy of equalization of educational expenditures in eliminating poverty and inequality will depend critically on the interaction of these expenditures with various sociological effects. Research on some of these questions is currently under way.

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Notes

- Solon (1992) and Zimmerman (1992) show that this correlation is actually closer to .45 when one controls for various types of measurement error.
- 2. The role of education in determining economic status has been shown in many studies. Card and Krueger (1992) is a recent analysis that shows educational quality can significantly affect future earnings. Kozol (1991) documents how inadequate local funding has affected education quality in poor school districts throughout the United States. Per capita student expenditure differences on the order of 75 percent to 100 percent between adjacent inner city and suburban schools are common throughout the United States. Local finance accounts for approximately 50 percent of all public expenditures on elementary and secondary schools.
- 3. In order for the model to exhibit uniform positive feedbacks, it would be necessary that a given family's utility be increasing in the income of other families. This will generally not hold, in two senses. First, families may respond to higher income by segregating themselves from poorer families, causing those families to lose any positive spillovers. Second, an increase in income among some families within a neighborhood will change their preferences over taxes, which could change the equilibrium tax rate in such a way as to hurt the other families in the neighborhood.
- 4. The model can exhibit multiple long-run equilibria for families in the sense that a family's average income depends on the sample path realization for the economy.
- 5. We abstract from preferences over neighborhood location and size. Incorporation of these factors will make heterogeneous communities more likely but will have no qualitative effect on our results. We also abstract from any effect of the level of parent income on the marginal utility of offspring income, which is an implication of some of the sociological work on the "culture of poverty." When this cross-derivative is positive, the likelihood with which the incomes of the rich diverge from those of the poor is increased.
- 6. Throughout, $Prob(x \mid y)$ denotes the conditional probability of x given y.
- 7. This formulation rules out any family-specific investment. Introduction of this effect would have no qualitative effect on our results concering persistent inequality between families located in different neighborhoods but would provide a way of explaining within-neighborhood inequality persistence.
- 8. Crane (1991) shows that a negative relationship exists between the high school dropout rate and the number of professional workers in a community, illustrating one way in which community factors can affect the human capital produced by a given level of expenditure. This effect is distinct from the impact of the community income distribution on individual productivity, which relates to the way in which an individual transforms training into an occupation.
- 9. Income state \$\overline{Y}_m\$ communicates with state \$\overline{Y}_n\$ if there is positive probability that a family with income \$\overline{Y}_m\$ attains \$\overline{Y}_n\$ in the future. The transient states of a stochastic process are those that the process will, with probability one, enter only a finite number of times.
- 10. #(I) appears because neighborhood size affects the per capita cost of education.
- 11. Bénabou (1993, 1994) and Durlauf (1994) show how certain parameterizations of family preferences can yield economies in which equilibrium neighborhoods supported by rental or house prices differences will emerge in economies of the type under study. For the United States, it is common for wealthy communities to implement zoning restrictions to ensure that house price differentials will actually exclude the poor; Hamilton (1975) and Wheaton (1991) provide conditions where zoning restrictions are necessary for house price differences to produce economic segregation.
- 12. This problem also applies when house prices are used to segregate families. Existence results derived in Westhoff (1977), where the rich and the poor each wish to live apart from one another, require strong additional restrictions on the joint distribution of utility functions and income beyond those we impose and are based on a much different specification of the economy.
- 13. Metaphorically, entering a neighborhood requires one to invest a high percentage of income into a house that cannot be resold except at a prohibitively large loss.
- 14. This restriction means that no family can be voted out of a neighborhood that it has already entered.
- 15. The complex interaction structure induced by the presence of neighborhood effects and endogenous neighborhood formation renders the mapping from restrictions on the conditional probabilities to technologies and preferences too complicated to characterize explicitly, so the current approach maximizes generality.

16. The possibility of growth requires that the productivity of increases in education among sufficiently wealthy families obeys a condition such as

$$\frac{O(r+1,\zeta_{i,t})-O(r,\zeta_{i,t})}{e_{r+1}-e_r} \geq \gamma > 1.$$

By choosing γ sufficiently large and $u(\cdot)$ sufficiently concave, it is straightforward to show that there exist probability laws for the productivity shocks among offspring which produce expected growth in the lowest income of offspring relative to parents. (Jones and Manuelli, 1990, derive an analogous result in the context of a growth model without productivity shocks.) In order for heterogeneous neighborhoods to experience growth in the lowest income across families, it is necessary to further restrict preferences so that rich families do not prefer such low taxes (in order to avoid redistribution) that the lowest income among offspring fails to grow. One way to do this is to restrict the utility function so that Cu'(C) is decreasing in C, which would mean that within a neighborhood, the preferred tax rate is increasing in family income. Since these preference restrictions apply only to utility evaluated at high consumption and offspring income levels, they are compatible with the restrictions necessary for equation (18) to hold.

Assumption 5 may be replaced with a condition that states that average or median income grows across generations in neighborhoods with incomes above some threshold. Such a condition will hold, for example, if preferences are modified so that the marginal utility of offspring income is increasing in parent income to such an extent that wealthy families only enter neighborhoods where their offspring will experience positive expected income growth.

17. Cooper (1992) shows how incentives for the rich to redistribute to the poor exist if the productivity of each worker depends on the education level of the society. See also Fernandez and Rogerson (1992) for an analysis of the effects of different school financing rules on the level of education across communities. Bénabou (1992) gives an interesting analysis of growth consequences of economic stratification.

References

Becker, G., and N. Tomes. (1979). "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility." *Journal of Political Economy* 87, 1153–1189.

Becker, G., and N. Tomes. (1986). "Human Capital and the Rise and Fall of Families." *Journal of Labor Economics* 4, S1-S47.

Bénabou, R. (1992). "Heterogeneity, Stratification, and Growth." Mimeo, MIT.

Bénabou, R. (1993). "Workings of a City: Location, Education, and Production." *Quarterly Journal of Economics* 108, 619–652.

Bénabou, R. (1994). "Education, Income Distribuion, and Growth: The Local Connection." Working paper 4798, National Bureau of Economic Research.

Brittain, J. (1977). The Inheritance of Economic Status. Washington, DC: Brookings Institution.

Card, D., and A. Krueger. (1992). "Does School Quality Matter? Returns to Education and the Characteristics of Public Schools in the United States." *Journal of Political Economy* 100, 1–40.

Cooper, R., and A. John. (1988). "Coordinating Coordination Failures in Keynesian Models." Quarterly Journal of Economics 103, 441–464.

Cooper, S. (1992). "A Positive Theory of Income Redistribution." Mimeo, Stanford University.

Cooper, S., S. Durlauf, and P. Johnson. (1994). "On the Evolution of Economic Status Across Generations." American Statistical Association, Business and Economic Section, *Papers and Proceedings* 50–58.

Corcoran, M., R. Gordon, D. Laren, and G. Solon. (1989). "Effects of Family and Community Background on Men's Economic Status." Working paper 2896, National Bureau of Economic Research.

Crane, J. (1991). "The Epidemic Theory of Ghettos and Neighborhood Effects on Dropping Out and Teenage Childbearing." *American Journal of Sociology* 96, 1226–1259.

Datcher, L. (1982). "Effects of Community and Family Background on Achievement." Review of Economics and Statistics 64, 32–41.

de Bartolome, C. (1991). "Equilibrium and Inefficiency in a Community Model with Peer Group Effects." *Journal of Political Economy* 98, 110–133.

- Durlauf, S. (1994). "Neighborhood Feedbacks, Endogenous Stratification, and Income Inequality." Mimeo, University of Wisconsin.
- Fernandez, R., and R. Rogerson. (1992). "Income Distribution, Communities and the Quality of Public Education: A Policy Analysis." Mimeo, Boston University.
- Galor, O., and J. Zeira. (1993). "Income Distribution and Macroeconomics." *Review of Economic Studies* 60, 35-53
- Glomm, G., and B. Ravikumar. (1992). "Public vs. Private Investment in Human Capital: Endogenous Growth and Income Inequality." *Journal of Political Economy* 100, 818–834.
- Hamilton, B. (1975). "Zoning and Property Taxation in a System of Local Governments." *Urban Studies* 12, 205-211.
- Jones, L., and R. Manuelli. (1990). "A Convex Model of Equilibrium Growth." Journal of Political Economy 98, 1008–1038.
- Kozol, J. (1991). Savage Inequalities. New York: Crown.
- Loury, G. (1981). "Intergenerational Transfers and the Distribution of Earnings." Econometrica 49, 843-867.
- Schelling, T. (1971). "Dynamic Models of Segregation." Journal of Mathematical Sociology 1, 143-186.
- Solon, G. (1992). "Intergenerational Income Mobility in the United States." *American Economic Review* 82, 393-408.
- Streufert, P. (1991). "The Effect of Underclass Isolation on Schooling Choice." Mimeo, University of Wisconsin, Madison.
- Westhoff, F. (1977). "Existence of Equilibria in Economies with a Local Public Good." *Journal of Economic Theory* 14, 84–112.
- Wheaton, W. (1991). "Land Capitalization, Tiebout Mobility and the Role of Zoning Restrictions." Mimeo, MIT. Wilson, W. J. (1987). *The Truly Disadvantaged.* Chicago: University of Chicago Press.
- Zimmerman, D. (1992). "Regression Towards Mediocrity in Economic Stature." *American Economic Review* 82, 409–429.