Universal Dataset Encoder: A Generalized VQC Approach to Approximate Encoding

First Degree Thesis

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By

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Certificate

This is to certify that the thesis entitled, "*Universal Dataset Encoder: A Generalized VQC Approach to Approximate Encoding*" and submitted by Sejal SARADA ID No. 2020B4A71849G in partial fulfillment of the requirements of BITS F421T Thesis embodies the work done by him under my supervision.

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Abstract

Bachelor of Engineering (Hons.)

Universal Dataset Encoder: A Generalized VQC Approach to Approximate Encoding

by Sejal Sarada

Quantum State Preparation remains constrained by circuit-specific methodologies that limit computational efficiency and generalizability across datasets. This research proposes a novel Variational Quantum Circuit approach utilizing a hardware-efficient ansatz capable of preparing multiple quantum states through a unified circuit architecture. Through comprehensive experimental validation, the proposed approach demonstrates significant advancements in amplitude-encoding of classical real-valued datasets. We employ a gradient-descent optimization strategy with a fidelity-based cost function to train the parameters to the Variational circuit with adjustable layers. The research systematically achieves two primary contributions: Firstly, generating single quantum states with enhanced computational efficiency compared to existing techniques; and Secondly, successfully producing multiple quantum states using a single trainable circuit ansatz with high computational fidelity. The findings show a potential paradigm for more adaptable dataset encoding methodologies, presenting a foundational framework for further work in the field of Quantum State Preparation.

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Chapter 1

Introduction

1.1 Quantum State Preparation: Foundations and Importance

Quantum state preparation is a fundamental process in quantum computing, enabling the encoding of classical data into quantum states. This step is critical for leveraging the computational advantages of quantum mechanics, including superposition, entanglement, and interference. These properties allow quantum systems to explore vast solution spaces efficiently, powering applications in diverse fields such as quantum machine learning, quantum optimization, and quantum simulations.

1.1.1 Significance of Quantum State Preparation

The ability to prepare precise quantum states directly influences the effectiveness of quantum algorithms. For example:

- **Quantum Machine Learning**: Many quantum algorithms require datasets to be encoded as quantum states before training models or performing classification tasks. Encoding high-dimensional datasets, such as those used in image recognition or financial modeling, often defines the scalability of the entire process.[1, 3, 5]

- **Quantum Chemistry**: Accurately simulating molecular systems hinges on the preparation of quantum states that approximate molecular wavefunctions, enabling calculations of chemical properties and reaction dynamics.[9]

- **Optimization Problems**: Algorithms like the quantum approximate optimization algorithm (QAOA) rely on problem-specific quantum states, prepared in superposition, to explore solution spaces and identify optimal configurations.[2]

Quantum state preparation is not just a pre-processing step; it is an enabler for achieving quantum computational advantage. However, as the size and complexity of problems grow, the efficiency and fidelity of state preparation become limiting factors.

1.1.2 Challenges in Current Research

Despite its significance, state preparation faces three primary challenges:

1. **High Resource Requirements**: Many existing methods rely on deep quantum circuits or numerous ancilla qubits, making them computationally expensive and impractical for near-term quantum devices.[1, 2]
2. **Scalability Issues**: Methods often fail to generalize across datasets or applications, requiring new architectures for different problem domains.[4, 6]
3. **Hardware Limitations**: Noise, decoherence, and gate errors in noisy intermediate-scale quantum (NISQ) devices exacerbate the difficulty of implementing resource-intensive techniques.[2]

These limitations create barriers to applying quantum algorithms in real-world settings, highlighting the need for innovative approaches that combine scalability, efficiency, and flexibility.

1.2 Advances and Trends in Quantum State Preparation

1.2.1 Deterministic vs. Learning-Based Approaches

Research in quantum state preparation has developed along two primary paradigms: deterministic methods and learning-based methods. Both approaches have strengths and limitations that inform their applicability to different problems.

Deterministic Methods Deterministic methods aim to provide provable guarantees for preparing specific quantum states. Grover and Rudolph's [1] (2002) work, "Creating Superpositions That Correspond to Efficiently Integrable Probability Distributions," is a seminal contribution in this space. Their algorithm efficiently prepares quantum superpositions representing probability distributions that can be integrated in polynomial time. This deterministic framework guarantees high fidelity but often comes at the cost of high circuit depth and significant computational resources. Other deterministic techniques, such as quantum Fourier transforms and amplitude amplification, also provide accuracy but face similar scalability challenges.[2] These methods are typically problem-specific, making them less adaptable to diverse datasets or generalized use cases.

Learning-Based Methods Learning-based methods, such as variational quantum circuits (VQCs), optimize state preparation through parameterized quantum gates and classical optimization. Unlike deterministic methods, these approaches do not guarantee exact results but provide practical advantages, particularly on NISQ devices. Schuld et al.[3] (2020) demonstrated how variational approaches can adapt to noisy environments while achieving high fidelity for data encoding. Benedetti et al.[4] (2019) highlighted the flexibility of VQCs in preparing quantum states for machine learning tasks. Table 1.1 gives a broad look at the comparison between these two.

The practical limitations of deterministic methods, particularly in resource-constrained environments, have shifted the focus of quantum state preparation research toward learning-based techniques.

1.2.2 The Case for Amplitude Encoding

Among various quantum encoding strategies, including basis encoding and angle encoding, amplitude encoding stands out for its efficiency and versatility. Amplitude encoding maps classical data onto the amplitudes of quantum states, achieving exponential data compression by representing 2^n classical values with nnn qubits. This makes it ideal for high-dimensional datasets encountered in quantum machine learning and optimization problems.[1, 7] For example, Mitarai et al. [6] (2018) demonstrated the utility of amplitude encoding in hybrid quantum-classical learning models, while Havlíček et al. [5] (2019) highlighted its role in quantum-enhanced feature mapping for kernel-based classifiers. These studies emphasize the scalability and practicality of amplitude encoding in diverse applications.

1.2.3 Comparative Analysis of Recent Advances

Recent research has proposed several innovative methods to address the challenges of quantum state preparation. Below is an expanded summary of prominent contributions, analyzing their methodologies and trade-offs.

Table 1.2 highlights the strengths of learning-based amplitude encoding approaches, particularly in adapting to NISQ constraints.

Table 1.1: Key Comparison of Deterministic and Variational Methods in Quantum State Preparation

Aspect	**Deterministic Methods**	**Variational Methods**	**Remarks/Examples**
Accuracy	High (provable guarantees for specific states)	High (empirical fidelity, varies with optimization)	Deterministic methods like Grover and Rudolph (2002) provide exact state preparation, while variational approaches depend on convergence [1,3].
Resource Requirements	High (deep circuits, ancilla qubits)	Moderate (shallow circuits, hardware-efficient ansatz)	Deterministic approaches are impractical for NISQ devices, whereas variational methods are optimized for shallow depths [2,3].
Scalability	Limited (dataset-specific)	Broad (generalizable across datasets)	Variational approaches scale better due to their adaptability to different datasets and noise [4].
Robustness to Noise	Poor (sensitive to hardware imperfections)	High (designed for NISQ devices)	Variational methods are inherently better suited for noisy environments, unlike deterministic algorithms [2,5].
Suitability for NISQ Devices	Poor	High	Variational methods, including VQCs, are practical for current quantum hardware [3,6].

1.3 Problem Statement

Quantum state preparation is a critical process in quantum computing, enabling the encoding of classical data into quantum states for use in algorithms across fields such as optimization, quantum machine learning, and quantum chemistry. Despite its importance, current state preparation methods face significant challenges that limit their scalability, efficiency, and applicability on noisy intermediate-scale quantum (NISQ) devices. These challenges can be categorized as follows:

1. **Excessive Resource Requirements**:Deterministic methods, such as Grover and Rudolph's algorithm for preparing quantum superpositions of probability distributions [1], offer provable guarantees of accuracy. However, they demand deep quantum circuits and substantial ancilla qubits, which are impractical for NISQ hardware due to noise, decoherence, and gate errors [2]. Learning-based approaches, such as those proposed by Schuld et al. [3] and Benedetti et al. [4], reduce resource requirements but can still involve significant computational costs, particularly for large datasets.

2. **Single-State Limitation**: Most existing methods focus on preparing a single quantum state tailored to a specific dataset. Techniques such as the quantum kernel-based encoding approach by Havlíček et al. [5] or the hybrid methods explored by Mitarai et al. [6] require re-optimization or new architectures for each dataset. This limitation reduces their generalizability, making them inefficient for tasks that involve batch processing or multi-state encoding, such as quantum machine learning models that operate on multiple inputs simultaneously.

3. **Absence of a Unified Framework**: Current methodologies lack a standardized framework for designing circuits and selecting optimal parameters to encode diverse datasets effectively. While amplitude encoding offers exponential compression of data [7], its practical implementations remain fragmented and application-specific. This gap complicates the implementation process and creates barriers to the broader adoption of quantum state preparation techniques.

These limitations hinder the practical scalability of quantum algorithms. For instance, in quantum machine learning, inefficient encoding can negate potential quantum speedups [8], while in quantum chemistry, resource-intensive state preparation restricts simulations to small molecular systems [9]. Without addressing these foundational challenges, quantum computing risks being constrained in solving high-dimensional or real-world problems.

To overcome these barriers, this research proposes the **Universal Dataset Encoder**, a variational quantum circuit (VQC) framework designed to address these critical gaps. This approach offers:

- A hardware-efficient ansatz with reduced resource requirements, adaptable to NISQ constraints.
- A generalizable architecture capable of preparing single and multiple quantum states using a unified circuit.
- A systematic methodology for amplitude encoding, balancing scalability, fidelity, and resource efficiency.

By addressing the limitations of current methods, this work aims to advance quantum state preparation, creating a pathway for the broader applicability of quantum computing.

1.4 Proposed Approach: Universal Dataset Encoder

To address the limitations of current quantum state preparation methods—excessive resource requirements, restricted scalability, and the lack of a generalizable framework—this research introduces the UNIVERSAL DATASET ENCODER (UDE). The UDE is a flexible and efficient variational quantum circuit (VQC) framework designed to prepare single and multi-state amplitude-encoded quantum states. It represents a significant departure from existing approaches by combining hardware efficiency, multi-state preparation capabilities, and generalizability within a single unified circuit architecture.

Key Features and Methodology

1. **Hardware-Efficient Ansatz**:At the heart of the UDE is a hardware-efficient ansatz, designed to minimize the depth of quantum circuits while retaining high expressivity. This is achieved through the use of parameterized quantum gates arranged in adjustable layers, allowing the circuit to scale with the complexity of the input data. The ansatz incorporates shallow-depth architectures that are particularly suited to noisy intermediate-scale quantum (NISQ) devices, where decoherence and gate errors impose strict limits on circuit execution.

   This design ensures that the UDE is compatible with current quantum hardware constraints, enabling it to prepare quantum states with high fidelity even on systems with limited qubit counts and connectivity. By focusing on shallow yet expressive circuits, the UDE addresses the inefficiencies of deterministic methods, which typically require deep, resource-intensive implementations.

2. **Multi-State Preparation via Ancilla Qubits**: A key innovation of the UDE is its ability to prepare multiple quantum states simultaneously using a single circuit architecture. This is achieved by incorporating ancilla qubits and utilizing measurement-based control strategies to partition the quantum system into distinct state subspaces.

   In tasks that require encoding multiple datasets or constructing batch states for quantum machine learning, this feature eliminates the need for separate circuits and re-optimization processes. For example, the UDE can prepare 2-qubit, 4-state configurations or 3-qubit, 9-state configurations, effectively reducing resource redundancy while maintaining strong fidelity. This multi-state encoding capability represents a significant improvement over existing single-state methods, which are often tailored to specific datasets and fail to generalize to broader applications.

3. **Amplitude Encoding for High-Dimensional Data**: Amplitude encoding is leveraged in the UDE to achieve exponential data compression, allowing 2^n classical data points to be represented using only n qubits. The UDE incorporates trainable quantum gate parameters, optimized via hybrid quantum-classical feedback loops, to encode data into the amplitudes of quantum states with high accuracy. By iteratively adjusting the gate parameters, the UDE ensures that the target quantum state closely approximates the desired distribution, achieving high fidelity in the process. This approach addresses the challenges of implementing amplitude encoding on NISQ devices, such as balancing fidelity with circuit depth, by adapting the circuit structure to the complexity of the data.

Performance and Results The Universal Dataset Encoder has been rigorously tested in both single-state and multi-state preparation scenarios, demonstrating its robustness and scalability:

- **Single-State Preparation**: The UDE achieved a fidelity of 1.0 in single-state preparation tasks for values to be encoded taken from a real Time-series Dataset. This highlighted its ability to encode classical data into quantum states with perfect accuracy. This performance underscores the circuit's expressivity and the effectiveness of its variational optimization process.
- **Multi-State Preparation**: The UDE was validated for encoding multi-state systems for which the data to be encoded was real-valued hand-written vectors, with the following results:
  - **2-Qubit, 4-State Preparation**: Achieved an average fidelity of 0.89, demonstrating efficient multi-state encoding with minimal resource overhead.
  - **3-Qubit, 9-State Preparation**: Achieved comparable fidelity of 0.87, showcasing the scalability of the approach to larger state configurations.

These results highlight the UDE's ability to handle both single and multi-state preparation tasks effectively, making it a versatile solution for diverse quantum applications. The observed fidelity metrics illustrate the framework's robustness in balancing accuracy with hardware constraints, even in resource-limited environments.

Table 1.2: Comparative Analysis of Recent Advances in Quantum State Preparation

Reference	**Method**	**Encoding Type**	**Single/ Multi-State**	**Circuit Depth**	**Fidelity**	**Key Limitations**
Grover & Rudolph (2002)	Deterministic preparation via efficient integrable distributions	Amplitude	Single	High	1.0	High computational overhead and circuit depth incompatible with NISQ devices
Schuld et al. (2020)	VQC optimization for data encoding	Basis	Single	Moderate	0.85	Requires preprocessing; limited scalability to larger datasets
Mitarai et al. (2018)	Hybrid quantum-classical learning	Amplitude	Single	Moderate	0.89	Noise sensitivity; does not generalize well to multi-state configurations
Quantinuum (2023)	Resource-saving modular protocols	Amplitude	Multi	Adjustable	$\sim$0.9	Requires custom gates; effectiveness varies based on algorithm specifics
Walsh Series Loader (2024)	Efficient decomposition of Walsh series	Amplitude	Single	Low	0.92	Limited applicability to complex or entangled states
Sparse Quantum State Prep. (2023)	Variational Ansatz for correlated systems	Hybrid	Single/Multi	Adjustable	0.87	System-specific convergence issues and resource intensity for larger datasets

Chapter 2

Universal Dataset Encoder

2.1 Aim of the Encoder

In quantum algorithms that process a classical data represented by either a single or an array of real-valued N-dimensional vector d of dataset D, first it has to be encoded into the quantum state; an encoding that is linked to a potential quantum advantage is amplitude-encoding.

2.1.1 Single state preparation

Dataset D has a single N-dimensional vector d: single-state preparation. To encode d to the amplitude of a single n-qubits state $|Data\rangle$. More specifically, given $|j\rangle = |j_1 j_2 j_3 \dots j_n\rangle$ where j_k is the state of the kth qubit in computational basis and $j = \sum_{k=0}^{n} 2^{n-k} j_k$, the data in the quantum state is given by:

$$|Data\rangle = \sum_{j=0}^{N-1} d_j |j\rangle \text{\_\_\_\_\_\_\_\_\_\_\_\_}(1)$$

where $N = 2^n$ and d_j denotes the j^{th} element of the vector d. Also, here d is normalized i.e. $\sum_j (d_j)^2 = 1$. Our approach uses a l-depth VQC [hence composed of O(ln) gates] to try to approximate the ideal state $|j\rangle = |j_1 j_2 j_3 \dots j_n\rangle$. The depth is set to be O(1) O(poly(n)). Suppose now that, given an N-dimensional vector a, the state generated by a VQC, represented by the unitary matrix $U(\theta)$ is given by $U(\theta)|0\rangle^{\otimes n} = \sum_{j=0}^{N-1} a_j |j\rangle$. The goal is to train U() so that the following condition is satisfied:

$$a_j = \langle j|U(\theta)|0\rangle = d_j \forall j \in \{0, 1, \dots, N-1\} \text{\_\_\_\_\_\_\_\_\_\_\_\_}(2)$$

Hence, the goal is to train $U(\theta)$ so that $U(\theta)|0\rangle^{\otimes n} = |Data\rangle$

2.1.2 Multiple state preparation

Dataset D has an array (of dimension g) of N-dimensional vectors d_g, where d_g can be considered the feature vector for datapoint g: multiple-state preparation. To encode d_g to the amplitude of of an n-qubits state $|Data\rangle$. Similar to above,

$$|Data_g\rangle = \sum_{j=0}^{N-1} d_{g_j}|j\rangle \text{\_\_\_\_\_\_\_\_\_\_\_\_\_}(3)$$

where $N = 2^n$ and d_{g_j} denotes the j^{th} element of the vector d_g. Also, here d_g is normalized i.e.

$$\sum_j d_{g_j}^2 = 1$$

Due to multiply states needing encoding, we use the fundamental approach of mapping the basis states to Our approach uses a single l-depth VQC ansatz [hence composed of O(ln) gates] to encode each of the states required one-by-one. We use the fundamental approach of mapping the basis states of the qubit-system to all the states. Given g number of N-dimensional vectors a_g, the states generated by a VQC using x ancillas(refer to section 2.2.2 for further details), represented by the unitary matrix $U(\theta)$ with *theta* the vector of parameters, are mapped as follows:

$$U(\theta)|0\rangle \rightarrow |a_1x_1\rangle \text{\_\_\_\_\_\_}(5)$$

$$U(\theta)|1\rangle \rightarrow |a_2x_2\rangle$$

$$U(\theta)|2\rangle \rightarrow |a_3x_2\rangle$$

$$\vdots$$

$$U(\theta)|g\rangle \rightarrow |a_gx_g\rangle$$

The goal is to train $U(\theta)$ so that the following condition is approximated:

$$a_{g_j} \approx \langle j|U(\theta)|g\rangle = d_{g_j} \forall j \in \{0, 1, \ldots, N-1\} \text{\_\_\_\_\_\_}(4)$$

Hence, the goal is to train $U(\theta)$ so that $U(\theta)|g\rangle \approx |Data_g\rangle \quad \forall g$

2.2 Proposed Circuit Architecture

The proposed circuit leverages a Variational Quantum Circuit (VQC) to achieve high-fidelity state preparation with tunable parameters and adjustable layers. The circuit is structured with

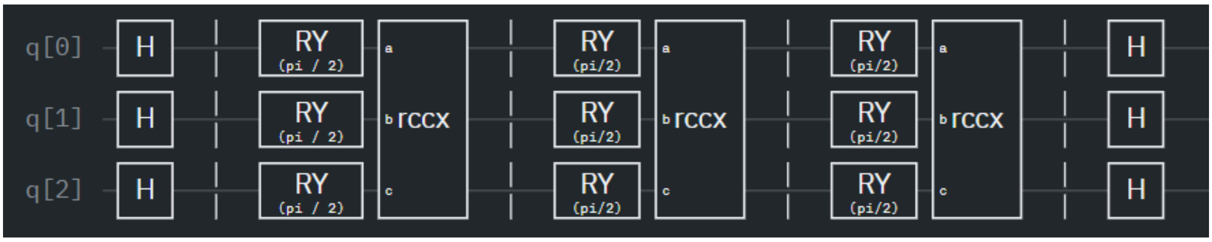
Hadamard layers at both the input and output, ensuring uniform superposition initialization and balanced readouts. This approach is tailored for both single-state preparation and multi-state preparation, with modularity to accommodate a range of quantum applications. Below, the design choices and their justifications are detailed, supported by relevant theoretical insights and experimental results.

2.2.1 Circuit Overview

The circuit begins with a Hadamard layer, transforming the qubits into an equal superposition state. This is followed by a sequence of R_y rotation gates and entanglement layers, which encode the target state. A final Hadamard layer completes the circuit, ensuring a symmetric structure for optimization.

1. **Single-State Preparation**:The circuit for single-state preparation focuses on minimal entanglement layers, optimizing for simplicity and reduced depth.

FIGURE 2.1: Circuit for Single State Preparation



2. **Multi-State Preparation**:In the multi-state scenario, additional entanglement layers and ancilla qubits are introduced to manage correlations between states.

FIGURE 2.2: Multiple for Single State Preparation

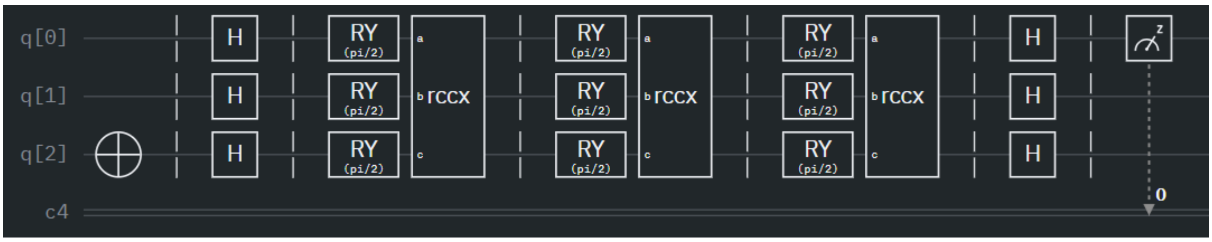
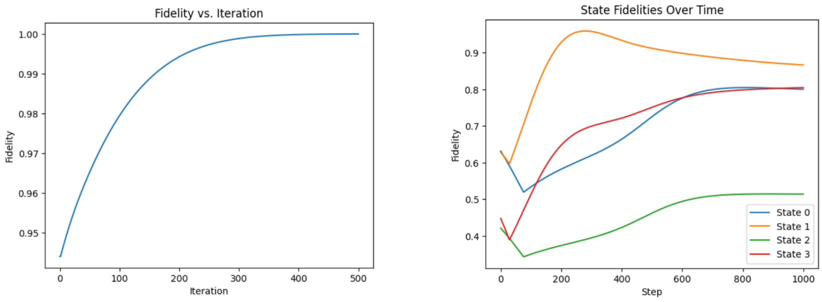


Figure 2.1 and Figure 2.2 illustrate the quantum circuits for single-state and multi-state preparation, respectively.



(A) Single-state prep (B) Multi-state prep

FIGURE 2.3: Fidelity plots when using CNOT gates

2.2.2 Design Choices and Justifications

Several critical design decisions were made to enhance the circuit's performance, supported by theoretical analysis and experimental evidence.

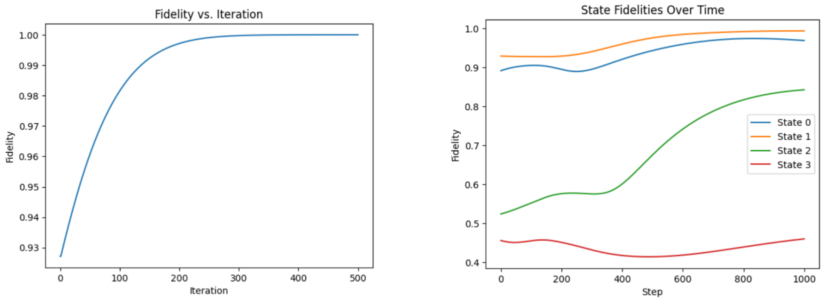
1. Use of only R_y Gates and not R_x or R_z: The choice of R_y gates for parameterized rotations is motivated by the need to encode non-negative real-valued amplitudes. Unlike R_z gates, which modify the phase of quantum states, R_y gates efficiently rotate the state vector in the $X - Z$ plane to achieve the desired amplitude distribution. Taking inspiration from Kouhei Nakaji et al. [13], since the encoding does not require any phase modifications, the use of R_z gates was deemed unnecessary.

2. CNOT vs. CRX Gates: A significant design consideration involved the choice of entangling gates. The circuit uses CRX gates instead of the traditional CNOT gates for entanglement. CRX gates enable smoother parameterization and better control of quantum correlations during training.
   Experimental results (see Figure 2.3, 2.4) highlight the advantages of CRX gates:

   - For single-state preparation, both CNOT and CRX gates achieve high fidelity; however, CRX gates converge marginally faster.
   - For multi-state preparation, CRX gates exhibit significantly better convergence, reaching fidelity ≈ 1 with fewer training epochs compared to CNOT gates. This improvement stems from the CRX gates' ability to encode continuous correlations between states.

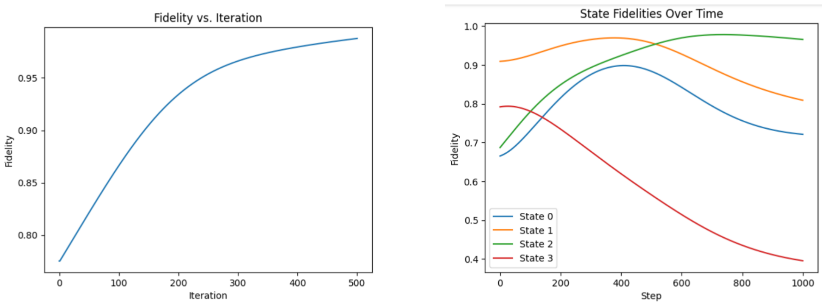
3. Entanglement Topology Three types of entanglement topologies were explored:

   - Complete Entanglement: Provides full connectivity between qubits but introduces high circuit depth and noise susceptibility.
   - Circular Entanglement: Balances connectivity and circuit efficiency, ensuring each qubit is entangled with its nearest neighbors.



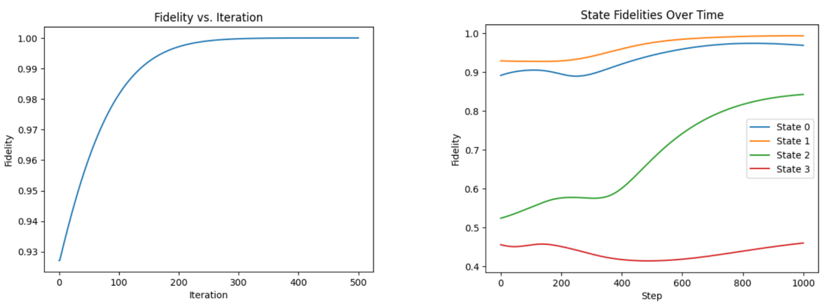
(A) Single-state prep (B) Multi-state prep

FIGURE 2.4: Fidelity plots when using CRX gates



(A) Single-state prep (B) Multi-state prep

FIGURE 2.5: Fidelity plots when using Complete Entanglement

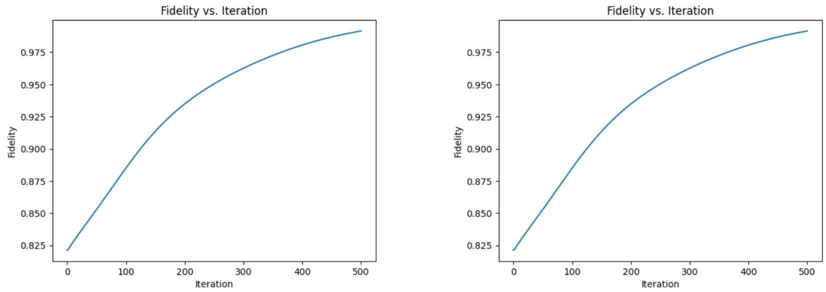


(A) Single-state prep (B) Multi-state prep

FIGURE 2.6: Fidelity plots when using Circular Entanglement

- Alternate Entanglement: Reduces depth further but sacrifices entanglement richness.

Simulation results (Figure 2.5, 2.6, 2.7) demonstrate that circular entanglement achieves the best trade-off, offering high fidelity with reduced circuit complexity. This topology minimizes decoherence effects while maintaining sufficient entanglement for accurate state preparation.



(A) Single-state prep (B) Multi-state prep

FIGURE 2.7: Fidelity plots when using Alternate Entanglement

4. Ancilla Qubits for Multi-State Preparation:To handle the additional complexity of multi-state preparation, ancilla qubits are employed to facilitate intermediate operations and manage quantum correlations. This approach ensures scalability without compromising the overall fidelity. They are introduced for 2 primary reasons:

   - To maintain orthogonality between initial and final quantum states prepared, even when the given data to be encoded is non-orthogonal. Let us prove this: Mathematically, taking inner product on both sides of (5), $\langle 0|\theta\rangle|\langle\theta|1\rangle = 0$.
     Hence, $\langle a_1x_1|a_2x_2\rangle = 0$
   - To encoded all given states when $g > N = 2^n$

   Therefore, for the Multiple state preparation circuit, use x+n number of qubits totally,where x qubits are measured out at the end, leaving an n-qubit quantum state as required.

2.2.3 Finalized Circuit Description

The finalized circuit integrates the above design choices to create an optimized architecture capable of efficient and accurate state preparation. Key components of the circuit include:

1. Initialization and Finalization with Hadamard Layers:These layers ensure that all qubits begin in an equal superposition state and are measured symmetrically, improving the stability and reliability of the optimization process.

2. Rotation Layers Using Ry Gates:Parameterized Ry gates encode the target amplitudes, with tunable parameters optimized during training to achieve high fidelity.
   Experimental results highlight the advantages of CRX gates

3. Circular Entanglement with CRX Gates Circular entanglement ensures efficient connectivity between qubits with minimal circuit depth. The parameterized CRX gates allow for fine-grained control over the degree of entanglement, enhancing the circuit's expressive power.

4. Ancilla Qubits for Multi-State Preparation In multi-state preparation scenarios, ancilla qubits act as intermediaries, enabling the decomposition and reconstruction of complex quantum states. This addition ensures scalability without compromising fidelity.

The finalized architecture achieves a balance between circuit depth, fidelity, and scalability, making it suitable for a wide range of quantum computing applications. Figure 2.1 and 2.2 provides a detailed schematic of the finalized circuit, annotating the key components and their roles in the design.

2.3 Proposed Approach

2.3.1 Single State Preparation

2.3.1.1 Condition for perfect encoding:

Given Data as non-negative real-valued N-dimensional vector d of dataset D, we generate a VQC circuit with parameters θ as $U(\theta)$. During this generation our aim is $U(\theta)|0\rangle \rightarrow |d\rangle$. Considering the condition

$$a_j = \langle j|U(\theta)|0\rangle = d_j \forall j \in \{0, 1, \ldots, N-1\} \_\_\_\_\_\_\_\_ (2)$$

our aim mathematically becomes: $\min_\theta L(\theta) = 1 - \langle d|u(\theta)|0\rangle$

2.3.1.2 Optimization of parameters:

We use Gradient-descent approach for classical optimization of Theta. In order to obtain maximum fidelity, we experimented with different cost-functions and pytorch optimizers. Cost functions looked at are fidelity-cost, mse-cost and l1-cost

$$k = U(\theta)|a\rangle$$

$$\text{(fidelity cost)} \quad L(\theta) = \langle d|k\rangle$$

$$\text{(mse cost)} \quad L(\theta) = \frac{1}{N}\sum_{i=1}^{N}(|d_i| - |k_i|)^2$$

$$\text{(l1 cost)} \quad L(\theta) = \frac{1}{N}\sum_{i=1}^{N}||d_i| - |k_i||$$

Following are our experimental results:

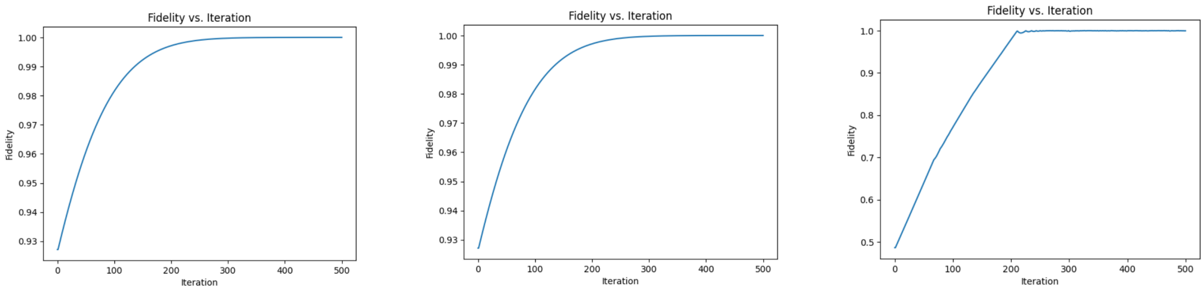


FIGURE 2.8: Graphs for Fidelity convergence using Fidelity-cost, MSE-cost and L1-Cost respectively

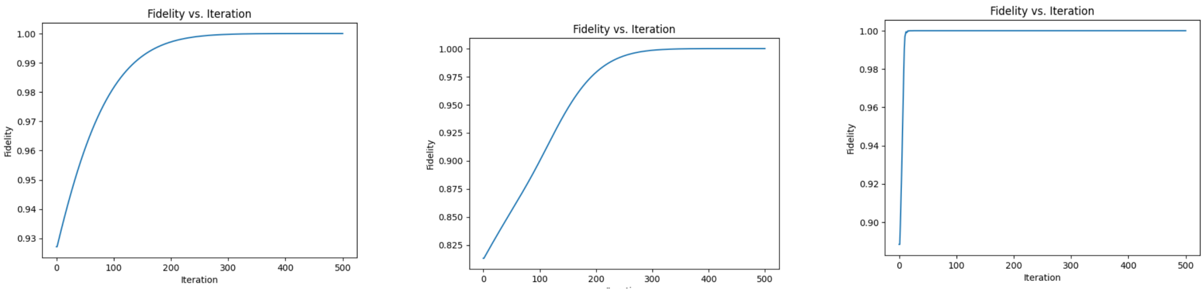


FIGURE 2.9: Graphs for Fidelity convergence using Adam, AdamW and Rprop Optimizers respectively

Observations from the Figures 2.8 for cost function and 2.9 for optimizer used lead to the conclusion: cost-function as fidelity-cost and optimizer as R-prop give us the best and fastest convergence.

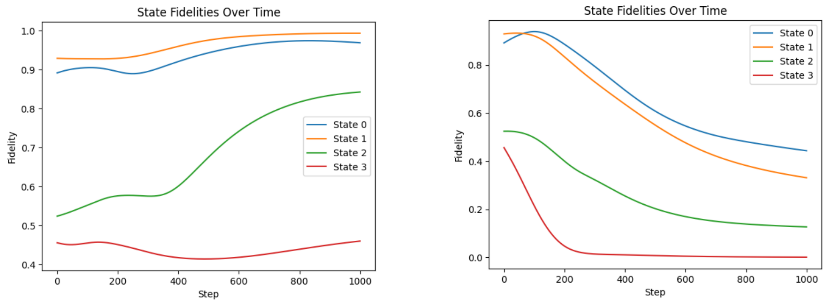
2.3.2 Multiple state preparation

2.3.2.1 Condition for perfect encoding:

Given Data a non-negative real-valued N-dimensional vector d as an element of entire dataset D, we generate a VQC circuit with parameters θ as $U(\theta)$ and measurement operator as V for measuring ancilla at the end. During this generation our aim is $U(\theta)|g\rangle \rightarrow |a_g x_g\rangle$; $V|a_g x_g\rangle \rightarrow |d_g\rangle$ where $|a_g x_g\rangle = \alpha_g|0\rangle|d_g\rangle + \beta_g|1\rangle|d_g\rangle$ Considering the condition

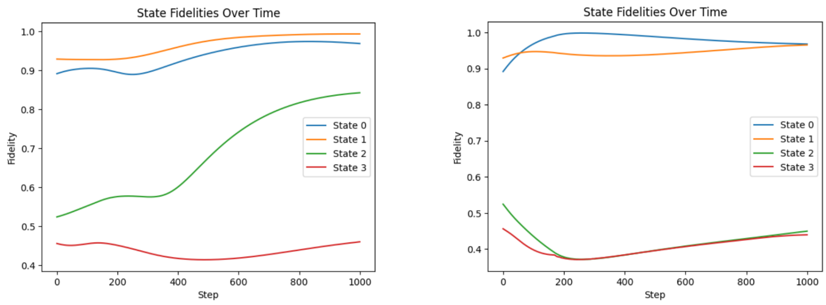
$$a_{g_j} \approx \langle j|U(\theta)|g\rangle = d_{g_j} \forall j \in \{0, 1, \ldots, N-1\} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (4)$$

our aim mathematically becomes: $\min_\theta L(\theta) = \sum_g \alpha_g(1 - \langle d_g|VU(\theta)|g\rangle)$



(A) (B)

FIGURE 2.10: Fidelity convergence using Fidelity-cost and MSE-cost respectively



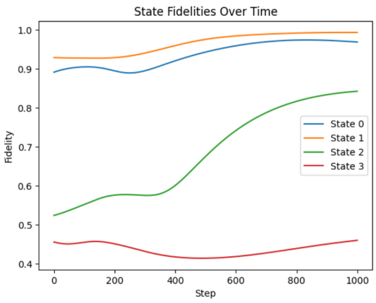
(A) (B)

FIGURE 2.11: Fidelity convergence using loss as Average over all states and Maximum over all states respectively

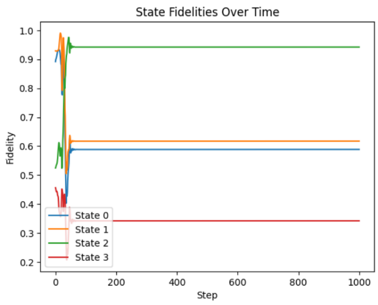
2.3.2.2 Optimization of parameters:

We use Gradient-descent approach for classical optimization of θ. In order to obtain maximum fidelity, we experimented with different cost-functions and pytorch optimizers. Cost functions considered are fidelity-cost, mse-cost and l1-cost, as well as taking the average of losses of all states to be prepared vs. only the maximum loss as cost-function while optimizing. Following are our experimental results:

Observations from the Figures 2.10, 2.11 and 2.12 lead to the conslusion: cost-fuction as fidelity-cost taken as average of all losses and optimizer as Adam gives us the best and fastest convergence.



(A)



(B)

FIGURE 2.12: Fidelity convergence using Adam and Rprop optimizers respectively

Chapter 3

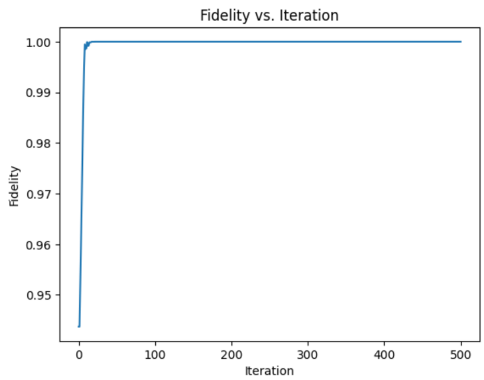
Implementation and Results

3.1 Perfect Amplitude Encoding of Single Quantum State

Our approach efficiently prepares the perfect encoding for a single state, which is first tested with the following hand-drawn states, and then with the snippet of a real time-series dataset. Following are our experimental results:

1. Figure 3.1: Fidelity vs. number of optimizing Iterations for 2-qubit encoding hand-drawn states, where the fidelity converges to 1.0

FIGURE 3.1: single-state 2-qubit encoding



2. Figure 3.2: Fidelity vs. number of optimizing Iterations for 3-qubit encoding hand-drawn states, where the fidelity converges to 1.0

3. Figure 3.3: Fidelity vs. number of optimizing Iterations for 2-qubit states encoding Time-series dataset, where the fidelity converges to 1.0

FIGURE 3.2: single-state 3-qubit encoding

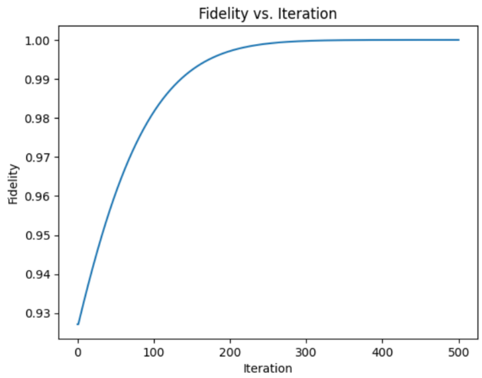
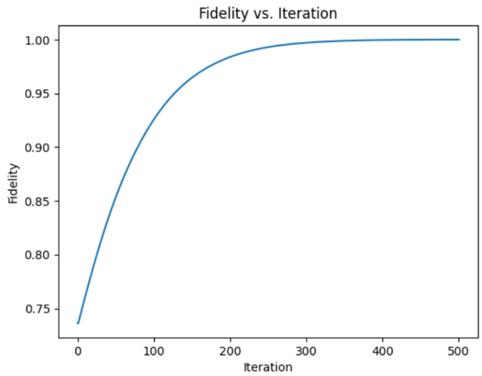


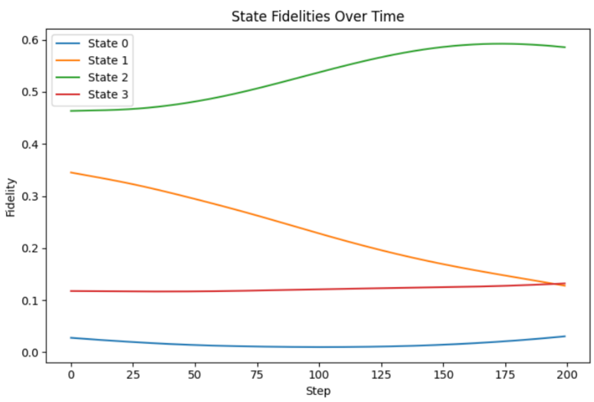
FIGURE 3.3: single-state 2-qubit Time-Series Data encoding



3.2 Approximate Amplitude Encoding of Multiple Quantum States

We first try to implement our circuit with a QGAN approach to optimization [13] of parameters(Figure 3.4). This approach fails to give good results and is hence discarded.

FIGURE 3.4: QGAN approach to 2-qubit Multi-state prep



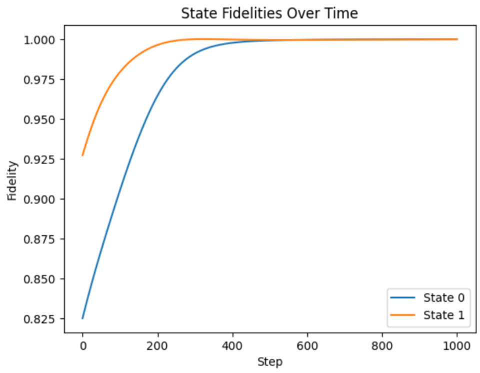
We also notice the problem of optimization being stuck in local minima with oscillating cost

noticed in some graphs during experiments. We hence make use of pytorch scheduler ReduceLROnPlateau to converge our fidelity to its maximum values.
Our approach efficiently prepares a good approximate encoding of multiple states using a single ansatz. Following are our experimental results:

3.2.1 2-Qubit States

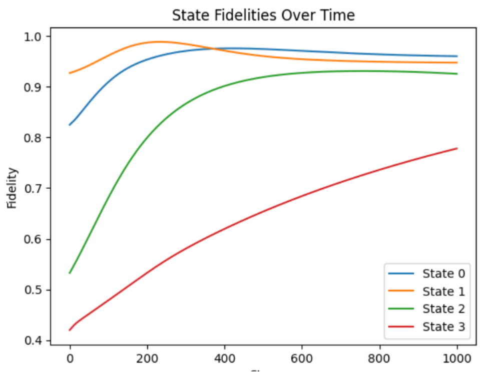
1. Figure 3.5: Fidelity vs. number of optimizing Iterations for 2-qubit encoding 2 hand-drawn states

FIGURE 3.5: 2 states 2-qubit encoding



2. Figure 3.6: Fidelity vs. number of optimizing Iterations for 2-qubit encoding 4 hand-drawn states

FIGURE 3.6: 4 states 2-qubit encoding



3. Figure 3.7: Fidelity vs. number of optimizing Iterations for 2-qubit encoding 6 hand-drawn states

3.2.2 3-Qubit States

1. Figure 3.8: Fidelity vs. number of optimizing Iterations for 3-qubit encoding 4 hand-drawn states

FIGURE 3.7: 6 states 2-qubit encoding

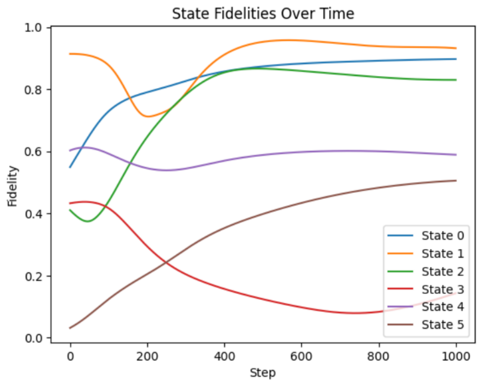
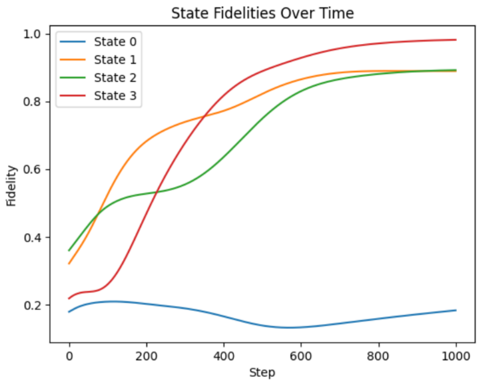
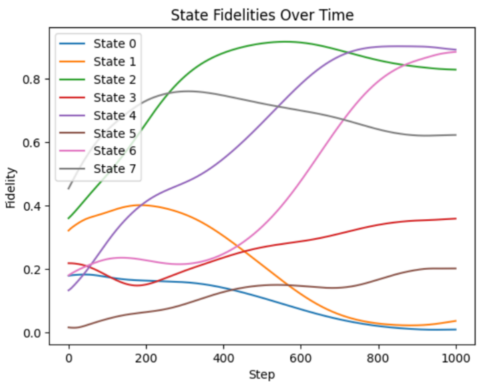


FIGURE 3.8: 4 states 3-qubit encoding



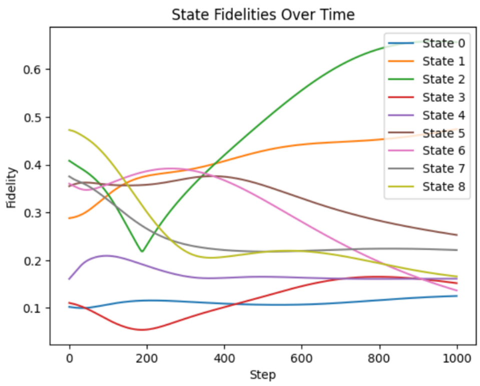
2. Figure 3.9: Fidelity vs. number of optimizing Iterations for 3-qubit encoding 8 hand-drawn states

FIGURE 3.9: 8 states 3-qubit encoding



3. Figure 3.10: Fidelity vs. number of optimizing Iterations for 3-qubit encoding 12 hand-drawn states

FIGURE 3.10: 9 states 3-qubit encoding



Chapter 4

Conclusion

This study introduces the Universal Dataset Encoder (UDE), a versatile variational quantum circuit framework designed to tackle key challenges in quantum state preparation. The UDE demonstrates the ability to achieve high-fidelity amplitude encoding for both single and multi-state quantum states while maintaining low circuit depth. This makes it particularly suitable for implementation on noisy intermediate-scale quantum (NISQ) devices. Experimental results highlight the framework's robustness and scalability, with fidelity reaching near-perfect levels for single-state preparation and consistently exceeding 0.85 for multi-state encoding across varying datasets.

The significant contributions of this work include:

1. Hardware-Efficient Ansatz: Development of shallow-depth circuits utilizing circular entanglement and parameterized CRX gates, optimizing expressivity while remaining practical for current quantum hardware.

2. Multi-State Encoding: Introduction of a unified circuit architecture capable of preparing multiple quantum states efficiently, reducing resource redundancy.

3. Advanced Optimization Techniques: Integration of specialized cost functions and adaptive optimizers to address convergence issues and enhance training reliability, particularly in multi-state scenarios.

These results underscore the UDE's potential to advance quantum algorithms by addressing one of the primary bottlenecks in quantum computing: efficient and scalable state preparation.

Future Work:
Several directions for future research are identified to build on the foundation established by this work:

1. Optimizing Circuit Design: Explore strategies to refine the number of circuit layers, balancing fidelity and resource efficiency to better adapt to the constraints of NISQ devices.

2. Advancing QGAN Applications: Investigate enhancements to Quantum Generative Adversarial Networks (QGANs) to improve their performance in multi-state preparation tasks and achieve more reliable optimization.

3. Hardware Implementation and Noise Mitigation: Conduct extensive tests on real quantum hardware, incorporating noise mitigation strategies such as randomized compiling or error suppression techniques to improve practical performance.

4. Improving Multi-State Encoding Fidelity: Develop novel approaches to achieve higher fidelity for multi-state preparation, focusing on maintaining low circuit depth while enhancing precision.

5. Mapping Probability Distributions: Extend the scope of the UDE to encode full probability distributions, enabling its application in areas like quantum sampling and probabilistic modeling.

By addressing these aspects, future work can further enhance the scalability and versatility of the UDE, enabling its application in a wider range of quantum computing problems and real-world use cases.

Bibliography

[1] Grover, L. K., Rudolph, T. (2002). Creating superpositions that correspond to efficiently integrable probability distributions. arXiv:quant-ph/0208112

[2] Preskill, J. (2018). Quantum computing in the NISQ era and beyond. Quantum, 2, 79

[3] Schuld, M., et al. (2020). Circuit-centric quantum classifiers. Phys. Rev. A, 101(3), 032308.

[4] Benedetti, M., et al. (2019). Parameterized quantum circuits as machine learning models. Quantum Sci. Tech., 4(4), 043001.

[5] Havlíček, V., et al. (2019). Supervised learning with quantum-enhanced feature spaces. Nature, 567, 209-212.

[6] Mitarai, K., et al. (2018). Quantum circuit learning. Phys. Rev. A, 98(3), 032309.

[7] Nielsen, M. A., Chuang, I. L. (2010). Quantum Computation and Quantum Information. Cambridge University Press.

[8] Biamonte, J., Wittek, P., Pancotti, N., Rebentrost, P., Wiebe, N., Lloyd, S. (2017). Quantum machine learning. Nature, 549(7671), 195-202.

[9] Cao, Y., Romero, J., Aspuru-Guzik, A. (2019). Potential of quantum computing for quantum chemistry. Chemical Reviews, 119(19), 10856-10915.

[10] Quantinuum (2023). Resource-saving modular protocols for state preparation. Retrieved from The Quantum Insider.

[11] Walsh Series Loader (2024). Efficient decomposition of Walsh series for quantum state preparation. Physical Review Letters.

[12] Sparse Quantum State Preparation (2023). Variational Ansatz for strongly correlated quantum systems. Retrieved from arXiv.

[13] Nakaji, K., Yamamoto, N. Quantum semi-supervised generative adversarial network for enhanced data classification. Sci Rep 11, 19649 (2021). https://doi.org/10.1038/s41598-021-98933-6

[14] Zoufal, C., Lucchi, A. Woerner, S. Quantum Generative Adversarial Networks for learning and loading random distributions. npj Quantum Inf 5, 103 (2019). https://doi.org/10.1038/s41534-019-0223-2