Assignment No.3

RajaSekhar Jala

Download all python codes from

https://github.com/Sekharjala/Assignment_3/codes

and pdf from

https://github.com/Sekharjala/Assignment_3/Assignment3.pdf

1 Quadratic Forms Q:2.5

Question : Find the area of the region in the first quadrant enclosed by x-axis, line $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = 0$ and the circle $\mathrm{x}^T\mathbf{x} = 4$

2 Solution

Given equation of a circle is

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.1}$$

can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

where $\mathbf{c}$ is the center.
Comparing equation (2.0.2) with the circle equation given,

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.3}$$

$$f = \mathbf{u}^T\mathbf{u} - r^2 \tag{2.0.4}$$

$$\implies \mathbf{c} = \begin{pmatrix}0\\0\end{pmatrix} \quad f = -4 \tag{2.0.5}$$

$$r = \sqrt{\mathbf{u}^T\mathbf{u} - f} = \sqrt{4} \tag{2.0.6}$$

$$r = 2 \tag{2.0.7}$$

From equation (2.0.7), circle is having center at 0 0 with radius of 2 units.The equation of the diagnal is x-axis is and the directional vector is

$$\begin{pmatrix}2\\0\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix} \tag{2.0.8}$$

The direction vector of line formed with O and A is $\begin{pmatrix}2\\0\end{pmatrix}$.
The direction vector of the given line

$$\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = 0 is \begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{2.0.9}$$

The angle between line (2.0.9) with OA ,

$$\cos\theta = \frac{\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\begin{pmatrix}2\\0\end{pmatrix}}{\left\|\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\right\|\left\|\begin{pmatrix}2 & 0\end{pmatrix}\right\|} = \frac{\sqrt{3}}{2} \tag{2.0.10}$$

$$\theta = 30^\circ \tag{2.0.11}$$

Using equation (2.0.7) and (2.0.11),the area of the sector is obtained as,

$$\frac{\theta}{360^\circ}\pi r^2 = \frac{30^\circ}{360^\circ}\pi(2)^2 = \frac{\pi}{3} \tag{2.0.12}$$

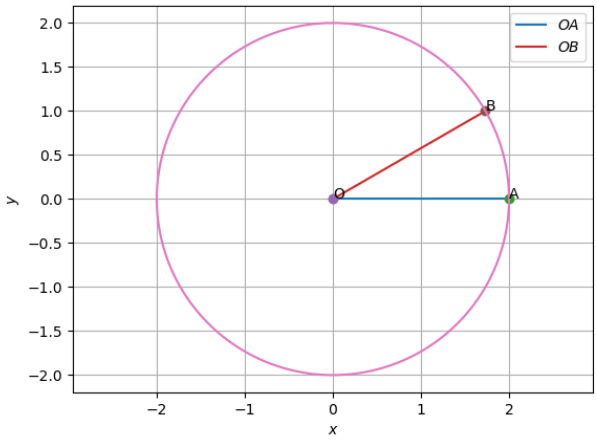

Fig. 0: Area Under the Curve circle

Ref:Tangents Normals
If a line

$$L : \mathbf{x} = \mathbf{a} + \lambda\mathbf{m} \tag{2.0.13}$$

touches a conic section at one point $\mathbf{q}$ then

$$\mathbf{m}^T(\mathbf{V}\mathbf{q} + \mathbf{u}) = 0 \tag{2.0.14}$$

From equation (2.0.1) $\mathbf{V}$ is $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ and $\mathbf{u}=\begin{pmatrix}0\\ 0\end{pmatrix}$
From equation of x-axis, (2.0.1) and (2.0.14) point of intersection is $\mathbf{A}$ is given by

$$\begin{pmatrix}0 & 2\end{pmatrix}(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\mathbf{A}+\mathbf{u}) = 0 \qquad (2.0.15)$$

$$\begin{pmatrix}0 & 2\end{pmatrix}\mathbf{A} = 0 \qquad (2.0.16)$$

$$\implies \mathbf{A} = \begin{pmatrix}2\\ 0\end{pmatrix} \qquad (2.0.17)$$

similarly
from eq (2.0.9),(2.0.1) and (2.0.14) point of intersection is B is given by

$$\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\mathbf{B}+\mathbf{u}) = 0 \qquad (2.0.18)$$

$$\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{B} = 0 \qquad (2.0.19)$$

$$\implies \mathbf{B} = \begin{pmatrix}\sqrt{3}\\ 1\end{pmatrix} \qquad (2.0.20)$$