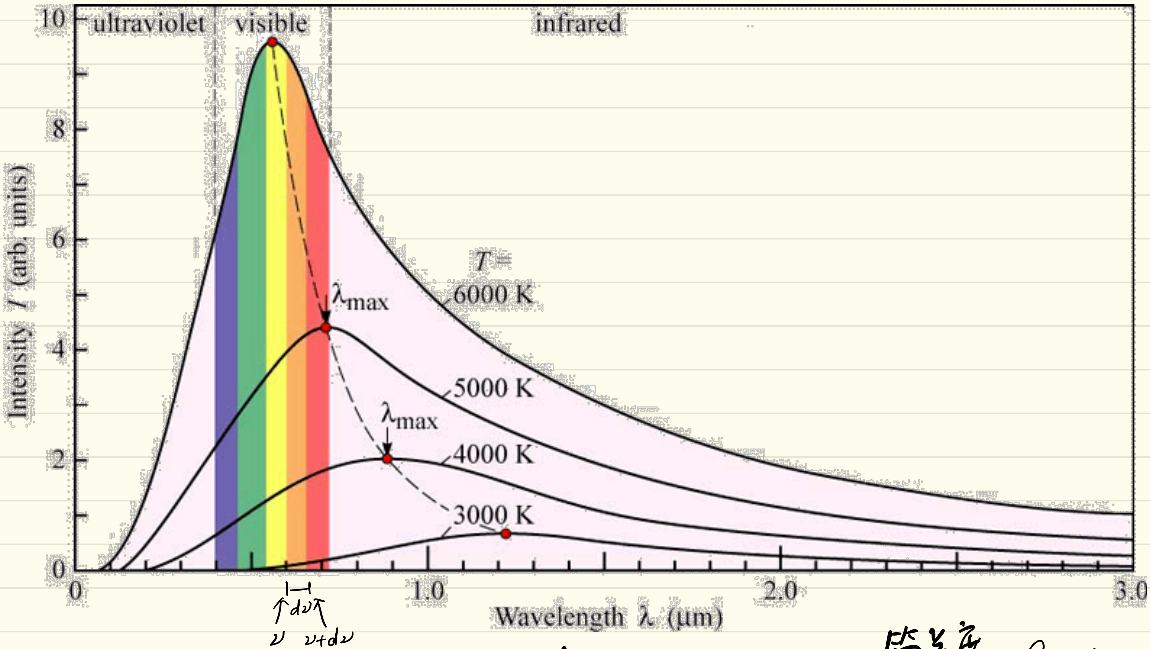
# §7.5 热辐射的普朗克理论

$u \propto T^4$

$u = u(\nu)$

$\uparrow$

?




波长

↳ $\nu$

简并度 $g(\nu)$

$\nu \longrightarrow \nu + d\nu$

$\varepsilon(\nu)$

频率

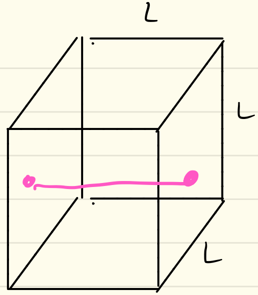
$\longleftarrow$

波长短

频率高

热辐射用电磁波波动微分方程描述

$$\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}+\frac{\partial^2 A}{\partial z^2}=\frac{1}{c^2}\cdot\frac{\partial^2 A}{\partial t^2}$$




取 L×L×L 立方体内的热辐射为研究对象

周期性边条件 —— 来自 $\boxed{\text{均匀性}}$ 和 $\boxed{\text{各向同性}}$ 的要求

$$\begin{cases} A(x+L,\ y,\ z)=A(x,\ y,\ z)\\ A(x,\ y+L,\ z)=A(x,\ y,\ z)\\ A(x,\ y,\ z+L)=A(x,\ y,\ z)\end{cases}$$

$\vec{r}=(x,\ y,\ z)$

波矢

微分方程通解：$A=A_0\sin(\vec{k}\cdot\vec{r}-\omega t)$

$\omega=2\pi\cdot\nu$

$k_1=\frac{2\pi}{L}n_1$ ; $k_2=\frac{2\pi}{L}n_2$ ; $k_3=\frac{2\pi}{L}n_3$

$k=\sqrt{k_1^2+k_2^2+k_3^2}=\frac{\omega}{c}$

$k=\frac{2\pi}{L}\sqrt{n_1^2+n_2^2+n_3^2}$

$\nu\to\nu+d\nu.$

$\nu=\frac{c\cdot k}{2\pi}=\frac{c}{L}\cdot\sqrt{n_1^2+n_2^2+n_3^2}$

$n_1,\ n_2,\ n_3=0,\pm1,\pm2,\cdots$

$$\nu = \frac{c}{L}\sqrt{n_1^2+n_2^2+n_3^2} \qquad n_1, n_2, n_3 = 0, \pm 1, \pm 2 \cdots$$

e.g. $10^5$

2) 在

有多少种 $n_1$ $n_2$ $n_3$ 组合满足

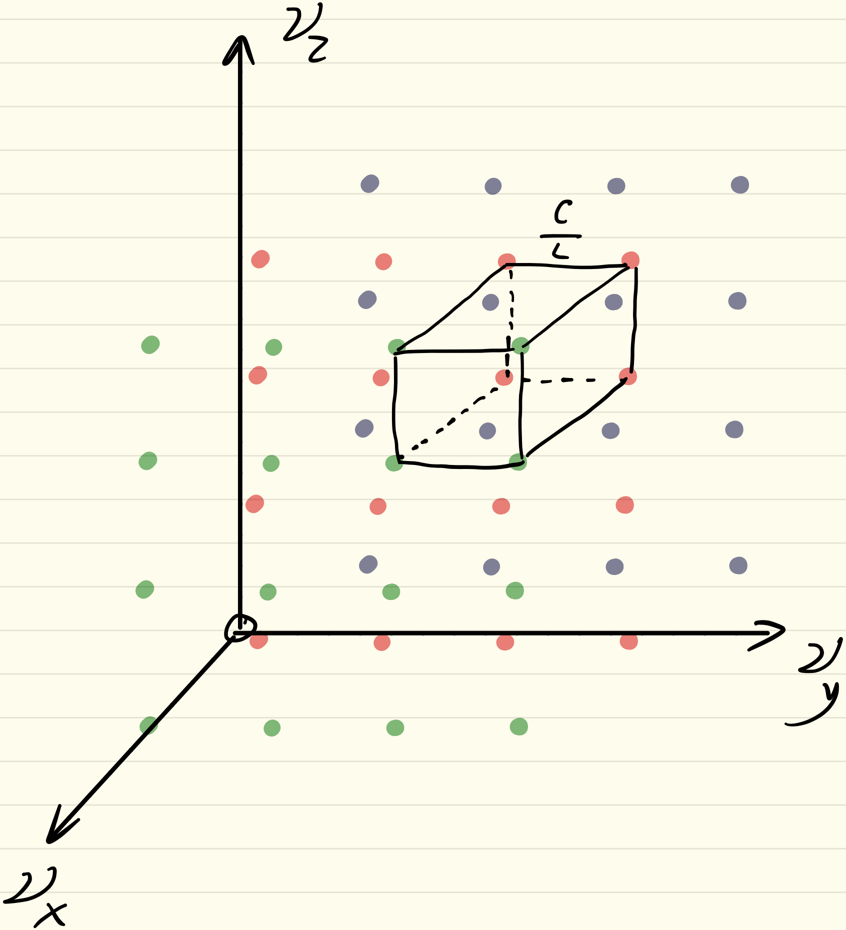
$$10^6 \text{ Hz} \leq \nu < 1.001 \times 10^6 \text{ Hz}$$

$$\nu = \frac{c}{L}\sqrt{n_1^2+n_2^2+n_3^2}$$

$$\begin{cases} \nu_x = \frac{c}{L} n_1 \\ \nu_y = \frac{c}{L} n_2 \\ \nu_z = \frac{c}{L} n_3 \end{cases}$$

$(\nu, \nu+d\nu)$范围

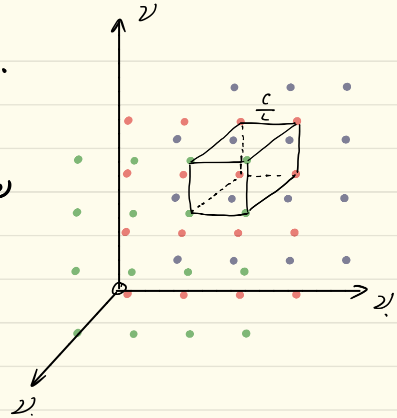
$$\frac{c}{L} \ll d\nu$$



$\nu_1 = \frac{c}{L} n_1 \qquad \nu_2 = \frac{c}{L} n_2 \qquad \nu_3 = \frac{c}{L} n_3 .$

$$\nu = \sqrt{\nu_1^2 + \nu_2^2 + \nu_3^2}$$

$\nu \to \nu + d\nu$




小立方体体积 $= \left(\frac{c}{L}\right)^3$

球壳体积 $= 4\pi \cdot \nu^2 \cdot d\nu$

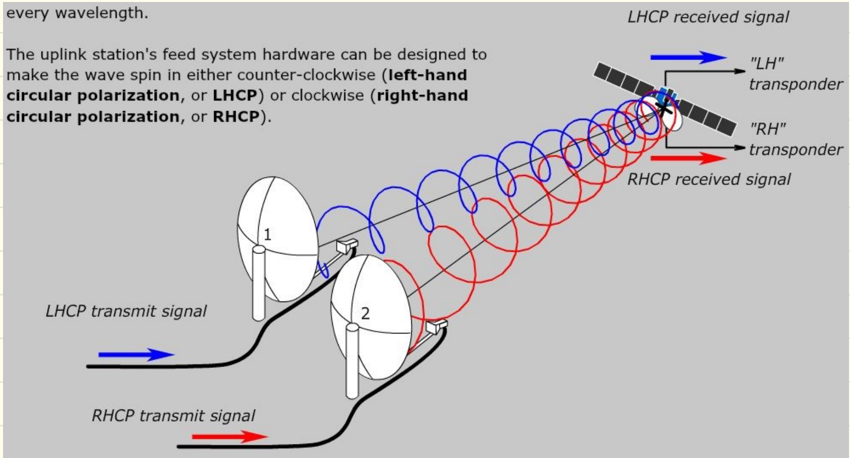
$\nu \to \nu + d\nu$ 范围点个数 $\dfrac{4\pi\nu^2 \cdot d\nu}{\left(\frac{c}{L}\right)^3} = 2\times \dfrac{4\pi L^3}{c^3}\nu^2 d\nu$

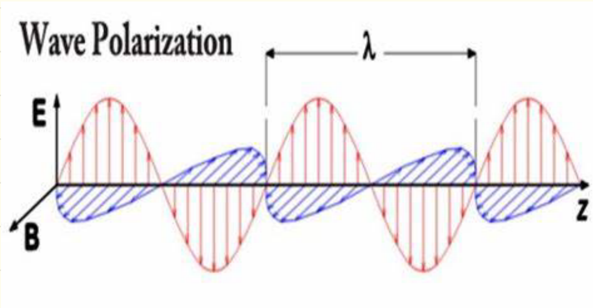
极化

$$G(\nu) = \frac{8\pi L^3}{c^3}\nu^2 d\nu = \frac{8\pi V}{c^3}\nu^2 \cdot d\nu$$

$$g(\nu) = \frac{8\pi V}{c^3} \cdot \nu^2$$

# 电磁波的极化


every wavelength.
The uplink station's feed system hardware can be designed to make the wave spin in either counter-clockwise (**left-hand circular polarization**, or **LHCP**) or clockwise (**right-hand circular polarization**, or **RHCP**).
LHCP received signal
"LH" transponder
"RH" transponder
RHCP received signal
1
2
LHCP transmit signal
RHCP transmit signal


Wave Polarization
λ
E
B
Z

# 瑞利-金斯公式

$$u(\nu, T) = \frac{g(\nu) \cdot \varepsilon(T)}{V}$$

能量均分定理.

$$\varepsilon(T) = kT$$

代入得 $u(\nu, T) = \left(\frac{8\pi V}{c^3} \cdot \nu^2 \cdot kT\right) / V = \frac{8\pi k \nu^2 T}{c^2}$

$$u(T) = \int_0^{+\infty} u(\nu, T) \cdot d\nu = \int_0^{+\infty} \frac{8\pi kT}{c^2} \nu^2 \cdot d\nu = +\infty$$

# 普朗克的量子理论

* 能量均分定理不适用

* 能量只能取最小单位的整数倍 $\varepsilon_n = n \cdot h\nu$

平均能量 $\bar{\varepsilon} = \dfrac{\text{总能量}}{\text{总状态数}} = \dfrac{\sum_{n=0}^{+\infty} \varepsilon_n \cdot \bar{a}_n}{\sum_{n=0}^{+\infty} \bar{a}_n}$

MB分布

$\bar{a}_n = g_n \cdot e^{-\alpha-\beta\varepsilon_n}$

得 $\bar{\varepsilon} = \dfrac{\sum_{n=1}^{\infty} \varepsilon_n g_n \cdot e^{-\beta\varepsilon_n}}{\sum_{n=1}^{\infty} \cdot g_n \cdot e^{-\beta\varepsilon_n}}$

配分函数.

$$Z = \sum_{n=0}^{+\infty} e^{-\beta \varepsilon_n} = \sum_{n=0}^{+\infty} e^{-n\beta \cdot h\nu} = \underbrace{\sum_{n=0}^{+\infty} \left(e^{-\beta h\nu}\right)^n}_{\text{等比数列求和}}$$

$$= \frac{1}{1-e^{-\beta h\nu}}$$

平均能量：$\bar{\varepsilon} = -\frac{\partial}{\partial \beta} \ln Z = -\frac{1}{Z} \cdot \frac{\partial Z}{\partial \beta}$

$$= -\frac{1}{Z} \cdot \left(\frac{1}{1-e^{-\beta h\nu}}\right)^2 \cdot h\nu \cdot e^{-\beta h\nu}$$

$$= -(1-e^{-\beta h\nu}) \cdot \left(\frac{1}{1-e^{-\beta h\nu}}\right)^2 \cdot h\nu \cdot e^{-\beta h\nu}$$

$$= \frac{h\nu}{e^{\beta h\nu}-1}$$

有别于能量均分定理的 $\bar{\varepsilon} = kT$

Taylor展开

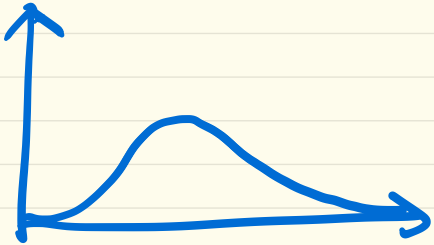
$e^{\beta h\nu} \rightarrow 1 + \beta h\nu + \cdots$ 则 $\bar{\varepsilon} = kT$

$$u(\nu, T) = \frac{g(\nu) \cdot \varepsilon(\nu, T)}{V}$$

$$= \frac{8\pi}{c^3} \cdot \frac{h\nu^3}{e^{\frac{h\nu}{kT}} - 1}$$

$\nu \to 0$ 时 $u \to 0$ （洛比塔法则）

$\nu \to +\infty$ 时 $u \to 0$



$$u(T) = \int_0^{+\infty} u(\nu, T) \cdot d\nu \propto T^4$$