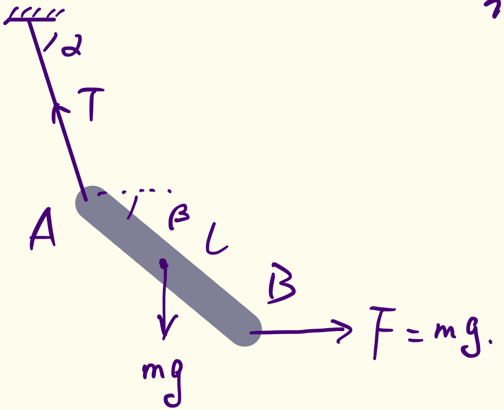
作业3.




① 分析$\beta$角时，若以AB杆为研究对象，有力平衡，力矩平衡.

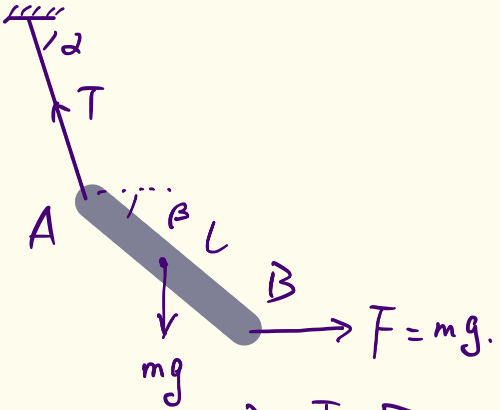
力矩平衡选A点为转动中心.

（此时未知的T和T方向，都和力矩无关）

$$M = -mg\cdot\frac{L}{2}\cdot\cos\beta + F\cdot L\cdot\sin\beta$$

平衡 $\Rightarrow M = 0$

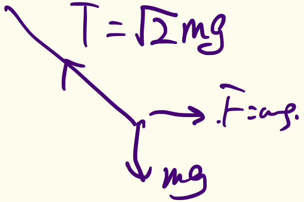
$$\frac{1}{2}\cos\beta = \sin\beta \Rightarrow \tan\beta = \frac{1}{2}$$




② 以AB杆对象，有力平衡条件。

水平方向合力 $F_x = F - T\cos\alpha$

竖直 $F_y = mg - T\sin\alpha$




平衡条件 $F_x=0$；$F_y=0$

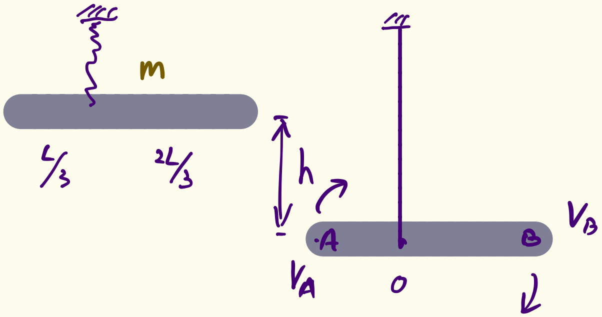
解出 $\alpha$ 和 $T$。

注意：平衡条件和杆的形状和力的作用点位置无关。也可知 $T=\sqrt{2}mg$.

$\alpha = 45^\circ$

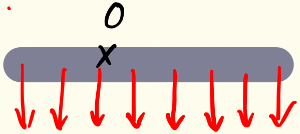
2. 绳无弹性. → $\frac{L}{3}$ 处完全非弹性碰撞.

碰撞过程机械能不守恒.



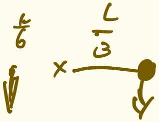

选 $\frac{L}{3}$ 点为转动中心. 则有碰撞前后角动量守恒.

碰撞前速度.




$v=\sqrt{2gh}.$

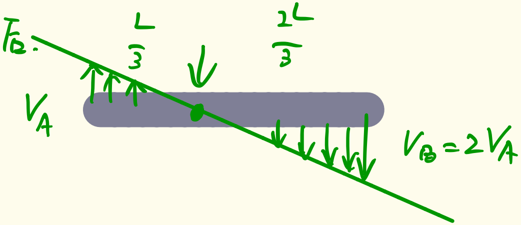
由自由落体计算得



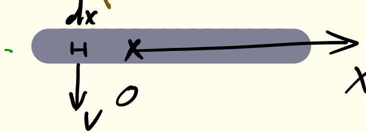

碰撞前角动量.

$$\boxed{J_0=\frac{1}{3}mv\cdot\frac{L}{6}-\frac{2}{3}mv\frac{L}{3}=-\frac{1}{6}mvL}$$

碰撞后:




或




$$J_0=\int_{-\frac{L}{3}}^{\frac{2}{3}L}-m\frac{dx}{L}\cdot v\cdot x=-\frac{mvL}{6}$$

碰撞后角动量用角速来描述方便.

$J_1 = I\omega$.

$I$ 为绕 $\frac{l}{3}$ 处转动的转动惯量.

$$I = \frac{1}{12}ml^2 + m\cdot\left(\frac{1}{6}l\right)^2 \quad (\text{平行轴定理})$$

$$= \frac{1}{9}ml^2$$

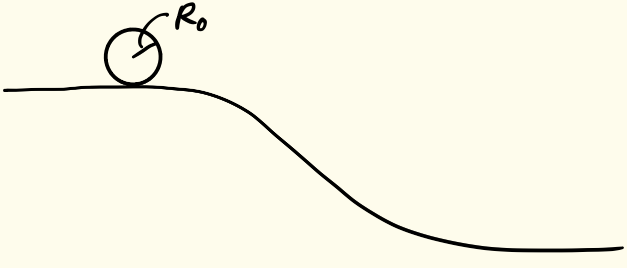
角动量守恒 $J_0 = J_1$

$$-\frac{1}{6}ml\sqrt{2gh} = \frac{1}{9}ml^2\cdot\omega$$

得 $\omega = -\frac{3}{2}\frac{\sqrt{2gh}}{l}$

$V_A = -\frac{1}{3}l\omega = \frac{1}{2}\sqrt{2gh}$ $\quad V_B = \frac{2}{3}l\omega = \sqrt{2gh}$.

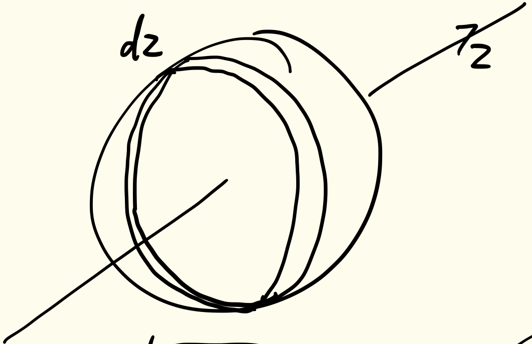
① 球绕其中心轴转动的转动惯量.



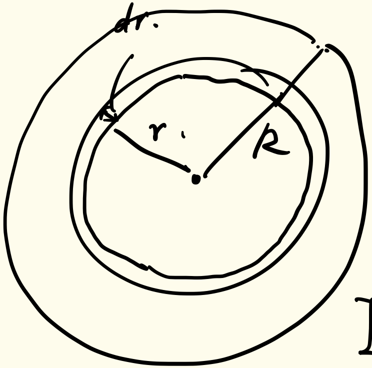

圆盘转动惯量.

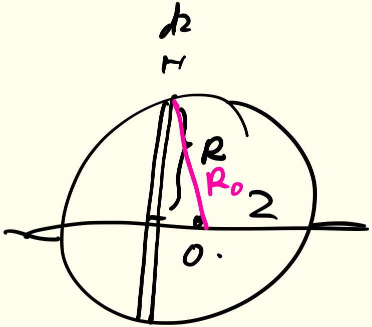
$$I_{盘}=\int_0^R r^2\,\frac{m}{\pi R^2}\cdot 2\pi r\cdot dr$$

$$=\frac{1}{2}mR^2$$




$$R=\sqrt{R_0^2-Z^2}$$




$$m=M\cdot\frac{3}{4\pi R_0^3}\cdot\pi R^2\cdot dz$$

$$=\frac{3M}{4R_0^3}(R_0^2-Z^2)\cdot dz$$

$$I_{球}=球转动惯量\int_{-R_0}^{R_0}\frac{1}{2}m\cdot R^2=\int_{-R_0}^{R_0}\frac{3M}{8R_0^3}(R_0^2-Z^2)^2\,dz$$

沿坡面滑下的过程机械能守恒

$$Mgh = \frac{1}{2} m v_c^2 + \frac{1}{2} I_{球} \omega^2$$

其中，纯滚动，$\omega R_0 = V_C$。

代入可解出 $V_C$。