ECE4710J Assignment 1

Name: Zhou Haoquan
ID:519370910059

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1 Fundamental Linear Algebra

a)

$$B = \begin{bmatrix} 2 & 2 & 2 \\ 5 & 8 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 10 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

c)

We first multiple A and B to get

$$AB = \begin{bmatrix} 9 & 12 & 4 \\ 7 & 12 & 15 \\ 0 & 0 & 100 \end{bmatrix}$$

now we can assume v_2 is

$$v_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

by multiplying two matrices above, we can write the equations

$$\begin{cases} 9x + 12y + 4z = 80 \\ 7x + 12y + 15z = 80 \\ 0x + 0y + 100z = 100 \end{cases}$$

which gives us the result

$$v_2 = \begin{bmatrix} \frac{11}{2} \\ \frac{53}{24} \\ 1 \end{bmatrix}$$

2 Calculus

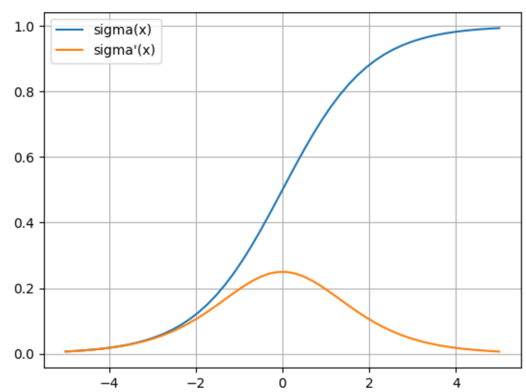
a)

$$\sigma(-x) = \frac{1}{1+e^x}$$
$$1-\sigma(x) = 1-\frac{1}{1+e^{-x}} = \frac{e^{-x}}{1+e^{-x}} = \frac{1}{e^x+1}$$
$$\Rightarrow \sigma(-x) = 1-\sigma(x)$$

b)

$$\frac{d}{dx}\sigma(x) = \frac{e^{-x}}{1+2e^{-x}+e^{-2x}} = \frac{1}{e^x+e^{-x}+2}$$
$$\sigma(x)(1-\sigma(x)) = \frac{1}{1+e^{-x}}\frac{1}{1+e^x} = \frac{1}{e^x+e^{-x}+2}$$
$$\Rightarrow \frac{d}{dx}\sigma(x) = \sigma(x)(1-\sigma(x))$$

c)



3 Minimization

We take the derivative of $f(c)$ and get

$$f'(c) = \frac{2}{n}\sum_{i=1}^{n}(c-x_i)$$

Now we want this derivative to be zero, that is

$$nc - \sum_{i=1}^{n} x_i = 0 \Rightarrow c = \frac{\sum_{i=1}^{n} x_i}{n}$$

Then we want to prove that this is a minimum, we calculate the second derivative of $f(c)$

$$f^{''}(c) = 2 > 0 \Rightarrow f^{''}(\frac{\sum_{i=1}^{n} x_i}{n}) = 2 > 0$$

Now we can verify that

$$c = \frac{\sum_{i=1}^{n} x_i}{n}$$

is indeed the minimum.

4 Probability

To make it easy, we assume that there are 100 women in this age. Then we can tell according to the probability that 1 of them have breast cancer and 99 of them are healthy.

Then for the real patient, it contributes $1 \times 0.8 = 0.8$ positive reuslt and $1 \times 0.2 = 0.2$ negative result.

For the healthy ones, it contributes $99 \times 9.6\% = 9.504$ positive result and $99 \times 90.4\% = 89.496$ negative result.

Now we have this woman tested positive, which means that she can have $\frac{0.8}{0.8+9.504} = 7.764\%$ to actually have breast cancer.

Use Bayes's Rule can get the same result.

5 Statistics

We calculate the posibility based on whether the students are Canadian or not. This obeys the Binomial Distribution. We try to approximate Binomial Distribution using Normal Distribution.

Then we can calculate the standard deviation of the corresponding Normal Distribution. We can then use the conclusion that the probability for a normal random variable lies in two standard deviations is 95% in Normal Distribution. We can then find that if the $\sigma = 6.1$, two standard deviations can almost contain 95% of the random varaibles. Then we can conclude that the standard deviation is about **b) 6.1**.