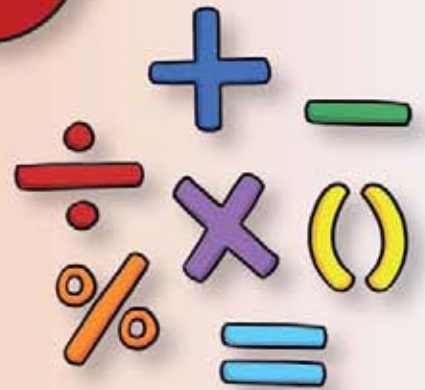


Clever



Keeping Maths Simple



H Botha • E du Plessis • I Nel • G Stols



7

Learner's
Book

Clever Keeping Maths Simple

Grade 7 Learner's Book

H Botha

E du Plessis

I Nel

G Stols



Clever Keeping Maths Simple Grade 7 Learner's Book

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Introduction

Welcome to this Mathematics Grade 7 *Learner's Book*.

Have you ever wondered how Mathematics could be useful to you? Do you think that you could use Mathematics outside the classroom?

Do you know that Mathematics is used in different ways every day?

For example:

Mathematics is used in **MUSIC**.

Mathematics is used in **MANY TECHNICAL CAREERS**.

Mathematics is used in **COOKING YOUR FAVOURITE FOOD**.

Mathematics is used in **COMPUTERS, CELL PHONES AND GAMES**.

Mathematics is used when **GOING TO THE SHOP TO BUY YOUR FAVOURITE THINGS**.

Just about everything we do uses Mathematics! So we can say that Mathematics is a language that makes use of symbols and notations to describe numerical, geometrical and graphical relationships in order to help us to give meaning to the world we live in.

To help you to develop the essential mathematical skills that you need to deal with mathematical situations competently, this Mathematics Grade 7 *Learner's Book* will guide you to:

- develop the correct use of the language of Mathematics
- develop number vocabulary, number concept and calculation and application skills
- communicate, think, reason logically and apply the mathematical knowledge gained
- investigate, analyse, represent and interpret information
- pose and solve problems
- build an awareness of the important role that Mathematics plays in real-life situations.

This Mathematics Grade 7 *Learner's Book* covers five main content areas:

- Numbers, Operations and Relationships
- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
- Data Handling

The authors and publisher wish you all the best in your study of Mathematics in Grade 7.

Topic

1

Whole numbers

In this topic you will learn to:

- multiply by multiples of 10, 100, 1 000 and 10 000
- multiply and divide by 5, 25 and 125
- use multiplication and division as inverse operations
- order, compare and represent numbers to at least 9-digit numbers
- recognise and represent prime numbers to at least 100
- round off numbers to the nearest 5, 10, 100 or 1 000
- recognise and use the commutative, associative and distributive properties of whole numbers
- recognise and use the additive property
- recognise and use the multiplicative property
- revise addition and subtraction of whole numbers to at least 6 digits
- revise multiplication and division of at least 4-digit numbers by 2-digit numbers
- estimate answers, perform calculations with all four operations and use your calculator
- use different techniques to perform and check written and mental calculations such as:
 - estimation
 - addition, subtraction and multiplication in columns
 - long division
 - rounding off and compensating
 - using a calculator
- revise work done on multiples, factors and prime factors
- list prime factors of numbers to at least 3-digit whole numbers
- find the lowest common multiple (LCM) and highest common factor (HCF) of numbers to at least 3-digit whole numbers
- solve problems that involve ratio and rate
- solve problems that involve percentages and decimal fractions in financial contexts.

**What you already know**

1. Arrange from smallest to largest:
765 439 493 765 34 567 756 943 493 756 954 376
2. Arrange from largest to smallest:
675 480 840 576 485 067 675 408 804 765 485 076
3. Complete the number sequence:
 - a) $37 + 50 \Rightarrow \square + 50 \Rightarrow 137 + 50 \Rightarrow \square + 50 \Rightarrow \square + 50 \Rightarrow \square$
 - b) $3\,452 - 25 \Rightarrow \square - 25 \Rightarrow 3\,402 - 25 \Rightarrow \square - 25 \Rightarrow \square - 25 \Rightarrow \square$

4. Fill in $<$, $>$ or $=$:
- $234\ 876 \square 243\ 675$
 - $30\ 000 \square (30 \times 1\ 000)$
 - $987\ 231 \square (900\ 000 + 8\ 000 + 200 + 30 + 1)$
 - $(15 \times 100) \square 15\ 000$
 - 10 eights \square eighty
5. Copy and complete:
- $527\ 132\ 489 = (5 \times \square) + (2 \times \square) + (7 \times \square) + (1 \times \square) + (3 \times \square) + (2 \times \square) + (4 \times \square) + (8 \times \square) + (9 \times \square)$
 - The digit 1 in the number 527 132 489 represents 100 000.
 - What number does the 5 represent?
 - What number does the 7 represent?
 - What number does the 3 represent?
 - Round off the number to the nearest 5.
 - Round off the number to the nearest 10.
 - Round off the number to the nearest 100.
 - Round off the number to the nearest 1 000.

Unit 1 Mental calculations



Important words

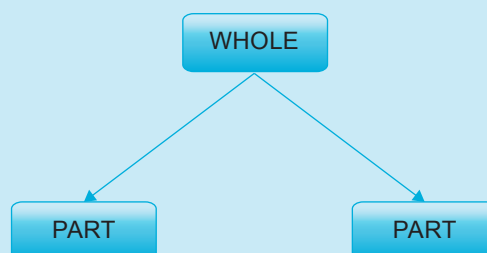
number bond	a picture showing the relationship between a number and the combination of its parts
inverse	the opposite of an operation
compensate	to make up or adjust for a change by changing something else
notation	a form of representing numbers

Number bonds

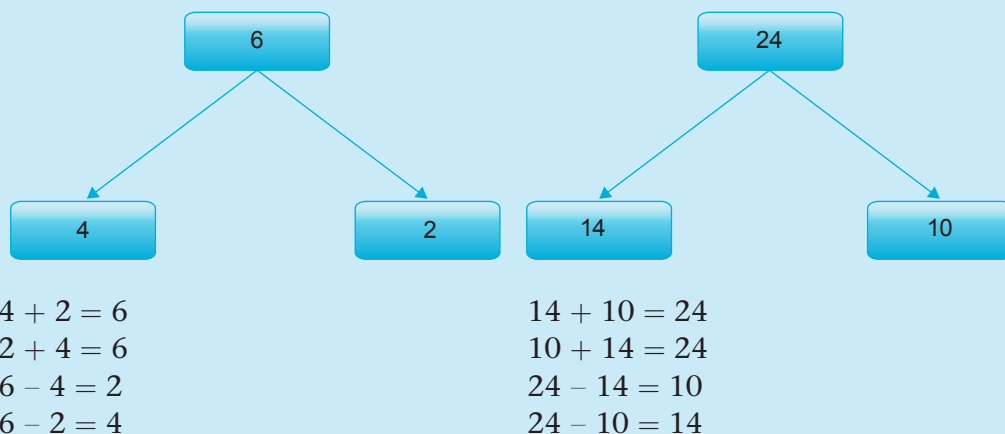
A **number bond** shows how a whole number is made up of parts. This means that number bonds show the relationship between the numbers that make up a whole number.

Example

Remember that different parts make up a whole. So, we can put parts together to make a whole (add), or we can take parts away from a whole to find the other parts (subtract).



Example



Times tables

An interval includes all the numbers between the two numbers given. We can say that counting in intervals is similar to multiplying.

Example

Count in intervals of 4.
Only write down the first four numbers.

Solution

4; 8; 12; 16; ...

Example

Write down the first four numbers of the $4 \times$ table.

Solution

4×1 ; 4×2 ; 4×3 ; 4×4 ; ... = 4; 8; 12; 16; ...

The answers in the two examples are the same. We call these lists of numbers multiples of 4, because all these numbers are divisible by 4 without a remainder. We can use **notation** to represent these multiples of 4, for example, $M_4 = 4; 8; 12; 16; \dots$

Multiplication is another method of adding the same numbers over and over again.

Example

Write 4×2 and 2×4 as the sum of numbers.

Solution

$$\begin{aligned} 4 \times 2 &= 2 + 2 + 2 + 2 \\ &= 8 \end{aligned}$$

$$\begin{aligned} 2 \times 4 &= 4 + 4 \\ &= 8 \end{aligned}$$

Get to know your multiplication tables up to at least 12×12 .
 Use this 12×12 multiplication grid. It will help you to solve multiplication and division problems. You can also use the multiplication grid to find multiples of numbers.

Example

Use the 12×12 multiplication grid to find the answer to 9×12 .

Solution

Look at the number grid. Find 9 in the first column. It is circled.

Then find 12 in the first row. It is circled.

At the point where these two meet, you will find your answer.

Therefore: $9 \times 12 = 108$.

\times	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Example

Use the 12×12 multiplication grid to find the multiples of 3.

Solution

$M_3 = 3; 6; 9; 12; \dots$ (Found in column 3 or row 3.)

Imagine that you need to multiply numbers that are larger than the numbers given in the 12×12 multiplication grid. Break the numbers up, then do the multiplication. This will simplify the calculation and make it easier to work out. Let's look at an example.

Example

Calculate 15×14 .

Solution

$$\begin{aligned}
 15 \times 14 &= 15 \times 2 \times 7 \text{ (because } 14 = 2 \times 7\text{)} \\
 &= 30 \times 7 \\
 &= 210
 \end{aligned}$$

Using multiplication to do division

Multiplication and division are **inverse** operations. This means that multiplication is the opposite of division. It also means that you can use multiplication to check the answer to a division problem, and vice versa.


Example

Calculate by reading the values from the table below:

a) 12×9

b) $108 \div 12$

c) $108 \div 9$



A girl is standing next to a large blackboard that displays a multiplication table. She is pointing with a piece of chalk to the number 12 in the top row, which is circled. The table is a 12x12 grid with numbers 1 through 12 in both the rows and columns. The values in the cells are the products of the row and column numbers.

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Solution

a) 108

b) 9

c) 12

Example

What number must be multiplied by 8 to get 72?

Solution

Instead of writing: $\square \times 8 = 72$, write $72 \div 8 = \square$.

The answer is: $72 \div 8 = 9$.

Use this method to solve more complex problems.

Example

What number must be multiplied by 872 to get 11 336?

Solution

The answer is more than 10 times 872. This is because 10×872 is equal to 8 720.

Next calculate 11×872 , 12×872 , etc. Do this until you find the answer.

Simplify this process by using division as the inverse operation of multiplication.

Instead of writing: $\square \times 872 = 11\,336$, write $11\,336 \div 872 = \square$.

Now use long division, short division or your calculator to solve the problem.

The answer is: $11\,336 \div 872 = 13$.

Multiplying and dividing by 10, 100, 1 000 and 10 000

When you multiply by 10, 100, 1 000 or 10 000, use a short method. Simply *include* as many zeros to the end of the given number as there are zeros in the numbers 10, 100, 1 000 or 10 000.

Example

1. Calculate 25×10 .
2. Calculate 85×100 .

Solution

1. $25 \times 10 = 250$ (Include one zero.)
2. $85 \times 100 = 8\,500$ (Include two zeros.)

When you divide by 10, 100, 1 000 and 10 000, use a short method. Simply *remove* as many zeros from the end of the given number as there are zeros in the numbers 10, 100, 1 000 or 10 000.

Example

1. Calculate $500 \div 10$.
2. Calculate $2\,300 \div 100$.

Solution

1. $500 \div 10 = 50$ (Remove one zero.)
2. $2\,300 \div 100 = 23$ (Remove two zeros.)

Multiplying by multiples of 10, 100 and 1 000

Suppose you want to multiply a number by 40. This calculation becomes easier if you first break 40 (which is a multiple of 10) into two parts, $40 = 4 \times 10$. First multiply the given number by 4. Then, multiply the answer by 10.

Example

Calculate 52×20 by breaking down one of the numbers.

Solution

$$\begin{aligned} 52 \times 20 &= 52 \times 2 \times 10 \\ &= 104 \times 10 \\ &= 1\,040 \end{aligned}$$

Example

Calculate 31×500 by breaking down one of the numbers.

Solution

$$\begin{aligned} 31 \times 500 &= 31 \times 5 \times 100 \\ &= 155 \times 100 \\ &= 15\,500 \end{aligned}$$

You can also use different combinations of multiplication and division to simplify calculations. You will explore this in the next section.

Multiplication by 5, 25 and 125

Remember, $5 = 10 \div 2$. So to multiply a number by 5:

- multiply the number by 10
- divide the answer by 2.

Example

Calculate 12×5 .

Solution

$$\begin{aligned} 12 \times 5 &= 12 \times 10 \div 2 \\ &= 120 \div 2 \\ &= 60 \end{aligned}$$

In a similar way, to multiply a number by 25, first multiply the number by 100, then divide the answer by 4. We can do this because $25 = 100 \div 4$.

Example

Calculate 44×25 .

Solution

$$\begin{aligned} 44 \times 25 &= 44 \times 100 \div 4 \\ &= 4\,400 \div 4 \\ &= 1\,100 \end{aligned}$$

To multiply a number by 125: First multiply the number by 1 000, then divide the answer by 8. Remember, $125 = 1\,000 \div 8$.

Example

Calculate 16×125 .

Solution

$$\begin{aligned} 16 \times 125 &= 16 \times 1\,000 \div 8 \\ &= 16\,000 \div 8 \\ &= 2\,000 \end{aligned}$$



How can I
divide by 5, 25
and 125?
I must remember that
multiplication and
division are inverse
operations!

Doubling and halving techniques

We can use **doubling** to multiply two numbers.

Example

Calculate 18×35 . Use the doubling method.

Solution

$$\begin{aligned} 1 \times 35 &= 35 \\ 2 \times 35 &= 70 \\ 4 \times 35 &= 140 \\ 8 \times 35 &= 280 \\ 16 \times 35 &= 560 \end{aligned}$$

Now we know that $16 \times 35 = 560$. But we need to know the answer to 18×35 (we need another two 35s). So we need to add ($2 \times 35 = 70$) to 560:

$$560 + 70 = 630$$

Therefore:

$$18 \times 35 = 630$$

Building up and breaking down complex numbers

Do you remember how to break down numbers? For example, $15 = 5 \times 3$. Therefore:

$$\begin{aligned} 645 \div 15 &= 645 \div 5 \div 3 \quad (15 = 5 \times 3) \\ &= 129 \div 3 \\ &= 43 \end{aligned}$$

Exercise 1

- Use your 12×12 multiplication grid to answer the following questions:
 - Look at the last digit in the multiples of 5. What do you notice?
 - Study the multiples of 9. What happens to the first digit of each number? What happens to the last digit of each number?
 - Find the common multiples of 2 and 5.
 - Look at your 12×12 multiplication grid. Colour all the square numbers.
 - What other patterns do you see?
- Calculate. Do not use a calculator:

a) 11×12	b) 25×364	c) 7×70
d) 60×88	e) 9×10	f) 80×29
g) $3 \times 1\,000$	h) $35 \times 8\,000$	i) $10\,000 \times 28$
j) $97 \times 30\,000$	k) 328×125	l) $100 \times 75\,476$
m) 150×11	n) $84\,008 \times 25$	o) $50 \times 857\,042$
- Calculate. Do not use a calculator:

a) $90 \div 5$	b) $4\,000 \div 50$	c) $70\,000 \div 400$
d) $58\,000 \div 100$	e) $98\,500 \div 50$	f) $7\,200 \div 60$
g) $7\,500 \div 50$	h) $14\,600 \div 100$	i) $12\,500 \div 25$
j) $3\,800 \div 5$	k) $12\,000 \div 125$	l) $936\,000 \div 300$
m) $850\,000 \div 500$	n) $1\,200\,000 \div 25$	
o) $340\,070\,000 \div 10\,000$		
- Use the inverse operations to find the unknown number in each case:
 - A number is multiplied by 25. You get an answer of 2 100. Calculate the number.
 - You want a total of R70 000 in your bank account. How many times do you need to deposit R125?
 - When you divide a number by 7 000, you get an answer of 60. What is the number?
 - Mary divides her savings between her 12 friends. Each friend gets R150. How much money did Mary have in her saving account?
 - A shop owner buys 125 ovens of the same kind. He spends R30 000 in total. How much did each oven cost?
- Use the inverse operations to check your answers in Question 4.

Unit 2 Ordering and comparing whole numbers



Important words

ascending order	arranging numbers from smallest to largest
descending order	arranging numbers from largest to smallest
factors	numbers that can divide exactly into another number
sequence	an ordered list of numbers

Place value of numbers

The place value and the value of a digit in a number are different. Consider the number 4 328. There are four digits: 4, 3, 2 and 8. Each digit has a place value and a value. If we write this number in a place value table, it will look like this:

Millions M	Hundred Thousands HTh	Ten Thousands TTh	Thousands (Th)	Hundreds (H)	Tens (T)	Units (U)
1 000 000	100 000	10 000	1 000	100	10	1
			4	3	2	8

Example

Consider the number 4 328. Find the place value of each digit. Now find the value of each digit.

Solution

The place value of 4 is thousands (1 000) but the value of 4 is 4 000 ($4 \times 1\,000$).

The place value of 3 is hundreds (100) but the value of 3 is 300 (3×100).

The place value of 2 is tens (10) but the value of 2 is 20 (2×10).

The place value of 8 is units (1) and the value of 8 is 8 (8×1).

When you write a number in expanded notation, you write the number as the sum of the values of all the digits.

Example

Write 4 328 in expanded notation.

Solution

$$4\,328 = 4 \times 1\,000 + 3 \times 100 + 2 \times 10 + 8 \times 1 \text{ or}$$

$$4\,328 = 4\,000 + 300 + 20 + 8$$

When you multiply a number by 10, the number gets bigger. Move all the digits of the number **one** place to the left. Write a zero at the end of the number. When you multiply a number by 100, move the digits of the number two places to the left. Write **two** zeros at the end of the number, and so on.

Let's see how this works in a place value table.



The place value of every column is ten times the place value of the column on its immediate right-hand side. That means that 10 is 10×1 , 100 is 10×10 ; 1 000 is 10×100 , and so on!

Example

Calculate:

1. 364×10

2. 364×100

3. $364 \times 10\,000$

Solution

	M	HTh	TTh	(Th)	(H)	(T)	(U)
	1 000 000	100 000	10 000	1 000	100	10	1
					3	6	4
1.				3	6	4	0
2.			3	6	4	0	0
3.	3	6	4	0	0	0	0

Rounding off numbers

You round off numbers to simplify calculations and to help you make sense of numbers. For example, you can round off numbers to the nearest 5, 10, 100, 1 000, and so on.

Rounding off a number to the nearest 5

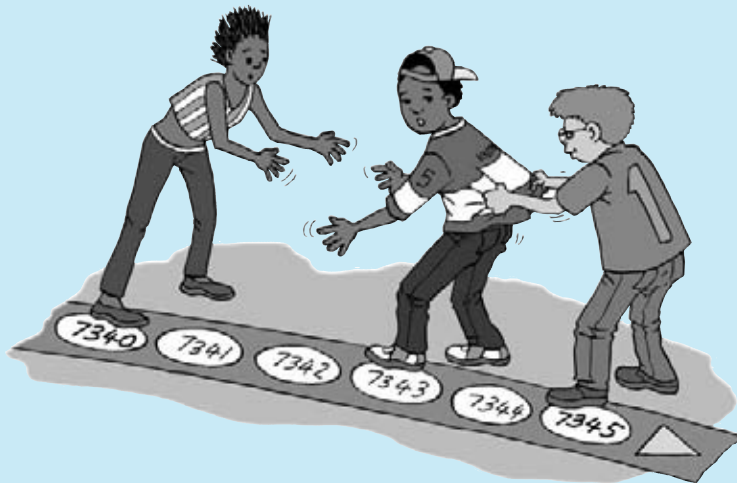
If the last digit of a number that must be rounded off is 3, 4, 5, 6 or 7, the last digit of the rounded number will become a five.

Example

Round off 7 343 to the nearest 5.

Solution

7 345



Look at the number line. 7 343 is closer to 7 345 than to 7 340.

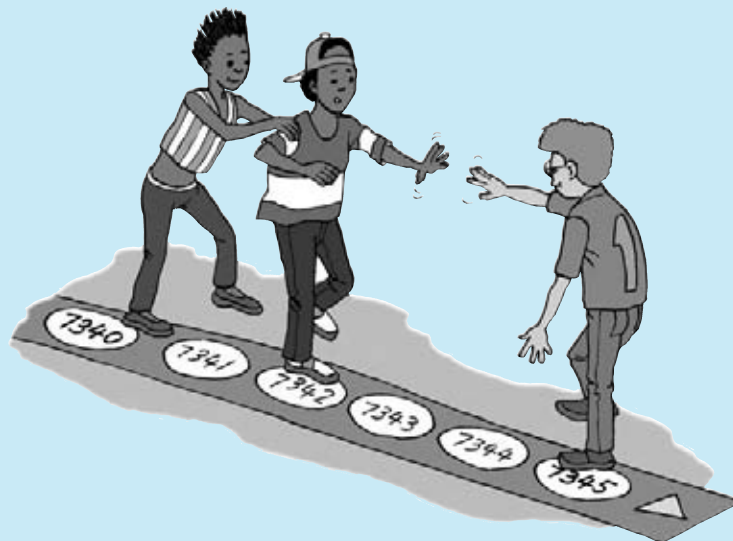
If the last digit of a number that must be rounded off is 0, 1 or 2, the last digit of the rounded number will become a zero.

Example

Round off 7 342 to the nearest 5.

Solution

7 340



Look at the number line. 7 342 is closer to 7 340 than to 7 345.

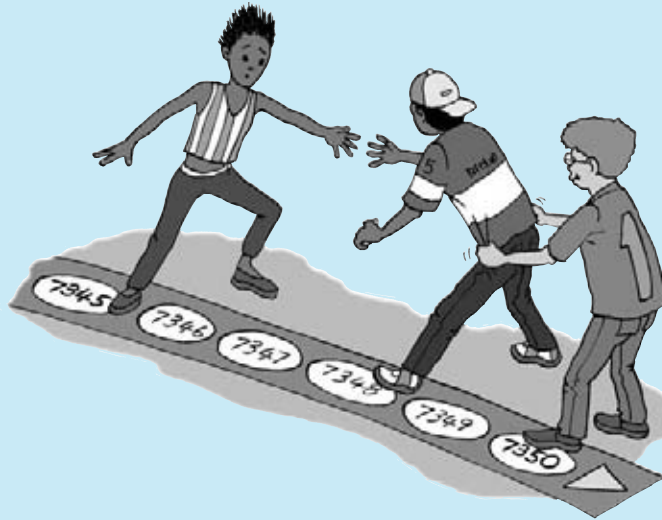
If the last digit of a number that must be rounded off is 8 or 9, the last digit of the rounded number will become a zero and we add one to the tens digit.

Example

Round off 7 348 to the nearest 5.

Solution

7 350

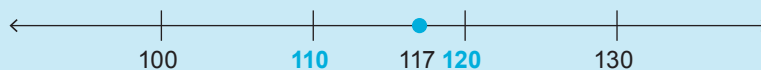


Look at the number line. 7 348 is nearer to 7 350 than to 7 345.

Rounding off a number to the nearest 10, 100, 1 000 and 10 000

Use a number line to round off numbers as you have done in the previous examples.

Example



Round off 117 to the nearest 10.

Solution

117 rounded off to the nearest 10 is 120. 117 is closer to 120 than to 110.

To round off a number to the nearest 10, work with the digit in the **units** column.

- If this digit is 5 or more, round the number up.
- If this digit is less than 5, round the number down.

Example

Round off 5 349 to the nearest 10.

Solution

5 349 rounded off to the nearest 10 is 5 350.

When you round off numbers to the nearest 100, use the digit in the **tens** column:

- If this digit is 5 or more, round the number up to the nearest 100.
- If this digit is less than 5, round the number down.

Notice the two zeros at the end.

Example

Round off 3 483 to the nearest 100.

Solution

3 483 rounded off to the nearest 100 is 3 500.

To round off a number to the nearest 1 000, use the digit in the **hundreds** column.

To round off a number to the nearest 10 000, use the digit in the **thousands** column.

Example

Round off 341 653 to the nearest 1 000. Then round off 341 653 to the nearest 10 000.

Solution

341 653 rounded off to the nearest 1 000 is 342 000.

341 653 rounded off to the nearest 10 000 is 340 000.

Rounding off and compensating

We can use rounding off to make the numbers in a calculation easier to work with. The answer we get is then an estimate. This means the answer is close to the correct answer, but not 100% correct, because we changed the original numbers slightly. To find the correct answer we can use rounding off and compensating as a calculation strategy. This means that we round off numbers and then compensate to make up for what will be lost or gained when rounding off.

Example

Use rounding off and compensating to simplify $58 + 29$.

Solution

$58 + 29 = (58 + 2) + (29 - 2)$	Round off 58 to 60 by adding 2. Then subtract 2 from the other number to compensate for this change.
$= 60 + 27$	
$= 87$	

Example

Use rounding off and compensating to simplify 5×88 .

Solution

$$\begin{aligned} 5 \times 88 &= (5 \times 90) - (5 \times 2) \\ &= 450 - 10 \\ &= 440 \end{aligned}$$

Round off 88 to 90. Then compensate for what was gained.

Check that your mental and written calculations are correct.

Judge how reasonable your answers are.

To judge how reasonable an answer is, use estimation by rounding off, doubling or halving. To check an answer, use a calculator, inverse operations or rounding off and compensating.



The division by 2 compensated for the earlier extra multiplication by 2. I sometimes use compensation without even thinking about it.

Example

Your calculation shows that the distance between your classroom and the principal's office is 3 000 metres. Is this reasonable?

Solution

No, because 3 000 metres is 3 kilometres. That is too far!

Exercise 2

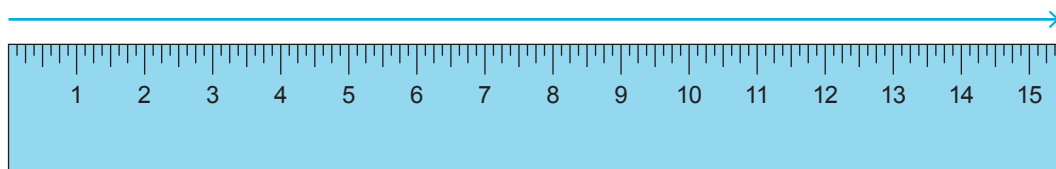
- Use rounding off and compensating of numbers to calculate the following:
 - $324 + 68$
 - $257 + 147$
 - $94 - 51$
 - $187 - 78$
 - 95×120
 - 52×40
- Solve the following problems:
 - Daniel bought a bicycle that was marked down from R1 595 to R1 385.
 - Estimate the amount he saved.
 - Is your answer realistic? Explain.
 - Calculate the actual amount he saved.
 - Use any method to check that your answer is correct.
 - Patience needs new curtain material for her house. Each drop is 233 cm long. Patience needs 18 drops.
 - Estimate the total length of material Patience needs to buy.
 - Is your answer realistic? Explain.

- iii) Calculate the actual length of material she needs to buy.
- iv) Check that your answer is correct.



Comparing numbers

You can **compare the sizes of numbers** in different ways. For example, you can present the numbers on a number line. Here, the numbers increase as we move from left to right on the number line.



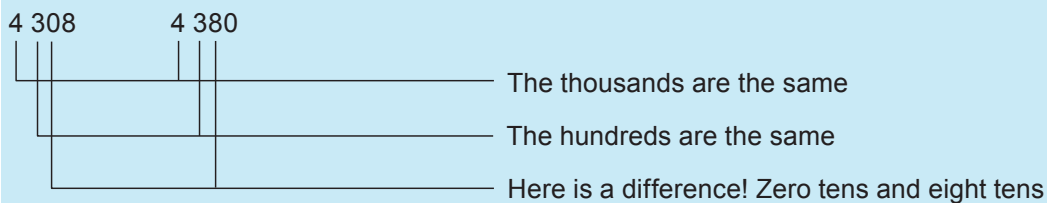
You can also compare the sizes of numbers by ordering or grouping all the numbers with the same number of digits. Then compare those numbers one digit at a time.

Example

Arrange the numbers 4 308, 438 and 4 380 in ascending order.

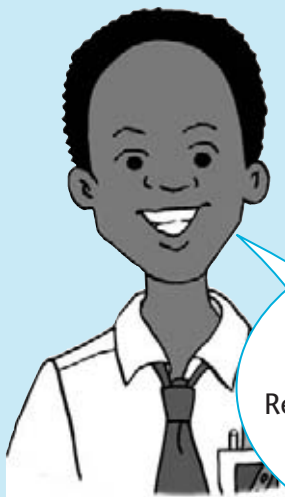
Solution

438 is a 3-digit number. 438 is smaller than the other two 4-digit numbers. Both 4 308 and 4 380 are 4-digit numbers. Compare the digits of these two numbers from left to right:



4 308 has zero tens while 4 380 has eight tens. So 4 308 is smaller than 4 380.

The numbers in ascending order are: 438; 4 308; 4 380.

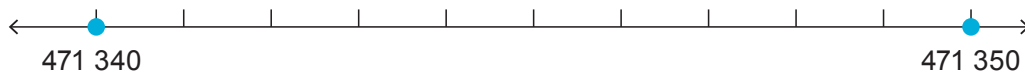


Learning to order and compare numbers helps you to see the sizes of numbers. Remember to estimate answers, understand calculations and check your answers.

Exercise 3

- Write down the place value of each bold digit:
a) 1 **2** 34 b) 1 **2** 34 c) 1 **2** 34 d) 1 **2** 34
- Write down the value of each bold digit:
a) 15 **2** 38 b) 92 **8** 57 c) 128 **4** 77 d) **4** 26 715
e) 898 **3** 23 f) 592 **8** 68 g) 4 **6** 27 154 h) 23 **4** 56
- Draw your own place value table. Represent these numbers in the table:
a) 6 tens
b) 2 hundreds + 4 tens + 2 units
c) 5 thousands + 8 hundreds + 3 tens + 6 units
d) 7 ten thousands + 5 hundreds + 4 units
e) 3 millions + 4 hundred thousands + 9 hundreds

4. Write in expanded notation:
- a) 3 056 b) 4 895 c) 11 285
d) 125 898 e) 3 262 554 f) 2 006 107
5. Write the numbers in words:
- a) 197 653 b) 60 056 c) 2 003 508
6. For each of the following numbers, do the following:
- i) Make the largest number possible. Use all the digits of the number.
ii) Make the smallest number possible. Use all the digits of the number.
- a) 1 259 b) 3 284 c) 10 986 d) 2 376 590
7. a) Find the missing numbers:
- i) 2 821; 2 828; 2 835; □; 2 849
ii) 328; 331; 336; 343; 352; □; 376
iii) 9; 27; 81; □; 729
- b) Look at the number lines. Find the number halfway between each indicated number:
- i)



ii)



8. Which number is larger?
- a) Eight hundred and five or eight hundred and fifty?
b) Three thousand four hundred and ten or three thousand one hundred and four?
c) One hundred thousand one hundred or one hundred thousand and eighty?
d) Five million eight hundred and sixty thousand or five million six hundred and eighty thousand?
9. Fill in $>$, $<$ or $=$ to make the number sentences true:
- a) $5\,826 \square 5 \times 1\,000 + 8 \times 100 + 2 \times 10 + 6$
b) $9 \times 1 + 49 + 1 + 300 + 450 + 3 \times 100 \square 3\,859$
c) $6\,928 \square \frac{1}{2}$ of 13 860
d) $22\,893 \square 22 \text{ thousands} + 8 \text{ hundreds} + 3 \text{ units} + 45 \times 2$
e) $4\,000\,000 + 567 \times 10\,000 + 2 \times 100 \square 4\,567\,200$
10. Arrange the following numbers in:
- a) Ascending order: 45 653; 32 654; 45 632; 23 456
b) Descending order: 4 300; 6 241; 510; 62 410; 43 000; 200; 50 101
11. a) Consider the list of numbers: 40, 41, 42, 43, 44, 45, 46, 47, 48, 49.
- i) Write down the prime numbers.
ii) Write down the composite numbers.
- b) Write down the prime numbers between 90 and 100.
c) Write down all the composite numbers from 75 to 80.

12. Solve:

- a) There are 55 348 people who need medical aid in Gauteng.
Round off the number of people to the nearest ten thousand.

Key
1 figure = 1 000 people



- b) There are 5 894 cities around the world that have HIV programmes.
Round off the number of cities to the nearest thousand.
- c) A charity spends R547 123 on meals for people in need in the Eastern Cape.
Round off this amount to the nearest hundred thousand.
- d) A plane travelled 69 899 km. Round off the kilometres to the nearest five.

Unit 3 Properties of whole numbers



Important words

properties of numbers specific attributes or laws that apply to numbers

The three number properties (or rules) are the **commutative**, **associative** and **distributive** properties. Here are three examples that show the properties.

1. $230 + 70 = 70 + 230$ and $25 \times 4 = 4 \times 25$ (Commutative property)

2. $230 + 70 + 50 = (230 + 70) + 50$
 $= 230 + (70 + 50)$ (Associative property)

3. $5 \times 23 = 5 \times (20 + 3)$ (Distributive property)
 $= (5 \times 20) + (5 \times 3)$
 $= 100 + 15$
 $= 115$

Commutative property for addition and multiplication

The commutative property allows you to change the *order* of the numbers you add or multiply. You will still get the same answer.

Example $917 + 73 = 990$ and $73 + 917 = 990$ So: $917 + 73 = 73 + 917$	Example $3\,280 \times 3 = 9\,840$ and $3 \times 3\,280 = 9\,840$ So: $3\,280 \times 3 = 3 \times 3\,280$
--	--

Work with a friend to answer these questions:

- Does the commutative property apply to more than two numbers? Explain.
- Can you apply the commutative property to subtraction and division? Support your answers with examples.

Associative property for addition and multiplication

The associative property (or rule) allows you to *group* numbers when you add or multiply. You still get the same answer.

Example Calculate $350 + 632 + 18$. Solution $(350 + 632) + 18 = 982 + 18 = 1\,000$ $350 + (632 + 18) = 350 + 650 = 1\,000$ So: $(350 + 632) + 18 = 350 + (632 + 18)$	Example Calculate $60 \times 4 \times 3$. Solution $(60 \times 4) \times 3 = 240 \times 3 = 720$ or $60 \times (4 \times 3) = 60 \times 12 = 720$ So: $(60 \times 4) \times 3 = 60 \times (4 \times 3)$
---	---

Which grouping made the addition easier?

Work with a friend to answer the question:
Can you apply the associative property to subtraction and division?
Support your answers with examples.

Distributive property for multiplication over addition and subtraction

The distributive property (or rule) allows you to *redistribute* numbers. You still get the same answer.

<p>Example</p> $15 \times 32 = 15 \times (30 + 2)$ $= (15 \times 30) + (15 \times 2)$ $= 450 + 30$ $= 480$	<p>Example</p> $23 \times 18 = 23 \times (20 - 2)$ $= (23 \times 20) - (23 \times 2)$ $= 460 - 46$ $= 414$
---	---

Identity properties

Addition

The number **0** is the identity element for addition. The identity element for addition means that when you add **0** to any number, the number keeps its identity or value. For example, $430 + 0 = 430$.

Multiplication

The number **1** is the identity element for multiplication. This means that when you multiply any number by **1**, the number keeps its identity or value. For example, $3\,428 \times 1 = 3\,428$.

Exercise 4

- Different number properties are used in the following calculations. Write down the name of each property used:
 - $4\,200 + 543 + 800 = 4\,200 + 800 + 543$
 - $874 \times 3 = 3 \times 874$
 - $750 \times 102 = 750 \times (100 + 2)$
 - $(2 \times 31\,975) \times 5 = 31\,975 \times (2 \times 5)$
 - $89 \times 9 = 9 \times (90 - 1)$
 - $86\,737 + 4\,386 = 4\,386 + 86\,737$
- Use the commutative, associative or distributive property of numbers to make each number sentence true.
 - $1\,416\,219 + 2\,508\,000 = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$
 - $45 \times 8 = (40 + \underline{\hspace{1cm}}) \times 8$
 - $(5 \times 4) \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \times (5 \times 75)$
 - $7\,665 + \underline{\hspace{2cm}} = 1\,234 + \underline{\hspace{2cm}}$
 - $57 \times 5 = 5 \times (60 - \underline{\hspace{1cm}})$

- f) $(8\,719 + 719) + 2 = \underline{\hspace{2cm}}$
 g) $\underline{\hspace{2cm}} \times 1\,389 = \underline{\hspace{2cm}} \times 218$
 h) $862 + (941 + 87) = (862 + \underline{\hspace{1cm}}) + \underline{\hspace{1cm}}$
 i) $2 \times 719 = \underline{\hspace{1cm}} \times (\underline{\hspace{1cm}} - \underline{\hspace{1cm}})$
 j) $6\,402\,000 + 324 = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$
 k) $24 \times \underline{\hspace{1cm}} = 250 \times (20 + \underline{\hspace{1cm}})$
 l) $(154 \times 2) \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \times (2 \times 3)$



Did you know?

$1 \times 8 + 1 = 9$
 $12 \times 8 + 2 = 98$
 $123 \times 8 + 3 = 987$
 $1\,234 \times 8 + 4 = 9\,876$
 $12\,345 \times 8 + 5 = 98\,765$
 $123\,456 \times 8 + 6 = 987\,654$
 $1\,234\,567 \times 8 + 7 = 9\,876\,543$
 $12\,345\,678 \times 8 + 8 = 98\,765\,432$
 $123\,456\,789 \times 8 + 9 = 987\,654\,321$

3. Simplify the following calculations. Use any of the three number properties. Show all your calculations. Write down the property you used.
- a) 53×5 b) $12\,300 \times 2 \times 3$ c) $40 + 212 + 60$
 d) 25×19 e) $\frac{1}{2} \times 420 \times 2$ f) 330×28
4. Fill in the missing number. Use the identity properties.
- a) $50\,813 \times \underline{\hspace{1cm}} = 50\,813$
 b) $4\,850 + 0 = \underline{\hspace{1cm}}$
 c) $\underline{\hspace{1cm}} + 10\,380 = 10\,380$
 d) $1 \times \underline{\hspace{1cm}} = 102\,000$

Unit 4 Calculations with whole numbers



Important words

reflect

think about what you have already learnt

Addition and subtraction

When we add or subtract numbers, we add or subtract the digits with the same place values. When we add and the answer is 10 or more, we carry it over to the next column on the left. When we subtract we may sometimes need to borrow from the column on the left.

Example

Calculate:

- a) $129 + 83$ b) $892 - 77$

Solution

a) $129 + 83$

	H	T	U
	¹ 1	¹ 2	9
+		8	3
	2	1	2

b) $892 - 77$

	H	T	U
	8	⁸ 9	¹ 2
-		7	7
	8	1	5

Multiplication

In the previous units, you have learnt number facts that you can use to simplify calculations with whole numbers and to develop number sense.

Multiplying 2×6 means: 2 times 6 or 2 groups of 6 or 2 sixes. You can write this as 2×6 or $(2)(6)$ or $2 \cdot 6$. We can think of multiplication as:

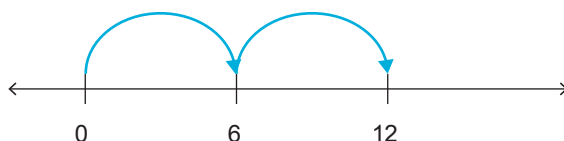
- repeated addition because $2 \times 6 = 6 + 6$

x x x x x x	x x x x x x
-------------	-------------

- an array of rows and columns in a rectangular form: 2 rows and 6 columns

x	x	x	x	x	x
x	x	x	x	x	x

- adding in intervals of 6 on a number line: We start at 0 and jump in intervals of 6. Two such jumps will end at 12. We say we add in intervals of 6.



Remember that $6 \times 0 = 0$.

Multiplying in columns

325×72

	TTh	Th	H	T	U
			3	2	5
			\times	7	2
			6	5	0
+	2	2	7	5	0
	2	3	4	0	0

$\therefore 325 \times 72 = 23\,400$

Division

Division can be represented as: $12 \div 6$ or $\frac{12}{6}$ or $6 \overline{)12}$.

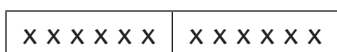
We can think of division as:

- dividing 12 into 6 groups:

x x	x x	x x	x x	x x	x x
-----	-----	-----	-----	-----	-----

Here, the size of each group is 2.
This is also called the sharing idea.

- dividing 12 into groups of size 6:



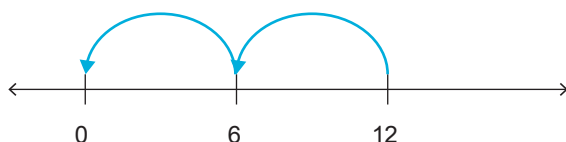
There are 2 groups of size 6.

This is also called the grouping idea.

- repeated subtraction in intervals of 6 on a number line:

Start at 12 and calculate how many times you can subtract 6 on a number line until you get to 0. You can subtract 6 two times.

This is also a form of grouping.



Remember that $0 \div 6 = 0$ but $6 \div 0$ is meaningless.

Long division

- Let us revise the procedure of long division (also called the division algorithm):

$\begin{array}{r} 48 \\ 12 \overline{) 576} \end{array}$ (12 cannot go into 5, but 12 can go into 57, 4 times. Write the 4 above the 7 of 57.)

(–) $\begin{array}{r} 48 \\ 96 \end{array}$ (The 4 in the answer is multiplied by 12 to find 48.)

96 (Subtract: $576 - 480$, to find 96.)

(–) $\begin{array}{r} 96 \\ 96 \end{array}$ (12 goes into 96, 8 times. Write the 8 above the 6 of 576. Say $8 \times 12 = 96$.)

0 (Subtract: $96 - 96$, to find 0.)

Therefore: $\frac{576}{12} = 48$. There is no remainder.

Rules of division

Divisible by	When	Example	Divisible by	When	Example
2	Last digit is even	86; 764; 2 348	7	When double the last digit subtracted from remaining digits is divisible by 7	357: $35 - 14 = 21$ which is divisible by 7
3	Sum of digits is divisible by 3	8 751 because $8 + 7 + 5 + 1 = 21$	8	Last 3 digits are divisible by 8	8 816 $(816 \div 8 = 102)$

4	Last 2 digits are divisible by 4	32 532 ($32 \div 4 = 8$)	9	Sum of digits is divisible by 9	32 688 because $3 + 2 + 6 + 8 + 8 = 27$
5	Last digit is 0 or 5	35; 764 060	10	Last digit is 0	30; 600; 78 870
6	Divisible by 2 and 3	76 446	11	First, add the alternate digits (starting at the units). Then add the remaining digits. Subtract the second number from the first number. If the answer is 0 or a multiple of 11, then the number is divisible by 11	22 704: $(2 + 7 + 4) - (2 + 0) = 13 - 2 = 11$

Estimation

Estimation lets you work out two values between which the answer to a calculation will fall.

Example

Between which two values will 23×76 lie?

Solution

The answer must be greater than: $20 \times 70 = 1\,400$ (both numbers are rounded down).

But less than: $30 \times 80 = 2\,400$ (both numbers are rounded up).

Estimation means to make a rough calculation or guess of the result of a calculation. To do this, we use rounding off.



Example

Estimate 23×76 .

Solution

$$\begin{aligned} 23 \times 76 &\approx 25 \times 75 && \text{(Round off both numbers to the nearest 5.)} \\ &= 25 \times 25 \times 3 && \text{(Break down the number 75 to simplify the} \\ &= 625 \times 3 && \text{multiplication.)} \\ &= 1\,875 \end{aligned}$$

or

$$\begin{aligned} 23 \times 76 &\approx 20 \times 80 && \text{(Round off both numbers to the nearest 10.)} \\ &= 1\,600 \end{aligned}$$

Use your calculator and find the actual answer to 23×76 .

Which estimation (rounding off to the nearest 5 or rounding off to the nearest 10) gave you a better estimation?



Did you know?

There is an order in which you need to do operations. You can use the acronym **BODMAS** to help you remember the order.

B	O	D	M	A	S
↓	↓	↓	↓	↓	↓
Brackets	Orders	Division	Multiplication	Addition	Subtraction

Remember: If there are no brackets in a calculation, you first do any multiplication and division operations. Note that you do these from left to right. The fact that division appears before multiplication in BODMAS does not matter. Then, you do any addition and subtraction, also from left to right.

Orders refers to exponents and roots. You will learn all about exponents in Topic 2.

Example

$$36 \div 3 \times 4 = 12 \times 4 = 48 \quad \checkmark$$

$$36 \div 3 \times 4 = 36 \div 12 = 3 \quad \times$$

$$6 - 5 + 1 = 1 + 1 = 2 \quad \checkmark$$

$$6 - 5 + 1 = 6 - 6 = 0 \quad \times$$

Exercise 5

1. Use your number sense to make the following calculations easier. Show all the steps you used:

a) $179\,000 + 27\,000 + 821\,000$

c) $7\,000 \div 125$

e) $7\,500 \div 25$

g) $2\,484 \div 9$

i) $4\,840 \times 125$

k) $3\,240 \times 25$

b) $8\,540 - 240 - 40$

d) 220×34

f) $2\,982 + 818 + 94 + 106$

h) $1\,250 \times 49$

j) 250×150

l) $7\,450 \div 5$

2. i) First estimate the answer of the calculation. Then use your calculator to find the exact answer. Draw a two-column table in your workbook. The first column is for the 'Estimated answer'. The second column is for the 'Calculator answer'.
- ii) Compare the estimated and calculated answer in each case. Did your estimation give you an approximate answer?
- a) 3×54 b) 6×129 c) 10×132
d) 8×66 e) 78×5 f) 53×20
g) 66×72 h) 334×97 i) 715×315
3. a) A total of 3 642 people attended a school's soccer tournament. Each ticket cost R20. How much did the school earn from the ticket sales?



- b) Elsie earns nine times more money than Busi. Busi earns R2 300 a month. How much does Elsie earn?
- c) Each box of chips contains 180 packets of chips. If you have 10 000 packets of chips, how many boxes can you fill?
4. Calculate the following:
- i) using long division
ii) by breaking down numbers.
- a) $5\,832 \div 8$ b) $2\,412 \div 18$ c) $72\,000 \div 90$
5. The following is given: $32 \times 104 = 3\,328$. Use the following techniques to check that this calculation is correct:
- a) Estimate two numbers between which the answer will lie
b) Doubling of numbers
c) The distributive property
d) Inverse operations
e) Compensation of numbers
f) A calculator.

Unit 5 Multiples and factors

Multiples and factors

Multiples of a given number are all the numbers into which the given number will divide without a remainder.

Example

The multiples of 6 are given as: $M_6 = 6; 12; 18; 24; \dots$

Factors are all the numbers that can divide exactly into another number.

Example

The factors of 6 are given as: $F_6 = 1; 2; 3$ and 6

A **prime number** is a number that can only be divided by 1 and itself.

We can also say a prime number has only two factors, namely 1 and itself. Numbers that have more than two factors are called **composite numbers**. Note that the number 1 is neither a prime number nor a composite number as it has only one factor, namely 1.

Example

Name the first six prime numbers.

Now name the first six composite numbers.

Solution

The first six prime numbers are: 2; 3; 5; 7; 11; 13; ...

The first six composite numbers are: 4; 6; 8; 9; 10; 12; ...

What do you notice when you combine the prime numbers, the composite numbers and the number 1?

Prime factors are factors of a number that are prime numbers. You can use a factor tree to break down a composite number into its prime factors.

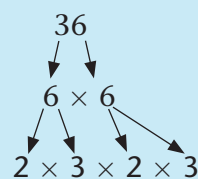
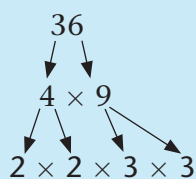
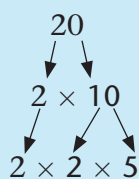
Example

Find the prime factors of 20 and 36.

Solution

Factors:

Prime factors:



Many numbers have more than one factor tree. In the previous example, we had two factor trees for 36. Try to draw two other factor trees for 36!

Example

Find the prime factors of 20.

Solution

You can also find the prime factors of a number by dividing the number by the prime numbers, starting with 2. Let's do it for the number 20.

2	20	2 goes into 20, 10 times
2	10	2 goes into 10, 5 times
5	5	Neither 2 nor the next prime number 3 can go into 5. The next prime number 5 goes into 5, once
	1	

Therefore, the prime factors for 20 are $2 \times 2 \times 5$.

Use the second method to find the prime factors of 36.

The highest common factor (HCF) and lowest common multiple (LCM)

You can use prime factors to find the highest common factor (HCF) of two or more numbers.

You use the HCF when you do calculations with fractions.

Example

Find the HCF of 12 and 32.

Solution

Prime factors of 12 = $2 \times 2 \times 3$	Circle the common factors for both numbers. To find the HCF you multiply these common factors. Do this by writing the common factor once .
Prime factors of 32 = $2 \times 2 \times 2 \times 2 \times 2$	
\therefore The HCF for 12 and 32 is $2 \times 2 = 4$	

You can use multiples or prime factors to find the **lowest common multiple** (LCM) of numbers.

Example

Find the LCM of 6 and 8 using:

1. multiples
2. prime factors.

Solution

1. $M_6 = 6; 12; 18; 24; 30; \dots$

$M_8 = 8; 16; 24; 32; 40; \dots$

The LCM of 6 and 8 is the lowest multiple that appears in both sets.

The LCM is 24.

2. Prime factors of $6 = 2 \times 3$

Prime factors of $8 = 2 \times 2 \times 2$

Count the number of times each prime factor appears in both factorisations. For each prime number, take the largest of those counts. Write down the prime number as many times as it appeared. You calculate the LCM by multiplying together all the prime factors you have written down.

The LCM of 6 and 8 is $2 \times 2 \times 2 \times 3 = 24$.

Exercise 6

- Draw a factor tree for the following numbers. Determine the prime factors.
 - 25
 - 18
 - 21
 - 35
 - 45
 - 12
- Find the HCF for the two given numbers:
 - 18 and 54
 - 8 and 180
 - 35 and 155
 - 9 and 135
 - 32 and 80
 - 63 and 28
 - 35 and 140
 - 150 and 300
 - 624 and 960
- Use multiples of numbers to find the LCM for each set of numbers:
 - 6 and 9
 - 6 and 5
 - 2; 9 and 18
 - 12 and 48
 - 23 and 69
 - 65; 260 and 520
- Use prime factors to find both the HCF and LCM for each set of numbers:
 - 240 and 924
 - 36, 90 and 72
 - 25, 125 and 750

Unit 6 Solving problems



Important words

expenses

money paid for food and services such as electricity

income

money you earn from working or selling goods

percentage

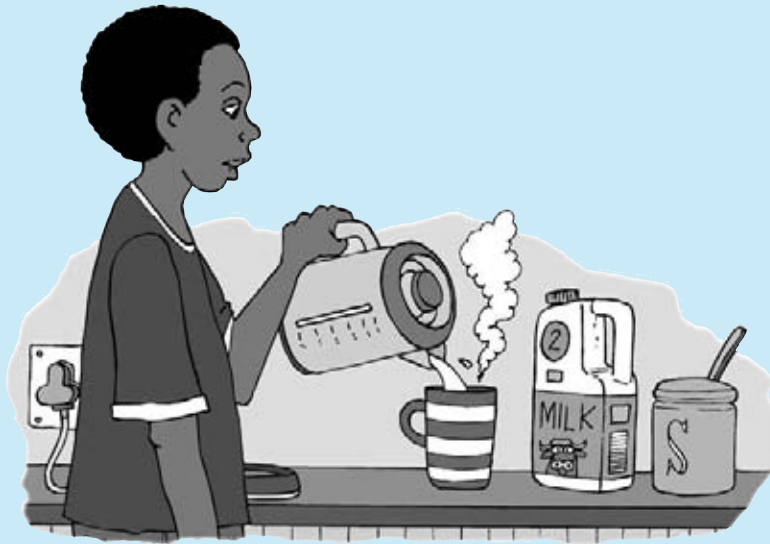
a number out of a hundred, for example, $\frac{25}{100} = 25\%$

Ratio

We use **ratio** when we compare two or more quantities of the same kind. A ratio can be expressed in two ways. Let's work through an example.

Example

Andile likes his coffee made in a certain way. He likes 80 ml of milk and 160 ml of coffee. Andile's cup holds 240 ml. The ratio of milk to coffee is 80 : 160. To write this ratio in its simplest form, you divide both sides by 80. The simplified ratio is 1 : 2. You can also write this ratio in fraction form: $\frac{1}{2}$.



Andile is very careful when he makes his coffee! The order of the numbers in the ratio is important. If Andile swaps the ratio around, he will pour twice as much milk as he wants. He will also pour half of the coffee that he wants.

Andile's friend offers him coffee and pours 200 ml of coffee in a large mug. How many millilitres of milk should she pour in Andile's mug if she keeps to the same ratio as in the previous example? What is the total amount of coffee-milk mix in Andile's mug?

For 200 ml of coffee, she must add 100 ml of milk to obtain 300 ml of the mix.

You can also use ratios in calculations when the total is known. Remember, you must work with the same units of measurement. In this case, both the milk and coffee are measured in millilitres. For example, if you have 450 ml of a coffee-milk mix in a large mug, the ratio of milk to coffee is still 1 : 2. You can calculate the total amount of milk and coffee that was poured into your mug as follows:

Milk: $\frac{1}{3}$ of 450 ml = 150 ml

Coffee: $\frac{2}{3}$ of 450 ml = 300 ml

Check your answers:

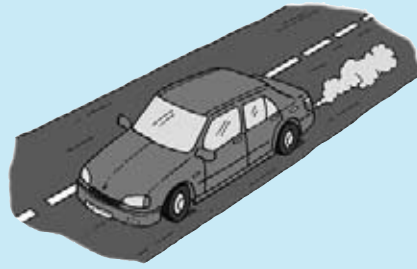
150 ml of milk + 300 ml of coffee = 450 ml
coffee : milk mix.

Rate

Rate is a kind of ratio where you compare two quantities of different kinds.

Example

A nurse in the hospital is paid a certain rate for her work, for example, R100 per hour (R100/h).



A car's speed is expressed in kilometres per hour (km/h).

Each country has its own money system.

For example, in South Africa, we use the rand and in the United States, they use the dollar. An exchange rate lets us compare two currencies, even though they use different systems.

The following are examples of exchange rates.

In each case, we express how much 1 unit of foreign currency costs in rand. For example,

1 United States dollar = R7,77

1 British pound = R12,61

1 Euro = R10,31

1 Canadian dollar = R7,86



Study the rates of the countries. Which country will be the most expensive country to visit by a South African tourist? Find out what the current exchange rates are for these countries.

Exercise 7

1. Solve the following problems.

They are based on ratio.

- In a small hospital, there are four times as many adult patients as there are child patients. What is the ratio of children to adult patients?
- Simon went fishing on Saturday and Sunday. On Saturday he caught 10 trout and 4 catfish. On Sunday he caught 16 trout and 8 catfish. On which day did Simon catch a higher ratio of trout to catfish?
- Mother uses 250 g broccoli, 750 g cauliflower and 50 g carrots to prepare a vegetable dish.



- i) Calculate the total amount of vegetables needed for the dish.
 - ii) Find the ratio of the vegetables.
 - iii) She wants to prepare this dish for a family gathering and decides to use 3 150 g of vegetables in total. How many grams of carrots does she need to buy?
2. Solve the following problems. They are based on rate.
- a) Nurses are paid R125 per hour (R125/h). Calculate how much Nurse Betty must get paid if she works a 12-hour shift.
 - b) A trailer rental company gives you two options. You can hire a trailer for R55 per hour or R472 per eight hours. Which is the cheaper option?
 - c) A butcher charges R168 for 12 kg of meat. How much does she charge per kilogram?
 - d) There are 20 flowers in a bouquet. A florist needs to make 40 bouquets. How many flowers does the florist need?
 - e) A man travels from Pretoria to Ballito, which is north of Durban. This is a distance of 680 km. He leaves his house at 06:00 and does not stop anywhere. He arrives in Ballito at 13:00. What is his average speed for this trip?
 - f) On a highway, Susan travels at a speed of 125 km/h between two rest stops. It took her one hour and 30 minutes to travel this distance. How far apart are the two rest stops?
 - g) An object moves at a speed of 15 metres per second (m/s). It covers a distance of 165 metres. How many seconds did it take the object to cover this distance?
 - h) You want to visit London and stay at a guest house for five days. The exchange rate is R12,61/pound, and the daily rate at the guest house is 115 pounds. Calculate the cost of your boarding (in rands).



Learn more about finance

Finance is about earning and managing money. It is part of our daily lives. It includes aspects such as profit and loss, budgets, accounts, loans and earning interest when you save money. One day you will need to manage your own money.

You make a **profit** when you sell a product for more than what you paid for it.

Example

Cedric buys a bicycle for R499. He decides to sell it for R51 more than he paid for it. What percentage profit did he make?

Solution

$$\begin{aligned} \text{R51 profit on R499} &= \frac{\text{R51}}{\text{R499}} \times \frac{100\%}{1} \\ &= 10,22\% \text{ profit} \end{aligned}$$

You make a **loss** when you sell a product for less than what you paid for it.

Example

Tshedi buys a T-shirt for R75. Her sister really likes the T-shirt, so Tshedi sells it to her for R50.

- a) What is Tshedi's loss in rands?
- b) What is Tshedi's percentage loss?

Solution

- a) $\text{Loss} = R75 - R50 = R25$
- b) $\text{Percentage loss} = \frac{R25}{R75} \times \frac{100\%}{1}$
 $= 33,33\% \text{ loss}$

Sometimes a shop will offer a **discount** on certain products. A discount is usually expressed as a percentage. For example, the shop might be running a 50% off sale on all winter clothes.

Example

A shop sells ladies jeans at a discount of 25%. Lizzy wants to buy a pair of jeans marked at R145. She only has a R100 note in her purse.

- a) What is the price of the pair of jeans after the discount?
- b) Will she be able to buy the jeans? If she can buy the jeans, how much change will she get? If she cannot buy the jeans, how much more money does she need before she can do so?



Solution

- a) $\frac{25}{100} \times \frac{R145}{1} = R36,25$

$$\begin{aligned}\text{The discounted price for the pair of jeans} &= R145 - R36,25 \\ &= R108,75\end{aligned}$$

- b) No. Lizzie needs another R8,75 to buy the pair of jeans.

Many shops allow you to open an **account**. An account lets you pay for something over time, rather than paying the whole amount at once. Each month, the shop will send you a statement that tells you how much you need to pay that month, how much you still owe and how much credit you still have available to spend.

The next example shows a statement that has been sent to Mrs L. Simelane.

Example

Mrs L Simelane 234 Fourth Avenue Pretoria 0182			CREDIT AVAILABLE 3 881,00		Pay your total due every month to have credit when you need it
			Date: 09/04/12		Mrs L Simelane
			Account number: 7000100100075415017		700200300
			STATEMENT		Instalment: 415,00
					Overdue: 0,00
					Total Due: 0,00
					Amount enclosed:
DATE	REF NO:	DETAILS	AMOUNT	BALANCE	Use the account number above as your reference number if you are paying electronically, and please ensure your bank links your payment to the following bank account: BFF BRANCH CODE 2614405 ACCOUNT NUMBER 5044224156 PLEASE NOTE: Even if you don't receive a statement every month, you must pay total due to avoid late payment charges
29/03/12		OPENING STATEMENT BALANCE		1 157,00	
		ATM PAYMENT	600,00	557,00	CREDIT LIMIT You now have: R4 563 You qualify for:
09/04/12		LIFESTYLE MEMBERSHIP FEE	25,00	532,00	For credit limit increases, account enquiries and address or telephone number changes call: 0860 133 445 (South Africa, Lesotho and Swaziland)
CLOSING STATEMENT BALANCE				532,00	Office hours: Mon–Fri: 08:00–17:00

A monthly **budget** is essentially a plan that helps you work out how much you expect to earn and spend in future. A budget is a planning tool to help you to make sure that you do not suddenly find that you are spending more than you are earning. A budget can also help you to plan for special events, such as a holiday. Let's look at an example of a budget.

Example of a monthly budget for a family

EXPENSE	RAND
INCOME	
Salary	7 500
Other	4 500
Total income	12 000
HOME EXPENSES	
Rent	2 000
Water & electricity	650
Phone	130
TV licence	20
Maintenance	100
Total home expenses	2 900
TRANSPORTATION	
Vehicle payment	1 500
Insurance	500
Fuel	600
Repairs	300
Total transport	2 900
HEALTH	
Medical aid	1 200
Medicine	800
Total health	2 000
DAILY LIVING	
Groceries	3 000
Clothing	200
Cleaning	150
Education	300
Pet care	100
Entertainment	400
Total daily living	4 150
TOTAL EXPENSES	11 950
TOTAL INCOME	12 000
NETT	50

A **loan** is a type of debt. The borrower initially receives an amount of money from, for example, a bank, and has to repay that money. In this case, the bank usually charges interest on the loan. For this reason, **interest** is known as the cost of borrowing money.

Example

John bought a car for R45 000. He paid a 10% deposit of R4 500. He borrowed the outstanding amount of R40 500 from his bank at a fixed rate of 10% per year. The bank agreed that John can repay them over two years. What will John's monthly payment be?



Solution

The bank calculated the interest as follows:

$$\begin{aligned} 10\% \text{ of R40 500} \times 2 \text{ years} &= \frac{10}{100} \times 40\,500 \times 2 \\ &= \text{R8 100} \end{aligned}$$

The bank added this interest to the amount John borrowed. The total amount John owed the bank was R48 600. John promised to repay the bank in two years (24 months). His monthly payment will be $\frac{\text{R48 600}}{24} = \text{R2 025}$ per month.

Exercise 8

(You may use your calculator in this exercise.)

1. Solve the following problems. They are based on profit and loss.
 - a) The initial price of a shirt is R124. It is sold on discount for R115. Calculate the percentage loss.

- b) Tumi buys a car for R65 000 and then sells it for R55 000.
Calculate his percentage loss.
 - c) Carol buys a house for R200 000 and then sells it for R300 000.
Calculate Carol's percentage profit.
 - d) Nathan buy 10 pairs of shoes at R50 a pair and then sells each pair for R65 a pair. Calculate his percentage profit.
 - e) Gogo decides to open a Spaza shop. She buys a number of items for R275. To advertise and rent a store costs a further R100. If Gogo sells her items for R375:
 - i) what is Gogo's profit or loss
 - ii) what should Gogo do to have a better sales plan?
2. Look at the example of a monthly budget on page 36. Set up a monthly budget for your family.
 3. Solve the following problems. They are based on accounts.
 - a) Use the following information and design your own invoice for a client. The labour to build the cabinet is R55. You buy the following items to build the cabinet:

30 × 20 mm bolts @ R7,99 each	30 × 4 mm nuts @ R5,45 each
2 × 2 m long pieces of plywood @ R45 each	5 m of material @ R12,65 a metre
2 × Superglue @ R17,20 a tube	1 × handle @ R9,99
2 m of PVC tube @ R15,75 a metre	1 × varnish @ R45 a tin

- b) Seshni works as a waitress at a restaurant. The customers at Table 17 order the following:



Write out the account for Table 17. Include a 10% tip for the waitress.

4. Solve the following problems. They are based on loans and interest.
- a) Calculate the interest on the following amounts for one year if the interest is set at 5% per year.
 - i) R2 421,00
 - ii) R5 000,00
 - iii) R156 000,21
 - b) A motorcycle costs R25 250 cash. Suresh would like to buy the motorcycle, but he needs to borrow the money from his parents. They charge him 10% interest per year.



- i) How much would Suresh pay back if he paid his parents back in one year?
 - ii) How much would Suresh pay back if he paid his parents back in two years?
 - iii) How much can Suresh save in 24 months if he paid cash for the motorcycle?
 - c) Mandy goes shopping and buys a few dresses, shoes and underwear. The total bill comes to R2 525,21. Mandy has to repay the bill over 12 months. Look at option i) and ii) and decide which would be the better deal.
 - i) For the first six months, Mandy makes interest-free payments. For the next 6 months, Mandy pays 20% interest on outstanding amounts.
 - ii) For the 12 months, Mandy pays 9,8% interest on the total amount due.

Topic

2

Exponents

In this topic you will learn to:

- compare and represent whole numbers in exponential form
- recognise and use the appropriate laws of operations using numbers involving exponents and square and cube roots
- perform calculations involving all four operations using numbers in exponential form
- solve problems in contexts involving numbers in exponential form.

**What you already know**

1. Represent $4 \times 4 \times 4 \times 4 \times 4$ in a shorter way.
2. Write 2 387 in expanded notation using powers of 10.

Unit 1 Introducing numbers in exponential form

**Important words**

expanded notation	a number written as the sum of each digit's value
prime numbers	numbers that are only divisible by 1 and itself (without a remainder)
whole numbers	0; 1; 2; 3; ...

More about numbers in exponential form

In the first topic, you learnt that multiplication is just repeated addition. For example, 3×5 is the same as $5 + 5 + 5$. In this case, multiplication is a shorter way of writing out a longer addition sum.

In the same way, the exponential form gives us a short way to write a long multiplication calculation. So for example, $2 \times 2 \times 2 = 2^3$ and $3 \times 3 \times 3 \times 3 \times 3 = 3^5$.

The exponential form is especially helpful when you work with repeating factors or when you work with large numbers.



Example	Solution
Write in exponential form.	
1. 10	1. $10 = 10^1$
2. 100	2. $100 = 10 \times 10 = 10^2$
3. 1 000	3. $1\,000 = 10 \times 10 \times 10 = 10^3$
4. 10 000	4. $10\,000 = 10 \times 10 \times 10 \times 10 = 10^4$
5. 100 000	5. $100\,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$
6. 1 000 000	6. $1\,000\,000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$

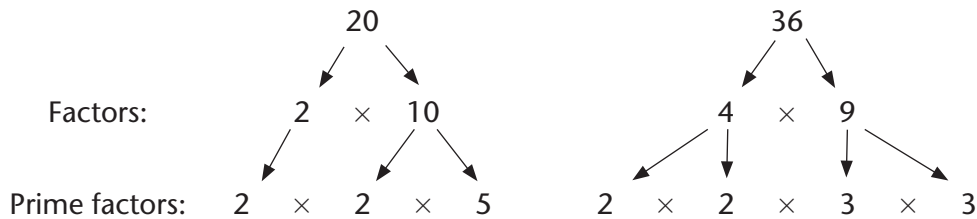
Example Represent $35 \times 35 \times 35 \times 35 \times 35 \times 35$ in exponential form. (Do not calculate the answer.)	Example What does 50^7 mean? (Do not calculate the answer.)
Solution $35 \times 35 \times 35 \times 35 \times 35 \times 35 = 35^6$	Solution $50^7 = 50 \times 50 \times 50 \times 50 \times 50 \times 50 \times 50$

Exercise 1

Represent each of the following in exponential form. Do not calculate the answer.

1. 5×5
2. $2 \times 2 \times 2 \times 2 \times 2$
3. $30 \times 30 \times 30 \times 30 \times 30$
4. $453 \times 453 \times 453 \times 453$

You can use the exponential form when you write the **prime factors** of numbers. In Topic 1 Unit 5 on page 28, you drew factor trees to find the prime factors of numbers. For example, the following factor trees show the prime factors of 20 and 36.



You can write the prime factors of 20 using exponential form:

$$20 = 2 \times 2 \times 5$$

$$= 2^2 \times 5^1$$

You can also write the prime factors of 36 using exponential form:

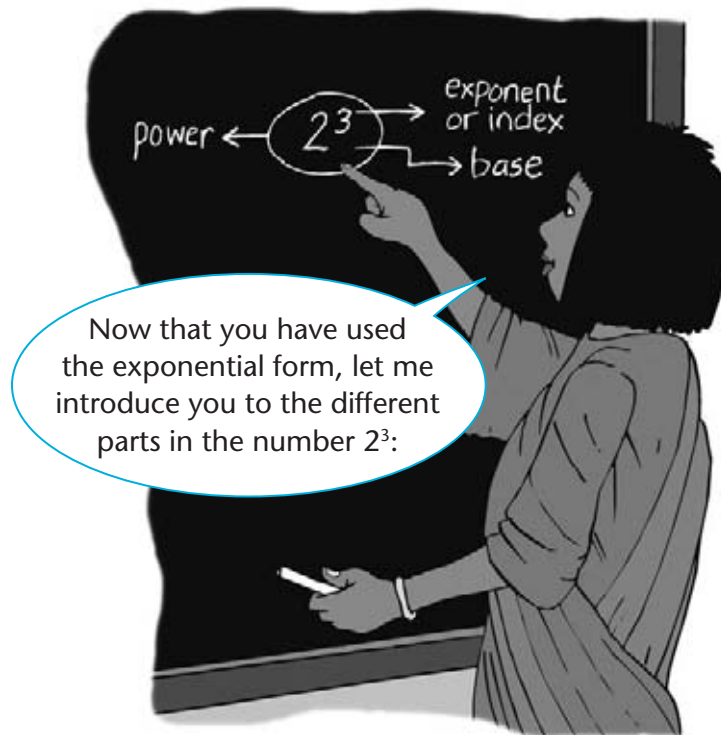
$$36 = 2 \times 2 \times 3 \times 3$$

$$= 2^2 \times 3^2$$

Exercise 2

1. Write the prime factors of the following numbers in exponential form:
 - a) The prime factors of $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
 - b) The prime factors of $1\,125 = 3 \times 3 \times 5 \times 5 \times 5$
 - c) The prime factors of $19\,208 = 2 \times 2 \times 2 \times 7 \times 7 \times 7 \times 7$
2. Draw a factor tree for the following numbers. Write the prime factors in exponential form.
 - a) 16
 - b) 72
 - c) 50

The parts of an exponential number



You read 2^3 as: '2 to the power of 3' or '2 raised to the power of 3'. The exponent shows how many times the base is multiplied by itself. You can also say the exponent shows the number of factors that are multiplied, for example: $2^3 = 2 \times 2 \times 2$.

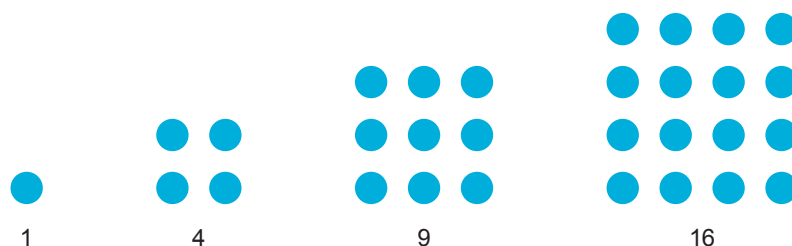
Remember, $20 = 2 \times 2 \times 5$. In exponential form we write the prime factors of 20 as $2^2 \times 5$. The factor 5 appears only once. You do not need to write the exponent 1. So instead of writing 5^1 you write 5. When a number does not have an exponent the exponent is 1.

Remember:

- $100^1 = 100$
- $12^2 = 12 \times 12 = 144$ (and not $12 \times 2 = 24$)
- $1^3 = 1 \times 1 \times 1 = 1$ (and not $1 \times 3 = 3$)

More about square numbers and square roots

A square number is a number that can be represented by dots in the shape of a square. Consider the following examples.



In this unit, you will learn more about square numbers and cube numbers. You will learn how to represent them in exponential form.

To find the **square of a number** you multiply the number by itself. Look at the table below. It will help you to find the first four square numbers.

The number	Multiplied by itself	Exponential form	The square number
1	1×1	1^2	1
2	2×2	2^2	4
3	3×3	3^2	9
4	4×4	4^2	16

Example

Let's study the square number 9.

How do you find the square number 9?

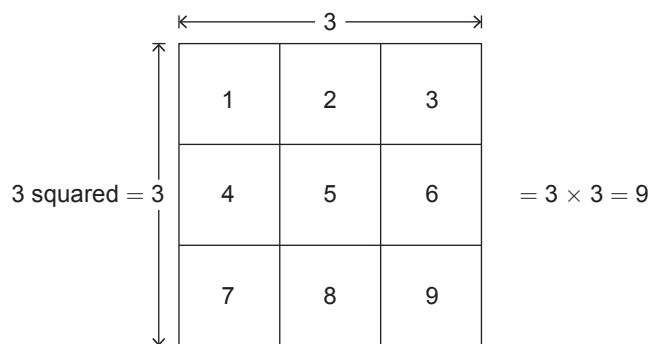
Solution

Start with the number 3 and multiply it by itself: 3×3 or 3^2 .

You read this as follows:

- The square of 3 is 9.
- 3 squared is 9.
- 3 to the power of 2 is 9.

Look at the diagram below. It shows the geometrical meaning of 3^2 . Do you notice that the figure is a square? It is also a 2-dimensional figure.



Example

Find the square of 25.

Solution

$$\begin{aligned}
 25^2 &= 25 \times 25 \\
 &= 625
 \end{aligned}$$

Exercise 3

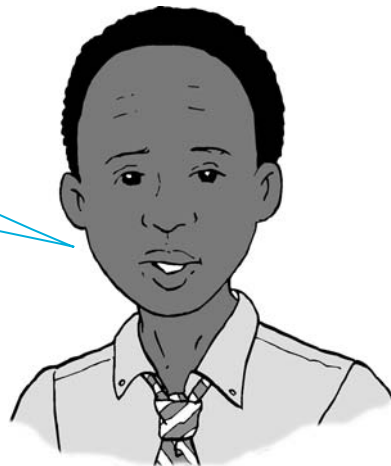
1. Copy and complete the following table:

The number	Multiplied by itself	Exponential form	The square number
5			
6			
7			
8			
9			
10			
11			
12			
13			

2. Write down a list of the first 13 square numbers.

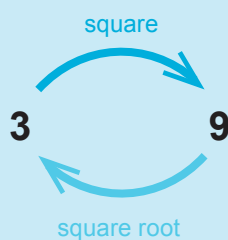
Square roots

Finding the **square root** of a number is the inverse process of finding the square of a number. This means that you need to find the original number that was multiplied by itself.



Example

If the square of 3 is 9, then the square root of 9 is 3. We write: $\sqrt{9} = 3$. The following illustration shows the relationship between squaring a number and finding the square root of a number.



[Source: <http://www.mathsisfun.com/square-root.html>]

<p>Example Find the square of 25.</p> <p>Solution $25^2 = 25 \times 25$ $= 625$</p>	<p>Example Find the square root of 25.</p> <p>Solution $\sqrt{25} = 5$ because $5 \times 5 = 25$.</p>
<p>Example Find $\sqrt{100}$. Explain your answer.</p> <p>Solution $\sqrt{100} = 10$ because $10 \times 10 = 100$, or because $10^2 = 100$.</p>	<p>Example Find $\sqrt{4}$. Explain your answer. Ask yourself, what number must I multiply by itself to get 4?</p> <p>Solution $2 \times 2 = 4$, therefore $\sqrt{4} = 2$.</p>

Exercise 4

Find the square roots. Give a reason for each answer.

1. $\sqrt{100}$ 2. $\sqrt{64}$ 3. $\sqrt{81}$ 4. $\sqrt{121}$

More about cube numbers and cube roots

The **cube of a number** is the number multiplied by itself three times. Look at the table below. It shows the first three cube numbers. It also shows how to represent the cube numbers in exponential form.

The number	Multiplied by itself three times	Exponential form	The cube number
1	$1 \times 1 \times 1$	1^3	1
2	$2 \times 2 \times 2$	2^3	8
3	$3 \times 3 \times 3$	3^3	27

Example

Let's study the cube number 27. How do you find the cube number 27?

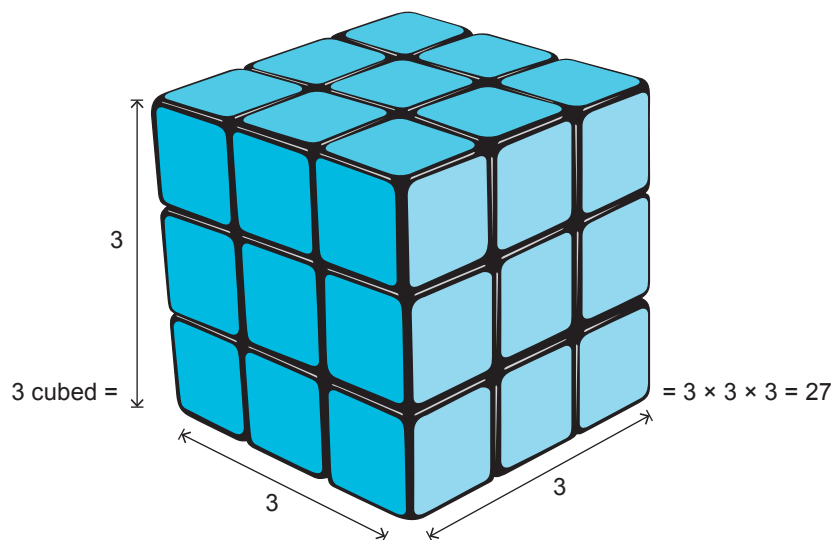
Here you say: $3 \times 3 \times 3 = 27$.

You can also write this as: $3^3 = 27$.

You read it as follows:

- The cube of 3 is 27.
- 3 cubed is 27.
- 3 to the power of 3 is 27.

Look at the drawing below. It shows the geometrical meaning of 3 cubed. The drawing is a 3-dimensional object. We call it a cube.



Example

Find the cube of 8.

Solution

$$8^3 = 8 \times 8 \times 8 = 512$$

Exercise 5

- Copy and complete the following table.

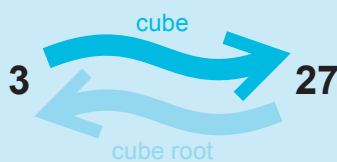
The number	Multiplied by itself three times	Exponential form	The cube number
4			
5			
6			

- Write down the list of the first six cube numbers.

Finding the **cube root** of a number is the inverse process of finding the cube of a number. Here, we know the cube number and need to find the original number that was multiplied by itself three times.

Example

The cube of 3 is 27. The cube root of 27 is 3. We write it as: $\sqrt[3]{27} = 3$. Look at the drawing. It shows the relationship between finding the cube and the cube root of a number.



When we write the square root symbol, we do not write the 2. However, for any other root, such as a cube root, we write in the number on the symbol. So, for example, we write the square root of 64 as $\sqrt{64}$, but we write the cube root of 64 as $\sqrt[3]{64}$.

Example

Find the cube root of 8.

Solution

Ask yourself which number multiplied by itself three times is equal to 8?

$$\sqrt[3]{8} = 2 \text{ because } 2 \times 2 \times 2 = 8.$$

Example

Determine $\sqrt[3]{64}$. Give a reason for your answer.

Solution

$$\sqrt[3]{64} = 4, \text{ because } 4 \times 4 \times 4 = 64.$$

Example

Find $\sqrt[3]{125}$. Give a reason for your answer.

Solution

$$\sqrt[3]{125} = 5 \text{ because } 5 \times 5 \times 5 = 125 \text{ or because } 5^3 = 125.$$

Exercise 6

Determine the following cube roots. Give a reason for your answer.

1. $\sqrt[3]{1}$
2. $\sqrt[3]{8}$

The exponential form is a short way for you to multiply equal numbers or to represent big numbers.

Example

Look at the table below. Work between the first two columns. You do not need to be able to calculate the value of the expansions. We have shown you the values so that you can see just how big the powers become!

Power	Expansion	Value of the expansion
32^2	32×32	1 024
10^3	$10 \times 10 \times 10$	1 000
21^4	$21 \times 21 \times 21 \times 21$	194 481
25^5	$25 \times 25 \times 25 \times 25 \times 25$	9 765 625

Exercise 7

1. Calculate:

- | | |
|--------------------------|--------------------------------|
| a) 2^3 | b) 2×3 |
| c) 3^2 | d) 2 raised to the fifth power |
| e) 4 cubed | f) 12 squared |
| g) the square root of 16 | h) the cube root of 8 |

2. A part of the 12-times multiplication table is shown. Complete the table. Write down what you notice about the numbers in the shaded blocks.

\times	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

3. Complete:

a) $64 = \square \times \square = \square^2$ b) $144 = \square \times \square = \square^2$
 c) $49 = \square \times \square = \square^2$ d) $81 = \square \times \square = \square^2$
 e) $121 = \square \times \square = \square^2$ f) $36 = \square \times \square = \square^2$

4. Determine the square number:

a) 9^2 b) 2^2 c) 5^2
 d) 1^2 e) 10^2 f) 13^2
 g) 3^2 h) 4^2

5. Determine the square root. Check your answers.

a) $\sqrt{36}$ b) $\sqrt{49}$ c) $\sqrt{1}$
 d) $\sqrt{9}$ e) $\sqrt{121}$ f) $\sqrt{81}$
 g) $\sqrt{169}$ h) $\sqrt{100}$

6. Complete:

a) $1 = \square \times \square \times \square = \square^3$
 b) $8 = \square \times \square \times \square = \square^3$
 c) $27 = \square \times \square \times \square = \square^3$
 d) $64 = \square \times \square \times \square = \square^3$
 e) $125 = \square \times \square \times \square = \square^3$
 f) $216 = \square \times \square \times \square = \square^3$

7. Determine the cubes. You can use your calculator but show all your calculations:

a) 3^3 b) 5^3 c) 2^3 d) 4^3

8. Determine the cube root. Check your answers.

a) $\sqrt[3]{27}$ b) $\sqrt[3]{64}$ c) $\sqrt[3]{8}$
 d) $\sqrt[3]{1}$ e) $\sqrt[3]{125}$ f) $\sqrt[3]{216}$

9. Expand the number. Do not calculate the final answer.

a) 23^5 b) 165^6 c) $3\,542^4$ d) $98\,547^3$

10. Represent the numbers in exponential form.

a) $82 \times 82 \times 82 \times 82 \times 82 \times 82 \times 82$
 b) $567 \times 567 \times 567 \times 567 \times 567$

11. Calculate:

a) 8^2 b) 8×2 c) 8×8 d) 2^8

12. Fill in: $>$, $<$ or $=$ to make the number sentence true.

a) $\sqrt{144} \square \sqrt{121}$

b) $4^2 \square 2^4$

c) $3^4 \square 4^3$

d) $\sqrt{3^2} \square 1^3$

Unit 2 Calculations using numbers in exponential form

Let's revise the **BODMAS** rule for operations:

- First, you do all operations in the **brackets**.
- Then do **orders**. This means all exponents or roots.
- Next, working from left to right, do all the **division and multiplication**.
- Finally, working from left to right, do all the **addition and subtraction**.

Example

Calculate $72 \div (3 + 5) - 2 \times (1 + 2)$.

Solution

$$\begin{aligned} 72 \div (3 + 5) - 2 \times (1 + 2) &= 72 \div 8 - 2 \times 3 \\ &= 9 - 6 \\ &= 3 \end{aligned}$$

Calculations with exponents and radicals

By now, you are familiar with numbers given in the forms: 2^3 or $\sqrt{16}$ or $\sqrt[3]{8}$. We call numbers like 2^3 exponents. We call numbers like $\sqrt{16}$ or $\sqrt[3]{8}$ **radicals**. A radical just refers to a number under a root (radical) sign. In BODMAS, exponents fall after brackets, but before multiplication and division.

Example

Calculate $4 \times 3^2 + 6 \times (5 + 2)$.

Solution

$$\begin{aligned} 4 \times 3^2 + 6 \times (5 + 2) &= 4 \times 9 + 6 \times 7 \\ &= 36 + 42 \\ &= 78 \end{aligned}$$

Example

Calculate:

1. $(7 - 4)^3$

2. $7^3 - 4^3$

Solution

1. $(7 - 4)^3 = 3^3 = 27$

2. $\begin{aligned} 7^3 - 4^3 &= (7 \times 7 \times 7) - (4 \times 4 \times 4) \\ &= 343 - 64 \\ &= 279 \end{aligned}$

$$(7 - 4)^3 \neq 7^3 - 4^3$$

Remember the BODMAS rule: First do the part in brackets, then the exponent part.

Example

1. $\sqrt{16 + 9}$
2. $\sqrt{16} + \sqrt{9}$

Solution

1. $\sqrt{16 + 9} = \sqrt{25} = 5$
2. $\sqrt{16} + \sqrt{9} = 4 + 3 = 7$

Remember the BODMAS rule:
First do the part in brackets, then the exponent part.

**Example**

Calculate $3^3 + 4^2$.

Solution

$$\begin{aligned} 3^3 + 4^2 &= (3 \times 3 \times 3) + (4 \times 4) \\ &= 27 + 16 \\ &= 43 \end{aligned}$$

Example

Calculate $\sqrt{36} + 54 - 5^2$.

Solution

$$\begin{aligned} \sqrt{36} + 54 - 5^2 &= 6 + 54 - 25 \\ &= 60 - 25 \\ &= 35 \end{aligned}$$

Example

Calculate $2^3 \times 2^2$.

Solution

$$\begin{aligned} 2^3 \times 2^2 &= (2 \times 2 \times 2) \times (2 \times 2) \\ &= 8 \times 4 \\ &= 32 \end{aligned}$$

Example

Calculate $\frac{3^5 + 7}{5^2 \times 10}$.

Solution

$$\begin{aligned} \frac{3^5 + 7}{5^2 \times 10} &= \frac{(3 \times 3 \times 3 \times 3 \times 3) + 7}{(5 \times 5) \times 10} \\ &= \frac{243 + 7}{25 \times 10} \\ &= \frac{250}{250} \\ &= 1 \end{aligned}$$

Example

Calculate $2^2 \times \frac{20}{4} - 6 \times 3 + 55$.

Solution

$$\begin{aligned} 2^2 \times \frac{20}{4} - 6 \times 3 + 55 &= 4 \times 5 - 6 \times 3 + 55 \\ &= 20 - 18 + 55 \\ &= 2 + 55 \\ &= 57 \end{aligned}$$

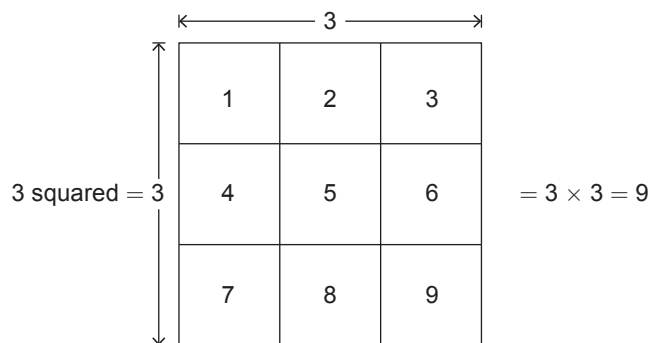
Exercise 8

- Use the identity properties and fill in the missing number:
 - $12^2 \times \underline{\hspace{1cm}} = 144$
 - $8^2 + 0 = \underline{\hspace{1cm}}$
 - $\underline{\hspace{1cm}} + 3^4 = 3^4$
 - $1 \times \underline{\hspace{1cm}} = 2^6$
- Use the number properties to complete the following:
 - $2^3 + 2^4 = 2^4 + \underline{\hspace{1cm}}$
 - $45 \times 4^3 = (40 + \underline{\hspace{1cm}}) \times 4^3$
 - $(5 \times 2^2) \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \times (5 \times 63)$
 - $(5^3 \times 3^2) \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \times (3^2 \times 4^2)$
 - $57 \times 7^4 = 7^4 \times (60 - \underline{\hspace{1cm}})$
 - $24 \times \underline{\hspace{1cm}} = 13^2 \times (20 + 4)$
 - $\underline{\hspace{1cm}} \times 3^3 = \underline{\hspace{1cm}} \times 3^4$
 - $7^2 \times 719 = \underline{\hspace{1cm}} \times (720 - \underline{\hspace{1cm}})$
- Calculate the answer using numbers in exponential form:
 - $1^5 + 9$
 - $80 - 7^2$
 - $4^2 \times 2$
 - 3×3^2
 - $\frac{8^2}{4}$
 - $\frac{6^3}{6^2}$
 - $\frac{12^2}{2^3}$
 - $9^2 \times 2^2$
 - 24×2^4
 - $10^2 - 9^2 + 4$
 - $2 \times 11^2 + 38$
 - $\frac{4^3 - 4}{15 \times 2^2}$
 - $25\,765 \times 10^2$
 - $400\,000 \div 10^5$
 - $13^2 \times 10^3$
 - $\frac{3^5 + 2}{7^2}$
 - $4 \times (6 - 2)^3 \div (3 - 1)^3 - 2$
- Calculate the answer using numbers in exponential and radical form:
 - $\sqrt{10\,000} \times 8$
 - $\sqrt{16} + \sqrt{144} \times 3$
 - $\frac{\sqrt{225}}{5^2 - 10}$
 - $\frac{\sqrt{64} - 2}{2}$
 - $\frac{5}{\sqrt[3]{125}}$
 - $9 \times \sqrt[3]{8} - 7 \times \sqrt[3]{8}$
 - $\frac{\sqrt{81} + \sqrt[3]{1}}{5}$
 - $7^3 - \sqrt{100} - 33$

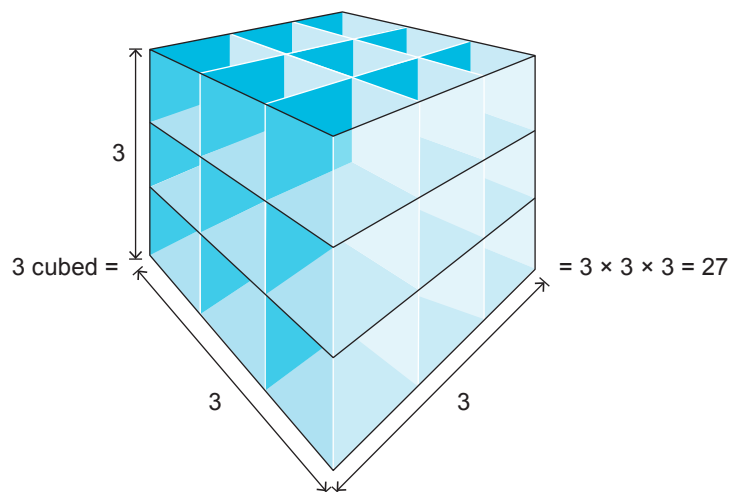
Unit 3 Solving problems

Where can you use exponents in everyday life?

In Unit 1 of this topic, we said 3^2 means 3×3 . We also explained that this represents the **area** of a square. You calculated the area of a square in Grade 6 by saying: Area = 3 blocks in a row \times 3 blocks in a column = 9 blocks.



In Grade 6, you also learnt how to calculate the **volume** of a box by saying: $\text{Volume} = 3 \times 3 \times 3 = 27$. On page 46, you learnt that you could write this calculation as $3^3 = 27$.



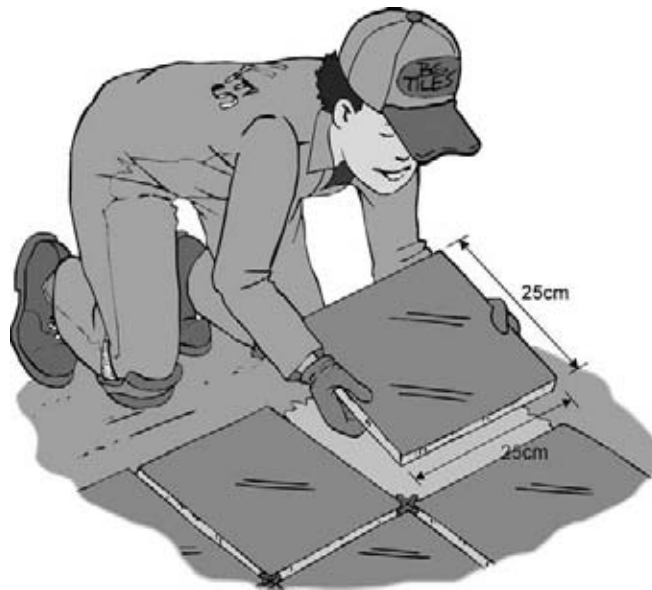
We also use exponents when we work with large numbers. In the next exercise you are going to work with large numbers, area and volume.

Exercise 9

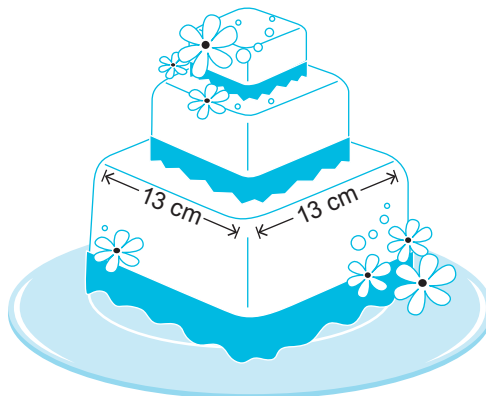
1. Write out the numbers indicated in each statement.
 - a) The distance between two towns is 6^3 kilometres.
 - b) The speed of light is about 57^3 miles per second. Do not calculate the answer.
 - c) The United States generates approximately $14^2 \times 1\,000\,000$ tons of waste in a year.
2.
 - a) The Tokyo Sky Tree is the world's tallest tower. It was opened on 22 May 2012 for the first time. High-speed lifts take visitors up to the observation decks. Calculate the height of the tower if it is given as $(25^2 + 9)$ m.
 - b) A man travels from Cape Town to Cairo in Egypt. This is a distance of approximately $2^3 \times 1\,000$ km. He then travels from Cairo to Alexandria in Egypt. This is a distance of 218 km. What distance did he travel in total?



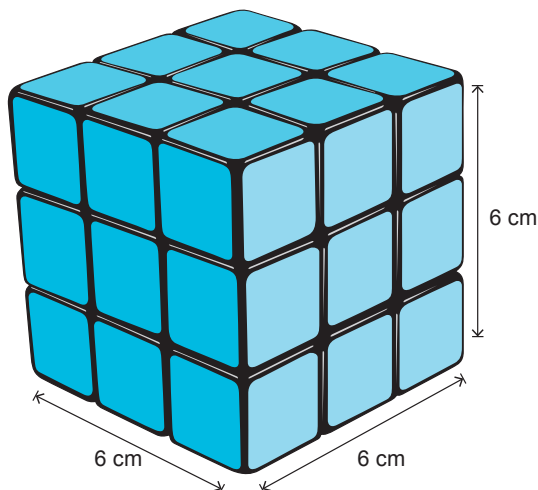
3. Ilza was admitted to hospital. Her bacteria count was 3^5 . After she took her medication for three days, the bacteria count decreased by 11^2 . Give the number of bacteria still in her body.
4. Fill in the missing numbers:
- a) The area of 1 floor tile = $25 \text{ cm} \times 25 \text{ cm} = \square \square$



- b) The area of the bottom layer of the wedding cake = $(13 \text{ cm})^2$
 = $\square \text{ cm} \times \square \text{ cm} = \square \text{ cm}^2$



- c) The volume of the Rubik's cube = $6 \text{ cm} \times 6 \text{ cm} \times 6 \text{ cm}$
 = $\square \square = \square \text{ cm}^3$



Topic

3 Construction of geometric figures

In this topic you will learn to:

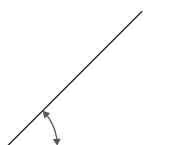
- measure angles accurately using a protractor
- classify angles in terms of acute, right, obtuse, straight, reflex angle and revolution
- accurately construct geometric figures using a compass, ruler and protractor, including angles (to one degree of accuracy), circles, parallel lines and perpendicular lines.



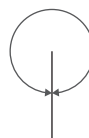
What you already know

Categorise the following angles as acute, right, obtuse, straight, reflex or a revolution.

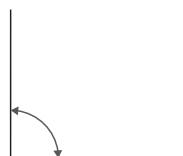
1.



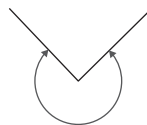
2.



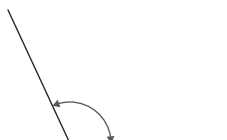
3.



4.



5.

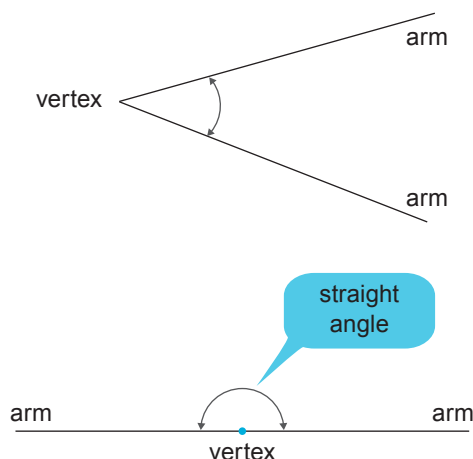


6.

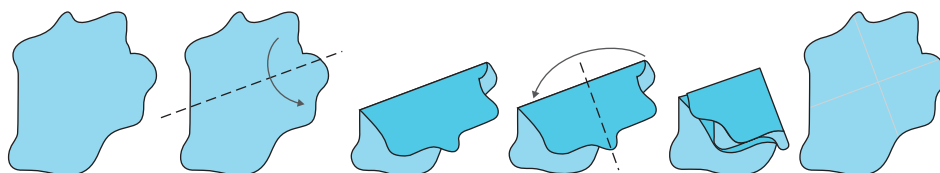


Unit 1 Measuring angles

In Grade 6 you learnt about angles. You found that an angle was formed by two rotating arms. For example, think about the two rotating arms of a clock. You will notice that the arms form an angle. The size of an angle is the size of the gap between the arms. The *vertex* of an angle is the point where the two arms meet. When the two arms form a straight line, the angle has a special name, a *straight angle*.

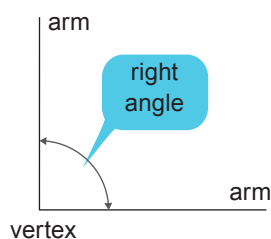


Another special kind of angle is a *right angle*. A right angle is equivalent to a quarter turn. You can use the corner and side of a sheet of paper (or this textbook) to check if shapes or objects have right angles. Look at the diagram below. It shows how to use a piece of paper to create a right angle.

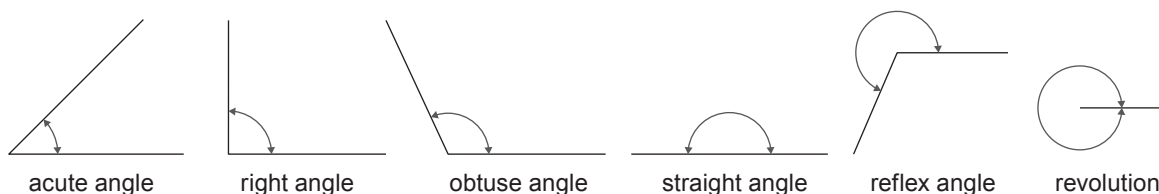


[Source: From website: http://thinkmath.edc.org/index.php/Right_angle#Paperfolding_to_construct_a_right_angle]

When you unfold the paper the creases will be *perpendicular* to each other. Here, you have made four equal right angles around a vertex.



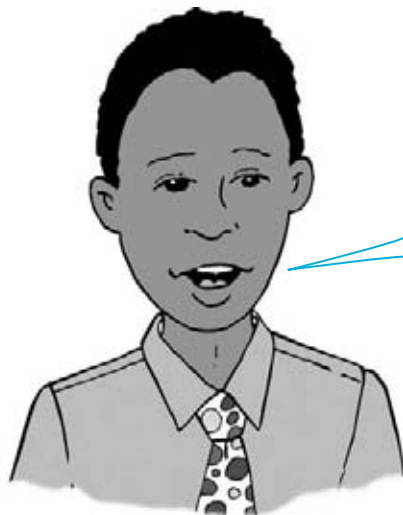
A *straight angle* is equivalent to a half turn. A *revolution* is equivalent to a full turn. There are six types of angles. You learnt about them in Grade 6. They include the straight angle and right angle.



Look at the sketches above. Can you see that:

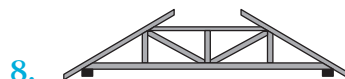
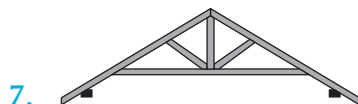
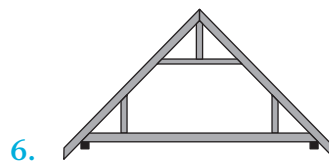
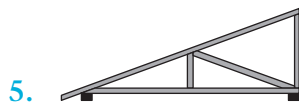
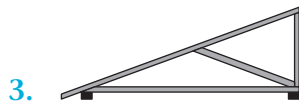
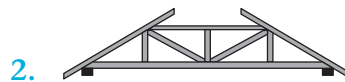
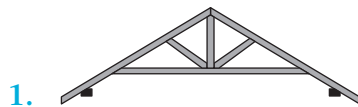
- an *acute angle* is smaller than a right angle
- an *obtuse angle* is greater than a right angle, but smaller than a straight angle
- a *reflex angle* is greater than a straight angle, but smaller than a revolution.

Exercise 1



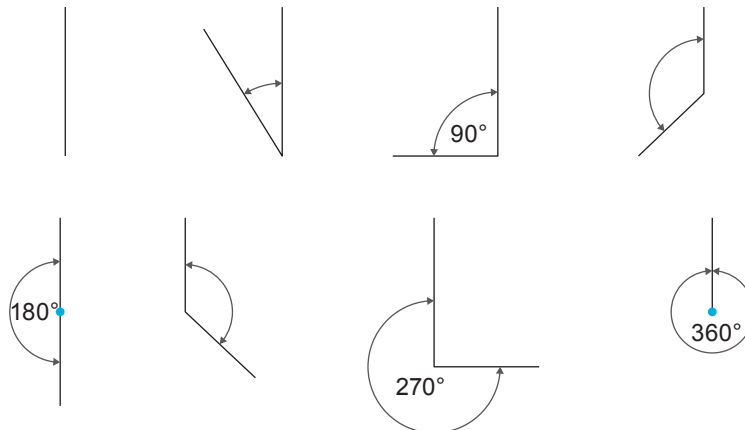
You will need a protractor, a ruler and a pencil to complete this exercise.

What kind of angles can you identify in the following pictures? Identify as many as possible.



Introduction to degrees

We measure angles in degrees. If the rotating arm of an angle completes a full circle it forms an angle of 360 degrees.



Notation

We use a little circle above and to the right of the number to show degrees. For example, 60° means 60 degrees.

Sometimes people use other units, for example radians, to measure an angle. However, degrees is used most often.



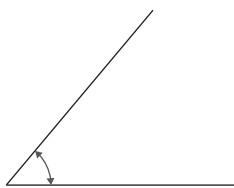
A right angle is exactly 90° . A straight angle is 180° . There are 360° in a circle. One degree is $\frac{1}{360}$ of a circle. The following diagram shows two examples of 1° angles. The size of the angle does not depend on the length of the rotating arms. These two angles are both 1° angles.



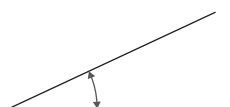
Exercise 2

Estimate the size of each angle. Say whether the angle is acute, right, obtuse, straight, reflex or a revolution.

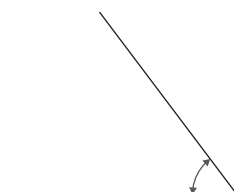
1.



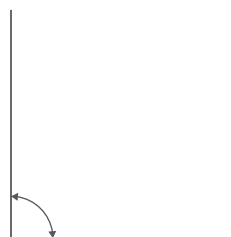
2.



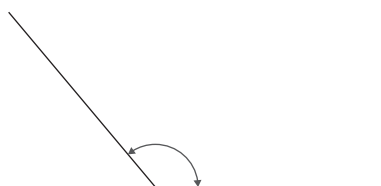
3.



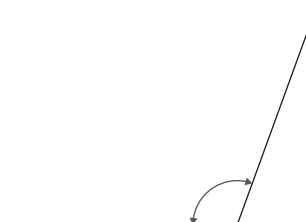
4.



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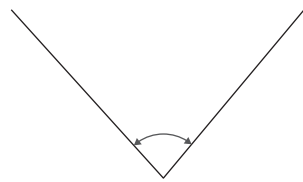
6.



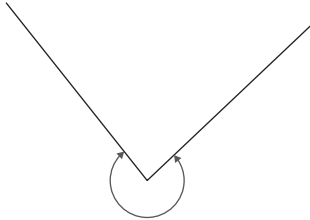
7.



8.



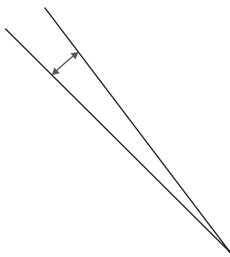
9.



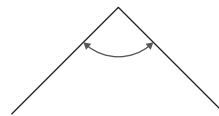
10.



11.



12.



How to measure angles

We measure the size of angles with a protractor. The scale on the protractor ranges from 0° to 180° (half-circle protractor). There are two scales on a 180° protractor. It has an inner and outer scale. You can read the scale from both sides.

You also get a 360° protractor. You use the 180° protractor because it fits easily into your pencil bag.

Follow these steps to measure an angle:

Step 1: Place the origin of the protractor on the vertex of the angle.

Remember, the vertex is the point at which the two arms meet.

