Project 4: Constraint Programming and Propositional Logic

LINFO1361 – Intelligence Artificielle

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Anatomy of a CP model

A CP model **describes** what is a valid solution to a problem

A CP model is composed of two (or three) sections :

- The variable description, with their domains
- The constraints applied on those variable
- For COP, an objective function (not in this assignment)

Then the solver is called and returns :

- SATISFIABLE if the model is satisfiable + a solution
- UNSATISFIABLE if the model is unsatisfiable

Sudoku: Description

What is Sudoku?

Sudoku is a game where a 9x9 grid with 9 3x3 sub-grids is partially filled with digits from 1 to 9.

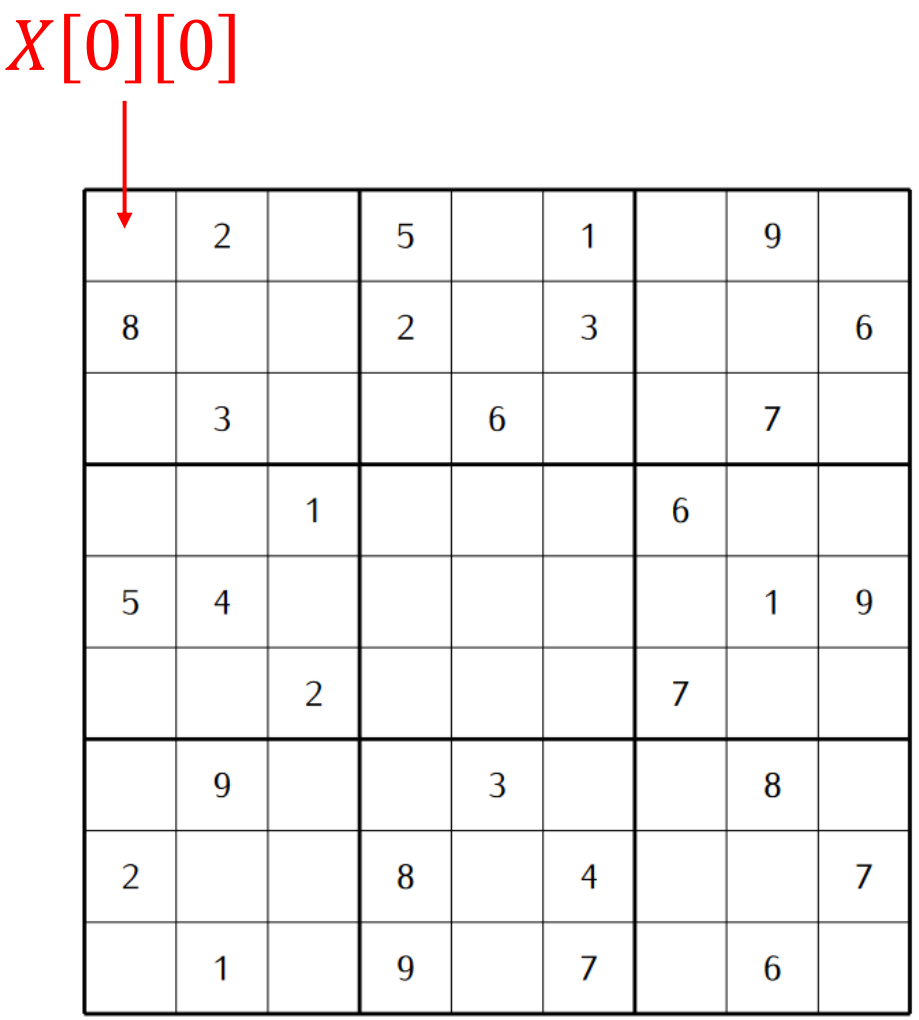
The **goal** of Sudoku is to **fill the empty cells** with digits from 1 to 9 by following **three rules**.

	2		5		1		9	
8			2		3			6
	3			6			7	
		1				6		
5	4						1	9
		2				7		
	9			3			8	
2			8		4			7
	1		9		7		6	

Sudoku grid example

Reading a CP Model

- One variable for each cell
- $X[i][j]$ represents the cell at row i and column j
- The **domain** of each variable is $\{1..9\}$
- A 9x9 array named *clue* contains the already fixed values
- What is the purpose of each constraint ?



Sudoku grid example

Writing a CP model

Now your goal is to write a CP model to solve the N-Amazons problem

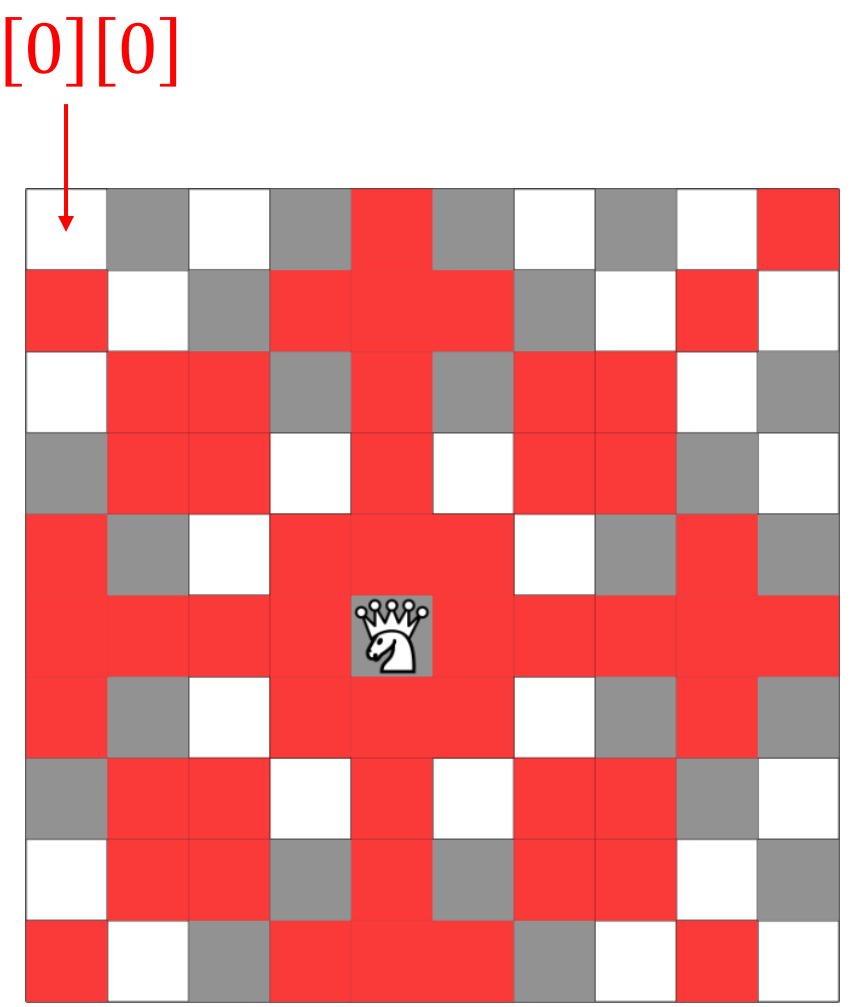
N-Amazons: Description

What is an Amazon?

Custom piece that possess the move of a **Queen** and of an **extended Knight** (4 move in one direction then 1 in another or 3 in one direction then 2 in another).

Valid Solution:

Places **N amazon pieces** on the **NxN board**, avoiding that any of the amazon attacks another one.



Example of the tiles that an Amazon threatens on a 10x10 board

Writing a CP model

First step : Variable choices

-> There is a **naive** way and a **smart** way

Second step : Constraint definition

-> How to avoid that there is two amazons on the same row ? On the same column ? ...

Input format

- The instances contain the value of N (number of row/column/amazons to place)
- They also contain the already placed amazons

    -> Some instance are unsatisfiable

    -> An amazon on position **(0, 0)** is at the **top left corner**

Output format

Your function should return :

- A boolean indicating if the instance is satisfiable or not
- If the instance is satisfiable : the solution represented by a NxN array, where a cell is equal to 1 if there is an amazon, 0 otherwise

! Try your model on every given instance : it may return a valid solution to one instance and an invalid one to one other.

SAT model

- Describe a problem with **literals** (boolean variables) and clauses in **conjunctive normal form** :

$(a \lor b \lor \neg c) \land (d \lor e)$ 

$(a \land b \land \neg c) \lor (d \land e)$ 

$(a \land b \lor \neg c) \land (d \lor e)$ 

$\neg (a \land b \lor c) \land (d \lor e)$ 

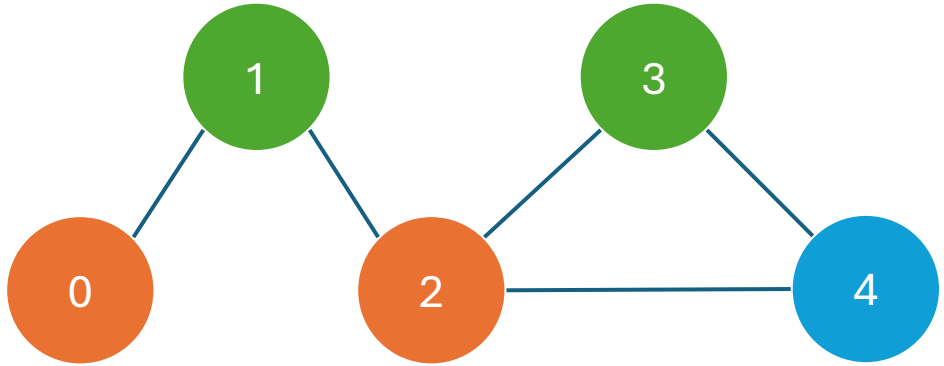
Graph coloring problem

What is Graph coloring problem?

A graph coloring problem contains a **graph** and a **maximal number of color**.

The goal is to assign a color to each vertex such as :

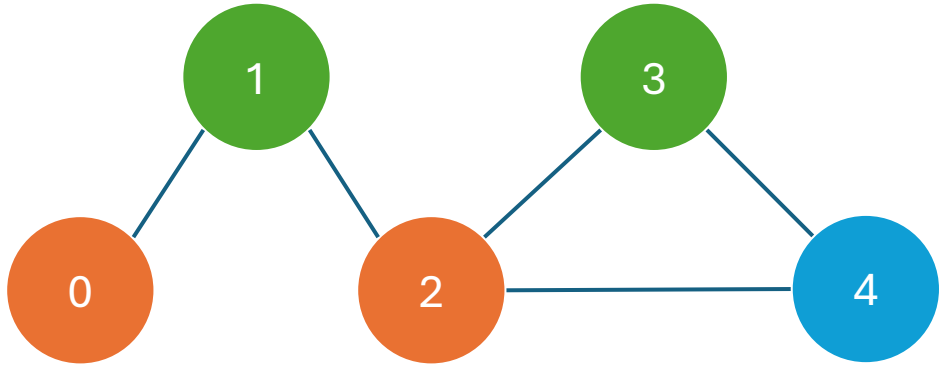
- Two neighbors **can’t** have the **same color**
- You can’t use more colors than the authorized **maximum**



Reading a SAT model

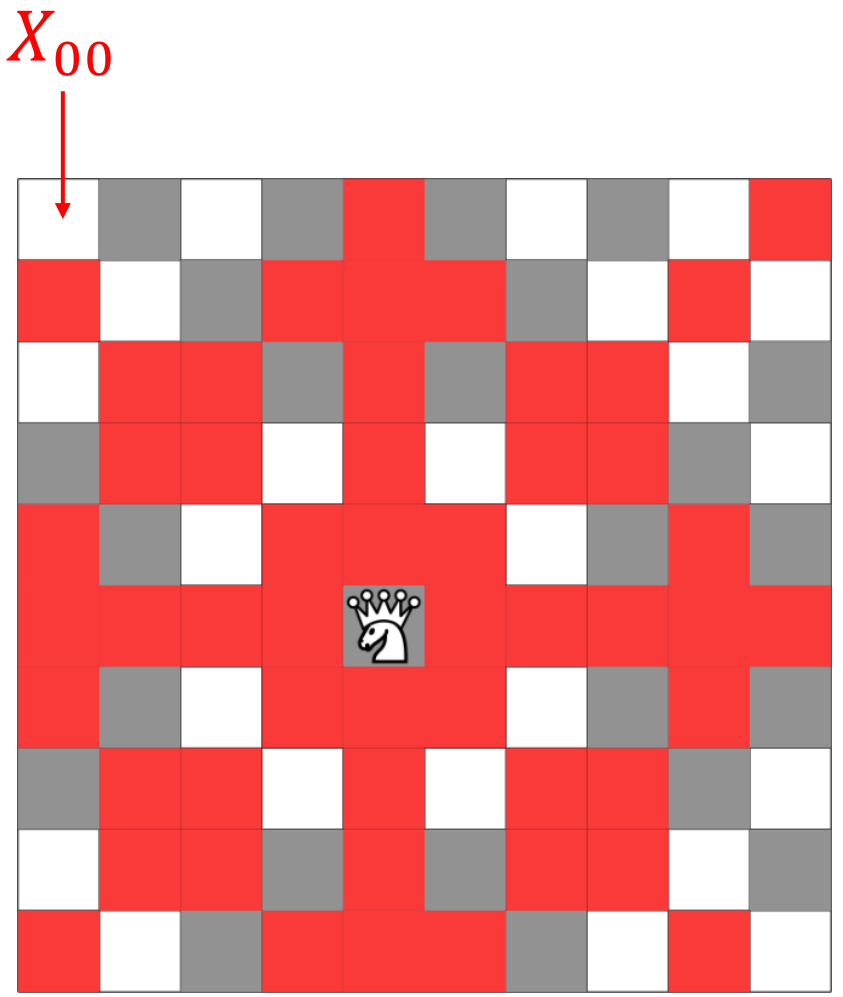
One literal for each vertex and color :
X_{ij} is true if and only if the color j is given to the node i

The model is given to you.
What is the purpose of each set of clauses ?



Writing a SAT model

NxN literals : X_{ij} is true iff there is an amazon at row i and column j

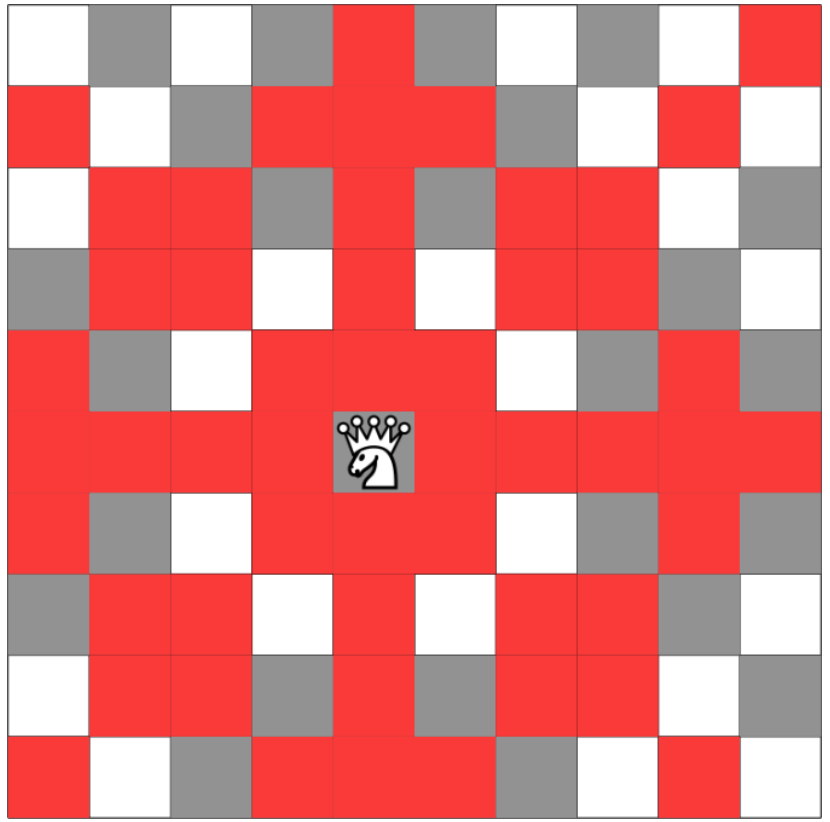


Example of the tiles that an Amazon threatens on a 10x10 board

Writing a SAT model

First step : List the sentences required to describe the problem in propositionnal logic.

And explain with your word what they mean



Example of the tiles that an Amazon threatens on a 10x10 board

Writing a SAT model

Second step : Develop the sentences in CNF (cf slide 10)

Use rules such as De Morgan's law :

$\neg(a \wedge b) \leftrightarrow \neg a \vee \neg b$ $\neg(a \vee b) \leftrightarrow \neg a \wedge \neg b$

Or

$$a \Rightarrow b \leftrightarrow \neg a \vee b$$

Or

$$a \vee (b \wedge c) \leftrightarrow (a \vee b) \wedge (a \vee c)$$

Writing a SAT model

Second step : Develop the sentences in CNF (cf slide 11)

Instead of writing :

$$(X_{00} \vee X_{01} \vee X_{02} \vee X_{03}) \wedge (X_{10} \vee X_{11} \vee X_{12} \vee X_{13}) \wedge (X_{20} \vee X_{21} \vee X_{22} \vee X_{23})$$

You can write :

$$\bigwedge_{i \in 0..2} \bigvee_{j \in 0..3} X_{ij}$$

Input format

- The instances contain the value of N (number of row/column/amazons to place)
- They also contain the already placed amazons

    -> Some instance are unsatisfiable

Output format

Your function should return :

- A list of Clause objects, representing the different clauses of your model
- The interface to MiniSAT will run it on your model and print the result

! Try your model on every given instance : it may return a valid solution to one instance and an invalid one to one other.

Grading details

Report: 11/15

- Reading a CP model: 1/15
- Writing a CP model: 4/15
- Theoretical question : 1/15
- Reading a SAT model: 1/15
- Writing a SAT model: 4/15

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- CP model : 2/15
- SAT model: 2/15