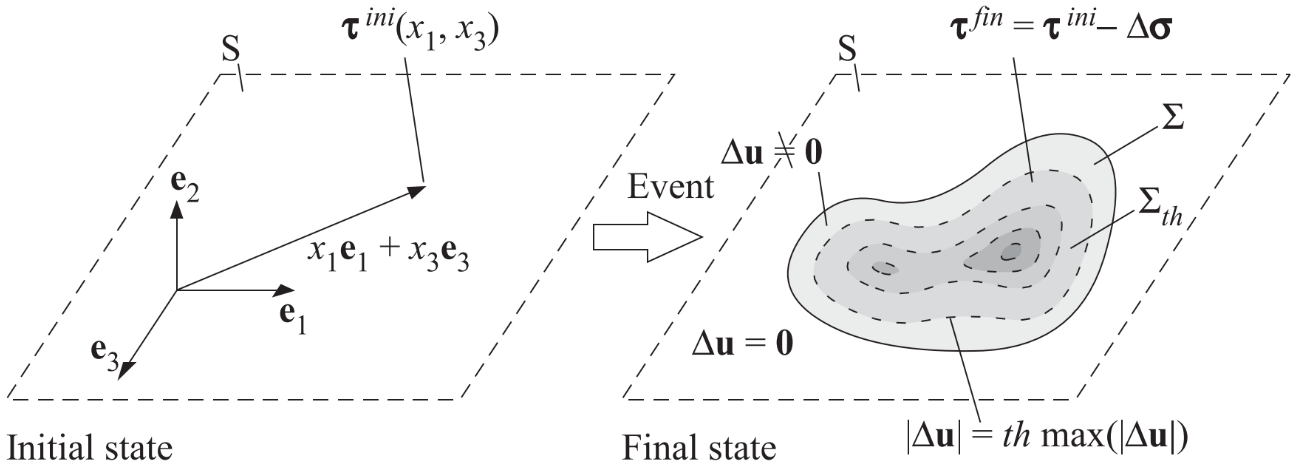
Stress Drop Reading Group

Week 1

July 16th, 2024

How to average stress drop?



Assumptions

(a) $\Delta u_2(x_1, x_3) = 0$ & $\Delta\vec{u} \perp \overrightarrow{e_2}$ (no fault opening)

(b) Slip in single direction ($\overrightarrow{e_1}$) is dominant.

(c) Planar fault S embedded in linear elastic infinite homogeneous medium

Stress drop (vector): $\Delta\vec{\sigma}(x_1, x_3) = \overrightarrow{\tau^i}(x_1, x_3) - \overrightarrow{\tau^f}(x_1, x_3)$

It is obvious that $\Delta\vec{\sigma}$ varies spatially on the fault plane. How to average?

Rupture domain: $\Sigma = \{\vec{x} \in S :\ |\Delta\vec{u}(\vec{x})| \neq 0\}$

Truncated domain: $\Sigma = \left\{\vec{x} \in S :\ |\Delta\vec{u}(\vec{x})| > th\ \times \max_S(|\Delta\vec{u}|)\right\}$ (for practical purpose)

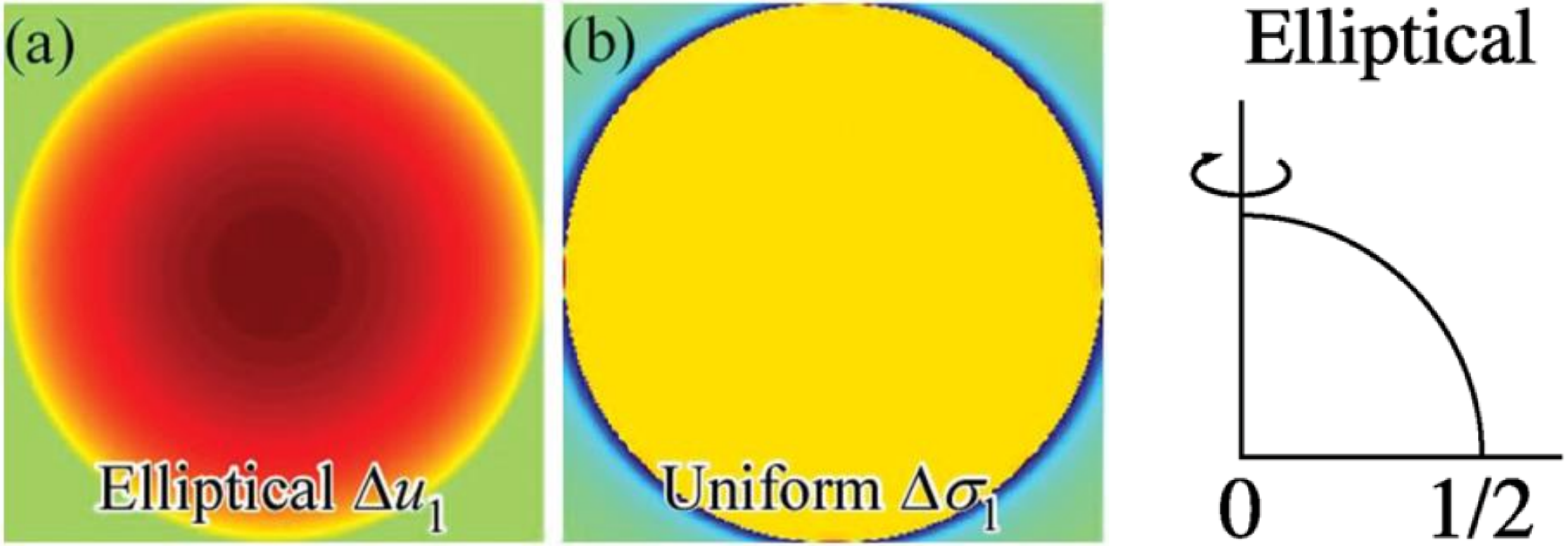
$$\Delta\bar{\sigma} = \frac{\int_S \Delta\vec{\sigma} \cdot \vec{w}\, dS}{\int_S \overrightarrow{e_1} \cdot \vec{w}\, dS}$$

→ How would we choose weight $\vec{w}$?

How to average stress drop?

(1) Moment-based stress drop, $\overline{\Delta\sigma_M}$

- Average of stress drop weighted by the slip distribution $\vec{E}^{12}$ due to uniform stress drop in the overall slip direction $\vec{e_1}$ over the same ruptured domain ($\vec{w} = \vec{E}^{12}$).
- Equal to seismologically estimated stress drop, $\frac{M_0}{\int_\Sigma E_1^{12} dS} \approx C\,\frac{M_0}{\rho^3} = C\,\frac{M_0}{A^{3/2}}$
- For circular rupture, $\overline{\Delta\sigma_M}$ emphasizes stress change in the middle of the ruptured domain.



How to average stress drop?

(2) Spatially averaged stress drop, $\overline{\Delta\sigma_A}$

- $\overline{\Delta\sigma_A} = \frac{1}{A}\int_\Sigma \Delta\sigma_1 dS \; (\vec{w} = \vec{e_1})$
- Equal to difference between average stress levels on the fault before and after earthquake.

(3) Energy-based stress drop, $\overline{\Delta\sigma_E}$

- Average of stress drop weighted by the final slip $\Delta\vec{u}$ at each point ($\vec{w} = \Delta\vec{u}$).
- Relevant stress drop for calculating radiation ratio, $\eta_R = E_R / \Delta W_0 = \frac{2\mu E_R}{\boldsymbol{\Delta\sigma} M_0}$
- $\overline{\Delta\sigma_E} \geq \overline{\Delta\sigma_M}$

Relation between $\overline{\Delta\sigma_E}$ and strain energy ΔW

Total strain energy: $\Delta W = \left(\frac{\tau^i + \tau^f}{2}\right)\overline{\Delta u}A$

Partial strain energy change: $\Delta W_0 = E_G + E_R$

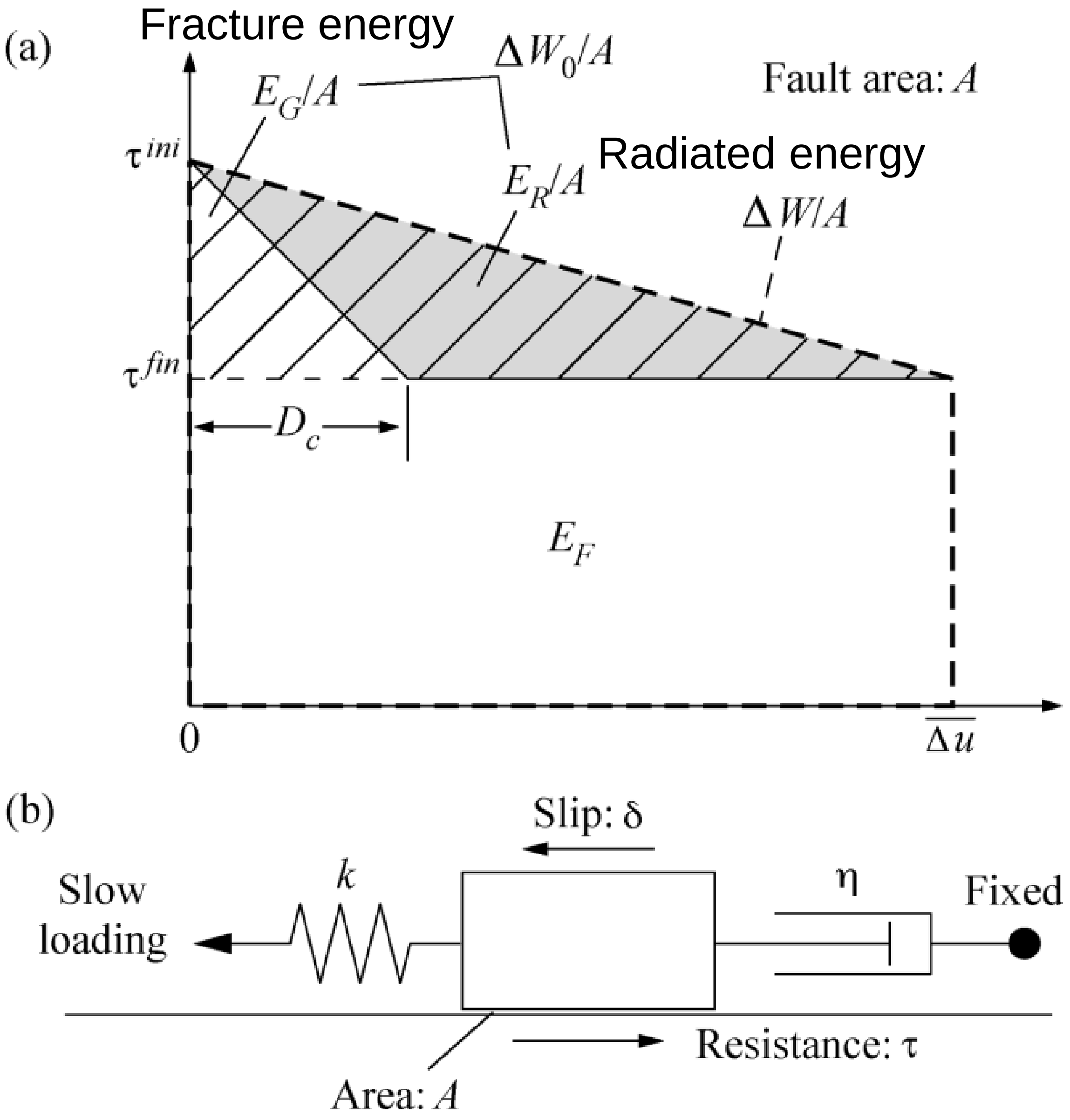
Radiation ratio (efficiency): $\eta_R = E_R/\Delta W_0 = \frac{2\mu E_R}{\Delta\sigma M_0}$

Introduce $\lambda \in [0,1]$ to represent continuous set of static solutions between initial ($\lambda = 0$) and final state ($\lambda = 1$) .

$$\overrightarrow{\delta_{vp}}(\lambda,\vec{x}) = \lambda\Delta\vec{u},\ \ \overrightarrow{\tau_{vp}}(\lambda,\vec{x}) = \lambda\vec{\tau}^f + (1-\lambda)\vec{\tau}^i$$

$$\Delta W = \int_0^1 \frac{dW}{d\lambda}d\lambda = \int_0^1\int_\Sigma \overrightarrow{\tau_{vp}}\cdot\frac{d\overrightarrow{\delta_{vp}}}{d\lambda}dSd\lambda$$

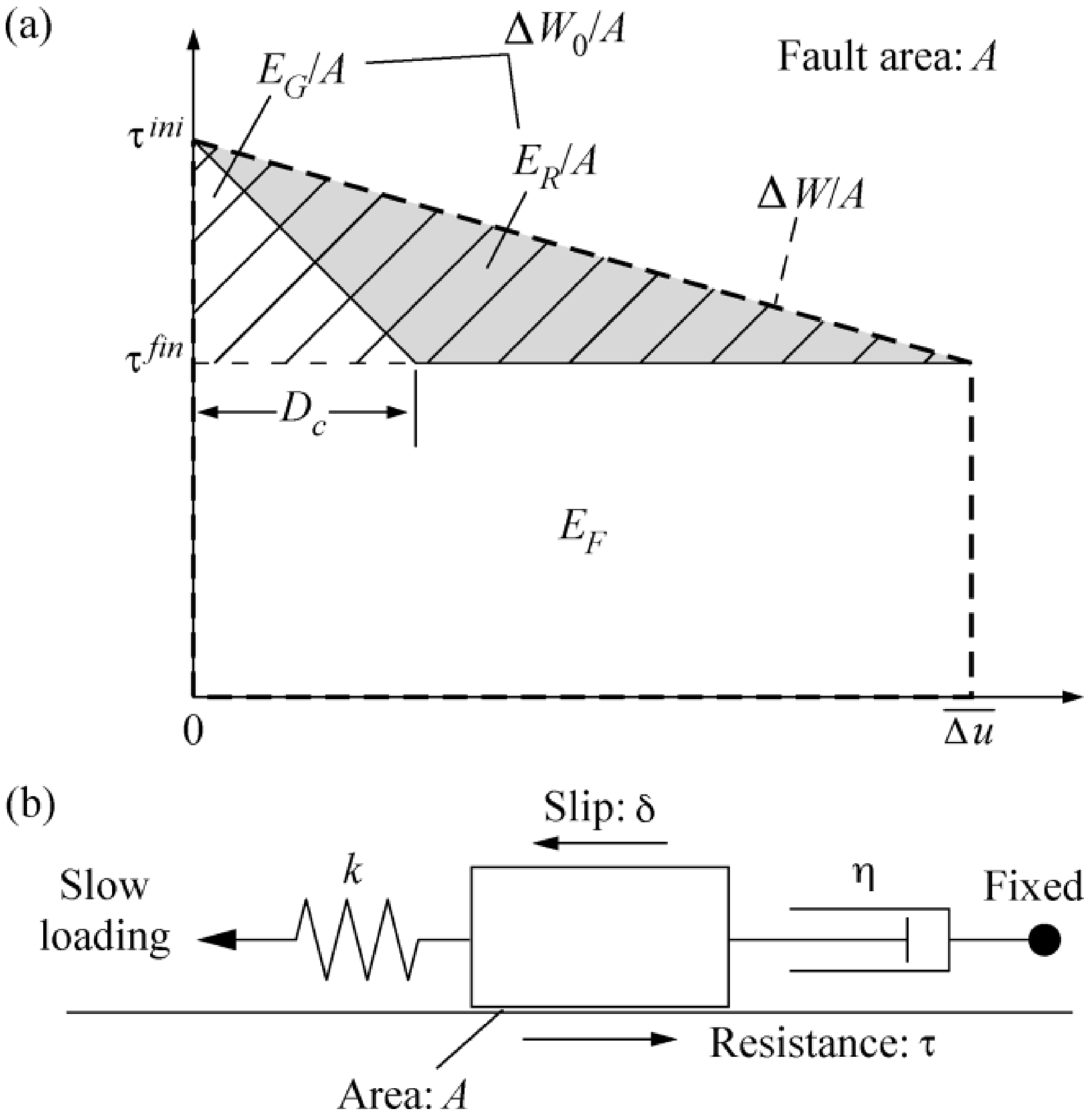
$$\Delta W = \int_\Sigma \left\{-\frac{1}{2}\Delta\vec{\sigma} + \vec{\tau}^i\right\}\cdot\Delta\vec{u}dS$$



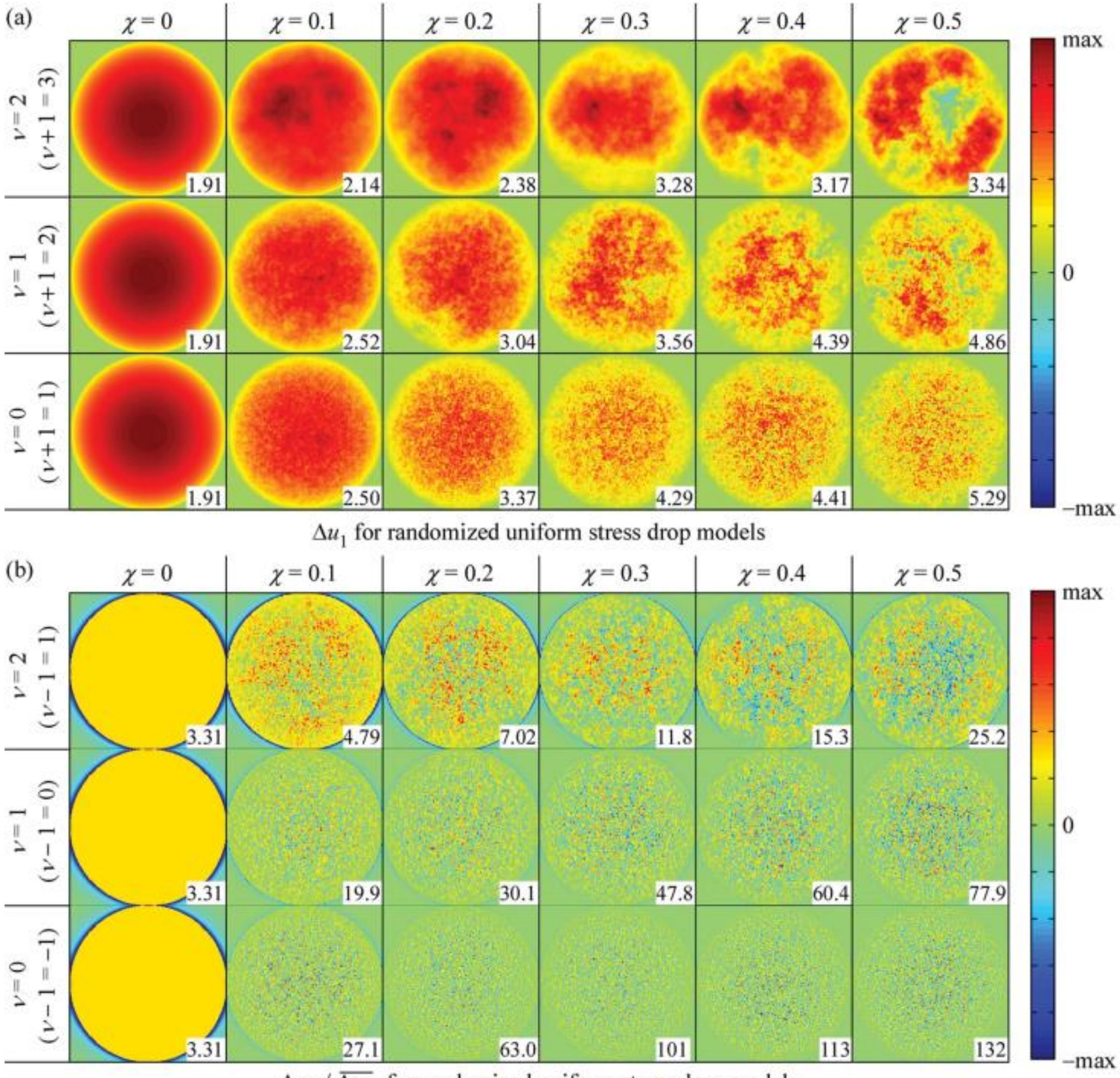
Relation between $\overline{\Delta\sigma_E}$ and strain energy ΔW

$$\Delta W_0 = \Delta W - \int_\Sigma \vec{\tau}^f \cdot \Delta\vec{u}\, dS = \frac{1}{2}\int_\Sigma \Delta\vec{\sigma} \cdot \Delta\vec{u}\, dS$$

$$\Delta W_0 = \frac{1}{2}\left[\frac{\int_\Sigma \Delta\vec{\sigma} \cdot \Delta\vec{u}\, dS}{\int_\Sigma \Delta u_1 dS}\right]\int_\Sigma \Delta u_1 dS = \frac{1}{2}\overline{\boldsymbol{\Delta\sigma_E}}\,\overline{\Delta u_1}A$$



Comparison of $\overline{\Delta\sigma}$ for heterogeneous ruptures

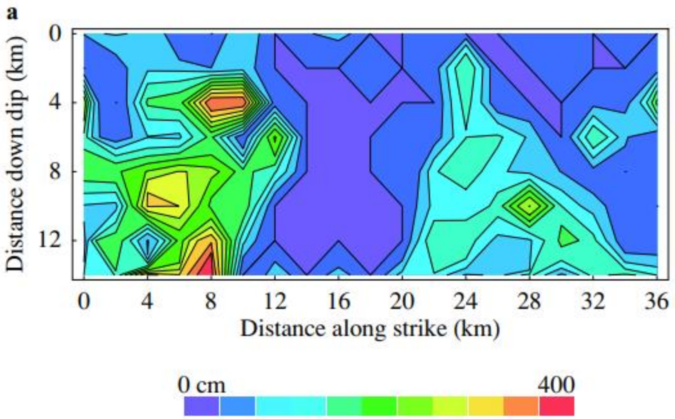


(a) Δu_1 for randomized uniform stress drop models

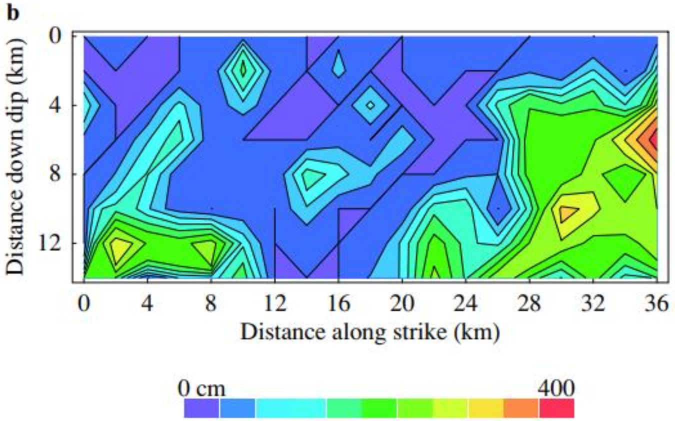
(b) $\Delta\sigma_1 / \overline{\Delta\sigma_M}$ for randomized uniform stress drop models

Stochastic slip model

- Lavallée et al. (2006)
- Heterogeneity of prestress or friction
- Realistic ground motion

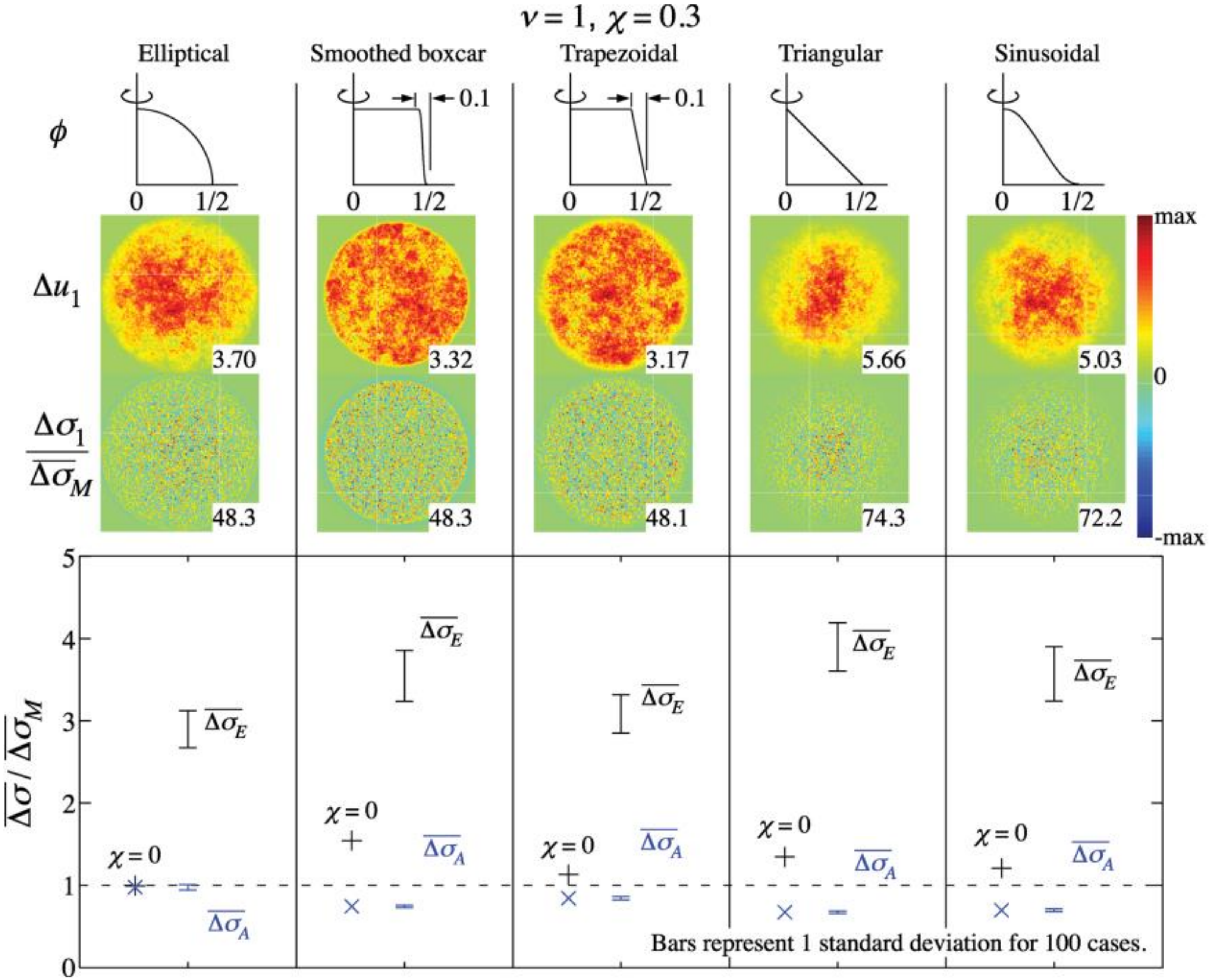


Dip slip



Strike slip

Comparison of $\overline{\Delta\sigma}$ for heterogeneous ruptures



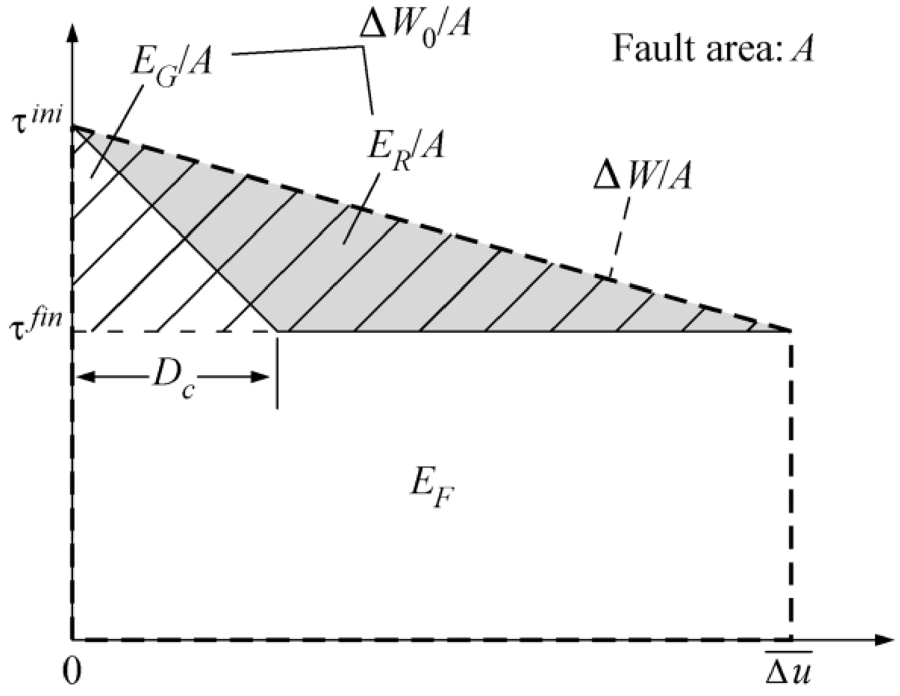
Energy partitioning and shear stress evolution

Radiated energy: $E_R = \frac{1}{2}\int_\Sigma \left(\vec{\tau}^i - \vec{\tau}^f\right) \cdot \Delta\vec{u}\, dS + \int_0^{t^{fin}} dt \int_\Sigma \frac{d\vec{\tau}}{dt} \cdot \vec{\delta} dS$

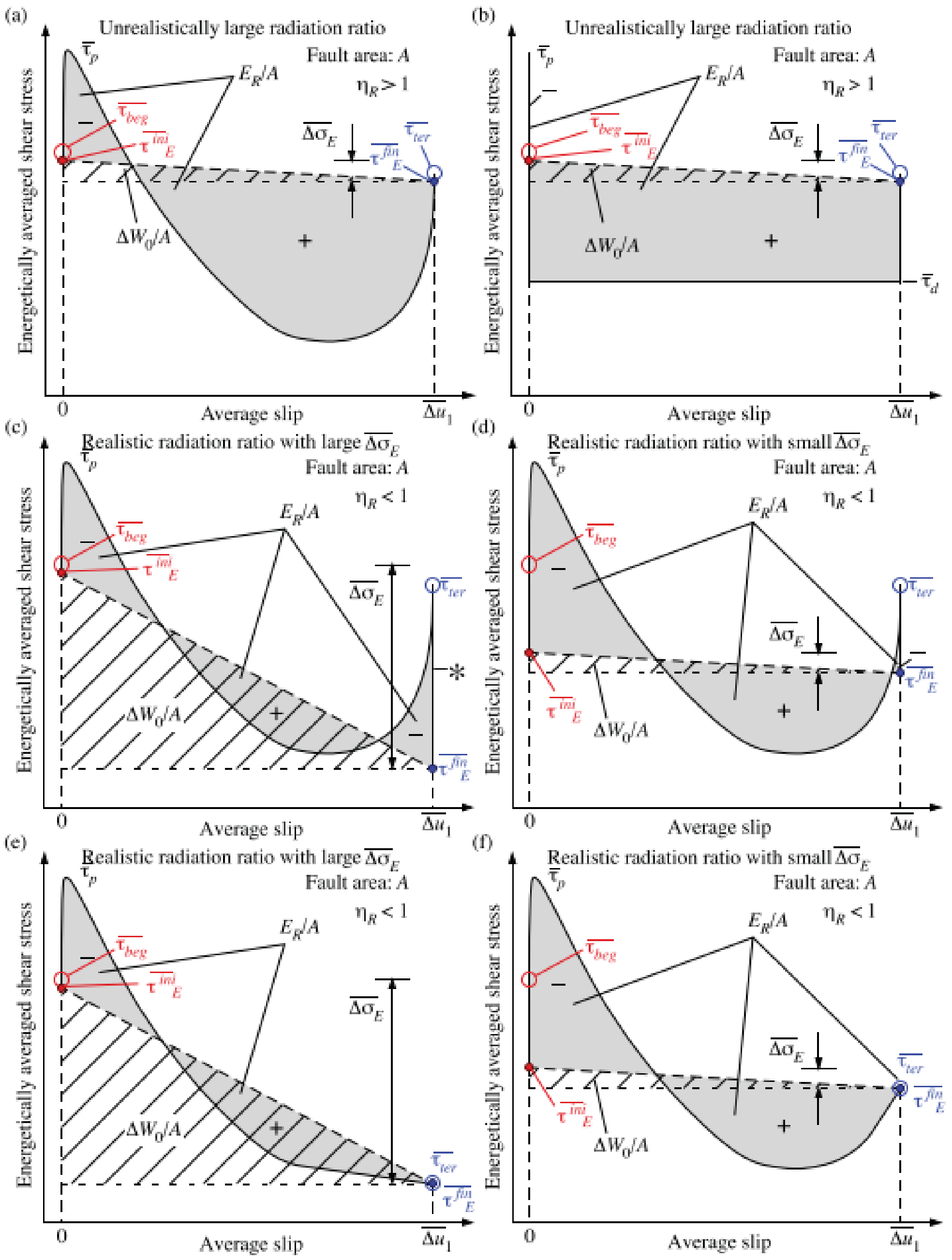
$$\approx \frac{1}{2}\int_\Sigma \left(\tau_1^i - \tau_1^f\right) \cdot \Delta u_1 dS + \int_0^{t^{fin}} dt \int_\Sigma \frac{d\tau_1}{dt} \delta_1 dS$$

Radiation ratio: $\eta_R = 1 + \frac{2\mu}{\overline{\Delta\sigma_E} M_0} \int_0^{t^{fin}} dt \int_\Sigma \frac{d\tau_1}{dt} \delta_1 dS$

→ Sign of $\frac{d\tau_1}{dt}$ determines whether $\eta_R > 1$ or $\eta_R < 1$

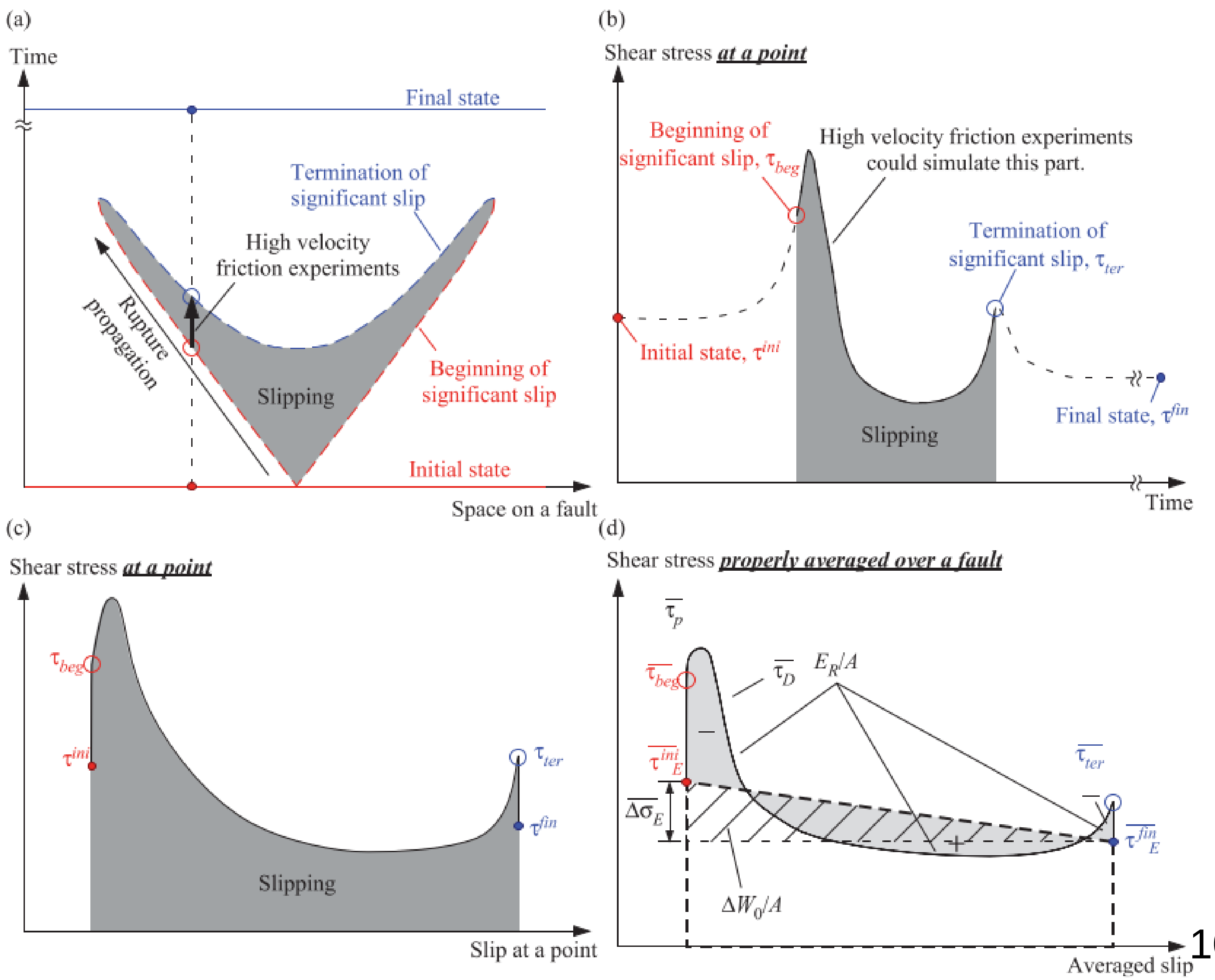


Energy partitioning and shear stress evolution



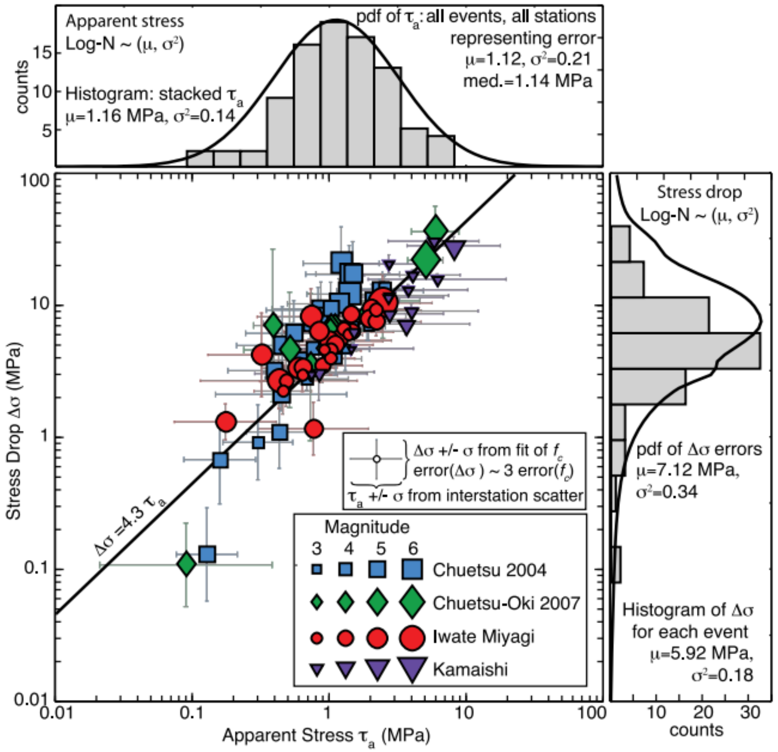
Overshoot: $\tau_d > \tau_f$

Undershoot: $\tau_d < \tau_f$



Apparent stress, σ_{app}

- $\sigma_{app} = \eta\bar{\sigma} = \frac{\mu E_R}{M_0}$ (Madariaga, 1977)
- Does not require an estimate of source dimension.
- Q) Can we use σ_{app} instead of $\Delta\sigma$? How much uncertainty in E_R?
- Q) How does source complexity (e.g., rupture directivity) influence E_R and σ_{app}?



Baltay et al. (2011)

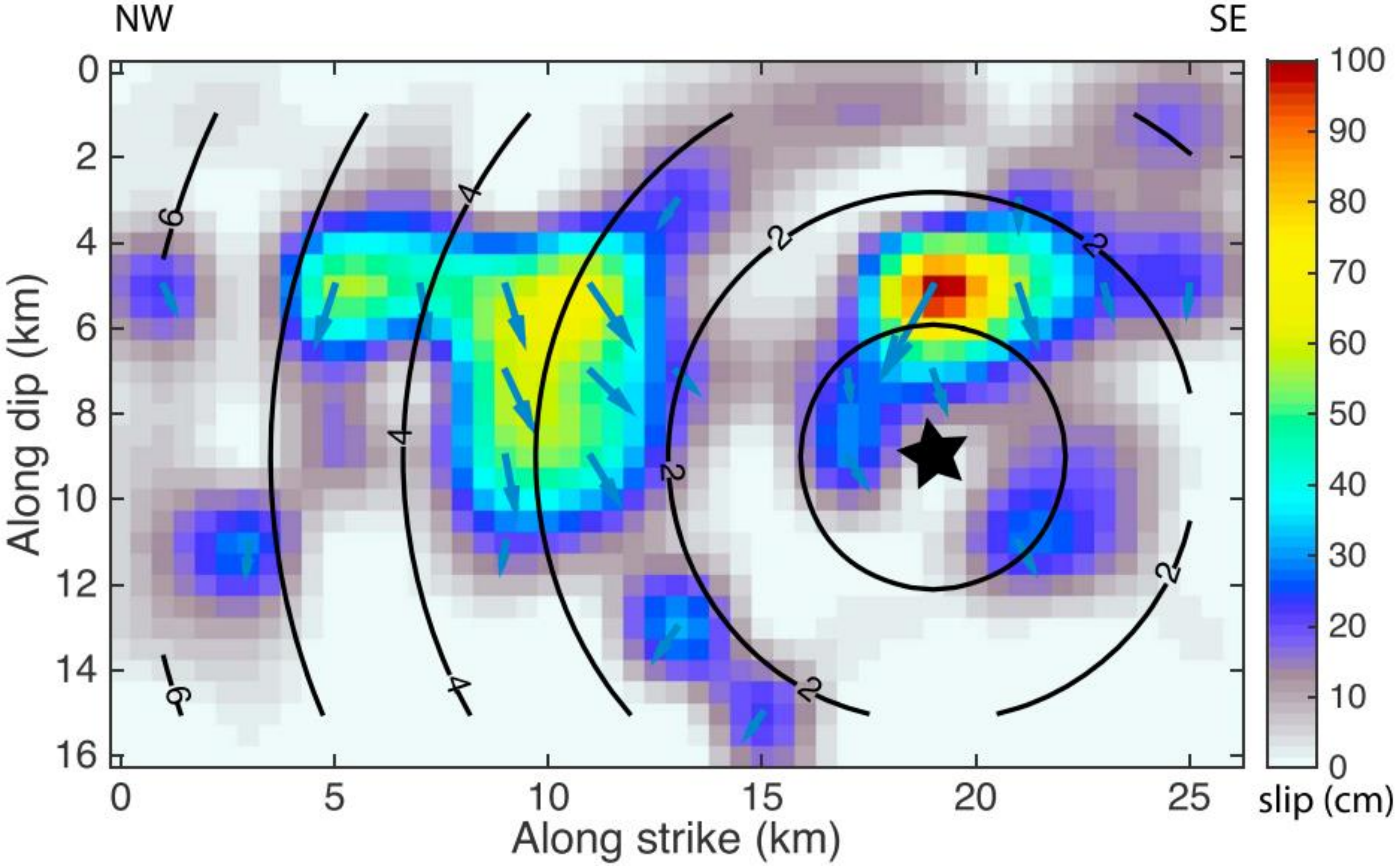
$$E_R = \frac{I}{4\pi^2\rho\beta^5}\int_0^\infty \left|\omega \cdot \dot{M}(\omega)\right|^2 d\omega$$

$\dot{M}(\omega)$: Displacement source spectra

I: Average mean-squared S-wave radiation pattern coefficient (2/5)

Example of earthquake with significantly varying rakes

2016 M_w 6.0 Amatrice, Italy, earthquake



Tinti et al. (2016)

Questions

1. Which stress drop average should we pursue to estimate in observational seismology?

   Would it depend on specific focus of the research?

2. Heterogeneity of stress drop on the fault leads to partially very low and high parts.

   Does it imply that small earthquakes that break those parts would have correspondingly low or high stress drops? Implication for self-similarity?

3. Considering the paradigm of elastic rebound theory, does the average stress drop matter

   or strong locked patch with locally high stress drop matter?