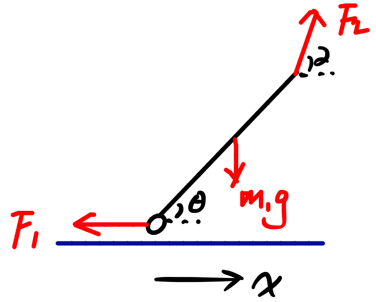
一、① 求广义力，可通过求势来求。

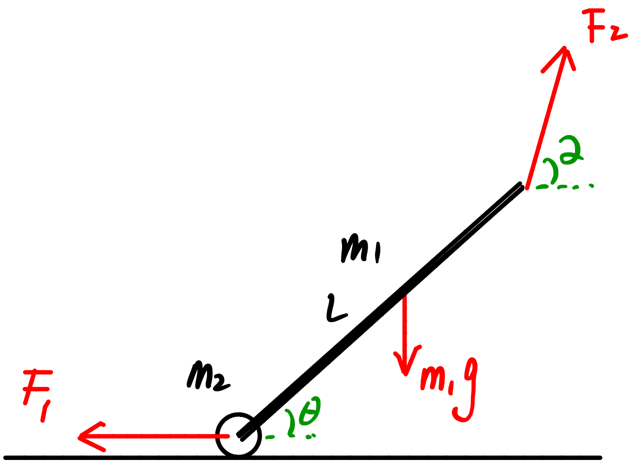


$$V = m_1 g \cdot \frac{L}{2} \cdot \sin\theta$$
$$+ F_1 \cdot x$$
$$-(F_2 \cdot \cos\alpha) \cdot (x + L \cdot \cos\theta)$$
$$-(F_2 \cdot \sin\alpha) \cdot L \cdot \sin\theta$$

广义力 $Q_x = -\frac{\partial V}{\partial x} = \cdots$; $Q_\theta = -\frac{\partial V}{\partial \theta} = \cdots$

② 虚位移 δx 和 $\delta\theta$，对应的虚功：$\delta W = Q_x \cdot \delta x + Q_\theta \cdot \delta\theta$

δx 和 $\delta\theta$ 相对独立。虚功原理：$\delta W = 0$。则 $Q_x = 0$，$Q_\theta = 0$



水平方向力平衡条件:

$$F = F_2 \cdot \cos\alpha$$

转动力矩平衡条件.

逆时针为正.

总力矩.

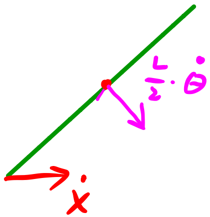
$$-m_1 g \frac{L}{2} \cdot \cos\theta - F_2 \cdot \cos\alpha \cdot L \cdot \sin\theta + F_2 \cdot \sin\alpha \cdot L \cdot \cos\theta = 0$$

③. 将 $L = T - V$ 代入拉格朗日方程. $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_s}\right) - \frac{\partial L}{\partial q_s} = 0$

柯尼希定理

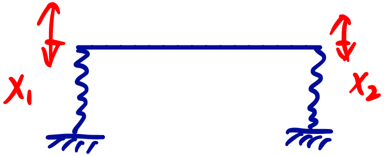
其中 $T = \frac{1}{2} m_2 \cdot \dot{x}^2 + \frac{1}{2} m_2 \cdot V_{\text{杆中心}}^2 + \frac{1}{2} I_{\text{杆中心}} \cdot \dot{\theta}^2$

其中 $V_{\text{杆中心}}^2 = V_{\text{杆中心}x}^2 + V_{\text{杆中心}y}^2$



$V_{\text{杆中心}x} = \dot{x} + \frac{L}{2}\dot{\theta} \cdot \sin\theta$

$V_{\text{杆中心}y} = \frac{L}{2}\dot{\theta} \cdot \cos\theta$.



广义坐标 x_1，x_2. 平衡位置为原点的竖直方向坐标。

动能（柯尼希定理）: $T = \frac{1}{2} m v_c^2 + \frac{1}{2} I_c \cdot \omega^2$

$$= \frac{1}{2} m \left(\frac{\dot{x}_1 + \dot{x}_2}{2}\right)^2 + \frac{1}{2} I_c \left(\frac{\dot{x}_2 - \dot{x}_1}{L}\right)^2$$

$\boxed{I_c = \frac{1}{12} m L^2}$

势能. $V = \frac{1}{2} k (x_1^2 + x_2^2)$

$L = T - V$，已经是 $x_1, x_2, \dot{x}_1, \dot{x}_2$ 的二阶多项式. 不需做 Taylor 展开了.

代入拉格朗日方程. $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_{1,2}}\right) - \frac{\partial L}{\partial x_{1,2}} = 0$

$$\frac{d}{dt}\left(\frac{1}{4}m\dot{x}_1+\frac{1}{4}m\dot{x}_2+\frac{1}{12}m\cdot\dot{x}_1-\frac{1}{12}m\dot{x}_2\right)-kx_1=0$$

$$\frac{1}{3}m\ddot{x}_1+\frac{1}{6}m\ddot{x}_2+kx_1=0. \quad \cdots ①$$

另一个方程可做 $x_1 \Leftrightarrow x_2$（对称性）得到

$$\frac{1}{6}m\ddot{x}_1+\frac{1}{3}m\ddot{x}_2+kx_2=0 \quad \cdots\cdots ②$$

$2\times① - ②$ 消去 $\ddot{x}_2$

$2\times② - ①$ 消去 $\ddot{x}_1$

$$\begin{cases}\frac{1}{2}m\ddot{x}_1+2kx_1-kx_2=0\\ \frac{1}{2}m\ddot{x}_2-kx_1+2kx_2=0\end{cases}$$

代入通解 $x_1=e^{-\lambda t}$.

$$\begin{cases}\left(\frac{1}{2}m\lambda^2+2k\right)\cdot x_1-kx_2=0.\\ \left(\frac{1}{2}m\lambda^2+2k\right)\cdot x_2-kx_1=0\end{cases}$$

对应行列式 $\begin{vmatrix} \frac{1}{2}m\lambda^2+2k & -k \\ -k & \frac{1}{2}m\lambda^2+2k \end{vmatrix} = 0$.

$$\left(\frac{1}{2}m\lambda^2+2k\right)^2 - k^2 = 0$$

$$\frac{1}{4}m^2\lambda^4 + 2mk\lambda^2 + 3k^2 = 0$$

$$m^2\lambda^4 + 8mk\lambda^2 + 12k^2 = 0.$$

$$(m\lambda^2+2k)(m\lambda^2+6k) = 0.$$

$$\lambda_1 = \pm\sqrt{\frac{2k}{m}} \qquad \lambda_2 = \pm\sqrt{\frac{6k}{m}}.$$