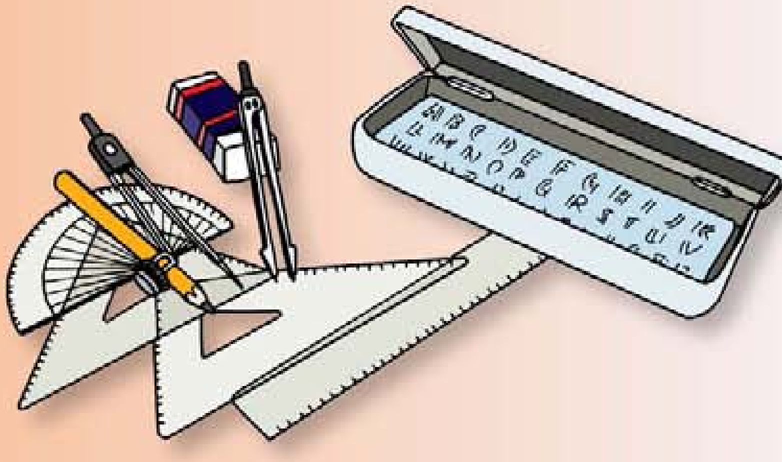
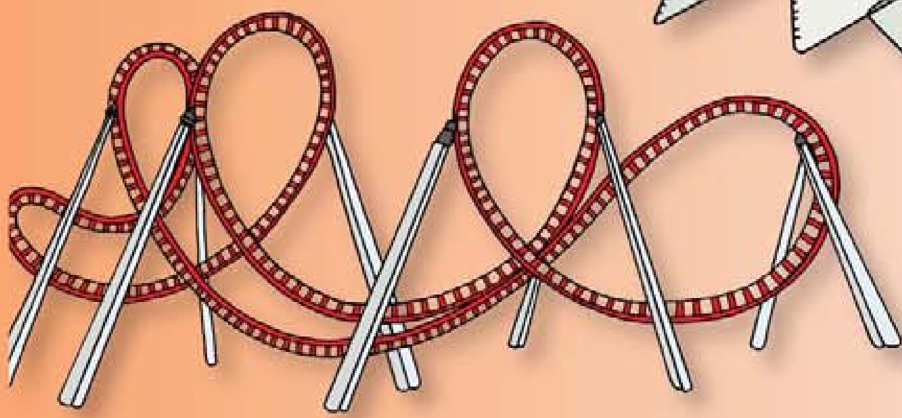
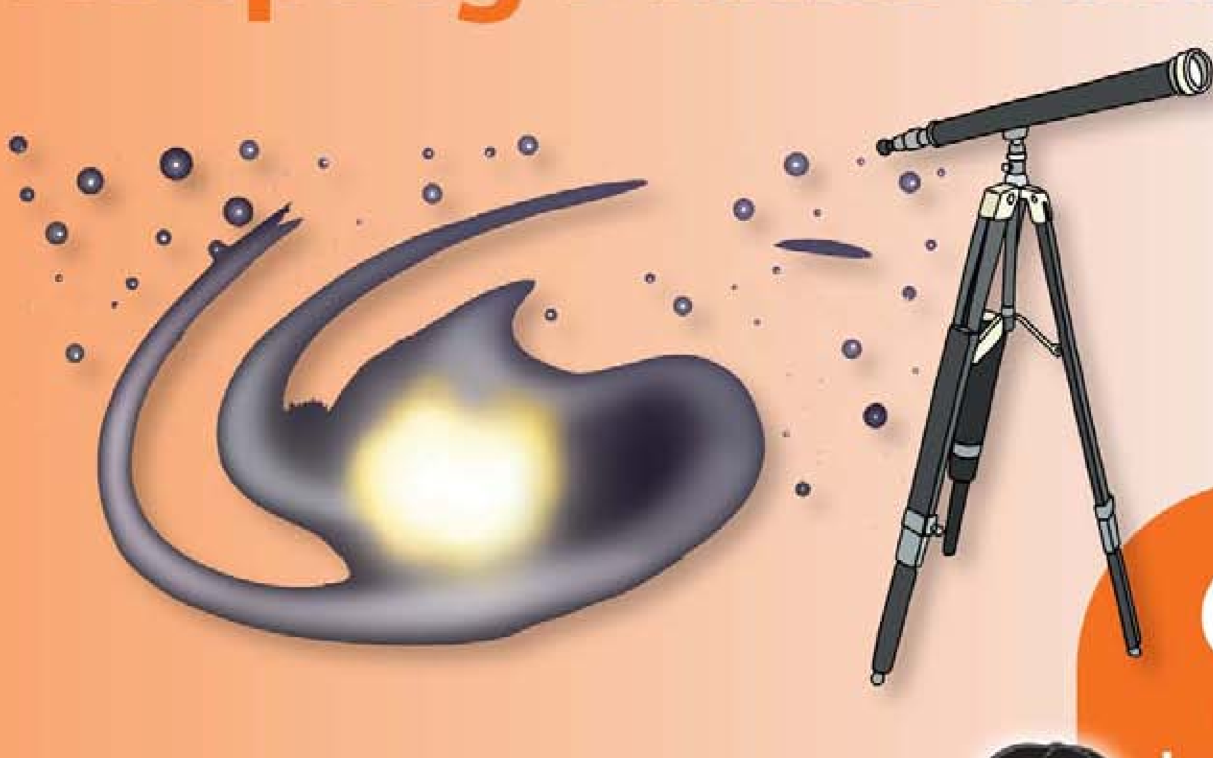
Clever Books





Clever

Keeping Maths Simple



9

Learner's Book

H Botha • E du Plessis • I Nel • G Stols

Clever Keeping Maths Simple

Grade 9

Learner's Book

H Botha

E du Plessis

I Nel

G Stols

MACMILLAN

Clever Keeping Maths Simple Grade 9 Learner's Book

First published 2013

13 15 17 16 14
0 2 4 6 8 10 9 7 5 3 1

Published by
Macmillan South Africa (Pty) Ltd
Private Bag X19
Northlands
2116
Gauteng
South Africa

Typeset by Ink Design
Cover illustration by Alex van Houwelingen
Cover design by Deevine Design
Illustrations by Ink Design, MPS, Catherine Kraetschmer, Alex van Houwelingen and Butch Stoltz

Photographs:
AAI Fotostock: page 315
Afripics: page 29, 58, 68, 168, 314, 337
INPRA: page 235, 339

Print ISBN: 978-1-4318-0372-9, e-ISBN: 9781431805150
WIP: 2049K000

Contents

Introduction

Welcome to this Mathematics Grade 9 *Learner's Book*.

Have you ever wondered how Mathematics could be useful to you? Do you think that you could use Mathematics outside the classroom?

Do you know that Mathematics is used in different ways every day?
For example:
Mathematics is used in ***MUSIC***.
Mathematics is used in ***MANY TECHNICAL CAREERS***.
Mathematics is used in ***COOKING YOUR FAVOURITE FOOD***.
Mathematics is used in ***COMPUTERS, CELL PHONES AND GAMES***.
Mathematics is used when ***GOING TO THE SHOP TO BUY YOUR FAVOURITE THINGS***.

Just about everything we do uses Mathematics! So we can say that Mathematics is a language that makes use of symbols and notations to describe numerical, geometrical and graphical relationships in order to help us to give meaning to world we live in.

To help you to develop the essential mathematical skills that you need to deal with mathematical situations competently, this Mathematics Grade 9 *Learner's Book* will guide you to:

- develop the correct use of the language of Mathematics
- develop number vocabulary, number concepts and calculations and application skills
- communicate, think, reason logically and apply the mathematical knowledge gained
- investigate, analyse, represent and interpret information
- pose and solve problems
- build an awareness of the important role that Mathematics plays in real-life situations.

This Mathematics Grade 9 *Learner's Book* covers five main content areas:

- Numbers, Operations and Relationships
- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
- Data Handling

The authors and publisher wish you all the best in your study of Mathematics in Grade 9.

Topic 1 Whole numbers

In this topic you will learn to:

- recognise, define and distinguish properties of the real number system
- revise calculations with all four operations on whole numbers
- revise calculation techniques
- use prime factorisation of numbers to find the LCM and HCF of numbers
- solve problems in contexts involving ratio, rate and proportion
- solve problems in financial contexts.

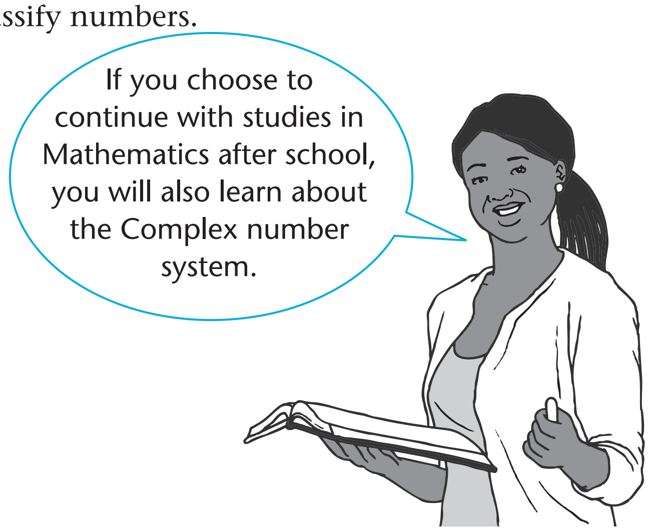
What you already know

1. Calculate: $69 + (4 \times 25 - 10) - 300$
2. Round 9 726 off to the nearest:
   a) 10 b) 100 c) 1 000
3. Use rounding off and compensation to calculate:
   a) $17 + 435$ b) 12×46
4. Calculate the actual answer of $929\,648 + 26\,873 - 499\,042$.

Unit 1 Properties of whole numbers

The real number system

The figure on the following page illustrates the real number system. It will help you to classify numbers.



The Real number system

Rational numbers

Includes Integers, Natural numbers and Whole numbers plus repeating and terminating decimals and fractions:

$-0{,}3$; $\frac{2}{3}$; $-\frac{5}{2}$; $2\frac{1}{2}$; $0{,}000\dot{6}$ and their opposites.

Integers

Includes all the Whole numbers and their opposites.
Positives and negatives: … -3; -2; -1; 0; 1; 2; 3; …

Natural or Counting numbers

These numbers can be shown with objects. They begin with 1; 2; 3; and continue forever.

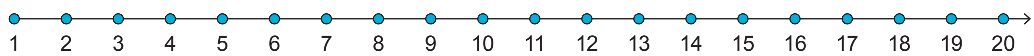
Whole numbers

Include the natural numbers plus zero.
0; 1; 2; 3 …

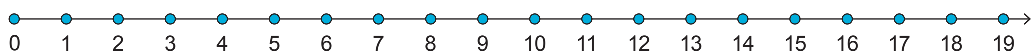
Irrational numbers

These numbers are represented by non-repeating, non-terminating decimals and their opposites.
Examples: π; $\sqrt{3}$; $\sqrt[3]{5}$; $-\sqrt{5}$

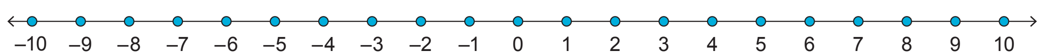
How do these numbers fit together? When we refer to a group of numbers, we can also talk about a set of numbers. The first set of numbers is the set of **Natural numbers** ($\mathbb{N}$): 1; 2; 3; 4; …



However, the natural numbers cannot provide an answer to calculations such as $a - a$. So the set was extended to a bigger set of numbers, called the **Whole numbers** ($\mathbb{N}_0$). The set of whole numbers consists of all the natural numbers, plus the number zero: 0; 1; 2; 3; ...

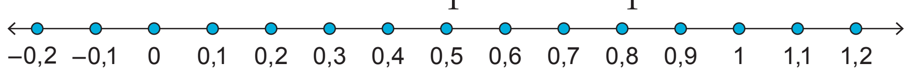


The whole numbers were then further extended to include negative numbers. This is the set of **Integers** ($\mathbb{Z}$): … -3; -2; -1; 0; 1; 2; 3; …



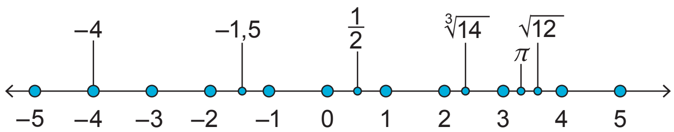
The next set of numbers enables us to do division. This is the set of **Rational numbers** ($\mathbb{Q}$). The rational numbers consist of the integers and fractions.

However, a fraction must be able to be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Examples of these fractions are: $\frac{1}{2}$; $\frac{4}{3}$; $2\frac{2}{3}$; $0{,}683$; $0{,}\dot{3}$. The number $0{,}\dot{3}$ is called a recurring decimal. In Unit 3, you will learn how to convert recurring decimals into common fractions. We can express every integer as a fraction, for example, $3 = \frac{3}{1}$ and $-6 = \frac{-6}{1}$.



We are not able to express all numbers as a fraction. For example, we cannot express any infinite, non-recurring decimal number as a fraction. These numbers include $\sqrt{+\text{ non-perfect square numbers}}$ and $\sqrt[3]{\text{non-perfect cube numbers}}$. We call these numbers **Irrational numbers** ($\mathbb{Q}'$). Examples of irrational numbers include $\pi = 3{,}1415927\ldots$; $\sqrt{12} = 3{,}464101\ldots$; $\sqrt[3]{14} = 2{,}410142\ldots$ Note that we often use 3,14 or $\frac{22}{7}$ as approximate values for π in calculations.

The set of irrational numbers is *not* an extension of the rational numbers. Instead, they form a separate set of numbers. Together, the rational numbers and the irrational numbers form the set of **Real numbers** ($\mathbb{R}$). Using real numbers, we can now include all number sets on a single number line.



Look again at the diagram on the previous page. The set of natural numbers is a subset of the whole numbers, which in turn is a subset of the integers, which in turn is a subset of rational numbers. All of these numbers, together with the irrational numbers, form part of the Real number system.

In summary, we have the following sets of numbers:
Natural numbers ($\mathbb{N}$): $\{1; 2; 3; \ldots\}$
Whole numbers ($\mathbb{N}_0$): $\{0; 1; 2; 3; \ldots\}$
Integers ($\mathbb{Z}$): $\{\ldots -3; -2; -1; 0; 1; 2; 3; \ldots\}$
Rational numbers ($\mathbb{Q}$): Integers + fractions in the form $\frac{a}{b}$; $b \neq 0$
Irrational numbers ($\mathbb{Q}'$): infinite, non-recurring decimal numbers
Real numbers ($\mathbb{R}$): rational numbers + irrational numbers

Exercise 1

1. Illustrate the real number system using a tree diagram.
2. Choose from the following lists all the:
   - i. Whole numbers
   - ii. Integers
   - iii. Rational numbers
   - iv. Irrational numbers.

   a) $-9;\ -\frac{7}{2};\ 5;\ \frac{2}{3};\ \sqrt{2};\ 0;\ 1;\ -4;\ -1$

   b) $-3\frac{1}{4};\ 3\pi;\ \sqrt{10};\ 3{,}4575\ \ldots;\ -9;\ \sqrt{25}$
3. Classify each of the following numbers into a set of numbers. A number may belong to more than one set. For example, $\sqrt[3]{-8}$ belongs to the set of real numbers, rational numbers and integers, because $\sqrt[3]{-8} = -2$.
   a) $2{,}373367\ldots$ b) $0{,}\dot{7}\dot{5}$ c) $-4{,}63$ d) $\sqrt{10}$
   e) -2 f) $0{,}3$ g) 8
4. Use your calculator to find approximate values for each number. Explain why these are approximate values.
   a) $\sqrt{3}$ b) $\sqrt{37}$ c) $\sqrt[3]{17}$ d) $\sqrt[3]{400}$

Unit 2 Calculations with whole numbers

Order of operations

If a calculation includes more than one operation, we need to perform the operations in a specific order. We first do any calculations in brackets. Then, we do any multiplication, including 'of', and division. We then do the multiplication and division from left to right. Finally, we work out any addition and subtraction, also from left to right.

Example

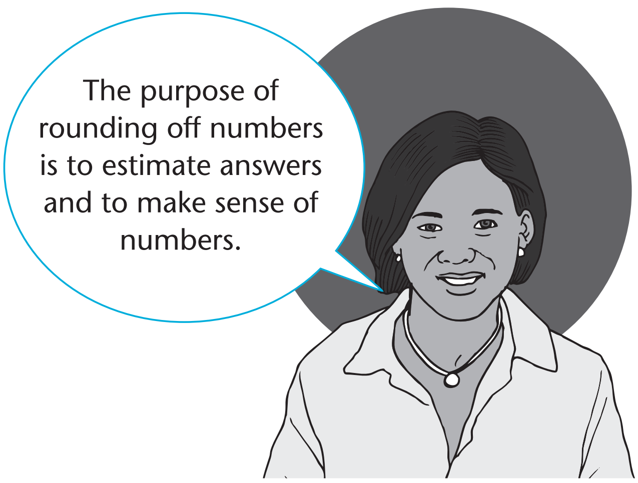
Calculate $75 + (6 \times 25 - 5) - 200$.

Solution

$$\begin{aligned} 75 + (150 - 5) - 200 &= 75 + 145 - 200 \\ &= 220 - 200 \\ &= 20 \end{aligned}$$

Rounding off numbers

Rounding off means we find another number close to the first number. The rounded number is often easier to work with, but it is also less accurate.



Example

- 8 726 lies nearer to 8 730 than to 8 720:
  Therefore, 8 7<u>2</u>6 rounded off to the nearest 10 is 8 730.
- 8 726 lies nearer to 8 700 than to 8 800:
  Therefore, 8 <u>7</u>26 rounded off to the nearest 100 is 8 700.
- 8 726 lies nearer to 9 000 than to 8 000:
  Therefore, <u>8</u> 726 rounded off to the nearest 1 000 is 9 000.

Rounding off and compensating

Rounding off and compensating is a technique that makes calculations easier. Remember, if we change one number to simplify the calculation, we need to change the other number(s) too. This technique makes the numbers easier to work with, but it does not change the value of the answer.

So we round off a number to make the calculation easier and then compensate to make up for what will be lost or gained when rounding off numbers.

Remember that $a + 0 = a$; $a - 0 = a$; $a \times 1 = a$; $a \div 1 = a$.

Example

Calculate the following:

a) $18 + 335$ (Round off 18 to 20 as it is easier to add 20.)

$= (18 + 2) + (335 - 2)$ $(+ 2 - 2 = 0)$
$= 20 + 333$
$= 353$

b) 15×26 (Easier to multiply by 30.)

$= (15 \times 30) - (15 \times 4)$
$= 450 - 60$
$= 390$

c) $97 - 62$ (Round off 97 to 100 as it is easier to subtract a number from 100.)

$= (97 + 3) - (62 + 3)$
$= 100 - 65$
$= 35$

or

$97 - 62$ (Round off 62 to 60 as it is easier to subtract 60.)

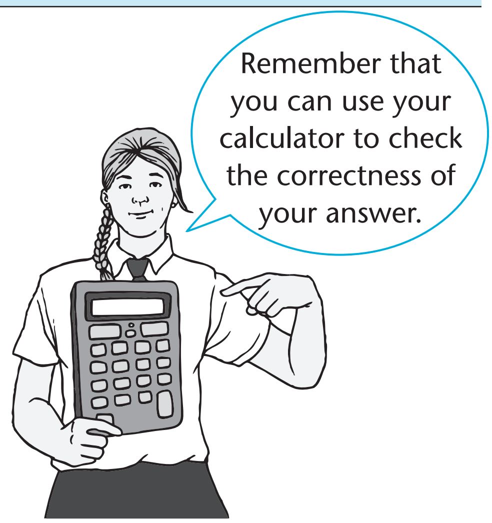
$= (97 - 2) - (62 - 2)$
$= 95 - 60$
$= 35$

d) $168 \div 12$ (Round off 168 to a number that can easily be divided by 12.)

$= (144 \div 12) + (24 \div 12)$
$= 12 + 2$
$= 14$

Add, subtract and multiply whole numbers and estimate answers

An estimate allows us to make a rough calculation. We use rounding to estimate an answer. To find the actual answer, we need to add, subtract or multiply in columns or do long division.



Example

Calculate the actual answer of 839 527 + 46 783 − 549 032.

Solution

$$\begin{array}{r} 839\,527 \\ +\quad 46\,783 \\ \hline 886\,310 \end{array} \qquad \begin{array}{r} 886\,310 \\ -\ 549\,032 \\ \hline 337\,278 \end{array}$$

The answer is 337 278.

Example

a) Estimate 236 × 873 by rounding off the numbers to the nearest 100.
b) Calculate the actual answer.
c) Use your calculator to check your answer.
d) Use division as the inverse operation to check the correctness of your answer.

Solution

a) $236 \times 873 \approx 200 \times 900 = 180\,000$

b)
$$\begin{array}{r} 236 \\ \times\quad 873 \\ \hline 708 \\ 16\,520 \\ +\ 188\,800 \\ \hline 206\,028 \end{array}$$

c) 206 028

d)
$$\begin{array}{r} 236 \\ 873\ \overline{)\,206\,028} \\ -\ 1\,746 \\ \hline 3\,142 \\ -\ 2\,619 \\ \hline 5\,238 \\ -\ 5\,238 \\ \hline 0 \end{array}$$

Exercise 2

1. Calculate:
   a) 23 × 11 − 23 × 10
   b) (32 ÷ 4) ÷ (36 ÷ 9)
   c) 60 ÷ 5 ÷ 4
   d) 48 − 12 − 10
   e) 85 − (27 + 52 ÷ 4)
   f) 2 × 12 ÷ 4 × 3

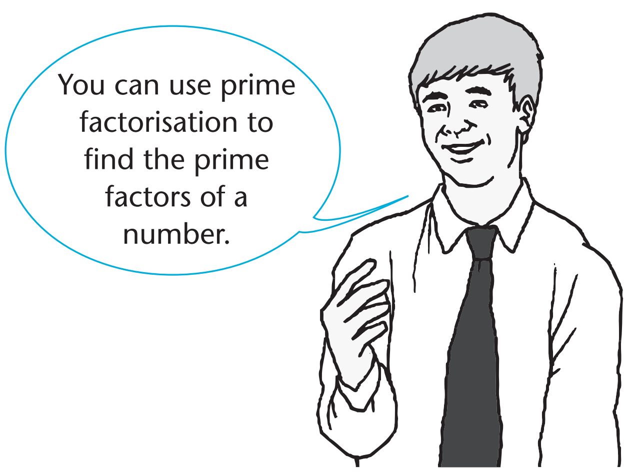
   Use your calculator for the following:
   g) (234 + 198) ÷ 3 + 33
   h) (264 − 181) × 27 × 39
   i) (804 + 42) × [1 380 ÷ (19 + 4)]

2. Calculate using rounding off and compensating. Use your calculator to check whether your answer is correct.
   a) $198 + 217$ b) $257 - 137$ c) 15×44
   d) 35×9 e) $396 \div 20$ f) $645 \div 15$
3. For each problem:
   i. estimate the answer by rounding off the numbers
   ii. calculate the actual answer
   iii. use your calculator to check the answer
   iv. use an inverse operation to check the correctness of your answer.
   a) 2 367 800 + 769 231 b) 274×363

Unit 3 Multiples and factors

Introduction

Terminology	Definition	Example
Multiples of a given number	The numbers into which the given number will divide without a remainder.	The multiples of 8 are 8; 16; 24; 32; …
Factors of a given number	Numbers that can divide exactly into the given number.	The factors of 6 are 1; 2; 3 and 6
A **prime number**	A number that has only two factors, namely **1** and **itself**. Numbers that have more than two factors are called **composite numbers**. The number **1** is neither a prime number nor a composite number, because it has only one factor, namely 1.	The first six prime numbers are 2; 3; 5; 7; 11; 13; … The first six composite numbers are: 4; 6; 8; 9; 10; 12; …
Prime factors	Factors of a number that are also prime numbers.	Prime factors of 12 are 2 and 3; the numbers 1, 4, 6 and 12 are also factors, but are not prime numbers.



Example

Use prime factorisation to write 1 728 as a product of its prime factors.

2	1 728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

The prime factors for 1 728 are $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^6 \times 3^3$.

The highest common factor

The **highest common factor** (HCF) of two or more numbers is the highest number that divides exactly into those numbers. We use prime factors to find the HCF. The HCF is very useful when we need to simplify fractions.

Example

Find the HCF of 27; 36; and 90.

Solution

Prime factors of 27 = $\mathbf{3} \times \mathbf{3} \times 3$

Prime factors of 36 = $2 \times 2 \times \mathbf{3} \times \mathbf{3}$

Prime factors of 90 = $2 \times \mathbf{3} \times \mathbf{3} \times 5$

$\therefore$ The HCF of 27, 36 and 90 is $3 \times 3 = 9$.

The lowest common multiple

The **lowest common multiple** (LCM) of two or more numbers is the smallest number that is a multiple of all the numbers. You can use multiples or prime factors to find the LCM.

Example

Find the LCM of 72 and 108 using multiples. (Use your calculator, but show all your calculations.)

Solution

Multiples of 72: 72; 144; **216**; …

Multiples of 108: 108; **216**; …

Therefore, the LCM of 72 and 108 is 216. This is the smallest number that is a multiple of both numbers.

Example

Find the LCM of 72 and 108 using prime factors.

Solution

We need to use the highest number of times each factor appears in the list of factors.

Prime factors of 72: $2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

Prime factors of 108: $2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

$\therefore$ The LCM of 72 and 108 is:

$$\begin{aligned} 2 \times 2 \times 2 \times 3 \times 3 \times 3 &= 2^3 \times 3^3 \\ &= 8 \times 27 \\ &= 216 \end{aligned}$$

Exercise 3

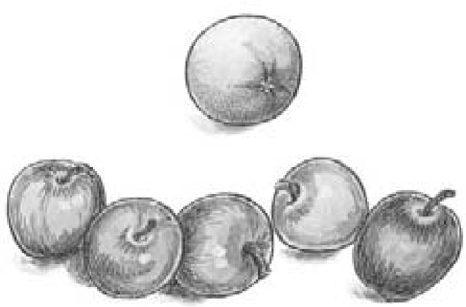
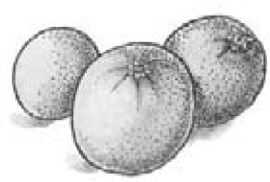
1. Find the HCF for the given numbers by using prime factorisation (you may use a calculator):
   a) 52 and 78 b) 144 and 90 c) 75; 120 and 150
   d) 212 and 159 e) 624 and 546 f) 220 and 284
2. Find the LCM for the given numbers either by listing their multiples or prime factors (you may use your calculator):
   a) 25 and 30 b) 72 and 120 c) 60 and 135
   d) 26 and 104 e) 24; 36 and 60 f) 35; 105 and 175

Unit 4 Solving problems

Important words

convert to change something into another form

Introduction

Ratio	Rate
A relationship between quantities that express the number of times one is larger or smaller than the other. For example, we can say that the ratio of oranges to apples is 1 to 5 or 1 : 5 or $\frac{1}{5}$.	Rate is a comparison of one quantity or measure in relation to a different quantity or measure. For example, if four people share twelve oranges, then there will be three oranges per person.
	

Ratio	Rate
We can also write: • oranges as a fraction of all the fruit is $\frac{1}{6}$ • apples as a fraction of all the fruit is $\frac{5}{6}$.	Speed is a common example of rate. For example, we measure the speed of a car in terms of how many kilometres the car is travelling per hour (km/h). A speed of 120 km/h is the speed limit on our national roads.
Ratios have no units. When we express a ratio between two quantities, they must both use the same units. So, the ratio between 1 cm and 15 mm is: $\frac{10\text{ mm}}{15\text{ mm}} = \frac{10}{15} = \frac{2}{3}$ or 2 : 3.	Rates have units because we compare different quantities. For example, if a person is paid R800 for working 8 hours, he or she receives a rate of R100/hour.

Ratio

Example

Write 18 days to 4 weeks as a ratio.

Solution

Before calculating the ratio, make sure that the units are the same.
Change 4 weeks to days:
$4 \times 7 = 28$ days

The ratio is 18 : 28 or 9 : 14. In fraction form, we have: $\frac{9\text{ days}}{14\text{ days}} = \frac{9}{14}$.

Rate

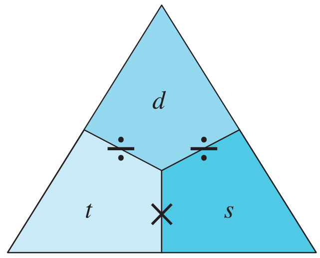
Rates allow us to express the relationship between two amounts that use different units of measurement, for example km per hour. However, when comparing rates, we need to make sure both rates use the same units of measurement. For example, 10 kilometres per hour is a very different speed to 10 kilometres per second.

Suppose we would like to compare how fast a person runs to the speed of a car. The units of measurement must be the same. If Lindiwe runs the 100 metres in 10 seconds, then her speed is 100 m/10s or 10 m/s.

When she drives her car, she might drive at a speed of 40 km/h. Which speed is fastest? We know that 10 m = 0,01 km. There are 60 seconds in a minute and 60 minutes in an hour. So there are 60×60 seconds in an hour = 3 600 seconds. Therefore:

$10\text{ m/s} = \frac{10\text{ m}}{1\text{ s}} = \frac{0{,}01\text{ km}}{\frac{1}{3\,600}\text{ h}} = 36\text{ km/h}$. So Lindiwe travels faster in her car.

We frequently encounter rate problems involving speed, distance and time. The following triangle can help us to work out one of these quantities in terms of the other two.



- distance = speed × time
- speed = $\frac{\text{distance}}{\text{time}}$
- time = $\frac{\text{distance}}{\text{speed}}$

Speed is usually given as constant speed or average speed.

Example

a) A car travelling at a constant speed travels 60 km in 18 minutes. How far will the car travel in 1 hour 12 minutes?

b) A car travelling at an average speed of 100 km/h covers a certain distance in 3 hours 20 minutes. At what speed must the car travel to cover the same distance in 2 hours 40 minutes?

Solution

a) Change time to hours:

$18 \text{ min} = \frac{18}{60} = \frac{3}{10} = 0{,}3 \text{ h}$

$1 \text{ h } 12 \text{ min} = 1 + \frac{12}{60} = 1 + \frac{2}{10} = 1 + 0{,}2 = 1{,}2 \text{ h}$

The car travels at $\frac{60 \text{ km}}{18 \text{ min}} = \frac{60}{0{,}3} \text{ km/h} = \frac{600}{3} \text{ km/h} = 200 \text{ km/h}$.

$$\begin{aligned}\text{Distance} &= \text{time} \times \text{speed}\\ &= 1{,}2 \text{ h} \times 200 \text{ km/h}\\ &= \frac{1{,}2 \text{ h}}{1} \times \frac{200 \text{ km}}{1 \text{ h}}\\ &= 240 \text{ km}\end{aligned}$$

b)
$$\begin{aligned}\text{Distance} &= \text{speed} \times \text{time}\\ &= 100 \text{ km/h} \times 3\tfrac{1}{3} \text{ h}\\ &= \frac{100}{1} \times \frac{10}{3}\\ &= \frac{1\,000}{3} \text{ km}\end{aligned}$$

$$\begin{aligned}\text{Speed} &= \frac{\text{distance}}{\text{time}}\\ &= \frac{1\,000}{3} \div 2\tfrac{2}{3}\\ &= \frac{1\,000}{3} \div \frac{8}{3}\\ &= \frac{1\,000}{3} \times \frac{3}{8}\\ &= 125 \text{ km/h}\end{aligned}$$

Direct and indirect proportion

When two quantities vary but stay in the same ratio, we say they are in **direct proportion**. For example, at one birthday party there were 2 boys and 4 girls (ratio 2 : 4). At another party, there were 8 boys and 16 girls (ratio 8 : 16). Although the quantities are different, the ratios are the same:

$\frac{2}{4} = \frac{8}{16} = \frac{1}{2}$

In other words, when expressed in their simplest form, both ratios are 1 : 2 or $\frac{1}{2}$. These are equivalent fractions. As one quantity increases, so the other quantity increases as well.

We say that:
- x is directly proportional to y if $\frac{x}{y}$ is a constant
- x and y are directly proportional if, as the value of x increases the value of y increases in the same proportion. Similarly, as the value of x decreases, the value of y decreases in the same proportion
- the direct proportion relationship is represented by a straight-line graph.

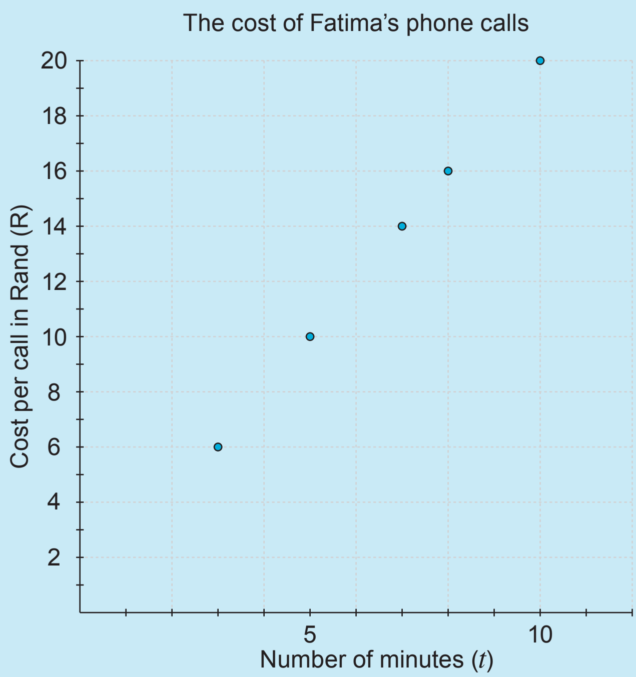
Direct proportion

Example

Fatima has a 'prepaid all day cost per minute' contract with a cellular phone company. When she received her first bill she listed five of the calls she made to help her to understand her bill.

Number of minutes (t)	10	3	8	5	7
Cost of phone call (R)	R20,00	R6,00	R16,00	R10,00	R14,00

Because Fatima likes Mathematics, she plotted the points on a graph. She noticed that she could draw a straight line through the points:

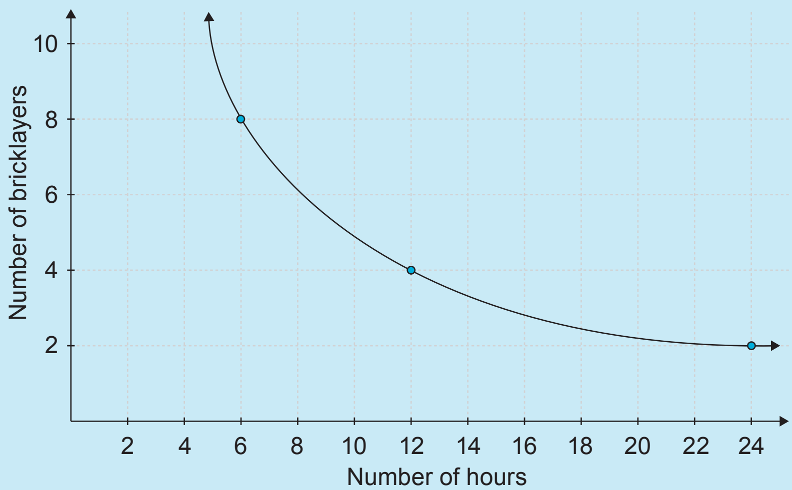


The next day she showed her graph to her teacher. Her teacher explained that this is an example of direct proportion. She asked Fatima to write down the ratio between the cost per call and the number of minutes. Fatima wrote: 20 : 10; 6 : 3; 16 : 8, and so on. Then she wrote the ratios in fraction form: $\frac{20}{10}$; $\frac{6}{3}$; $\frac{16}{8}$; and so on. She realised she could simplify all the ratios to $\frac{2}{1}$. Since all the ratios are equivalent, namely $\frac{2}{1}$ or just 2 : 1, we say x is directly proportional to y. So Fatima pays R2,00 for every minute she spends making a call. In general, we say that Fatima pays $2x$ rand for every x minutes.

Indirect proportion

Example

The following graph shows the relationship between the number of bricklayers and the length of time it takes to complete a job.



a) We can tabulate the values on the graph as follows:

Number of bricklayers	2	4	8
Number of hours	24	12	6

b) The ratios are: 2 : 24; 4 : 12 and 8 : 6. These are not equivalent ratios. So this is not a direct proportion relationship. However, if we multiply the two numbers in each case, the answer is always the same:
$2 \times 24 = 4 \times 12 = 8 \times 6 = 48$

c) As the number of bricklayers increases, the number of hours decreases. Similarly, as the number of bricklayers decreases, the number of hours increases.

d) We call this indirect proportion.

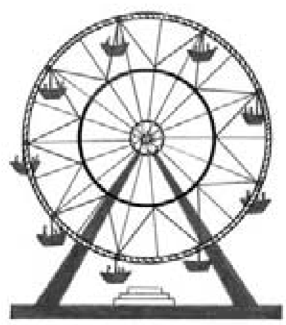
We say that:

- x is indirectly or inversely proportional to y if $x \times y$ is a constant. We write this as $x \times y = c$ or $y = \frac{c}{x}$.
- x and y are indirectly proportional if:
  - as the value of x increases, the value of y decreases
  - as the value of x decreases, the value of y increases.
- the indirect proportional relationship is represented by a non-linear curve.

Exercise 4

You may use your calculator, but show all calculations.

1. Write each of the following as a ratio in its simplest form. First make the units the same in each case:
   a) 90 sec to 7 min
   b) 48 min to 3 h
   c) 2,5 days to 80 hours
   d) 3 800 mm to 3 m
   e) 2,5 m to 75 cm
   f) 5 300 m to 2,5 km
2. Last month, Sophie spent 39 hours watching TV and 36 hours playing sport. During the same month, Fransina spent 42 hours watching TV and 36 hours playing sport. Who had the lower ratio of time spent watching TV to time spent playing sport?
3. At a wedding, there were 40 people from the groom's side and 56 people from the bride's side of the family.
   a) Find the ratio of the groom's family to the bride's family at the wedding.
   b) The cost of the catering for the wedding was R11 520. The bride and groom decided to divide this cost using the ratio calculated in a). How much did the groom and the bride each have to pay?
4. On average the human eye blinks once every 5 seconds. At this rate, approximately how many times will four learners blink in a single school day that is 6 hours long?
5. Abdul walks 5 km in 35 minutes. How far does he walk in 21 minutes?
6. Hezekiel runs 1 000 m at a rate of 3 minutes per kilometre. How long does he take to run 500 m?
7. Joe earns R25 per hour working at a pizza shop. How much will he earn over a weekend of two shifts, each six hours long?



8. In an amusement park the cost per ride depends on the number of rides.
   a) Use the following table to draw the graph of the number of tickets versus the price per ticket.

Number of tickets bought	1	2	4	8
Price per ticket	R20	R10	R5	R2,50

   b) What happens to the price per ticket as a person buys more tickets?
9. A taxi takes 6 hours to travel from Johannesburg to Durban at a speed of 100 km/h. How far is it from Johannesburg to Durban?

10. Mapoela walks 2 km in 30 minutes. At that rate, how far could she walk in $1\frac{1}{2}$ hours?
11. Ntate runs 12 km in 3 hours. How many hours will it take him to run 1 km?
12. A man completes a journey in 2 hours when driving at a constant speed of 120 km/h. How long will it take him to cover the same distance if he travels at a speed of 80 km/h?
13. While on holiday, your family drove 648 km at an average speed of 72 km/h. How long did it take them to cover the distance?
14. A yellow car and a red car left Johannesburg at the same time. After travelling in the same direction for two hours, the yellow car had travelled 20 km further than the red car. If the average speed of the red car was 80 km/h, find the speed of the yellow car.
15. It takes one housekeeper two hours to do the ironing at Mrs Xolo's house. How long will it take three housekeepers to do the same amount of ironing?
16. The speed of sound is 1 235 km/h. How far (in metres) will sound travel in 5 seconds?
17. Juan runs 4 km in 30 minutes. How long will it take him to run 80 km?

Finance

Revision

Your teacher will give you examples of the work you did in Grade 7 and Grade 8. Discuss these examples in class.

Terminology	Explanation
Profit	You make a profit when you sell a product for more than you paid for it.
Loss	You make a loss when you sell a product for less than you paid for it.
Discount	A seller offers a discount when they reduce the normal selling price by an amount. This is often expressed as a percentage.
Value added tax (VAT)	An extra amount paid as tax to the government. It is based on the selling price and is expressed as a percentage. The current VAT rate in South Africa is 14%.
Budget	A budget is a plan of your expected income and expenses.
Account	An account is a record of all transactions between a buyer and a seller.
Loan	A loan is a type of debt.
Interest	Interest is the cost of borrowing money from someone else.
Rent	A regular payment for the use of someone else's property.

Terminology	Explanation
Hire purchase	A hire purchase agreement is a method of buying goods by paying regular instalments over time. The buyer usually has to pay a deposit, which is a percentage of the selling price. The remaining cost is paid off every month until the full selling price is repaid. The monthly payments include interest. The buyer only owns the product once he or she has made the final payment.
Exchange rate	An exchange rate is the cost of one country's money in terms of another country's money.

Simple interest

Interest is the cost of borrowing money. If we borrow money from the bank, the bank charges us interest. If we save money with a bank, we are effectively lending the money to the bank. In this case, the bank pays us interest. In Grade 8, you learnt about **simple interest**. We calculate simple interest only on the initial amount of the loan.

Example

John borrowed R45 000 from the bank, to be repaid over 5 years. The bank charges simple interest at 8% per annum. How much interest will John pay over the 5 years?

Solution

Interest per year $= \frac{8}{100} \times \frac{\text{R45 000}}{1}$

$= \text{R3 600}.$

For 5 years: R3 600 $\times$ 5 $=$ R18 000

In the previous example, we calculated the simple interest as:
total interest $=$ interest rate $\times$ amount borrowed $\times$ number of years

We can express this as a formula:
$\text{SI} = \frac{r}{100}.P.n$
Or
$\text{SI} = \frac{P.n.r}{100}$
Here, SI is the total amount of simple interest, P is the principal amount (the amount borrowed), n is the number of years and r is the interest rate. If we express r as a decimal, we can also write this formula as:

$\text{SI} = P.n.i$

In this case, $i = \frac{r}{100}$.

Example
An amount of R800 is invested for three years. Interest is calculated using simple interest at a rate of r% per annum. The investment yields R168. Calculate the value of r.

Solution

$\text{SI} = \frac{Pnr}{100}$. Therefore:

$$\begin{aligned}100\text{ SI} &= Pnr\\100(168) &= 800(3)r\\16\,800 &= 2\,400r\\r &= 7\%\end{aligned}$$

Example
How long will it take for R3 000 invested at 6% per annum simple interest to grow to R4 260?

Solution
Interest = R4 260 − R3 000 = R1 260. Now, $\text{SI} = \frac{Pnr}{100}$. Therefore:
$100\text{ SI} = Pnr$
$100(1\,260) = 3\,000n(6)$
$126\,000 = 18\,000n$
$n = 7$ years

Compound interest

Compound interest works differently to simple interest. In this case, we now pay interest on both the original amount and the interest already added. This is the kind of interest banks charge when we borrow money.

Example
John borrows R3 000 from the bank. He must repay the loan after three years. The bank charges compound interest at 10% per annum. How much interest will John pay over the three years?

Solution

Interest after 1 year: $\frac{10}{100} \times 3\,000 = \text{R}300$.

Total amount due = R3 000 + R300 = **R3 300**.

Interest after 2 years: $\frac{10}{100} \times \mathbf{3\,300} = \text{R}330$.

Total amount due = R3 300 + R330 = **R3 630**.

Interest after 3 years: $\frac{10}{100} \times \mathbf{3\,630} = \text{R}363$.

Total amount due = R3 630 + R363 = R3 993. He pays R993 interest.

In the previous example, we calculated the amount of interest at the end of each year. We then added this amount to the original amount. The interest in the second year is then based on the amount owing at the beginning of that year. This includes the interest charged on the first year. The same is true for the third year.

This method can take a long time to work out, especially if the loan is taken out over many years. Fortunately, we have a formula to help us:

$A = P\left(1 + \frac{r}{100}\right)^n$

Here, A is the final amount, P is the principal amount, r is the interest rate and n the number of years.

Example

Calculate the compound interest on a loan of R3 450 at 6,5% per annum for five years.

Solution

$A = P\left(1 + \frac{r}{100}\right)^n = 3\ 450\left(1 + \frac{6,5}{100}\right)^5$

$\approx$ R4 726,80

The amount of compound interest is R4 726,80 − R3 450 = R1 276,80.

Commission

Many people earn money on commission. This means that the amount they earn depends on how much they sell. The commission is usually a percentage of the value of the sales.

A good example of people who earn commission is estate agents. An estate agent markets and sells houses on your behalf. In return, they are paid a percentage of the selling price if they sell the house. However, if they are not able to sell the house, they do not earn anything.

Example

Funiwe works at a clothing store. She makes 4% commission on everything she sells. Her sales for the week are as follows (in rand):

Day in week	Sales (in rand)
Monday	2 312
Tuesday	547
Wednesday	5 554
Thursday	6 581
Friday	398
Saturday	7 240

a) Work out her commission for the week.
b) What percentage of her total commission did she earn on Saturday?

Solution

a) Total sales = R22 632
Commission: 4% of R22 632 $= \frac{4}{100} \times \frac{22\ 632}{1}$
$=$ R905,28

b) $\frac{7\ 240}{22\ 632} \times \frac{100}{1} \approx 32\%$

Exercise 5

You may use your calculator in this exercise.

1. Zia has saved money and now wants to buy some items at an end-of-year sale. She finds many items offered at a discount. Calculate the amount of the discount and the new selling price on each of the following items:

   a) 15% off a R450 bicycle
   b) 20% off a R2 999 TV

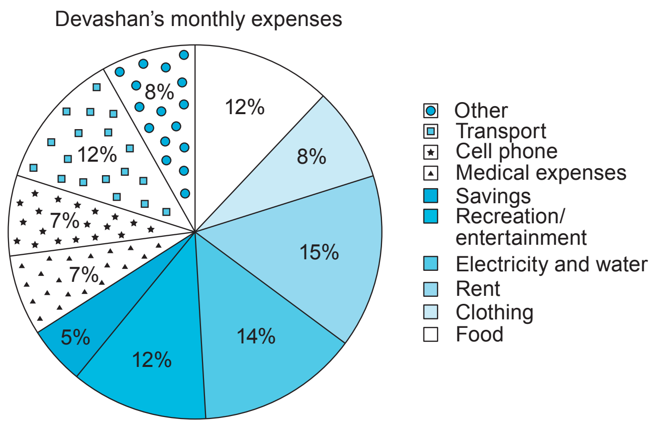
   Calculate the percentage discount on the following items:

   c) An R84,50 jacket selling for R75
   d) A R9 999 lounge suite selling for R1 000 less than normal
   e) Complete the following table. Did the store owners make a profit on these discounted prices?

Item	Cost price	Selling price	Profit or loss	Amount profit or loss
Bicycle	R305,00	R382,50		
TV	R2 500,00	R2 399,20		
Jacket	R75,00	R75,00		
Lounge suite	R4 500,00	R8 999,00		

   f) Why do shops have sales? Do you think a sale is a good idea for shop owners? Explain.

2. Devashan has written down all his expenses for a month. He used the figures to draw a pie chart.



   a) Devashan earns R7 650 per month. How much does he spend on the following items per month?
      i. Food
      ii. Medical expenses
      iii. Cellular phone
      iv. Other
   b) What expenses might be included in 'Other'?
   c) What is his largest expense?
   d) How would his budget change if Devashan was supporting a wife who was living with him?

3. The city council introduced a sliding scale to work out the cost of water. According to the scale, the cost of water is as follows:

Kilolitres	Cost in rand per kilolitre
0–6	Free
7–12	R4,55
13–18	R9,70
19–42	R14,38
43–72	R17,76
73+	R23,43

Mapule has a swimming pool at her house. The pool is 7 m long × 4 m wide × 1,5 m deep (remember 1 cubic metre = 1 000 litres = 1 kilolitre).

a) What does it cost to fill up the pool when it is empty?
b) If the pool was 30 cm deeper, what would it then cost to fill up?

4. To finance your education, you get a bursary valued at R10 000. The company agrees that, if you fail at any time, you must repay the bursary and they will charge you simple interest at an annual rate of 7%. If you fail, what amount will you owe after five years?

5. You deposit R2 500 in a savings account at an interest rate of 11%, compounded annually. What amount will you receive after:

a) 4 years
b) 8 years
c) 12 years?

6. Tshepang works at a clothing store and makes 3,5% commission on everything he sells. His sales for the week (in rand) are as follows:

Day	Sales (in rand)
Monday	2 311,87
Tuesday	546,99
Wednesday	5 553,98
Thursday	6 581,10
Friday	397,25
Saturday	7 239,65

a) Work out his commission for the week.
b) What percentage of his total commission did he earn on Saturday?

7. Complete the following hire purchase (HP) agreement:

Cash purchase price is	R12 600
Less 10% cash deposit or down payment	
Balance owing	
Hire purchase period	36 months
Finance charges (interest of 15% p.a.)	
Total amount due	
Monthly payment	
Total cost via hire purchase	

Topic 2 Integers

In this topic you will learn to:

- revise calculations with all four operations
- revise calculations involving squares, square roots, cubes, cube roots
- revise the commutative, associative and distributive properties of integers
- revise the additive and multiplicative inverses for integers
- solve problems in contexts involving multiple operations with integers.

What you already know

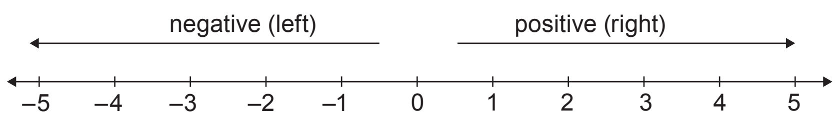
Calculate:

1. $-17 + (-6)$
2. $-10 + (+8)$
3. $-7 - 19$
4. $3 \times (-9)$
5. $\sqrt[3]{(-125)}$

Unit 1 Calculations using integers

The set of integers includes all negative and positive whole numbers:
… $-3; -2; -1; 0; 1; 2; 3;$ …

Adding integers



When we use a number line to add integers:

- we move to the right if the number is positive
- we move to the left if the number is negative.

Example

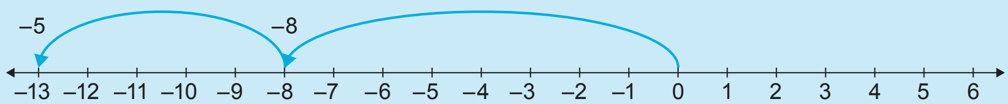
Calculate:

a) $-8 + (-5)$

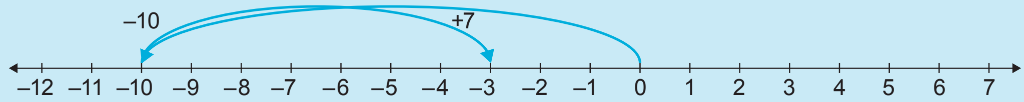
b) $-10 + (+7)$

Solution

a) We say: $-8 + (-5) = -13$ or in short $-8 - 5 = -13$



b) We say: $-10 + (+7) = -3$



When adding more than two integers, it is often easier to first add all the positive numbers, and then add all the negative numbers. Finally, add the two answers together to calculate the final answer. For example:

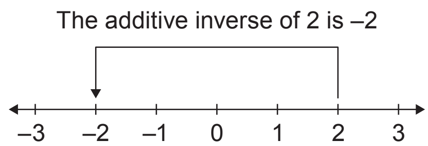
$$\begin{aligned} -13 + 6 + 18 - 65 + 56 - 12 &= (6 + 18 + 56) + (-13 - 65 - 12) \\ &= 80 - 90 \\ &= -10 \end{aligned}$$

Did you notice that we wrote 80 rather than +80? When there is no sign in front of a number, the number is positive. Check the answer using your calculator. Work from left to right.

Subtracting integers

The additive inverse of 15 is -15 and the additive inverse of -8 is 8. When we add a number and its additive inverse, the answer is always 0. We call this the additive identity.

For example, $85 - 15$ and $85 + (-15)$ both equal 70. So, subtracting 15 from 85 is the same as adding the additive inverse of 15 to 85.



Example

Subtract -5 from -14.

Solution

This means you need to calculate: $-14 - (-5)$.

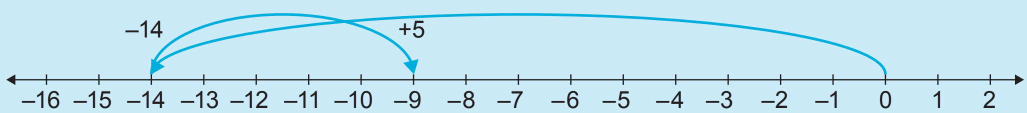
Subtracting is the same as adding the additive inverse.

Therefore:

$-14 - (-5) = -14 + (+5) = -9$

Or

$-14 + 5 = -9$



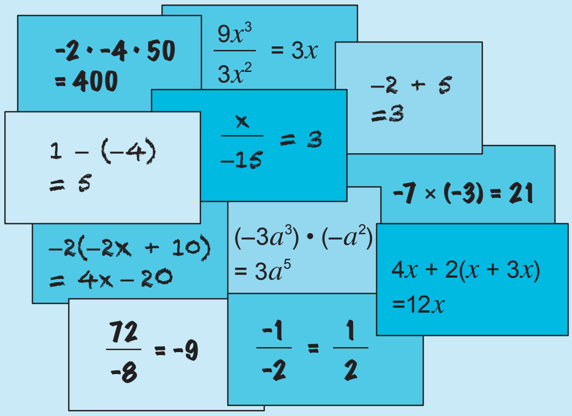
Multiplying and dividing integers

You learnt in Grade 7 that multiplication is the same as repeated addition. In Grade 8, you used repeated addition to find the product of a positive and negative number.

Example

$2 \times (-14) = (-14) + (-14) = -28$

The product of a positive and a negative number is a negative number.



By investigating patterns, you also discovered that $(-9) \times (-9) = 81$.
We can check this answer using division as the inverse operation of multiplication:

$81 \div (-9) = -9$

We know that:
- a positive integer × a positive integer equals a positive integer
- a negative integer × a negative integer equals a positive integer
- a positive integer × a negative integer equals a negative integer
- a negative integer × a positive integer equals a negative integer.

When we **divide** integers, we use the same rules as in multiplication to find the sign of the answer. Once again, we can check our answers using inverse operations.

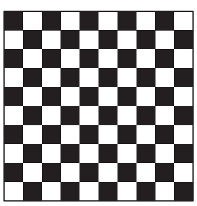
Example

If $(-7) \times 9 = -63$, then $(-63) \div 9 = -7$.

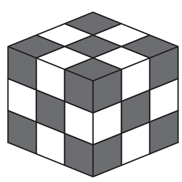
If $7 \times (-9) = -63$, then $(-63) \div -9 = 7$.

If $(-7) \times (-9) = 63$, then $63 \div (-9) = -7$.

Square and cube numbers and square and cube roots



$10^2 = 10 \times 10$
$= 100$



3^3 (cubed)
$= 3 \times 3 \times 3$
$= 27$

The geometrical representation of a square number is a two-dimensional (2D) square. The unit of measurement is units squared. The geometrical representation of a cubed number is a three-dimensional (3D) object, called a cube. The length, width and height of the cube are all equal. The unit of measurement is cubed units.

The square of a number and the square root of a number are opposite operations. Similarly, the cube of a number and the cube root of a number are opposite operations.

Example

a) If $(-3)^3 = (-3) \times (-3) \times (-3) = -27$ then $\sqrt[3]{(-27)} = -3$.
Check: $(-3) \times (-3) \times (-3) = -27$

b) If $7^2 = 7 \times 7 = 49$, then $\sqrt{49} = 7$
Check: $7 \times 7 = 49$

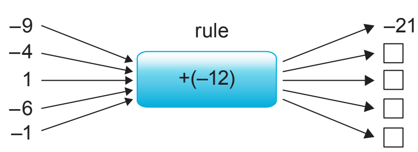
Example

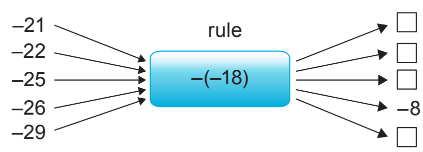
a) $\sqrt{49} = 7$. Remember, when there is no sign in front of the root sign, it means the root is positive.

b) $\sqrt{-121}$ is undefined in the real number system.

c) $(-12)^2 = (-12) \times (-12) = 144$
But $-12^2 = -(12 \times 12) = -144$

Exercise 1

1. Write down the opposite (additive inverse) of each of the following:
   a) 13　　b) 251
   c) −43　　d) −171
2. Find the sum of the numbers and their opposites (additive inverses).
   a) $13 + (-13) =$ ____　b) $-72 + (+72) =$ ____　c) $58 + (-58) =$ ____
3. Complete the following flow diagrams:
   a)


   b)


4. Add the following integers:
   a) $7 + (-2) + (+8) + (-13)$
   b) $25 + (-14) + (-30) + 18$
   c) $(-2x) + 5x + 9x + (-14x)$
   d) $-9ab + (-3ab) + 13ab + 2ab + (6ab)$
   e) $9a + (-4b) + (-7b) + b + (-5a)$
   f) $-y + 3x + (-7y) + 2y + (-5x) + x$
5. Subtract the following integers:
   a) $6x - (+7x)$　b) $-14m - (8m)$　c) $18p - (-16p)$
   d) $-23y - (-17y)$　e) $12 - (-2) - (-15)$　f) $15x - (-12x) - (4x)$
   g) $-6c - (-6c) - (-6c)$
   h) $3b - 8a - 19b + b - 12a$
   i) $-5xy - xy + 2x - 13x + 9xy - 10x$

6. Complete the following tables:

a)

x	5	3	1	−1	−3	−5	−7	−9	−11
$x + (-12)$									

b)

x	−15	−20	−25	−30	35	40	45	50
$x - (-20)$								

7. Calculate the following products and quotients:

a) $(-3) \times (-4) \times (-6)$ b) $5 \times (-12) \times (-3)$ c) $(-11)(2)(-4)$
d) $(-25)(-18)$ e) $-6a \times 15a \times 3a$ f) $-3x(-3x)(-5x)$
g) $-72 \div -8$ h) $132 \div -11$ i) $(-108) \div 12$
j) $-30p \div -6p$ k) $96r \div -8r$ l) $-48a \div 6a$
m) $\frac{64}{-16}$ n) $\frac{-1\,000}{-8}$ o) $\frac{-20\,000}{40}$

8. Use the correct order of operations to complete the following:

a) $-8 + 3 \times 2$ b) $17 + 15 \div (-5)$
c) $-45 \div (-3) - 6 + 4$ d) $16 - (13 - 5 \times 2) - 20$
e) $-14(4 - 14) + (-7 - 9)$ f) $-22 - [(-50) \div (-5)]$
g) $7 - (-8)(2) + 9(-12)$ h) $(-3)^2 + \sqrt[3]{-64} \times 25 \div -50$
i) $\sqrt[3]{(-1\,000)} - (-12) + 72 \div 2^3$ j) $\sqrt{625} \times 4 + 72 - 18 \div 3$
k) $0x - 22x - 32x - 42x - (4)(2x)$ l) $3(4y - 10y) \div (2y - 4y) - 4y$

Unit 2 Properties of integers

Introduction

We use the following properties of numbers to make calculations easier:

- the commutative property
- the associative property
- the distributive property
- the additive inverse
- the multiplicative inverse.

The commutative property

The commutative property tells us that the *order* in which we ***add*** or ***multiply*** numbers does not matter. The answer remains the same.

Example

a) $(-25) + (-15) = -40$ and $(-15) + (-25) = -40$
b) $(-75) \times 2 = -150$ and $2 \times (-75) = -150$

The associative property

The associative property tells us that we can *group together* any two numbers when ***adding*** or ***multiplying***. The answer remains the same.

Example

a) $(-9 + 8) + (-6) = -1 + (-6) = -7$ and $-9 + [8 + (-6)] = -9 + 2 = -7$

b) $(-35 \times -2) \times 3 = 70 \times 3 = 210$ and $-35 \times (-2 \times 3) = -35 \times -6 = 210$

The distributive property

The distributive property allows us to *redistribute* numbers over addition and subtraction. Once again, the answer remains the same. We can apply this property to multiplication over addition and multiplication over subtraction.

Example

a) $8 \times [(-9) + (-7)] = (8 \times -9) + (8 \times -7)$
$= -72 + (-56)$
$= -128$

Check:
$8 \times [(-9) + (-7)] = 8(-16)$
$= -128$

b) $-3[12 - (-9)] = (-3 \times 12) - (-3 \times -9)$
$= -36 - (27)$
$= -63$

Check:
$-3[12 - (-9)] = -3(12 + 9)$
$= -3 \times 21$
$= -63$

Additive inverse

We call **0** the identity element for addition. This is because when we add 0 to a number, the number remains the same. For example, $13 + 0 = 13$ and $-9 + 0 = -9$. The additive inverse also applies to subtraction. For example, $8 - 0 = 8$.

When we subtract integers, we use the additive inverse of the numbers. For example: the additive inverse of 15 is -15 and the additive inverse of -28 is 28. Therefore:
$15 + (-15) = 0$
$-28 + 28 = 0$

When we add any number and its additive inverse, the answer is always **0**.

Multiplicative inverse

We call **1** the identity element for multiplication. This is because when we multiply a number by 1, the number remains the same. For example:
$-55 \times 1 = -55$
$22 \times 1 = 22$

The multiplicative inverse also applies to division. For example, $4 \div 1 = 4$.

When we multiply a number by its multiplicative inverse, the answer is always 1. This is why we call 1 the multiplicative identity! Another name for the multiplicative inverse is the reciprocal. For example, the multiplicative inverse (the reciprocal) of 6 is $\frac{1}{6}$, because $\frac{6}{1} \times \frac{1}{6} = 1$.

A number multiplied by its multiplicative inverse is always equal to 1.

Exercise 2

1. Complete the following by using the commutative or associative property:
   a) $-160 + 240 = \_\_\_\_$ and $240 + (-160) = \_\_\_\_$
   b) $-50 \times 70 = \_\_\_\_$ and $70 \times -50 = \_\_\_\_$
   c) $(-45 \times -3) \times 10 = \_\_\_\_$ and $-45 \times (-3 \times 10) = \_\_\_\_$
   d) $-80 + (-40 + 20) = \_\_\_\_$ and $[-80 + (-40)] + 20 = \_\_\_\_$
   e) $-12x \times (11x \times \_\_\_\_) = 3\ 036x$ and $(-12x \times 11x) \times -23x = \_\_\_\_$
   f) $200y + \_\_\_\_ = -256y$ and $-456y + \_\_\_\_ = -256y$
   g) $13a \times -400a = \_\_\_\_$ and $-400a \times 13a = \_\_\_\_$
2. Use the distributive property to complete the following:
   a) $7 \times [12 + (-2)] = (7 \times \_\_\_\_) + (7 \times \_\_\_\_) = \_\_\_\_ + \_\_\_\_ = \_\_\_\_$
   b) $90(-70 + 500) = (\_\_\_\_ \times -70) + (\_\_\_\_ \times 500) = \_\_\_\_ + \_\_\_\_ = \_\_\_\_$
   c) $-6(12 - 5 + 8) = \_\_\_\_ + \_\_\_\_ + \_\_\_\_ = \_\_\_\_$
   d) $12(-9 - 8 - 2) = \_\_\_\_ + \_\_\_\_ + \_\_\_\_ = \_\_\_\_$
   e) $-2(4a - 18a + 23a) = \_\_\_\_ + \_\_\_\_ + \_\_\_\_ = \_\_\_\_$
   f) $-(-8ab + 12ab - 4ab - ab)$
   g) $-8x \times 38x = -8x \times (40x - \_\_\_\_)$
   $= (-8x \times 40x) - (-8x \times \_\_\_\_) = \_\_\_\_ - \_\_\_\_ = \_\_\_\_$
   h) $48y \times -9y = (50y - \_\_\_\_) \times -9y$
   $= (50y \times -9y) - (2y \times \_\_\_\_) = \_\_\_\_ - \_\_\_\_ = \_\_\_\_\_\_$

Unit 3 Solving problems

Where do we use integers?

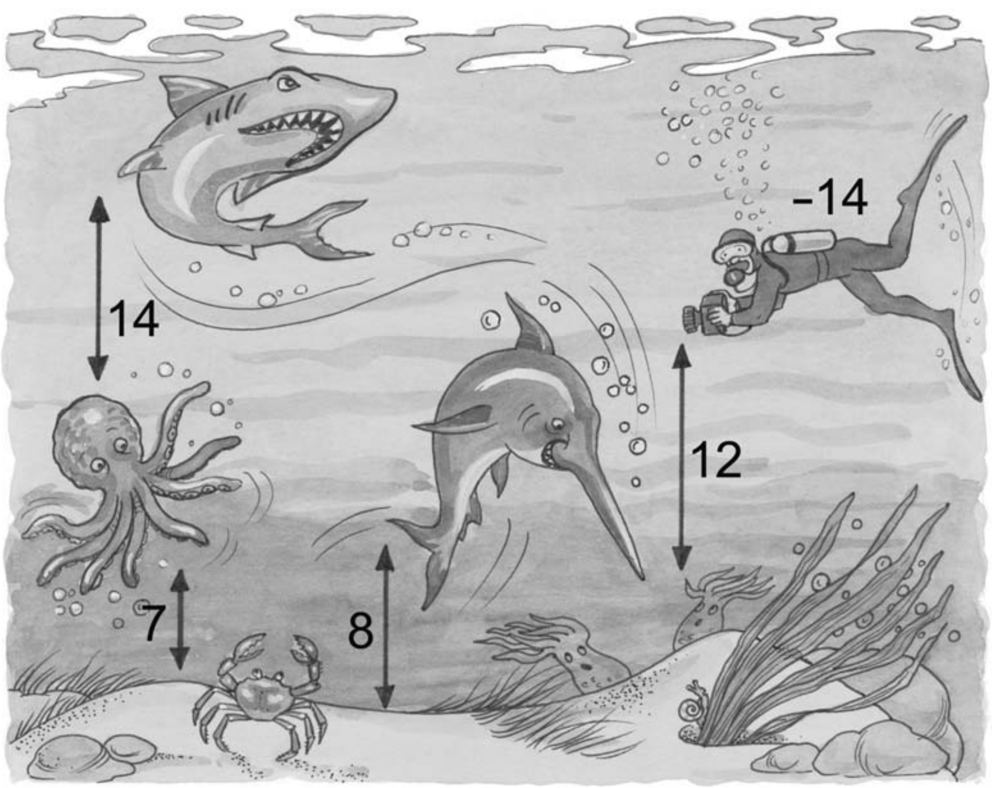
Example	Positive number	Negative number
Temperature	Above 0 °C	Below 0 °C
Altitude of places in the world	Above sea level	Below sea level
Business transaction	Profit	Loss
Budgets	Surplus	Debt
Bank statement	Credit	Debit

Companies list on public **stock exchanges** so that they can raise money for new projects or to expand. Anybody can then own a piece of this company by buying shares or stocks in the company. When we buy and sell shares in a company, we trade on the stock exchange.

A **bank account** is an agreement between a bank and an account holder. The account holder can pay money into an account (**deposit**) or take money out of the account (**withdrawal**). The bank keeps the money safe for the account holder. Each month, the bank produces a **statement**, which shows the transactions on the account. A transaction occurs when money is deposited or withdrawn from the account. The statement also shows the **balance** of the account, which is the amount of money currently in the account.

Exercise 3

1. Express each of the following as an integer:
   a) A debt of R420
   b) A profit of R8 400
   c) A temperature of 24 °C above zero
   d) 823 m below sea level
   e) A credit of R1 270
   f) A temperature of 12 °C below zero
   g) A surplus of R268
   h) 2 300 m below sea level
   i) A loss of R260
   j) A debit of R540
2. A diver swims at a depth of 14 m below the surface of the sea. Express each of the following positions as an integer:



   a) The sharp nose of the sword fish just in front of the diver
   b) The lowest point on the sand just to the right of the crab
   c) The bottom part of the octopus to the left of the crab
   d) The fin of the shark

3. Each place on Earth is a certain height above sea level (0 metres). For example:
   - The Dead Sea: -424 m
   - The Caspian Sea: -28 m
   - Mount Everest: 8 848 m
   - Mount Kilimanjaro: 5 893 m

   a) What is the meaning of these numbers?
   b) How much higher is Mount Everest above sea level than Mount Kilimanjaro?
   c) What is the difference in altitude between the Dead Sea and the Caspian Sea?

4. Patricia goes to the gym because she would like to lose some weight. Her hips measured 121 cm when she started. She tabulated the changes in her measurements over six weeks. The results are shown in the following table.

End of	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
Centimetre change	$-1{,}5$ cm	$+0{,}5$ cm	-1 cm	-2 cm	$+1$ cm	$-0{,}5$ cm

   a) What was her actual measurement at the end of each week?
   b) What was her final measurement at the end of the six weeks?
   c) Write down the change in her measurements over the whole period as an integer.

5. A well is a structure created in the ground by digging, boring or drilling to access groundwater in underground aquifers. For example, Woodingdean Well was dug by hand between 1858 and 1862 and is 392 m deep. It is thought to be the world's deepest hand-dug well.
   a) Write the depth of the well as an integer.
   b) If there is 260 m of water in the well, use integers to indicate the surface of the water in the well.
   c) If they stopped digging when the well was half the depth it is now, how deep would the well have been? Express your answer as an integer.

6. Below is Mr Mwambakana's bank account statement. Calculate his balance. Explain his financial position in your own words. If you were his financial advisor, what advice would you give him?

Credit Card Account	Date	Description	Amount (R)
Mr T. Mwambakana 10 North Street Southwin 0181	1/10	Balance brought forward	825,87 –
	2/10	Deposit Soshanguve ATM	1 130,00
	5/10	Pick n Pay	263,79 –
	8/10	Cash Withdrawal Pretoria North ATM	400,00 –
	14/10	Edgars Clothing	299,99 –
	14/10	Mr Price Clothes	139,98 –
	21/10	Cash Withdrawal Pretoria North ATM	250,00 –
	22/10	Spar	315,69 –
	24/10	Deposit Internet Acc 422 3257 6831	1 450,00
	25/10	Vodacom Direct	650,00 –
	27/10	Cash Withdrawal Soshanguve ATM	200,00 –
	29/10	Shoperama	299,00 –
	31/10	Pick n Pay	89,45 –
		Balance:	

Topic 3 Common fractions

In this topic you will learn to:

- revise addition, subtraction, multiplication and division of common fractions
- revise calculations with squares, cubes, square roots and cube roots of common fractions
- revise equivalent fractions between common fractions
- revise equivalent fractions between common fractions; decimal fractions and percentage forms of the same number.

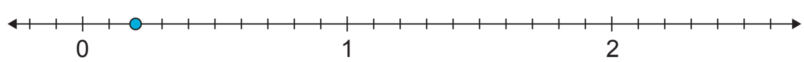
What you already know

1. Convert the common fraction to a mixed number: $\frac{12}{5}$
2. Convert the mixed number to a common fraction: $3\frac{3}{7}$
3. Calculate:
   a) $4\frac{3}{4}$ of 12 b) $\frac{1}{3} \div 12$

Unit 1 Calculation techniques

Convert between common fractions and mixed numbers

Common fractions that have a numerator smaller than the denominator lie between 0 and 1 on the number line and are called proper fractions. For example: $\frac{1}{5}$



A **mixed number** has a whole number part and a fraction part. We call fractions with a numerator bigger than the denominator **improper fractions**. We can always write improper fractions as mixed numbers. Mixed numbers always lie to the right of 1 on a number line, because they are greater than 1. Let's see how to convert between improper fractions and mixed numbers.

Example

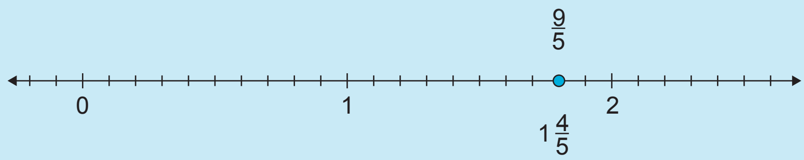
Convert the improper fraction $\frac{9}{5}$ to a mixed number.

Solution

We know that $\frac{9}{5} = \frac{5}{5} + \frac{4}{5} = 1 + \frac{4}{5}$. Therefore the answer is $1\frac{4}{5}$.

Example
Convert the mixed number $1\frac{4}{5}$ to an improper fraction.

Solution
$1\frac{4}{5} = 1 + \frac{4}{5} = \frac{5}{5} + \frac{4}{5} = \frac{9}{5}$
To find the total number of fifths, we multiply the 1 by 5. This gives us the number of fifths in 1 whole. Then, we add the other 4 fifths. This gives 9 fifths, so the answer is $\frac{9}{5}$.



Equivalent fractions and fractions in simplest form

Equivalent fractions are fractions that have the same value, even though they look different. To find equivalent fractions, we multiply the numerator and denominator by the same number. For example, we can multiply by $\frac{2}{2}$, because $\frac{2}{2} = 1$. We know that 1 is the multiplicative identity. This means that multiplying any number by 1 leaves the number unchanged.

Example
$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{3 \times 3}{5 \times 3} = \frac{3 \times 4}{5 \times 4} \ldots$
These calculations give the equivalent fractions: $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20}$. We use equivalent fractions when adding and subtracting fractions. Then, we simplify the answer when possible so that it is in simplest form.

Example
Find three equivalent fractions to $\frac{5}{8}$.

Solution
$\frac{5}{8} \times \frac{2}{2}, \frac{5}{8} \times \frac{3}{3}, \frac{5}{8} \times \frac{4}{4}$

$\frac{10}{16}, \frac{15}{24}, \frac{20}{32}$

We can now say that $\frac{5}{8} \underset{\times \mathbf{4}}{\overset{\times \mathbf{4}}{=}} \frac{20}{32}$

Now let's work backwards to simplify the fraction. We start with $\frac{20}{32}$. We already know that the simplest form of this fraction is $\frac{5}{8}$. But how do we calculate that? When we worked out the equivalent fractions, we *multiplied* both the numerator and denominator by the same number. Therefore, to simplify a fraction, we *divide* both the numerator and denominator by the same number.

To find the simplest form of a fraction, we need to find the highest common factor of the numerator and denominator.

The factors of 20: 1; 2; **4**; 5;10; 20
The factors of 32: 1; 2; **4**; 8; 16; 32
So the HCF of 20 and 32 is 4.
Therefore, we need to divide 20 and 32 by 4.

$$\frac{5}{8} = \frac{20}{32} \quad (\div \mathbf{4})$$

If we divide a numerator and denominator by the same number, we are dividing by 1. For example: $\frac{12}{15} = \frac{12}{15} \div 1 = \frac{12}{15} \div \frac{3}{3} = \frac{4}{5}$. (Remember: 1 = $\frac{3}{3}$)

Example

Write $\frac{132}{96}$ in its simplest form.

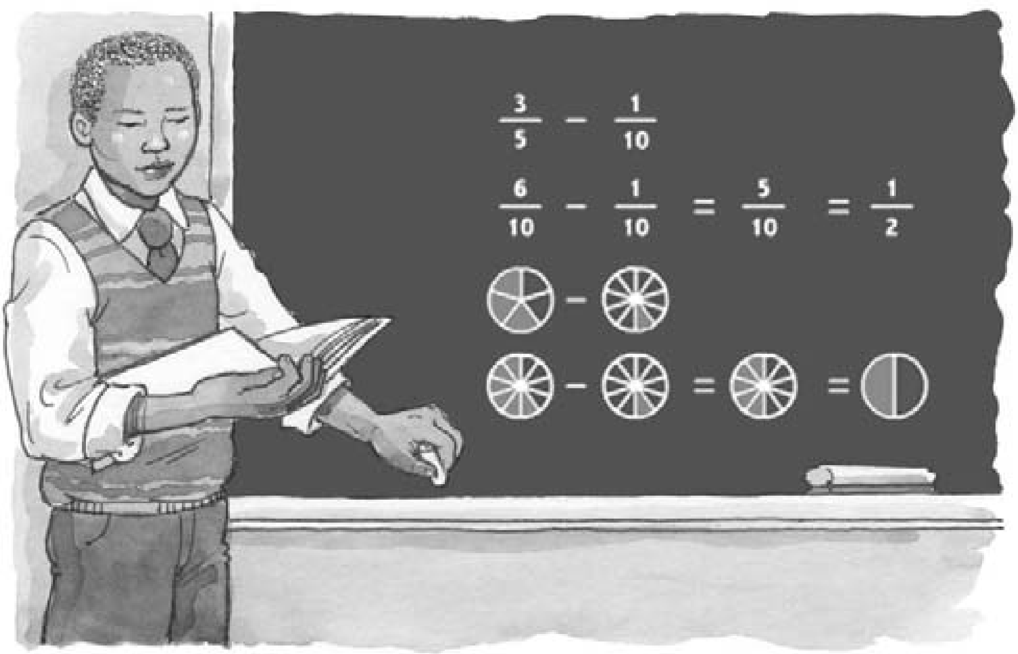
Solution

$\frac{132}{96} = \frac{11}{8}$ (Divide numerator and denominator by the HCF: 12)

Add, subtract, multiply and divide with common fractions

Add and subtract fractions

To add and subtract fractions, we first need to make their denominators the same. If the denominators are not the same, we find a common denominator by using the lowest common multiple (LCM). In this way, we write the fractions as equivalent fractions.



Example

Calculate: $\frac{1}{2} + \left[4\frac{3}{4} - \left(3\frac{1}{6} - 2\frac{1}{3}\right)\right]$

Solution

Method 1:

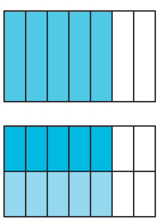
$$\begin{aligned}\frac{1}{2} + \left[\frac{19}{4} - \left(\frac{19}{6} - \frac{7}{3}\right)\right] &= \frac{6}{12} + \left[\frac{57}{12} - \left(\frac{38}{12} - \frac{28}{12}\right)\right]\\ &= \frac{6}{12} + \left(\frac{57}{12} - \frac{38}{12} + \frac{28}{12}\right)\\ &= \frac{6 + 57 - 38 + 28}{12}\\ &= \frac{53}{12}\\ &= 4\frac{5}{12}\end{aligned}$$

Method 2:

$$\begin{aligned}\frac{1}{2} + \left[4\frac{3}{4} - \left(3\frac{1}{6} - 2\frac{1}{3}\right)\right] &= \frac{1}{2} + \left[4\frac{3}{4} - \left(1\frac{1}{6} - \frac{1}{3}\right)\right]\\ &= \frac{1}{2} + \left[4\frac{3}{4} - \left(\frac{7}{6} - \frac{2}{6}\right)\right]\\ &= \frac{1}{2} + \left(4\frac{3}{4} - \frac{5}{6}\right)\\ &= \frac{1}{2} + \left(\frac{19}{4} - \frac{5}{6}\right)\\ &= \frac{6}{12} + \left(\frac{57}{12} - \frac{10}{12}\right)\\ &= \frac{53}{12}\\ &= 4\frac{5}{12}\end{aligned}$$

Multiply fractions

Remember that the word 'of' means you have to multiply. To understand $\frac{1}{2}$ of $\frac{5}{7}$ you can use a diagram. First, illustrate $\frac{5}{7}$ by dividing the whole figure into sevenths. Then, shade five of the sections (the first 5 columns in the figure). To find $\frac{1}{2}$ of the shaded area, divide the seven parts into two equal parts. Then, shade one part again. The whole is now divided into 14 equal parts of which 5 parts are double shaded.



Divide a whole into 7 equal parts. Then, shade 5 of the 7 parts to represent $\frac{5}{7}$.

Now calculate $\frac{1}{2}$ of the shaded area.

Divide the shaded area in two. Then, shade one of those parts in a darker shade. Now we have $\frac{1}{2}$ of $\frac{5}{7} = \frac{5}{14}$.

Mathematically, we multiply the numerators and multiply the denominators. Then we simplify the answer if we can. For example: $\frac{1}{2} \times \frac{5}{7} = \frac{1 \times 5}{2 \times 7} = \frac{5}{14}$.

Sometimes it is easier to first simplify the fractions before multiplying the numerators and the denominators.

Example

Calculate $5\frac{2}{3}$ of 9.

Solution

$$\begin{aligned} 5\frac{2}{3} \text{ of } 9 &= \frac{17}{3} \times \frac{9}{1} \quad \text{(Divide both 9 and 3 by the HCF, which is 3)} \\ &= \frac{17}{1} \times \frac{3}{1} \\ &= 51 \end{aligned}$$

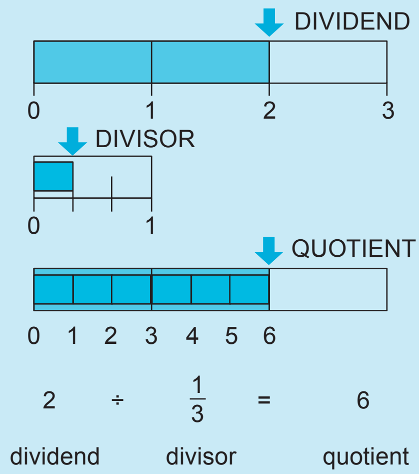
Divide fractions

Example

Calculate: $2 \div \frac{1}{3}$

Solution

We have two whole units and want to divide it into thirds. We know there are 3 thirds in one whole. So there will be 6 thirds in 2 units.



The next example uses the multiplicative inverse (or reciprocal) of a fraction. Remember, multiplying a number by its multiplicative inverse is always equal to 1. In the case of a fraction, we find the multiplicative inverse by swapping the numerator and the denominator. For example, the multiplicative inverse of $\frac{5}{6}$ is $\frac{6}{5}$, because $\frac{5}{6} \times \frac{6}{5} = 1$.

Example

Calculate: $\frac{1}{2} \div 3$

Solution

Think about sharing half a pizza among three people. This means each person will get a sixth of the pizza. Mathematically, we work this out as:

$$\begin{aligned} \frac{1}{2} \div \frac{3}{1} &= \frac{1}{2} \times \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

Example

Calculate: $5\frac{5}{6} \div \frac{5}{8}$

Solution

$$\begin{aligned}
5\tfrac{5}{6} \div \tfrac{5}{8} &= \frac{35}{6} \div \frac{5}{8} \\
&= \frac{35}{6} \times \frac{8}{5} \\
&= \frac{35 \div 5}{6 \div 2} \times \frac{8 \div 2}{5 \div 5} \\
&= \frac{7}{3} \times \frac{4}{1} \\
&= \frac{28}{3} \\
&= 9\tfrac{1}{3}
\end{aligned}$$

(Simplify fractions: HCF of 6 and 8 is 2 and HCF of 35 and 5 is 5)

Example

Calculate:

a) $6\frac{1}{4} - \left(11\frac{3}{8} + 5\frac{1}{2}\right) \div 5 \times 2$

b) $\frac{\sqrt{16}}{5} \div \frac{2^5}{5^2}$

Solution

a)
$$\begin{aligned}
&\frac{25}{4} - \left(\frac{91}{8} + \frac{11}{2}\right) \div 5 \times 2 \\
&= \frac{25}{4} - \frac{135}{8} \times \frac{1}{5} \times \frac{2}{1} \\
&= \frac{25}{4} - \frac{27}{4} \\
&= \frac{-2}{4} \\
&= -\frac{1}{2}
\end{aligned}$$

b)
$$\begin{aligned}
\frac{4}{5} \div \frac{32}{25} &= \frac{4}{5} \times \frac{25}{32} \\
&= \frac{1}{1} \times \frac{5}{8} \\
&= \frac{5}{8}
\end{aligned}$$

Exercise 1

1. Calculate the following (you may use your calculator, but show all calculations):

a) $6\frac{1}{2} + 4\frac{1}{3} + 1\frac{1}{2} + \frac{2}{3}$

b) $72\left(\frac{1}{6} + \frac{1}{4}\right)$

c) $\left(37 \times \frac{1}{8}\right) + \left(3 \times \frac{1}{8}\right)$

d) $\frac{63}{7} - \frac{26 - 2}{4}$

e) $\frac{12(9 + 1)}{4} - 5 \times 3$

f) $\left(1\frac{1}{5} + 3\frac{3}{5}\right) \times 1\frac{1}{4}$

g) $3\frac{1}{2} - 2\frac{1}{4} \div \frac{3}{4}$

h) $2\frac{5}{8} \div \frac{1}{2}$ of $1\frac{3}{4}$

i) $\frac{3}{8} \times 5\frac{2}{3} - \frac{1}{2} \times 1\frac{1}{5}$

j) $8\frac{1}{5} + 9\frac{3}{10} - 4\frac{1}{2}$

k) $\left(13 - 1\frac{1}{8}\right) \div 4\frac{3}{4}$

l) $1\frac{1}{3} \times 3\frac{1}{2} \div 4\frac{2}{3}$

m) $2\frac{2}{3} - \frac{2}{3} \times 2$

n) $4\frac{3}{7} + \left(2\frac{4}{21} + 1\frac{1}{3}\right)$

o) $\left(1\frac{1}{2} - \frac{7}{8}\right) + \frac{3}{4} \times 1\frac{1}{3}$

2. Calculate the following:

a) $\left(\frac{2}{5}\right)^2 + \sqrt[3]{27}$

b) $\left(\frac{1}{5}\right)^3 - \left(\frac{3}{5}\right)^2$

c) $\sqrt[3]{\frac{1\,000}{64}} \times \left(\frac{4}{3}\right)^2 + \frac{2}{3}$

d) $\frac{2^2}{3} - \frac{1^3}{-3}$ e) $\left(\frac{3}{4}\right)^2 \times \left(\frac{1}{-2}\right)^2$ f) $\left(\frac{1}{3}\right)^2 \div \left(\frac{-1}{3}\right)^2$

g) $\sqrt{\frac{100}{81} \times \frac{121}{25}}$ h) $\left(\frac{4}{3}\right)^3 \div \frac{8}{3} - \frac{2^3}{3}$ i) $\sqrt{\frac{16}{25} + \frac{9}{25}}$

Unit 2 Solving problems

A fraction is a number that represents part of a whole. Fractions are useful when you want to split things up or want to use only a part of a whole. For example, we work with fractions in recipes, when buying food, sewing, building a house, in medicine, and so on.

Example

Batseba invites friends over on Friday evening for a braai. There are 22 people altogether, but her recipe for potato and bean salad only serves four people. She has enough of everything in her house except for the salt. The recipe requires $\frac{1}{2}$ teaspoon of salt. Calculate how much salt she needs to be able to serve 22 people.

Solution

She needs to make the recipe $\frac{22}{4}$ times. Therefore, Batseba needs:

$$\frac{1}{2} \times \frac{22}{4} = \frac{11}{4}$$
$$= 2\tfrac{3}{4} \text{ teaspoons of salt}$$

Example

Nosisi's little sister has 7 of 10 collectable dolls. Therefore, she has collected $\frac{7}{10} \times \frac{100}{1} = 70\%$ of the range.

Example

A town had 864 mm of rain in a year. The next year, the figure dropped to 734 mm. Calculate the percentage decrease.

Solution

$$\text{Percentage decrease} = \frac{\text{amount decreased}}{\text{initial amount}} \times \frac{100}{1}$$
$$= \frac{130}{864} \times \frac{100}{1}$$
$$\approx 15\%$$

Exercise 2

1. In a Grade 9 class of 36 learners, three were absent because of flu. What fraction of the class was:
   a) absent b) present?
2. Ahsan spends $1\frac{1}{4}$ hours doing homework. Of this, she spends 25 minutes on Mathematics. What fraction of her homework time does she spend on Mathematics?
3. What fraction is:
   a) 36 sec of 5 min b) 36 sec of 5 hours c) $3\frac{5}{6}$ hours of 1 day
   d) 50 ml of 3 ℓ e) 50 ml of 3 kl f) 450 mm of 8 m
   g) 450 mm of 9 km h) 72c of R18 i) 125 g of 15 kg
4. Saju is 12 years old and his mother is 36 years old.
   a) What fraction is Saju's age of his mother's age?
   b) What fraction will Saju's age be of his mother's age in 54 years' time?
   c) Are the fractions in a) and b) equal? If not, which fraction is greater?
5. In an examination, Stephen scores 88 marks, Palesa 72 marks, Sarah 60 marks and Lebogang 76 marks. The examination is out of 100 marks.
   a) What fraction of the total marks did each receive?
   b) Arrange the fractions in descending order.
   c) Convert all marks to a mark out of 25.
6. Isaac and Naledi have a house on $\frac{1}{8}$ of a hectare of land. Of the land, $\frac{1}{3}$ of a hectare is planted with grass.
   a) How much of the land is not planted with grass?
   b) There are 10 000 m^2 in 1 hectare. How many square metres are not planted with grass?

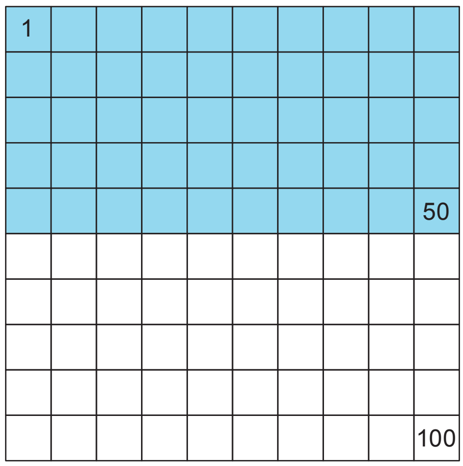
7. A plumber has a pipe $5\frac{3}{16}$ m long for a fitting. He only needs a pipe $3\frac{7}{8}$ m, so he cut the pipe. How much of the pipe does he cut off?
8. Saahir earned R$85\frac{1}{4}$ per hour working for eight hours on Friday. He also earned overtime pay, which is $1\frac{1}{2}$ times his regular rate of R$85\frac{1}{4}$. He worked four hours of overtime on Friday. How much pay did he earn altogether on Friday?

9. A fishing boat travelled $69\frac{3}{8}$ km in $3\frac{3}{4}$ hours. At what speed (in kilometres per hour) did the boat cruise?
10. Rebecca has a few 8 m long pieces of material. She wants to use this material to make and sell tablecloths. She wishes to make two sets, each with three tablecloths. Each tablecloth will be $3\frac{1}{4}$ m long.
    a) How many of these 8 m long material pieces does she need?
    b) How many metres of leftover material will she have?
    c) If she had to buy the material in one running length, how many metres would she need to buy?

11. What percentage did Joyce get for her test if she got 27 problems right out of a possible 45?
12. A couple enjoyed a meal at a restaurant. They then wanted to tip the waiter 12% of the amount of the bill. Their bill came to R$212\frac{1}{4}$. How much money will the waiter receive?
13. Calculate the percentage increase if the price of a cricket bat increases from R250 to R285.
14. A watch is advertised at R1 190. After Christmas, the price drops to R950. What is the percentage reduction in the price of the watch?

Unit 3 Equivalent forms of fractions

Per cent actually means **per 100**, or out of 100. In the figure below, 50% of the large square is blue. The large square is divided into 100 small squares, of which 50 are blue. This represents a fraction of $\frac{50}{100}$.



Example

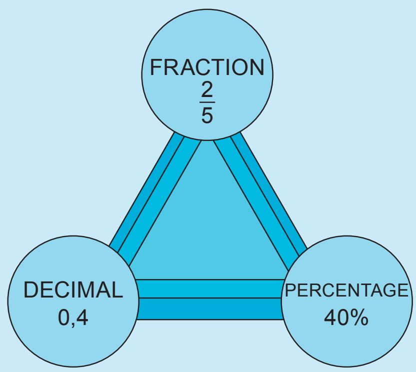
If you colour $\frac{2}{5}$ of the square above, what percentage will that be?

Solution

You need to write $\frac{2}{5}$ as an equivalent fraction with a denominator of 100.

Therefore: $\frac{2 \times 20}{5 \times 20} = \frac{40}{100}$. So $\frac{2}{5} = 40\%$.

We can also write this as: $\frac{2}{5} \times \frac{100}{1} = 40\%$.



Equivalent forms

Common fractions, decimal fractions and percentages are equivalent forms of the same number. We use equivalent forms, for example, when working out a percentage for a test. If you achieve $\frac{20}{25}$ for your Mathematics test, your teacher will write your result as 80%. Or, suppose you need to calculate a fraction of your pocket money. For example, you know you only have $\frac{3}{5}$ of R100,75 to spend on a night out with friends. Another example might be that you only need $\frac{3}{4}$ of a 3,5 m long piece of ribbon to tie around a present.

Example

Write $\frac{9}{4}$ as a percentage: $\frac{9 \times 25}{4 \times 25} = \frac{225}{100}$

$= 225\%$

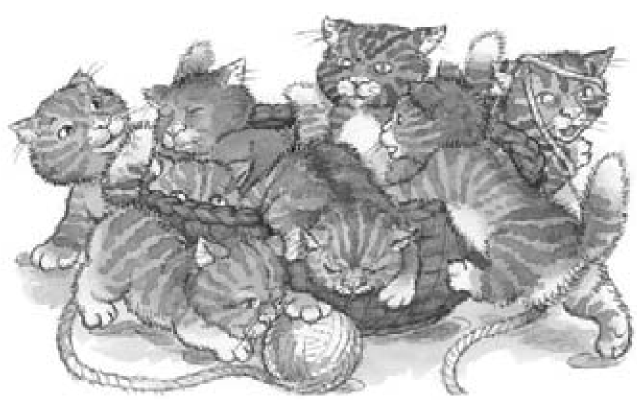
Write 225% as a decimal number: 2,25.

Exercise 3

1. Express each percentage as a fraction and simplify where possible:
   a) 7% b) 64% c) 0,4%
   d) 245% e) $9\frac{3}{8}\%$
2. Express these common fractions as percentages:
   a) $\frac{31}{50}$ b) $\frac{4}{25}$ c) $\frac{7}{20}$ d) $\frac{7}{8}$
3. What percentage is:
   a) 42 mm of 11 cm? b) 280 g of 35 kg? c) 1 600 ml of 5 ℓ?
4. Calculate the following:
   a) 18% of 840 ml b) 60% of R2 300 c) 24% of 175 kg
5. Complete the following table of equivalent notations.

Percentage	Fraction	Equivalent fraction	Decimal	Percentage	Fraction	Equivalent fraction	Decimal
5%	$\frac{5}{100}$		0,05		$\frac{50}{100}$		0,5
10%	$\frac{10}{100}$	$\frac{1}{10}$		60%		$\frac{3}{5}$	
20%			0,2	70%	$\frac{70}{100}$		
25%		$\frac{1}{4}$				$\frac{3}{4}$	0,75
30%	$\frac{30}{100}$			80%			0,8
		$\frac{2}{5}$	0,4	90%		$\frac{9}{10}$	

6. There are 45 cars parked in a parking lot, of which 10 are red.
   a) What fraction is the number of red cars of the total number of cars?
   b) What percentage of the cars is red?
7. Rithika's cat had eight kittens. Six of the kittens were male.
   a) What fraction of the kittens is female?
   b) What percentage of the kittens is female?
   c) What percentage is male?



Topic 4 Decimal fractions

In this topic you will learn to:

- revise place value and rounding off decimal fractions
- revise doing calculations using all four operations with decimal fractions
- revise doing calculations that involve powers and radicals of decimal fractions
- revise equivalent forms between common fractions, decimal fractions and percentages.

What you already know

1. Arrange the numbers in order of size, from smallest to largest:
   a) 0,02; 0,002; 0,202; 0,022   b) 6,006; 6,066; 6,666; 6,606
2. Fill in $<$, $>$ or $=$ between each pair of numbers:
   a) 0,398 * 0,113   b) 0,732 * 0,065   c) 0,814 * 0,767
3. Complete the extract from a number grid:

13,205	13,23	13,255							13,43
13,455		13,505	13,53				13,63		

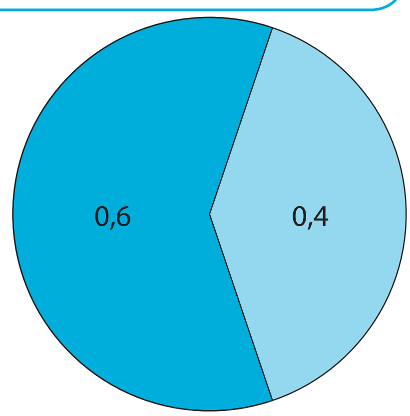
4. Complete the table by rounding off the given numbers:

Number	To one decimal place	To two decimal places	To three decimal places
18,3782			
4,37285			

5. Calculate:
   a) $3{,}11 + 2{,}02 + 93{,}01$ b) $1{,}44 + 0{,}712 + 6{,}08$
   c) $1\ 227{,}536 - 34{,}6094$ d) $1{,}059 - 0{,}0392$

Unit 1 Calculations with decimal fractions

The word decimal comes from the Latin word *decem*, which means 10. Our number system is a base 10 system. We use decimals when measuring length, capacity, mass and so on, but also when we work with money.



Decimal numbers are very similar to mixed fractions. All the digits to the left of the decimal comma represent a whole number or an integer. All the digits to the right of the decimal comma represent a common fraction where the numerator is smaller than the denominator. Take 8,125:

Integer part fractional part

$$8 + 0{,}125$$

$$8 + \frac{125}{1\,000}$$

The place value table

Consider the decimal table in the next example. As you move from right to left in the table, each digit becomes 10 times bigger. As you move from left to right, each column is a tenth of the previous column. Each digit in a number has a place value and a value. We use the place values of digits in decimal numbers to round off decimal numbers.

Example

In the number 68,492, the place value of **9** is hundredths $\left(\frac{1}{100}\right)$. The value of **9** is $\frac{9}{100}\left(9 \times \frac{1}{100}\right)$

$\times$ 10 $\times$ 10 $\times$ 10 $\times$ 10 $\times$ 10

Hundreds (H)	Tens (T)	Units (U)	,	tenths (t)	hundredths (h)	thousandths (th)
100	10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1\,000}$
	6	**8**	,	**4**	**9**	**2**

$\div$ 10 $\div$ 10 $\div$ 10 $\div$ 10 $\div$ 10

Converting between different forms

Example

1. Write the following decimal fractions as common fractions:
   a) 0,008 b) 0,0023 c) 0,00025
2. Write the following common fractions as decimal fractions:
   a) $\frac{54}{100}$ b) $\frac{3}{1\,000}$ c) $\frac{54}{10\,000}$

Solution

1. a) $0{,}008 = \frac{8}{1\,000}$ b) $0{,}0023 = \frac{23}{10\,000}$

   c) $0{,}00025 = \frac{25}{100\,000}$

2. a) $\frac{54}{100} = 0{,}54$ b) $\frac{3}{1\,000} = 0{,}003$ c) $\frac{54}{10\,000} = 0{,}0054$

Rounding off decimal numbers

When we round off a number, we reduce the number of digits in the number. At the same time, we try to keep the value of the number similar to the original value. The result is a less accurate number, but it is easier to use. For example, $0{,}\underline{4}92$ rounded off to the nearest tenth is 0,5,while $0{,}4\underline{9}2$ rounded off to the nearest hundredth is 0,49. It is often useful to round off values to estimate an answer to a calculation. That way, we have some idea of how accurate our answer is. In other words, rounding off helps us to check our answers.

Adding and subtracting decimal numbers

We add and subtract decimal numbers in the same way that we add and subtract whole numbers. In other words, we write the digits with the same place values in columns. Then, we add or subtract each column, borrowing or carrying when necessary.

Example

Calculate: 23,4589 – 17,0842

	$\cancel{2}^1$	^{13}	,	$\cancel{4}^3$	^{15}	8	9
–	1	7	,	0	8	4	2
		6	,	3	7	4	7

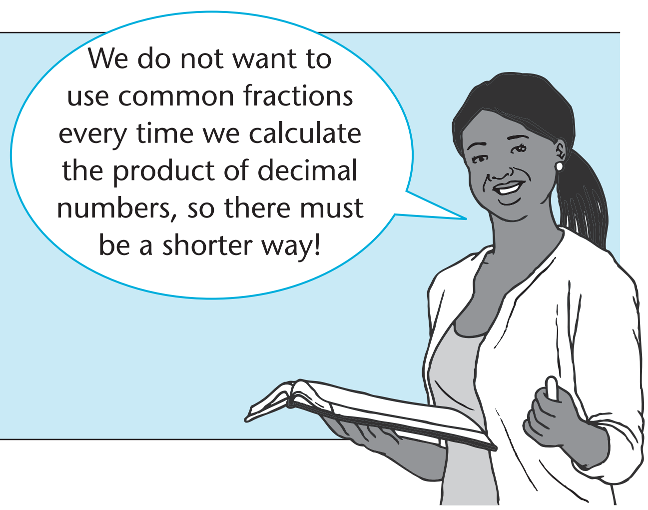
Multiplying decimal numbers

To multiply decimal numbers, first convert the decimal numbers to common fractions. Then, multiply the fractions in the normal way.

Example

$$0{,}4 \times 0{,}003 = \frac{4}{10} \times \frac{3}{1\,000}$$
$$= \frac{12}{10\,000}$$
$$= 0{,}0012$$

$$0{,}012 \times 0{,}11 = \frac{12}{1\,000} \times \frac{11}{100}$$
$$= \frac{132}{100\,000}$$
$$= 0{,}00132$$



Use your calculator to work out patterns such as the following. Can you explain what happens when we multiply decimal numbers?

$4 \times 3 = \mathbf{12}$	$12 \times 11 = \mathbf{132}$
$0{,}4 \times 3 = \mathbf{1{,}2}$	$1{,}2 \times 11 = \mathbf{13{,}2}$
$0{,}4 \times 0{,}3 = 0{,}\mathbf{12}$	$0{,}12 \times 11 = \mathbf{1{,}32}$
$0{,}4 \times 0{,}03 = 0{,}0\mathbf{12}$	$1{,}2 \times 0{,}11 = 0{,}\mathbf{132}$
$0{,}4 \times 0{,}003 = 0{,}00\mathbf{12}$	$0{,}12 \times 0{,}11 = 0{,}0\mathbf{132}$
	$0{,}012 \times 0{,}11 = 0{,}00\mathbf{132}$

The rules for multiplying decimal numbers:
- Ignore the decimal numbers and multiply the numbers as normal.
- The number of decimal places in the answer is equal to the sum of the decimal places in each number being multiplied.
- Insert the decimal comma after working out the answer.

For example, let's calculate $0{,}012 \times 0{,}11$.
- Calculate $12 \times 11 = 132$
- The answer has $3 + 2 = 5$ decimal places.
- Therefore, the answer is 0,00132.

Example

1. How many decimal places are there in the product of 8,283 and 2,05?
2. Calculate: $8{,}283 \times 2{,}05$ by converting both numbers to common fractions.
3. Check your answer in Question 2 using your calculator.
4. Estimate your answer by rounding off the numbers to the nearest unit.

Solution

1. 8,283 has three decimal places and 2,05 has two decimal places. The product will have five decimal places.
2. $$\begin{aligned} 8{,}283 \times 2{,}05 &= \frac{8\,283}{1\,000} \times \frac{205}{100} \\ &= \frac{1\,698\,015}{100\,000} \\ &= 16{,}98015 \end{aligned}$$
3. $8{,}283 \times 2{,}05 = 16{,}98015$
4. $\underline{8}{,}283 \times \underline{2}{,}05 \approx 8 \times 2 = 16$

Dividing decimal numbers

Let's first revise how to divide a decimal number by a whole number. Remember to put the comma of the quotient above the comma of the dividend.

Example

Calculate: $0{,}3488 \div 2$.

Solution

$$
\begin{array}{r}
0{,}1744 \\[-3pt]
2\overline{)\,0{,}3488} \\
-\underline{2} \\
14 \\
-\underline{14} \\
08 \\
-\underline{8} \\
08 \\
-\underline{8} \\
0
\end{array}
$$

Dividing a decimal by a decimal is a bit more complicated. There are two ways to solve this problem:

- Convert the decimal numbers to common fractions. Then calculate in the normal way.

Or

- Change the divisor to a whole number. If the divisor has one decimal place, then we need to multiply both numbers by 10. This keeps the answer the same, but we can divide by a whole number.

Example

Calculate: $0{,}72 \div 0{,}8$

Solution

- We can convert the decimal numbers to common fractions:

$$
\begin{aligned}
\frac{72}{100} \div \frac{8}{10} &= \frac{72}{100} \times \frac{10}{8} \\
&= \frac{9}{10} \\
&= 0{,}9
\end{aligned}
$$

- Make the divisor a whole number by multiplying both numbers by 10:

$$
\begin{aligned}
0{,}72 \div 0{,}8 &= 7{,}2 \div 8 \\
&= 0{,}9
\end{aligned}
$$

Example

a) Calculate $1{,}445 \div 0{,}17$ using long division.
b) Check your answer by converting the decimal numbers to common fractions.
c) Check your answer by using a calculator.

Solution

a) $1{,}445 \div 0{,}17 = 144{,}5 \div 17$ (Multiply both numbers by 100.)

$$
\begin{array}{r|l}
 & \;\;\;\;\;\;8\;,\;5 \\
\cline{2-2}
17 & 1\;4\;4\;,\;5 \\
 & -1\;3\;6 \\
\cline{2-2}
 & \;\;\;\;\;\;8\;\;\;5 \\
 & \;\;\;-8\;\;\;5 \\
\cline{2-2}
 & \;\;\;\;\;\;\;\;\;\;\;0
\end{array}
$$

b) $1{,}445 \div 0{,}17 = \frac{1\,445}{1\,000} \div \frac{17}{100}$

$= \frac{1\,445}{1\,000} \times \frac{100}{17}$

$= \frac{85}{10}$

$= 8{,}5$

c) $1{,}445 \div 0{,}17 = 8{,}5$

There is often more than one way to solve a problem. Practise all of them until you find the ones you prefer.

Example

Calculate: $0{,}0023 \div 0{,}46$

Solution

To make 0,46 (the divisor) a whole number, multiply it by 100. Then, we must also multiply 0,0023 by 100 so that the value of the expression does not change. Therefore:

$0{,}0023 \div 0{,}46 = 0{,}23 \div 46$

$$
\begin{array}{r|l}
 & 0\;,\;0\;0\;5 \\
\cline{2-2}
46 & 0\;,\;2\;3\;0 \\
 & \;\;\;-2\;3\;0 \\
\cline{2-2}
 & \;\;\;\;\;\;\;\;\;\;\;0
\end{array}
$$

Converting a recurring decimal number to a common fraction

A recurring decimal number is a decimal number in which the digits after the comma form a recurring pattern. The pattern never ends. We say the pattern continues to infinity. For example: $0{,}\dot{3}$ means 0,333…; $0{,}\dot{2}\dot{3}$ means 0,232323… and $3{,}\dot{2}4\dot{5}$ means 3,245245245… These numbers are rational numbers. This means we can convert them to common fractions in the form $\frac{a}{b}$; $b \neq 0$.

Example

Convert $0,\dot{4}\dot{5} = 0,454545\ldots$ to a common fraction.

Solution

Let $x = 0,454545$ .... (1)

$\therefore 100x = 45,454545$........................ (2)

Because two digits repeat, we multiply by 100. This moves one set of repeating digits to the left of the comma.

Now subtract: (2) – (1):

$100x - x = 45,4545454545\ldots - 0,45454545\ldots$

$99x = 45$

$x = \frac{45}{99}$

$= \frac{5}{11}$ (Divide by $\frac{9}{9} = 1$.)

$\therefore 0,\dot{4}\dot{5} = \frac{5}{11}$

Squares, cubes, square roots and cube roots

We know that the square of five is $5^2 = 5 \times 5 = 25$. The opposite operation enables us to find the square root of 25: $\sqrt{25} = 5$. Similarly, we can find the cube of four: $4^3 = 4 \times 4 \times 4 = 64$. This enables us to find the cube root of 64: $\sqrt[3]{64} = 4$. Now work through the following examples:

- $8^2 = 8 \times 8 = 64$ Implies that $\sqrt{64} = 8$
- $(0,8)^2 = 0,8 \times 0,8 = 0,64$ Implies that $\sqrt{0,64} = 0,8$
- $(0,08)^2 = 0,08 \times 0,08 = 0,0064$ Implies that $\sqrt{0,0064} = 0,08$
- $(0,1)^3 = 0,1 \times 0,1 \times 0,1 = 0,001$ Implies that $\sqrt[3]{0,001} = 0,1$
- $(0,2)^3 = 0,2 \times 0,2 \times 0,2 = 0,008$ Implies that $\sqrt[3]{0,008} = 0,2$
- $\sqrt{0,0121} = 0,11$ Check: $0,11 \times 0,11 = 0,0121$
- $\sqrt[3]{0,000027} = 0,03$ Check: $0,03 \times 0,03 \times 0,03 = 0,000027$

Exercise 1

1. Convert the following recurring decimals to common fractions:
   a) 0,333… b) 0,777777… c) $0,\dot{6}$
   d) $3,\dot{1}\dot{8}$ e) $2,\dot{3}1\dot{2}$
2. Write down the place value and the value of each digit in the following numbers:
   a) 348,92 b) 87,063 c) 4,3926
3. Write the following numbers in expanded form:
   For example: $21,472 = 20 + 1 + \frac{4}{10} + \frac{7}{100} + \frac{2}{1\,000}$
   a) 238,94 b) 67,381 c) 0,457239
4. Calculate the following (without using a calculator):
   a) $2,16 - 3,8 + 4,54$ b) $0,095 - 0,0095$
   c) $177,573 + 87,439 - 196,73$ d) $38,85 + 61,08 + 8,85$
   e) $7,041 + 12,403 + 149,5$ f) $23,7 - 14,737$
   g) $-0,058 \div 10$ h) $-0,03 \times 1\,000$
   i) $28,56 \times 3,12$ j) $285,6 \times 0,0312$
   k) $-21,63 \div 0,48$ (long division) l) $0,2015 \div 0,031$ (long division)

5. Copy and complete the following (without using a calculator):

a) $8 \times 6 =$ ____
$0,8 \times 6 =$ ____
$0,8 \times 0,6 =$ ____
$0,8 \times 0,06 =$ ____
$0,08 \times 0,06 =$ ____
$0,08 \times 0,006 =$ ____

b) $12 \times (-12) =$ ____
$1,2 \times (-12) =$ ____
$0,12 \times (-12) =$ ____
$0,12 \times (-1,2) =$ ____
$0,012 \times (-0,12) =$ ____
$0,012 \times (-0,012) =$ _____

c) $96 \div 8 =$ _____
$9,6 \div 0,8 =$ _____
$0,96 \div 0,8 =$ _____
$0,096 \div 0,8 =$ _____
$0,096 \div 0,08 =$ _____
$0,00096 \div 0,08 =$ _____

6. In the following questions:
   i. Find the number of decimal places in the answer.
   ii. Estimate your answer by rounding off the numbers.
   iii. Calculate the answer without using a calculator.
   iv. Check your answer by using a calculator.

a) $0,39 + 0,011$ b) $53,095 - 42,0095$ c) $-0,39 \times 0,0034$
d) $17,643 \times 3,02$ e) $0,75 \div 0,003$ f) $36,8097 \div 4,231$

7. Use your calculator to do the following calculations. Round off the numbers to estimate the answer. Then, compare the estimate with the true answer.

a) $-3,786 \times 7,0013$ b) $-4,862 \times -1,0507$ c) $349,651 \times 0,005$

8. Calculate the following:

a) $(0,2)^2 + \sqrt{0,36}$ b) $\sqrt{0,0001} - (0,05)^2$
c) $(0,004)^2 + \sqrt[3]{0,000000064}$ d) $(1,2)^2 \times (0,009)^2$
e) $(0,15)^2 \div \sqrt{0,000169}$ f) $(-0,01)^2 \div (0,1)^3$
g) $(-0,003)^2 + (0,2)^3$ h) $\sqrt[3]{0,027} \times \sqrt[3]{0,000001}$

Unit 2 Solving problems

We use decimals in science, laboratory experiments, currency exchange, sports statistics, weight, pressure, gravity, finances, preparing meals, taxes, doses of medicine, tipping a waiter, and so on. We can go on forever because decimals are all around us! You may use your calculator in this unit, but show all your calculations.

Example

Nhlanhla decided to buy a television set for R1 999,00 on hire purchase over two years. He has to pay a 10% deposit. The shop charges him 15% p.a. simple interest in finance charges.

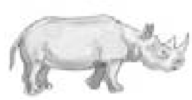
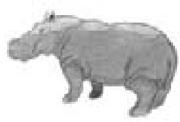
a) Calculate the monthly payment.
b) Calculate the total cost of the TV set on hire purchase.

Solution

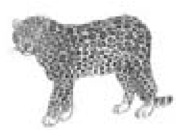
a)	Cash purchase price	R1 999,00
	Less 10% cash deposit	$\frac{10}{100} \times \frac{1\ 999,00}{1} = R199,90$
	Balance owing	R1 999,00 − R199,90 = R1 799,10
	Hire purchase period	2 years = 24 months
	Finance charges at 15% p.a. simple interest	$SI = \frac{1\ 799,10 \times 2 \times 15}{100} = R539,73$
	Total amount due after two years	R1 799,10 + R539,73 = R2 338,83
	Monthly payment	R2 338,83 ÷ 24 = R97,45
b)	Total cost of the TV set on hire purchase	R2 338,83 + R199,90 (deposit) = R2 538,73

Exercise 2

1. Researchers studied various animals at a nature reserve. They measured the average height and mass of the animals. This is what they found:

Animal	Height (in m)	Mass (in kg)
Rhino	2,05	2 268
Hippo	1,52	1 800
Elephant	3,45	3 200
Zebra	1,54	340
Lion	0,9	191
Leopard	0,86	75

a) List the animals, according to height, from the tallest to shortest. Write the name and height of each animal.
b) Calculate the total mass of all the animals.
c) Which is the lightest animal?
d) Which is the heaviest animal?
e) Calculate the difference in height between the tallest and shortest animal.

2. To save our natural resources, Rithika tries to conserve electricity. She knows the unit of electrical power is a kilowatt hour. One kilowatt is equal to 1 000 watts. To find the kilowatt hours, multiply the kilowatts by the number of hours. Electricity costs 38c per kilowatt hour. The fewer kilowatt hours she uses, the less she has to pay for electricity. Use the table to find the kilowatt hours used per day for each appliance. Then calculate the cost of electricity for each appliance.

Description	Time	Power rating (Watts)	Kilowatt hours	Cost in cents
Oven	2 h	1 000		
Electric frying pan	30 min	1 200		
Dishwasher	0,75 h	1 200		
Fridge	8 h	615		
Steam iron	4 h	1 750		
Energy saving globes	8 h	11		
Normal light globes	8 h	60		
Clock	24 h	2		
Total cost in rand				

3. In South Africa, the law states that 'the price you see is the price you pay'. This means that Value Added Tax (VAT) is included in the prices you see marked on products in the shops. Rupal buys the following items:

Item	**Cost in rand**
4 ℓ of milk	R16,80
2 ℓ ice cream	R29,99
1 kg frozen peas	R10,99
1 kg margarine	R12,79
800 g fish fingers	R18,99
1 kg rice	R11,69

a) How much does Rupal have to pay?
b) All items include VAT of 14%. How much VAT did he pay?
c) Suppose VAT increases from 14% to 16%. How much VAT would he pay now?
d) No VAT is charged on the margarine, milk and rice. How much VAT (14%) does he now pay overall?

4. Complete the following hire purchase agreement (see example on page 49):

Cash purchase price	R34 550,00
Less 10% cash deposit	
Balance owing	
Hire purchase period	48 months
Finance charges at 15% p.a. simple interest	
Total amount due	
Monthly payment	
Total cost on hire purchase	

5. The following table shows the exchange rates between the South African rand and different world currencies on 15 July 2012.

Currency	Exchange rate
British pound	0,0777
Euro	0,0988
Japanese yen	9,5923
US dollar	0,1200
Australian dollar	0,1184
Singapore dollar	0,1529
Swiss franc	0,1187

The table works in the following way:
- If you have R1 you can exchange it for 0,0777 of a British pound.
- If you have R1 you can exchange it for 9,5923 Japanese yen.

a) How much will the following cost (in rand)?
   i. 100 British pounds
   ii. 300 US dollars
   iii. 150 Singapore dollars

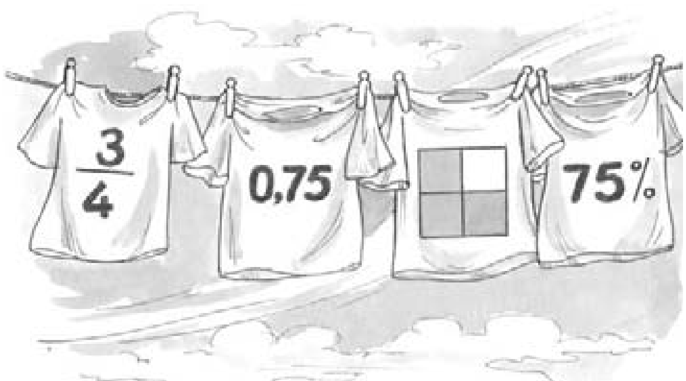
b) How many Japanese yen can you buy for R500?

c) Tshepang has been chosen to sing in the National Youth Choir. The choir is going on an overseas tour. Tshepang must pay for the following extra activities:
   - 10 British pounds for the underground trains in London
   - 65 British pounds for entrance fees to places of interest in London
   - 450 Euros to spend in Paris and Munich

   What is the rand value of the amount Tshepang will have to pay?

Unit 3 Equivalent forms

The figure below illustrates the different forms in which we can represent the same number. In this case, we have represented three-quarters as a common fraction, a decimal fraction, an illustration and as a percentage.



Example

Write $\frac{630}{125}$ as a decimal number by:

a) converting the denominator to 100

b) using division

c) using a calculator.

Solution

a) Find equivalent fraction with a denominator of 100:

$$\frac{630}{125} = \frac{630 \times 8}{125 \times 8} = \frac{5\ 040}{1\ 000} = \frac{504}{100} = 5{,}04$$

b)

$$\begin{array}{r|l} & 5\ ,\ 0\ \ 4 \\ \hline 1\ 2\ 5 & 6\ \ 3\ \ 0\ ,\ 0\ \ 0 \\ & -\ 6\ \ 2\ \ 5 \\ \hline & \quad\ \ 5\quad 0\ \ 0 \\ & \quad -\ 5\quad 0\ \ 0 \\ \hline & \qquad\qquad 0 \end{array}$$

c) 5,04

Example

Arrange the following fractions, decimals and percentages in descending order:

$\frac{33}{50}$; 0,72; 70%; 0,82; $\frac{17}{25}$

Solution

First convert all numbers to the same kind of number. In this case, we will convert all numbers to percentages:

66%; 72%; 70%; 82%; 68%

We can now place them in descending order:

82%; 72%; 70%; 68%; 66%

In their original form, the list is:

0,82; 0,72; 70%; $\frac{17}{25}$; $\frac{33}{50}$

Example

Write 2,228 as:

1. a common fraction with a denominator of 100
2. a percentage.

Solution

1. $\frac{2\ 228}{1\ 000} = \frac{2\ 228 \div 10}{1\ 000 \div 10}$

   $= \frac{222{,}8}{100}$ (Three digits after the comma represents thousandths)

2. $\frac{222{,}8}{100}$ means 222,8% (Or you can multiply 2,228 by 100)

Exercise 3

1. Arrange the following fractions, decimals and percentages in ascending order:
   a) 87,5%; 0,877; $\frac{17}{20}$; 87,05%; $\frac{22}{25}$
   b) $\frac{63}{10}$; 63%; 6,03; $\frac{635}{100}$
2. Fill in the empty blocks with the correct fraction, decimal or percentage:

Percentage	Decimal	Fraction
83%	0,83	$\frac{83}{100}$
40%		
		$\frac{3}{5}$
	0,34	
		$2\frac{9}{11}$
166%		
	0,08	
	1,25	

3. Kamala's shadow is 1,2 m long and her height is 1,6 m.
   a) What fraction is her shadow of her height?
   b) What percentage is her shadow of her height?
4. Moshe has an average of 64,8% in his July examination for Mathematics.
   a) How many marks did he get out of 100?
   b) Write his mark out of 100 as a fraction. Simplify the fraction.

Topic 5 Exponents

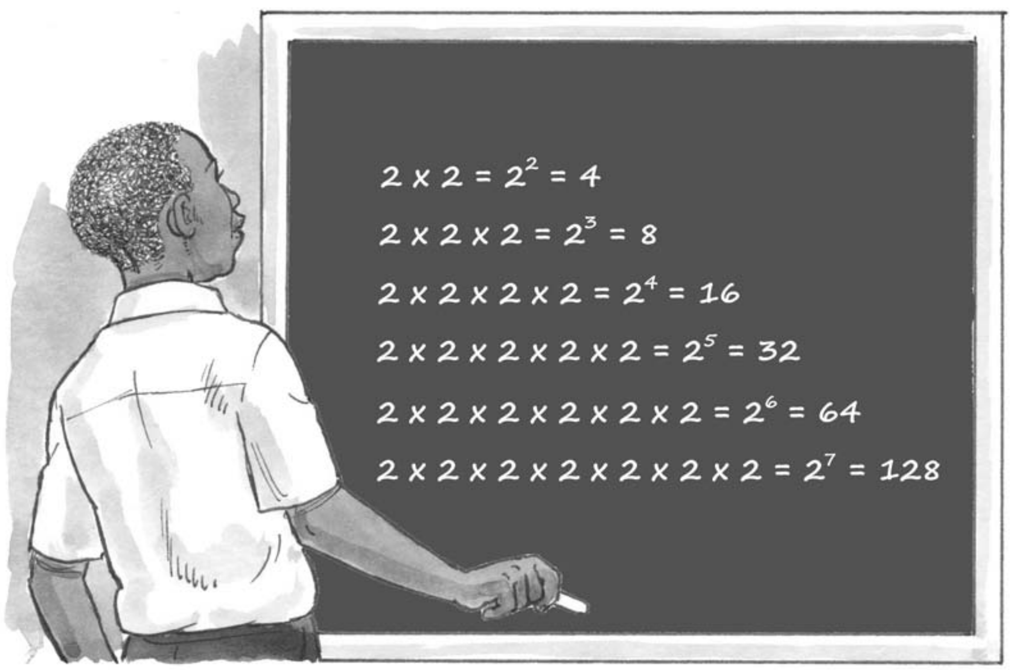
In this topic you will learn to:

- revise comparing and representing integers in exponential form
- revise comparing and representing numbers in scientific notation
- extend scientific notation to include negative exponents
- revise some general laws of exponents
- extend the general laws of exponents to include negative exponents
- perform calculations involving all four operations
- solve problems in contexts involving numbers in exponential form and scientific notation.

What you already know

1. Write in exponential form:
   a) $16 \times 16 \times 16$ b) $(-5) \times (-5)$ c) $z \times z \times z \times z$
2. Write in expanded form:
   a) 6^4 b) $(-11)^2$ c) $(x)^3$

Unit 1 Compare and represent numbers in exponential form



Represent integers in exponential form

Exponential form is a short way to write repeated multiplication.
For example:
$12 \times 12 \times 12 \times 12 \times 12 \times 12 = 12^6$.

power ← 12^6 → exponent or index

→ base

From the place value table in Topic 4, we can see the following about exponential forms and powers of 10:

$10 = 10^1$

$100 = 10 \times 10 = 10^2$

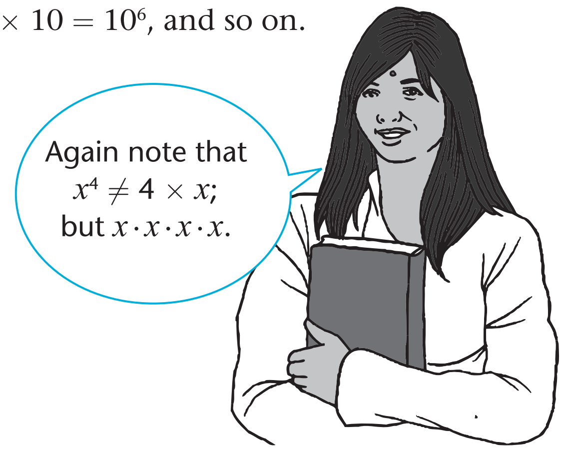
$1\ 000 = 10 \times 10 \times 10 = 10^3$

$10\ 000 = 10 \times 10 \times 10 \times 10 = 10^4$

$100\ 000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

$1\ 000\ 000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$, and so on.

Note that $10^6 \neq 10 \times 6$.



Instead of using numbers, we can also use variables to explain exponential notation.

$x = x^1$

$x \cdot x = x^2$

$x \cdot x \cdot x = x^3$

$x \cdot x \cdot x \cdot x = x^4$, and so on.

Example

The following table shows a few numbers written in exponential form. It then shows the same numbers written in expanded form and as a number. Notice how fast numbers in exponential form increase in value!

Exponential form	Expanded form	Number
88^2	88×88	7 744
88^3	$88 \times 88 \times 88$	681 472
88^4	$88 \times 88 \times 88 \times 88$	59 969 536

Example

1. These examples show repeated multiplication written in exponential form:
   a) $27 \times 27 \times 27 \times 27 \times 27 \times 27 = 27^6$
   b) $(-45) \times (-45) \times (-45) \times (-45) = (-45)^4$
      Note that this is not the same as -45^4, which is $-(45 \times 45 \times 45 \times 45)$.
   c) $y \times y \times y \times y \times y = y^5$
2. These examples show powers written in expanded form:
   a) $96^4 = 96 \times 96 \times 96 \times 96$
   b) $(-121)^3 = (-121) \times (-121) \times (-121)$
   c) $(xy)^6 = xy \times xy \times xy \times xy \times xy \times xy$

Compare integers written in exponential form

To compare integers in exponential form, we need to first calculate the value of the powers. Then we can compare the answers.

Example

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$ (−2 is raised to the fourth power)

$-2^4 = -(2 \times 2 \times 2 \times 2) = -16$ (Only 2 is raised to the fourth power)

$4^2 = 4 \times 4 = 16$

$2 \times 4 = 8$

In Grade 8, we established that, if the base is a *negative* number:

- an odd exponent means the answer is negative, for example: $(-1)^{1\,001} = -1$
- an even exponent means the answer is positive, for example: $(-1)^{1\,000} = 1$.

Example

$(-6)^2 = 36$	If we have -6^2, only 6 is the base and the negative is in front of the power. This means only the 6 is raised to the second power. The answer is $-(6 \times 6) = -36$.	$(-6)^2 \neq -6^2$
$(-6)^3 = -216$	If we have -6^3, only the 6 is the base and the negative is in front of the power. This means only the 6 is raised to the third power. The answer is $-(6 \times 6 \times 6) = -216$.	$(-6)^3 = -6^3$

Example

Replace the * with >, < or = to make the number sentence true:

1. $(-3)^4$ * $(4)^3$
2. $\sqrt[3]{-64}$ * $(-3)^3$

Solution

1. 81 * 64. Therefore $(-3)^4 > (4)^3$.
2. −4 * −27. Therefore $\sqrt[3]{-64} > (-3)^3$.

Numbers in scientific notation

Scientists have developed a shorter way to express very large and very small numbers.

We call this method scientific notation. Scientific notation helps us make sense of very large or small numbers, but also simplifies calculations. For example the speed of light is 300 000 000 m/sec and the mass of a dust particle is 0,000000000753 kg.

Study the following calculations. Use what you know about multiplying powers of 10 and decimals to find a pattern.

$3{,}2 \times 10^2 = 3{,}2 \times 100$ $= 320$	Therefore: $320 = 3{,}2 \times 10^2$
$5{,}43756 \times 10^3 = 5{,}43756 \times 1\ 000$ $= 5\ 437{,}56$	Therefore: $5\ 437{,}56 = 5{,}43756 \times 10^3$
$7{,}0087 \times 10^4 = 7{,}0087 \times 10\ 000$ $= 70\ 087$	Therefore: $70\ 087 = 7{,}0087 \times 10^4$

A number in **scientific notation** is a number between 0 and 10 multiplied by a power of 10. You can think of scientific notation as follows:

$\boxed{\textbf{Any number}} = \boxed{\textbf{a number between 0 and 10}} \times \boxed{\textbf{a power of 10}}$

$\mathbf{5\ 437{,}56 = 5{,}43756 \times 10^3}$

Example

These numbers are written in scientific notation:

$32 = 3{,}2 \times 10^1$ $\quad 320 = 3{,}2 \times 10^2$ $\quad 3\ 200 = 3{,}2 \times 10^3$

$5\ 000 = 5 \times 10^3$ $\quad 60\ 000 = 6 \times 10^4$ $\quad 2\ 800\ 000\ 000 = 2{,}8 \times 10^9$

The following table shows examples of very large numbers. The units are metres in each case.

1 trillion	$1 \times 10^{12} = 1\ 000\ 000\ 000\ 000$	Diameter of the largest star
	$1 \times 10^{11} = 100\ 000\ 000\ 000$	
	$1 \times 10^{10} = 10\ 000\ 000\ 000$	
1 billion	$1 \times 10^9 = 1\ 000\ 000\ 000$	Diameter of the sun
	$1 \times 10^8 = 100\ 000\ 000$	Diameter of Jupiter
	$1 \times 10^7 = 10\ 000\ 000$	Diameter of Earth
1 million	$1 \times 10^6 = 1\ 000\ 000$	
	$1 \times 10^5 = 100\ 000$	
	$1 \times 10^4 = 10\ 000$	Height of the highest mountain
1 thousand	$1 \times 10^3 = 1\ 000$	
	$1 \times 10^2 = 100$	
	$1 \times 10^1 = 10$	Height of the tallest dinosaur
	$1 \times 10^0 = 1$	Length of an adult's step

- 10^3 is also called kilo: 1 000 metres is 1 kilometre.
- 10^6 is also called mega: 1 000 000 bytes of memory on a cell phone is 1 megabyte.
- 10^9 is also called giga: 1 000 000 000 bytes of memory on a computer is 1 gigabyte.
- 10^{12} is also called tera: 1 terabyte is equal to 1 000 gigabytes.

Example

Is the following true or false? $5,239 \times 10^2 < 9,06 \times 10^3$

Solution

First calculate the numbers. Then, compare their sizes:

$5,239 \times 10^2 = 523,9$

$9,06 \times 10^3 = 9\,060$

Therefore: $5,239 \times 10^2 < 9,06 \times 10^3$

The statement is true.

Exercise 1

1. Write in exponential form. (Do not calculate the answer.)
   a) $-7 \times -7 \times -7$
   b) $56 \times 56 \times 56 \times 56 \times 56$
   c) $pq \times pq \times pq \times pq$
2. Write in expanded form:
   a) -8^4 b) $(-8)^4$ c) ab^6 d) $(ab)^6$
3. Calculate:
   a) 13^2 b) 13×2 c) 13×13 d) 2^{13}
4. Predict the sign of the answer in each case:
   a) $(-7)^8$ b) $(-74)^9$ c) $-(138)^4$
   d) $(359)^6$ e) $(-1\,000)^5$ f) $(-309)^1$
5. Compare the numbers. Replace the * with >, < or =:
   a) $12^2 * 6^3$ b) $9^2 * 2^9$ c) $5^2 * 5^3$
   d) $(-7)^4 * -7^4$ e) $(-12)^2 * -24$ f) $(-1)^4 * (-1)^{10}$
   g) $(-3)^3 * (-3)^5$ h) $6^5 * 5^5$ i) $(-2)^3 * -6$
   j) $3,45 \times 10^3 * 3,45 \times 10^2$
   k) $7,26 \times 10^2 * 2,76 \times 10^2$
   l) $9,2 \times 10^4 * 9,254 \times 10^4$
6. Write in scientific notation:
   a) 270 b) 2 700 c) 27 000
   d) 325 167 e) 70 000 f) 300 005
7. There are approximately 120 000 000 000 stars in our galaxy, called the Milky Way. Express the number of stars in scientific notation.
8. Write in ordinary notation:
   a) $3,0 \times 10^8$ b) $1,5 \times 10^{11}$ c) $4,6 \times 10^9$

Unit 2 Calculations using numbers in exponential form

Revision

In the following laws of exponents, m and n are natural numbers (positive numbers) and a and t cannot be 0.

Law	Example
1. $a^m \times a^n = a^{m+n}$	$2^3 \times 2^4 = 2^{3+4} = 2^7 = 128$ $x^3 \times x^4 = x^{3+4} = x^7$
2. $a^m \div a^n = a^{m-n}$	$3^5 \div 3^2 = 3^{5-2} = 3^3 = 27$ $x^5 \div x^2 = x^{5-2} = x^3$
3. $(a^m)^n = a^{mn}$	$(2^3)^2 = 2^{3\times 2} = 2^6 = 64$
4. $(a \times t)^n = a^n \times t^n$	$(3x^2)^3 = 3^3x^6 = 27x^6$
5. $a^0 = 1$	$a^4 \div a^4 = \frac{a^4}{a^4} = \frac{a \cdot a \cdot a \cdot a}{a \cdot a \cdot a \cdot a} = 1$ (Expand and cancel.) $a^4 \div a^4 = a^{4-4} = a^0$ (Law 2 of exponents) Therefore: $a^0 = 1 \qquad 5^0 = 1 \qquad (37)^0 = 1 \qquad (4x^2)^0 = 1$

Until now, we have only considered exponents that were natural numbers (positive numbers). We now extend the laws of exponents to include negative exponents.

You know that $2^5 \div 2^3 = 2^{5-3} = 2^2$. But what happens if the first exponent is smaller than the second exponent?

$$
\begin{aligned}
2^3 \div 2^7 &= \frac{2^3}{2^7} \\
&= \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} && \text{(Write the powers in expanded form.)} \\
&= \frac{1}{2 \times 2 \times 2 \times 2} && \text{(Cancel the common factors.)} \\
&= \frac{1}{2^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)
\end{aligned}
$$

However, from the laws of exponents, we have:

$$
\begin{aligned}
2^3 \div 2^7 &= 2^{3-7} && \text{(Law 2 of exponents)} \\
&= 2^{-4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)
\end{aligned}
$$

From equations (1) and (2), we have that:

$$\frac{1}{2^4} = 2^{-4}$$

or

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

What about:

$$
\begin{aligned}
(-3)^2 \div (-3)^5 &= \frac{(-3)^2}{(-3)^5} \\
&= \frac{(-3) \times (-3)}{(-3) \times (-3) \times (-3) \times (-3) \times (-3)} \\
&= \frac{1}{(-3) \times (-3) \times (-3)} \\
&= \frac{1}{(-3)^3} && \text{(Expand and cancel.)}
\end{aligned}
$$