Assignment No.3

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https://github.com/Sekharjala/Assignment_3/codes

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1 QUADRATIC FORMS Q:2.5

Question : Find the area of the region in the first quadrant enclosed by x-axis, line $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix}\mathbf{x} = 0$ and the circle $\mathbf{x}^T\mathbf{x} = 4$

2 SOLUTION

Given equation of a circle is

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.1}$$

can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

where $\mathbf{c}$ is the center.
Comparing equation (2.0.2) with the circle equation given,

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.3}$$

$$f = \mathbf{u}^T\mathbf{u} - r^2 \tag{2.0.4}$$

$$\implies \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad f = -4 \tag{2.0.5}$$

$$r = \sqrt{\mathbf{u}^T\mathbf{u} - f} = \sqrt{4} \tag{2.0.6}$$

$$r = 2 \tag{2.0.7}$$

From equation (2.0.7), the point at which circle touches x-axis is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.
The direction vector of line formed with O and A is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.
The direction vector of the given line

$$\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix}\mathbf{x} = 0\, is \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \tag{2.0.8}$$

The angle between line (2.0.8) with OA ,

$$\cos\theta = \frac{\begin{pmatrix} \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix}}{\left\|\begin{pmatrix} \sqrt{3} & 1 \end{pmatrix}\right\|\left\|\begin{pmatrix} 2 & 0 \end{pmatrix}\right\|} = \frac{\sqrt{3}}{2} \tag{2.0.9}$$

$$\theta = 30^\circ \tag{2.0.10}$$

Using equation (2.0.7) and (2.0.10),the area of the sector is obtained as,

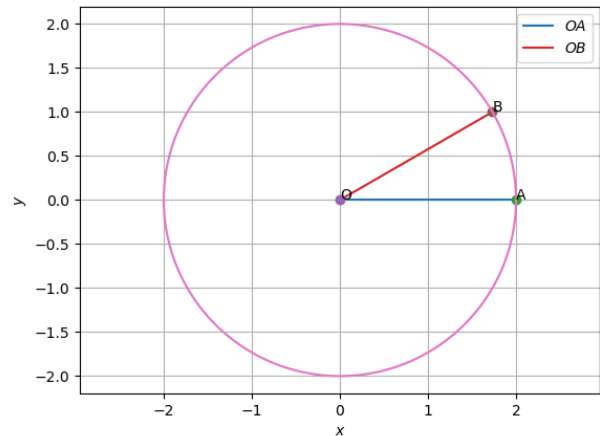$$\frac{\theta}{360^\circ}\pi r^2 = \frac{30^\circ}{360^\circ}\pi(2)^2 = \frac{\pi}{3} \tag{2.0.11}$$

Fig. 0: Area Under the Curve circle

Point $\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Point $\mathbf{B}$ is on the line $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix}\mathbf{x}=0$

$$\mathbf{B} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} \tag{2.0.12}$$

$$= \begin{pmatrix} 2\cos 30^\circ \\ 2\sin 30^\circ \end{pmatrix} \tag{2.0.13}$$

$$= \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \tag{2.0.14}$$