Assignment No.3

RajaSekhar Jala

Download all python codes from

https://github.com/Sekharjala/Assignment_3/codes

and pdf from

https://github.com/Sekharjala/Assignment_3/Assignment3.pdf

1 Quadratic Forms Q:2.5

Question : Find the area of the region in the first quadrant enclosed by x-axis, line $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x} = 0$ and the circle $x^T\mathbf{x} = 4$

2 Solution

Given equation of a circle is

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.1}$$

can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

where $\mathbf{c}$ is the center.
Comparing equation (2.0.2) with the circle equation given,

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.3}$$
$$f = \mathbf{u}^T\mathbf{u} - r^2 \tag{2.0.4}$$
$$\implies \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad f = -4 \tag{2.0.5}$$
$$r = \sqrt{\mathbf{u}^T\mathbf{u} - f} = \sqrt{4} \tag{2.0.6}$$
$$r = 2 \tag{2.0.7}$$

From equation (2.0.7), circle is having center at 0 0 with radius of 2 units.The equation of the diagnal is x-axis is and the directional vector is

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.8}$$

The direction vector of line formed with O and A is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.
The direction vector of the given line

$$\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x} = 0 is \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \tag{2.0.9}$$

The angle between line (2.0.9) with OA ,

$$\cos\theta = \frac{\begin{pmatrix} \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix}}{\left\|\begin{pmatrix} \sqrt{3} & 1 \end{pmatrix}\right\| \left\|\begin{pmatrix} 2 & 0 \end{pmatrix}\right\|} = \frac{\sqrt{3}}{2} \tag{2.0.10}$$
$$\theta = 30^{\circ} \tag{2.0.11}$$

Using equation (2.0.7) and (2.0.11),the area of the sector is obtained as,

$$\frac{\theta}{360^{\circ}}\pi r^2 = \frac{30^{\circ}}{360^{\circ}}\pi(2)^2 = \frac{\pi}{3} \tag{2.0.12}$$

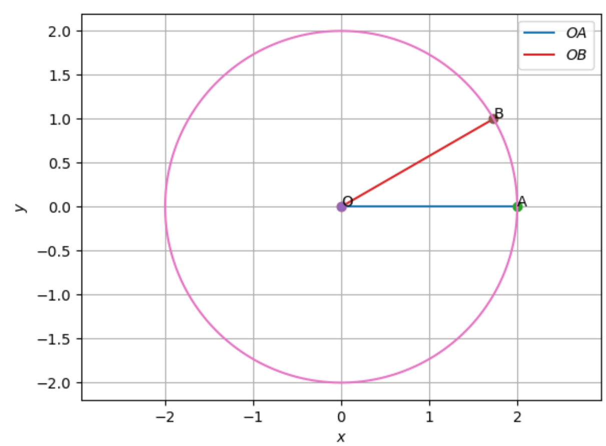

Fig. 0: Area Under the Curve circle

Ref:Tangents and Normal
If $\mathbf{V}^{-1}$ exists, given the normal vector $\mathbf{n}$, the tangent points of contact to (2.0.1) are given by

$$\mathbf{q}_i = \mathbf{V}^{-1}\left(\kappa_i\mathbf{n} - \mathbf{u}\right), i = 1, 2 \tag{2.0.13}$$
$$\text{where } \kappa_i = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\mathbf{n}^T\mathbf{V}^{-1}\mathbf{n}}} \tag{2.0.14}$$

$\mathbf{V}^{-1}$ exists

$$\mathbf{V}^{-1} = \frac{1}{1-0}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{2.0.15}$$

$$= \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{2.0.16}$$

from (2.0.14) and (2.0.8)

$$\kappa_i = \pm\sqrt{\frac{\begin{pmatrix}0 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}+4}{\begin{pmatrix}2 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}2\\0\end{pmatrix}}} \tag{2.0.17}$$

$$\kappa_i = \pm 1 \tag{2.0.18}$$

$$\mathbf{q}_i = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}(\pm 1\begin{pmatrix}2\\0\end{pmatrix}) \tag{2.0.19}$$

$$\implies \mathbf{q}_1 = \begin{pmatrix}2\\0\end{pmatrix}, \mathbf{q}_2 = \begin{pmatrix}-2\\0\end{pmatrix} \tag{2.0.20}$$

$\mathbf{q}_1$ lies in 1^{st} Quadrant

$$So, \mathbf{A} = \begin{pmatrix}2\\0\end{pmatrix} \tag{2.0.21}$$

Similarly from (2.0.14) and (2.0.9)

$$\kappa_i = \pm\sqrt{\frac{\begin{pmatrix}0 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}+4}{\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}}} \tag{2.0.22}$$

$$\kappa_i = \pm 1 \tag{2.0.23}$$

$$\mathbf{q}_i = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}(\pm 1\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}) \tag{2.0.24}$$

$$\mathbf{q}_1 = \begin{pmatrix}\sqrt{3}\\1\end{pmatrix}\mathbf{q}_2 = \begin{pmatrix}-\sqrt{3}\\-1\end{pmatrix} \tag{2.0.25}$$

$\mathbf{q}_1$ lies in 1^{st} Quadrant

$$So, \mathbf{B} = \begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{2.0.26}$$