Scaling / normalization of LORETA images

Usually LORETA files contain several images, not only one.

Example: A LORETA file can contain 4 images, corresponding to spectral density at 4 frequency bands (delta, theta, alpha, beta), where each image consists of 6239 voxels. This can be represented as $s(b,v)$, with $b=1\ldots4$ denoting the band (b=1 is delta; b=2 is theta; b=3 is alpha; b=4 is beta); and $v=1\ldots6239$ denoting the voxel.

There are three average spectral powers of interest:
1. At each frequency band, the average over all voxels (i.e. frequency-wise):

$$\bar{s}(b,\bullet)=\frac{1}{6239}\sum_{v=1}^{6239}s(b,v)$$

2. At each voxel, the average over all frequency bands (i.e. voxel-wise):

$$\bar{s}(\bullet,v)=\frac{1}{4}\sum_{b=1}^{4}s(b,v)$$

3. The grand-average over all frequency bands and voxels (i.e. subject-wise):

$$\bar{s}(\bullet,\bullet)=\frac{1}{(4\times6239)}\sum_{b=1}^{4}\sum_{v=1}^{6239}s(b,v)$$

The new transformed data will be denoted as $\sigma(b,v)$.

No-normalization or no-scaling makes use of the data without any additional transformation, i.e.:

$$\sigma(b,v)=s(b,v) \quad for \quad b=1\ldots4 \quad and \quad v=1\ldots6239$$

Subject-wise- normalization or subject-wise- scaling:

$$\sigma(b,v)=\frac{s(b,v)}{\bar{s}(\bullet,\bullet)} \quad for \quad b=1\ldots4 \quad and \quad v=1\ldots6239$$

i.e. all the data (the spectral density at each freq and at each vox) is scaled by the factor $\bar{s}(\bullet,\bullet)$.

Frequency-wise- normalization or Frequency -wise- scaling:

$$\sigma(b,v)=\frac{s(b,v)}{\bar{s}(b,\bullet)} \quad for \quad b=1\ldots4 \quad and \quad v=1\ldots6239$$

i.e. the image at each freq is scaled by the factor $\bar{s}(b,\bullet)$.

Voxel-wise- normalization or voxel-wise- scaling:

$$\sigma(b,v)=\frac{s(b,v)}{\bar{s}(\bullet,v)} \quad for \quad b=1\ldots4 \quad and \quad v=1\ldots6239$$

i.e. at each voxel, the 4 spectral values are scaled by the factor $\bar{s}(\bullet, v)$. This is equivalent to the classical scaling known as the "relative spectra" for EEG signals.

ANOTHER EXAMPLE: Such normalization or scaling can be applied to a LORETA file corresponding to an ERP, consisting of squared current density at each voxel. In this case, the index (subscript) "b" then corresponds to time, and may typically range $b = 1 \ldots 256$.

The normalization – scaling is optional. There are no universal guiding principles on their usage. However, this is an example where subject-wise scaling is useful:
In a group study with several subjects, suppose that for each subject there is a loreta file containing the spectral density for several frequency bands. These data have enormous variability even in a "homogenous" group of normal subjects. Roughly, a homogeneous group of normals will have same spectral shape, but some can have easily up to 5 times more power than others. Any statistical analysis will suffer from this source of very high variance. Subject-wise scaling eliminates this source of variance, which might not be of any physiological interest, because it might be due to variable scalp and skull conductivity from subject to subject.

All these scalings are legitimate; there is nothing illegal about using one or another. They all have their own interpretation and meaning:
1. subject-wise scales the all the data in the file with a single factor, and eliminates a global source of variability.
2. frequency-wise (or time-wise) scales the image at each frequency (or time), and eliminates the frequency by frequency variability.
3. voxel-wise scales spectrum or signal at each voxel, and eliminates the voxel by voxel variability (in the same way as relative power spectra eliminates the electrode by electrode variability of absolute power).

The concept of scaling/normalization of neuroimages can be found in:

S.J. Kiebel, A.P. Holmes. The general linear model. (2004) In: K.J. Friston, J. Ashburner, W.D. Penny (editors). Human Brain Function, 2nd edition, Part II – Imaging Neuroscience – Theory and Analysis, Elsevier science.

At the moment of this writing (2008-April), a draft version of this chapter was available at the SPM website:

http://www.fil.ion.ucl.ac.uk/spm/doc/books/hbf2/pdfs/Ch7.pdf