理论力学第二次作业

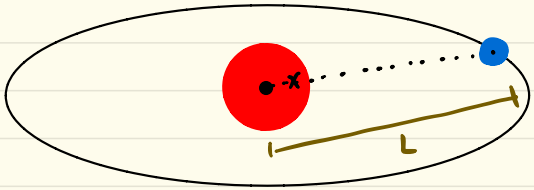
近似 $M_s \gg m_p$

1. 系外行星径向速度法

$V = 4.92\ m/s$

$T = 20.73\ day$

$M_s = 1.2\ M_\odot$



$$F = \frac{G M_s m_p}{L^2}$$

质心到行星距离 $d_p = \frac{M_s}{M_s + m_p} \cdot L$

…… 恒 …… $d_s = \frac{m_p}{M_s + m_p} \cdot L$

$$\frac{G M_s m_p}{L^2} = M_s \omega^2 \cdot d_s$$

$$\omega = \frac{2\pi}{T}$$

$$G(M_s + m_p) = \left(\frac{2\pi}{T}\right)^2 \cdot L^3$$

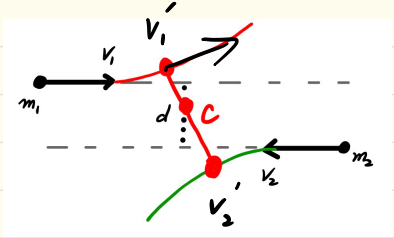
$$\approx \left(\frac{2\pi}{T}\right)^2 \frac{L^3}{M_s}$$

解出 L.

易. $V = \omega \cdot d_s$

$$\frac{m_p}{M_s} L \cdot \omega = V$$

解出 m_p



$F = k/r^2$ 排斥力

选质心静止参考系（质心运动速 $v_C = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}$）

在此参考系下，1 质点速率 $v_1 - v_C$

质心位置位于距离 1 质点路径 $\frac{m_2}{m_1+m_2} d$

最近点时速度方向与连线方向垂直，角动量守恒

$$m_1 v_1 \cdot \frac{m_2}{m_1+m_2} d = m_1 v_1' \cdot \frac{m_2}{m_1+m_2} \cdot c$$

1 质点受力 $F_1 = k/r^2$，对应的势 $V = +\frac{k}{r}$

机械能守恒

引入 r_1 …… 质心到 1 质点的距离，$F_1 = k\left(\frac{m_2}{m_1+m_2}\right)^2 / r_1^2$

$$r_1 = \frac{m_2}{m_1+m_2} \cdot r$$

$$V_1 = k\left(\frac{m_2}{m_1+m_2}\right)^2 / r_1$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + k\left(\frac{m_2}{m_1+m_2}\right)^2 \Big/ \left(\frac{m_2}{m_1+m_2} \cdot c\right)$$

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1'^2 + k\left(\frac{m_1+m_2}{m_2}\right)\Big/c$$

$$v_1' = \frac{v_1 d}{c}$$

代入得

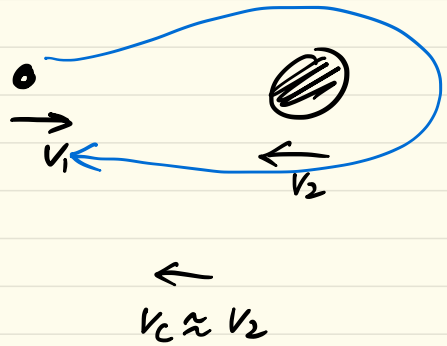
$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1\frac{v_1^2d^2}{c^2} + k\left(\frac{m_1+m_2}{m_2}\right)\Big/c$$

$$\frac{1}{2}m_1v_1^2c^2 - k\left(\frac{m_1+m_2}{m_2}\right)c - \frac{1}{2}m_1v_1^2d^2 = 0$$

$$C = \frac{k\left(\frac{m_1+m_2}{m_2}\right) \pm \sqrt{k^2\left(\frac{m_1+m_2}{m_2}\right)^2 + m_1^2v_1^4d^2}}{m_1v_1^2}$$

C显然为正，故取“+”号.

弹弓效应



质心静止参考系下.

地球速率 $v_1 + v_2$

如弹性碰撞. 后速率 $v_1 + v_2$

但方向相反.

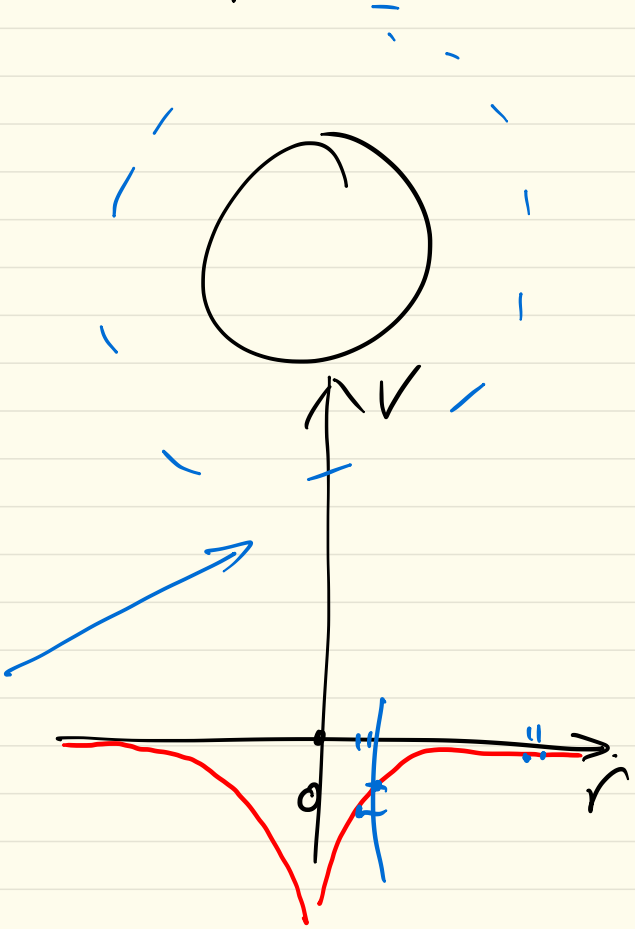
$$v_1 + v_2 + v_2 = v_1 + 2v_2$$

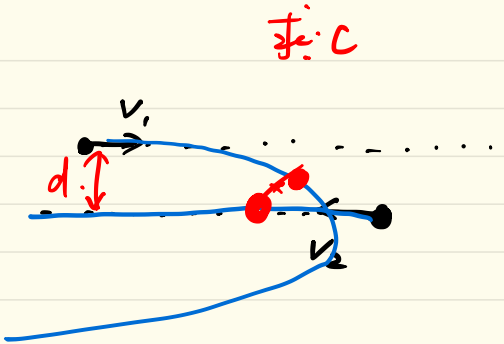
静止参考系下.

地球 $v_1 \longrightarrow v_1 + 2v_2$

洛希极限.

$$V = -\frac{GM_0}{r}$$





$M_J >> M_\oplus$

火箭

$$\frac{d[(M-\lambda t)\cdot V]}{dt} + \lambda\cdot(V-u_0) = -(M-\lambda t)g$$

$$M\frac{dV}{dt} - \frac{d(\lambda t\cdot V)}{dt} + \lambda V - \lambda u_0 = (M-\lambda t)g$$

$$M\frac{dV}{dt} - \lambda V - \lambda t\frac{dV}{dt} + \lambda V - \lambda u_0 = (M-\lambda t)g$$

$$\frac{dV}{dt} = \frac{\lambda u_0 + (M-\lambda t)g}{M-\lambda t}$$

积分得 $V = \cdots$