圆轨道稳定性

重力力矩 $-mg\frac{L}{2}\theta$

弹簧力矩: $kL^2\theta$

总力矩 $(kL^2 - mg\frac{L}{2})\theta$

总力矩为逆时针 则使θ减小.

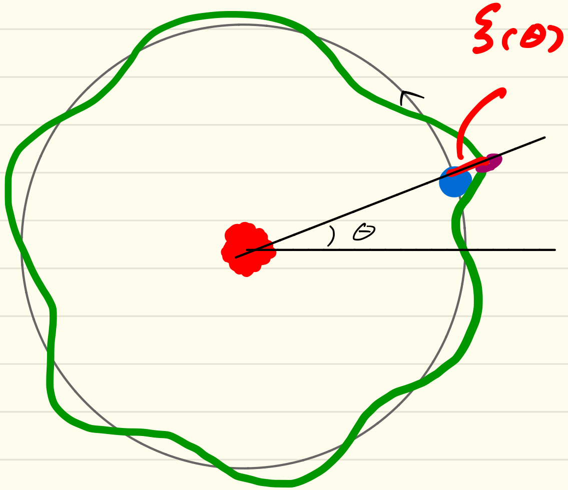
$\Rightarrow$ 稳定.

$kL^2 - mg\frac{L}{2} > 0$

即 $k > \frac{mg}{2L}$

$$u \equiv \frac{1}{r}$$

$$h^2 u^2 \left(\frac{d^2 u}{d\theta^2} + u \right) = -\frac{F}{m} \equiv P(u)$$



$$h^2 u^3 = P(u)$$

引入 $u = \underbrace{u_0}_{\text{圆轨道.}} + \underbrace{\xi(\theta)}_{\text{微扰.}}$

代入比耐公式.

$$h^2 (u_0+\xi)^2 \left(\frac{d^2(u_0+\xi)}{d\theta^2} + u_0 + \xi \right) = P(u_0+\xi)$$

$$h^2 (u_0+\xi)^2 \left(\frac{d^2 u_0}{d\theta^2} + \frac{d^2 \xi}{d\theta^2} + u_0 + \xi \right) = P(u_0+\xi)$$

$$\left(\frac{d^2u_0}{d\theta^2}+\frac{d^2\xi}{d\theta^2}+u_0+\xi\right)=\frac{P(u_0+\xi)}{h^2(u_0+\xi)^2}$$

对 $P(u_0+\xi)$ 做 Taylor 展开 $P(u_0+\xi)=P(u_0)+\frac{dP}{du}\xi+\mathcal{O}(\xi^2)$

$\frac{1}{(u_0+\xi)^2}$ 做 Taylor 展开 $\frac{1}{(u_0+\xi)^2}=\frac{1}{u_0^2}\left(1-2\frac{\xi}{u_0}+\mathcal{O}(\xi^2)\right)$

$$\frac{d^2u_0}{d\theta^2}+\frac{d^2\xi}{d\theta^2}+u_0+\xi=\left(P(u_0)+\frac{dP}{du}\xi+\mathcal{O}(\xi^2)\right)\cdot\frac{1}{u_0^2}\left(1-2\frac{\xi}{u_0}+\mathcal{O}(\xi^2)\right)$$

$$=\frac{P(u_0)}{u_0^2}+\frac{1}{u_0^2}\left[\frac{dP}{du}-2\frac{P(u_0)}{u_0}\right]\cdot\xi+\mathcal{O}(\xi^2)$$

圆轨道比耐公式

可消去.

$$\frac{d^2\xi}{d\theta^2} - \left[\left.\frac{dP}{du}\right|_{u=u_0} - 2\frac{P(u_0)}{u_0} + 1\right]\xi = 0$$