Assignment No.3

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Download all python codes from

https://github.com/Sekharjala/Assignment_3/codes

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https://github.com/Sekharjala/Assignment_3/ Assignment3.pdf

1 Quadratic Forms Q:2.5

Question : Find the area of the region in the first quadrant enclosed by x-axis, line $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = 0$ and the circle $x^T\mathbf{x} = 4$

2 Solution

Given equation of a circle is

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.1}$$

can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

where $\mathbf{c}$ is the center.
Comparing equation (2.0.2) with the circle equation given,

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.3}$$
$$f = \mathbf{u}^T\mathbf{u} - r^2 \tag{2.0.4}$$
$$\implies \mathbf{c} = \begin{pmatrix}0\\0\end{pmatrix} \quad f = -4 \tag{2.0.5}$$
$$r = \sqrt{\mathbf{u}^T\mathbf{u} - f} = \sqrt{4} \tag{2.0.6}$$
$$r = 2 \tag{2.0.7}$$

From equation (2.0.7), the point at which circle touches x-axis is $\begin{pmatrix}2\\0\end{pmatrix}$.
The direction vector of line formed with O and A is $\begin{pmatrix}2\\0\end{pmatrix}$.
The direction vector of the given line

$$\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = 0 is \begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{2.0.8}$$

The angle between line (2.0.8) with OA ,

$$\cos\theta = \frac{\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\begin{pmatrix}2\\0\end{pmatrix}}{\left\|\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\right\|\left\|\begin{pmatrix}2 & 0\end{pmatrix}\right\|} = \frac{\sqrt{3}}{2} \tag{2.0.9}$$
$$\theta = 30^{\circ} \tag{2.0.10}$$

Using equation (2.0.7) and (2.0.10),the area of the sector is obtained as,

$$\frac{\theta}{360^\circ}\pi r^2 = \frac{30^\circ}{360^\circ}\pi(2)^2 = \frac{\pi}{3} \tag{2.0.11}$$

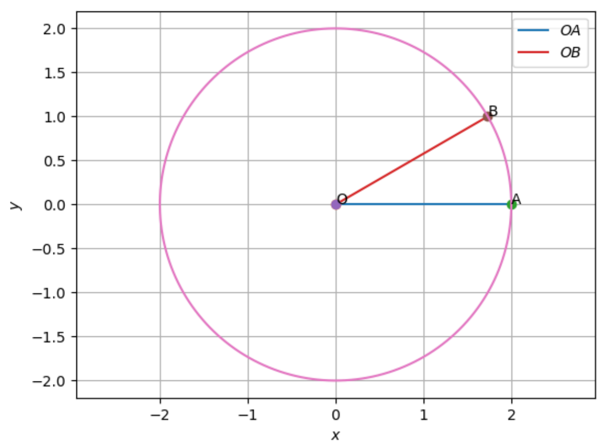

Fig. 0: Area Under the Curve circle

Point $\mathbf{A} = \begin{pmatrix}2\\0\end{pmatrix}$

Point $\mathbf{B}$ is on the line $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}$x=0 can be expressed as

$$\mathbf{x} = \mathbf{p} + \lambda\mathbf{m} \tag{2.0.12}$$

Where p is a vector and m is Directional vector of the line

$$\mathbf{x} = \begin{pmatrix}0\\0\end{pmatrix} + \lambda\begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{2.0.13}$$

from (2.0.1) substituting value of x we get

$$\left(\begin{pmatrix}0\\0\end{pmatrix}+\lambda\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}\right)^T\left(\begin{pmatrix}0\\0\end{pmatrix}+\lambda\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}\right)=4 \tag{2.0.14}$$

$$\lambda\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\times\lambda\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}=4 \tag{2.0.15}$$

$$\lambda^2\begin{pmatrix}4\end{pmatrix}=4 \tag{2.0.16}$$

$$\lambda=1 \tag{2.0.17}$$

substitute (2.0.17) in (2.0.13) point on circle

$$\mathbf{x}=\begin{pmatrix}0\\0\end{pmatrix}+\lambda\begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{2.0.18}$$

$$\mathbf{x}=\begin{pmatrix}0\\0\end{pmatrix}+1\begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{2.0.19}$$

$$\mathbf{x}=\begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{2.0.20}$$

so, B $=\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$