Gradient Descent

January 15, 2024

```
[1]: using LinearAlgebra
```

```
[2]: """
         x = gradient_descent(A, b, n)

     Use gradient descent (a rather inefficient method) to approximate the solution␣
      ↪of `A*x=b`
     using `n` iterations.
     """
     function gradient_descent(A, b, n)
         x0 = A \ b
         x = zero(b)
         for i in 1:n
             r = b - A*x
             println("iter $i   |e|=$(norm(x-x0))   |r|=$(norm(r))")
             # x1 = x +  *r
             # min!(|b - A*x1|^2)
             # min!(|r -  *A*r|^2)
             # solve!((r -  *A*r)' * (A*r))
             s = A * r
              = (r' * s) / (s' * s)
             x +=   * r
         end
         return x
     end
```

```
[2]: gradient_descent
```

```
[3]: A = randn(10, 10)
     A = A + A'
     A = A*A
```

```
[3]: 10×10 Matrix{Float64}:
      31.085      8.80099    -7.22036    …    4.44075    3.30729     3.0609
       8.80099   11.7012     -3.96008        -0.433471  -1.84626     6.18429
      -7.22036   -3.96008    42.3542         -4.59445    0.783553  -17.0228
       1.33746    1.55973     9.75757         3.85878   -1.96274     2.77541
      14.2905    11.6704     -2.33556         2.06972   -5.20206     2.98991
```

```
3.94271     2.26038      5.11624      …     6.12681     7.20081      0.461841
2.74136    -1.14242     -7.29027           2.79331    -1.8995       2.73448
4.44075    -0.433471    -4.59445          27.0575      0.660172     5.18242
3.30729    -1.84626      0.783553          0.660172   16.333       -3.8113
3.0609      6.18429    -17.0228            5.18242    -3.8113       15.5531
```

[4]:
```
b = randn(10)
```

[4]:
```
10-element Vector{Float64}:
 -0.5963705923824458
 -0.4432664799139341
  0.21764847612876542
 -0.051180011877580575
 -0.29391944871729186
 -0.3442958434252882
 -0.4270140761555792
 -0.12804845449736957
  0.04248396384135171
  0.361132711649667
```

[5]:
```
x0 = A\b
```

[5]:
```
10-element Vector{Float64}:
 -0.15167694413936955
 -1.3206882609412474
  0.9862721767909133
 -0.8617355404778875
  1.2693035242190542
 -0.9378460560305537
  0.2735760781208395
  0.0093332584217201
  0.9998243346972874
  1.789086044574857
```

[6]:
```
x = gradient_descent(A, b, 20)
norm(x - x0), norm(b - A*x)
```

```
iter 1    |e|=3.201349644008161    |r|=1.066780544032526
iter 2    |e|=3.192715397577798    |r|=0.5809169134923685
iter 3    |e|=3.180897439694364    |r|=0.5087606598711178
iter 4    |e|=3.168986257811709    |r|=0.48206700276467856
iter 5    |e|=3.1563217421463885   |r|=0.4670217875150276
iter 6    |e|=3.143922587446365    |r|=0.45725667284926247
iter 7    |e|=3.130947937429802    |r|=0.450407509440478
iter 8    |e|=3.1182960296600246   |r|=0.445300154746874
iter 9    |e|=3.1050550868916025   |r|=0.44124886663080043
iter 10   |e|=3.0921890889783796   |r|=0.437853713326544
iter 11   |e|=3.0787719182218445   |r|=0.4348670141983182
```

```
iter 12    |e|=3.065811614152817    |r|=0.43214324093088413
iter 13    |e|=3.0523543969539095   |r|=0.4295912245550948
iter 14    |e|=3.0394102149496933   |r|=0.4271576301635707
iter 15    |e|=3.0260033989715476   |r|=0.4248086664846064
iter 16    |e|=3.0131335484379047   |r|=0.4225242679576549
iter 17    |e|=2.999817992973106    |r|=0.42029129511651897
iter 18    |e|=2.9870460337660285   |r|=0.4181015587380413
iter 19    |e|=2.9738370756726655   |r|=0.41594933322698824
iter 20    |e|=2.961171248555185    |r|=0.4138307375613385
```

[6]: (2.9480739004311296, 0.4117427969705447)

[7]:
```
x = gradient_descent(A, b, 100)
norm(x - x0), norm(b - A*x)
```

```
iter 1     |e|=3.201349644008161    |r|=1.066780544032526
iter 2     |e|=3.192715397577798    |r|=0.5809169134923685
iter 3     |e|=3.180897439694364    |r|=0.5087606598711178
iter 4     |e|=3.168986257811709    |r|=0.48206700276467856
iter 5     |e|=3.1563217421463885   |r|=0.4670217875150276
iter 6     |e|=3.143922587446365    |r|=0.45725667284926247
iter 7     |e|=3.130947937429802    |r|=0.450407509440478
iter 8     |e|=3.1182960296600246   |r|=0.445300154746874
iter 9     |e|=3.1050550868916025   |r|=0.44124886663080043
iter 10    |e|=3.0921890889783796   |r|=0.437853713326544
iter 11    |e|=3.0787719182218445   |r|=0.4348670141983182
iter 12    |e|=3.065811614152817    |r|=0.43214324093088413
iter 13    |e|=3.0523543969539095   |r|=0.4295912245550948
iter 14    |e|=3.0394102149496933   |r|=0.4271576301635707
iter 15    |e|=3.0260033989715476   |r|=0.4248086664846064
iter 16    |e|=3.0131335484379047   |r|=0.4225242679576549
iter 17    |e|=2.999817992973106    |r|=0.42029129511651897
iter 18    |e|=2.9870460337660285   |r|=0.4181015587380413
iter 19    |e|=2.9738370756726655   |r|=0.41594933322698824
iter 20    |e|=2.961171248555185    |r|=0.4138307375613385
iter 21    |e|=2.9480739004311296   |r|=0.4117427969705447
iter 22    |e|=2.9355165780481065   |r|=0.40968328416084626
iter 23    |e|=2.9225320159971315   |r|=0.4076503306163291
iter 24    |e|=2.9100834293132833   |r|=0.40564241726075667
iter 25    |e|=2.8972114388963615   |r|=0.4036581854151253
iter 26    |e|=2.8848710459003537   |r|=0.4016964720786442
iter 27    |e|=2.8721109091108006   |r|=0.39975619821676284
iter 28    |e|=2.8598778886097467   |r|=0.3978364141988503
iter 29    |e|=2.8472287045888396   |r|=0.39593622178476406
iter 30    |e|=2.835102135893857    |r|=0.3940548188724221
iter 31    |e|=2.822562939187401    |r|=0.39219143884271573
iter 32    |e|=2.8105418693949424   |r|=0.39034539156840126
iter 33    |e|=2.798111675302646    |r|=0.38851601342825365
```

```
iter 34    |e|=2.78619514566692413    |r|=0.3867027040328067
iter 35    |e|=2.7738729686117827     |r|=0.38490488377682414
iter 36    |e|=2.7620600254637173     |r|=0.3831220264911326
iter 37    |e|=2.7498448868453433     |r|=0.3813536228151124
iter 38    |e|=2.7381345862677606     |r|=0.37959920917128326
iter 39    |e|=2.7260255179349975     |r|=0.37785833586663975
iter 40    |e|=2.714416927747524      |r|=0.3761305927993499
iter 41    |e|=2.702412973434721      |r|=0.37441558151733934
iter 42    |e|=2.6909051738815113     |r|=0.3727129380313787
iter 43    |e|=2.6790053896627803     |r|=0.371022308245725
iter 44    |e|=2.6675974734561407     |r|=0.36934336820857705
iter 45    |e|=2.6558009276849903     |r|=0.36767580245066395
iter 46    |e|=2.644491999719053      |r|=0.3660193219636849
iter 47    |e|=2.6327977727055893     |r|=0.3643736450675692
iter 48    |e|=2.6215869496101485     |r|=0.3627385133723874
iter 49    |e|=2.609994133174564      |r|=0.3611136748576268
iter 50    |e|=2.5988805428049764     |r|=0.3594988980428984
iter 51    |e|=2.587388239786738      |r|=0.35789395701060683
iter 52    |e|=2.5763710207045154     |r|=0.35629864398480776
iter 53    |e|=2.5649783444724346     |r|=0.3547127560665837
iter 54    |e|=2.5540566454467566     |r|=0.35313610639789755
iter 55    |e|=2.542762719434542      |r|=0.3515685124096962
iter 56    |e|=2.5319356989801345     |r|=0.3500098057279197
iter 57    |e|=2.520739656259434      |r|=0.3484598217597608
iter 58    |e|=2.510006482217264      |r|=0.3469184084816323
iter 59    |e|=2.4989074651126453     |r|=0.3453854172104162
iter 60    |e|=2.4882673142745655     |r|=0.3438607103978284
iter 61    |e|=2.4772644740203917     |r|=0.34234415345172253
iter 62    |e|=2.466716531795851      |r|=0.34083562164767206
iter 63    |e|=2.4558090282325904     |r|=0.33933499288115887
iter 64    |e|=2.4453524883539544     |r|=0.33784215379504307
iter 65    |e|=2.4345394896602452     |r|=0.3363569933571512
iter 66    |e|=2.4241735539226297     |r|=0.33487940829150814
iter 67    |e|=2.4134542363789753     |r|=0.33340929738785446
iter 68    |e|=2.4031781144103457     |r|=0.3319465663148429
iter 69    |e|=2.392551662190338      |r|=0.3304911225785994
iter 70    |e|=2.382364571247861      |r|=0.32904287978746577
iter 71    |e|=2.3718301762330705     |r|=0.32760175318607043
iter 72    |e|=2.361731341021973      |r|=0.3261676634334849
iter 73    |e|=2.3512882026370145     |r|=0.32474053264754993
iter 74    |e|=2.34127685514869       |r|=0.3233202877514764
iter 75    |e|=2.330924180213409      |r|=0.32190685697047844
iter 76    |e|=2.320999559579869      |r|=0.3205001727957193
iter 77    |e|=2.3107365621759937     |r|=0.31910016888186843
iter 78    |e|=2.300897914538201      |r|=0.31770678267178554
iter 79    |e|=2.29072381588819       |r|=0.31631995264940155
iter 80    |e|=2.2809703942761512     |r|=0.3149396206637172
iter 81    |e|=2.270884422632316      |r|=0.3135657294965153
```

```
iter 82    |e|=2.2612154868551575    |r|=0.3121982249199158
iter 83    |e|=2.251216877397466     |r|=0.3108370535430143
iter 84    |e|=2.241631693941965     |r|=0.3094821646333743
iter 85    |e|=2.231719688683158     |r|=0.30813350821073965
iter 86    |e|=2.2222175306194987    |r|=0.30679103665941587
iter 87    |e|=2.2123913783164038    |r|=0.3054547030408289
iter 88    |e|=2.202971525210055     |r|=0.3041244625206773
iter 89    |e|=2.1932304812801644    |r|=0.30280027087531236
iter 90    |e|=2.1838922191090315    |r|=0.30148208575485747
iter 91    |e|=2.174235545551566     |r|=0.3001698653612108
iter 92    |e|=2.164978166627645     |r|=0.2988635695659155
iter 93    |e|=2.1554051319484495    |r|=0.29756315874011907
iter 94    |e|=2.146227934843378     |r|=0.2962685947438464
iter 95    |e|=2.136737813983117     |r|=0.2949798398904983
iter 96    |e|=2.1276401034570975    |r|=0.29369685782224964
iter 97    |e|=2.118232177722309     |r|=0.2924196125936746
iter 98    |e|=2.109213264655952     |r|=0.29114806944633803
iter 99    |e|=2.0998868216525763    |r|=0.28988219399785703
iter 100   |e|=2.090946022981325     |r|=0.28862195292727094
```

[7]: (2.0817003565504, 0.2873673132574228)

[ ]: