非线性方程的求根

$$x^3 - 10x^2 + 30x - 25 = 0$$

$x = 0$: $f(0) = -25 . < 0$

$x = 2$: $f(2) = 8 - 40 + 60 - 25 > 0$

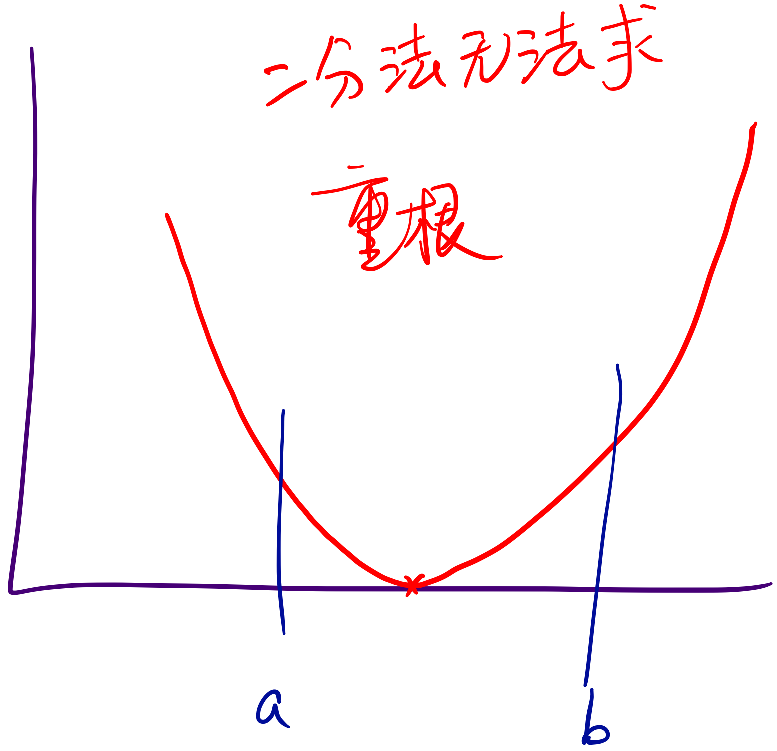
$[0, 2]$. $f(1) = 1 - 10 + 30 - 25 < 0$

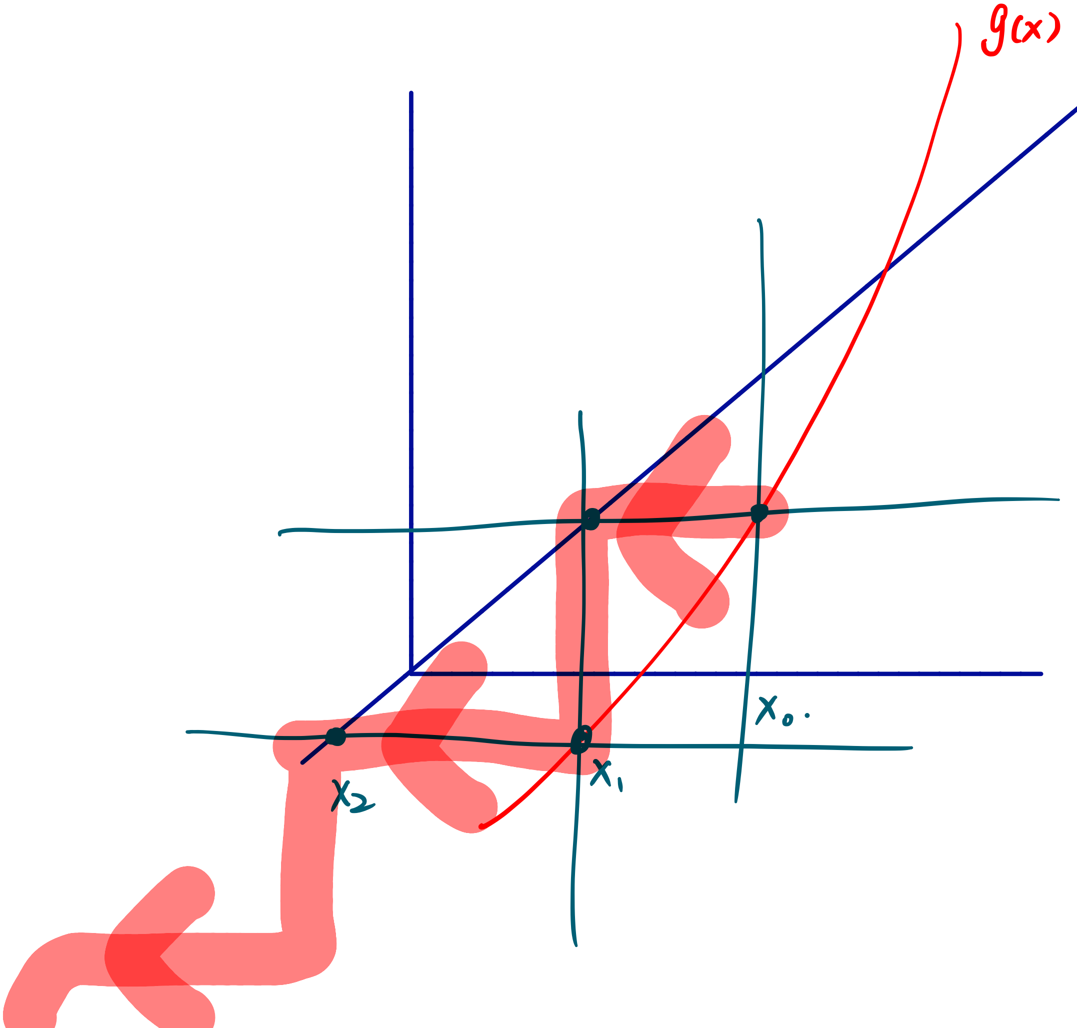
$f(0) < 0 .$ $f(1) < 0$ $f(2) > 0$

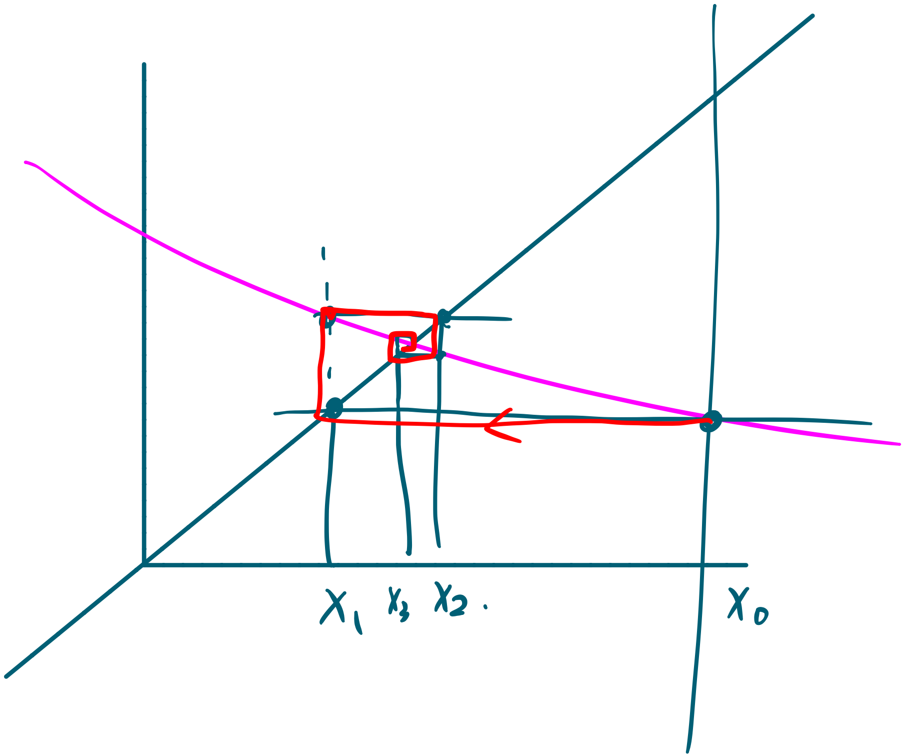
$[1, 2]$ $f(1.5) = 1.5^3 - 10 \times 1.5^2 + 30 \times 1.5 - 25 = \cdot\cdot$

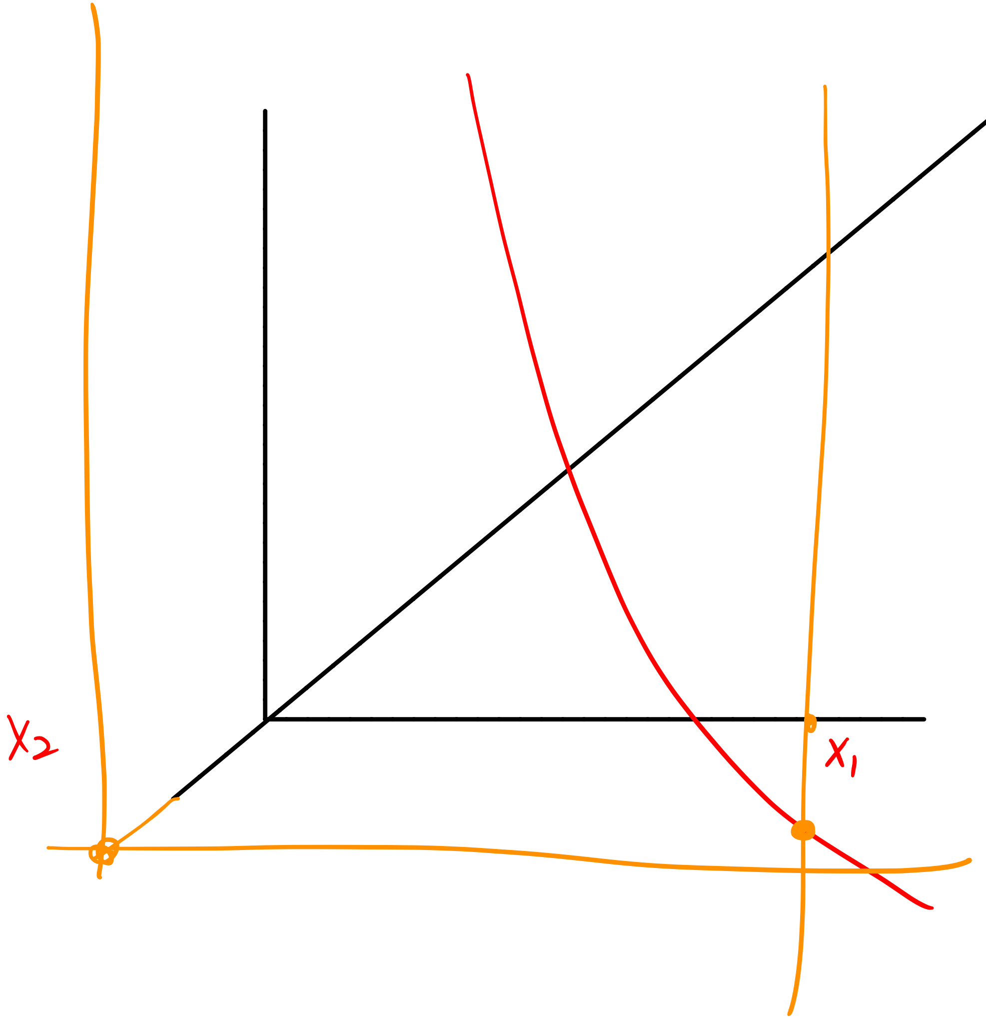
$f(1) < 0 .$ $f(1.5) > 0 .$ $f(2) > 0$

$[1, 1.5]$


二分法无法求
重根
a
b


g(x)
x₀.
x₁
x₂


x_1
x_3
x_2
x_0



x_2
x_1

$g(x) = e^{-x}$ 判断收敛性

$$g'(x) = -e^{-x}$$

$x \in [0, 1]$. $\frac{1}{e} < |g'(x)| < 1$

牛顿法

$$f(x) = x - x^{\frac{1}{3}} - 2 \quad , \quad \boxed{f(x) = 0}$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{x_k - x_k^{\frac{1}{3}} - 2}{1 - \frac{1}{3} x_k^{-\frac{2}{3}}}$$

$$= x_k - \frac{3x_k - 3x_k^{\frac{1}{3}} - 6}{3 - x_k^{-\frac{2}{3}}} = \frac{\cancel{3x_k} - x_k^{\frac{1}{3}} - \cancel{3x_k} + 3x_k^{\frac{1}{3}} + 6}{3 - x_k^{-\frac{2}{3}}}$$

线性.

$$\begin{cases} f(x+y) = f(x) + f(y) \\ f(ax) = af(x) \end{cases}$$

或等效于:

$$f(ax+by) = af(x) + bf(y)$$

线性代数

$$Ax_1 + Ax_2$$

$$= A(x_1+x_2)$$

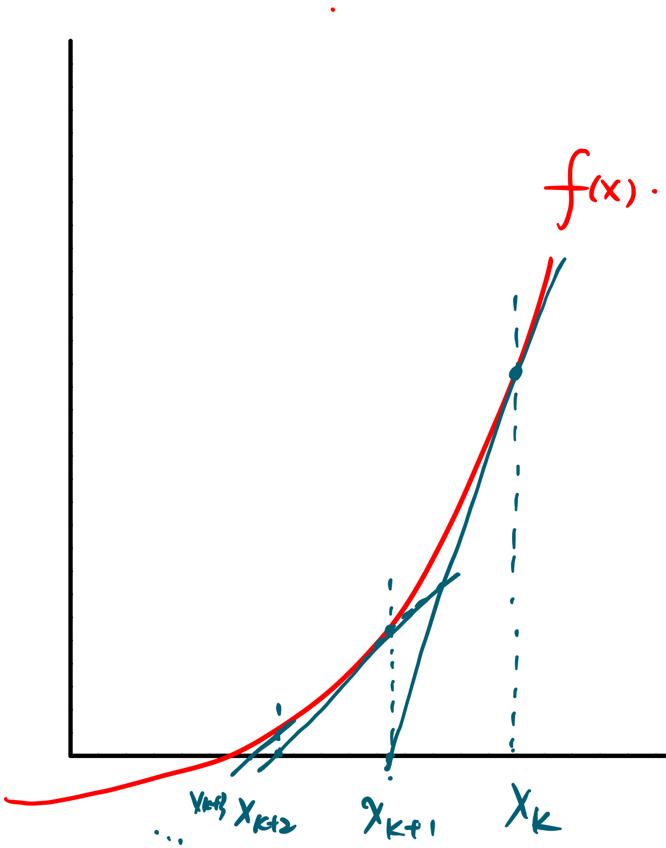
$$A(ax_1) = a(Ax_1)$$

微分方程

$$y' = f(x)$$

x_1, x_2 为解.

则 ax_1 也为解 x_1+x_2 也为解



f(x)
x_{k+3}
x_{k+2}
x_{k+1}
x_k
...