

Assignment 3

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Find Python Codes from below link

https://github.com/Sekharjala/Matrix-Matrix_2

and latex-tikz codes from

https://github.com/Sekharjala/Matrix-Matrix_2

1 EXAMPLES I

1.1 Question 14

Prove that the point $\begin{pmatrix} -1/14 \\ 39/14 \end{pmatrix}$ is the centre of the circle circumscribing the triangle whose angular points are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \quad (1.1.1)$$

2 SOLUTION

let assume that circumecentre of the triangle ABC is \mathbf{O}

$$\|\mathbf{A} - \mathbf{O}\| = \|\mathbf{B} - \mathbf{O}\| = \|\mathbf{C} - \mathbf{O}\| \quad (2.0.1)$$

$$\|\mathbf{A} - \mathbf{O}\|^2 - \|\mathbf{B} - \mathbf{O}\|^2 = 0 \quad (2.0.2)$$

Which can be simplified as

$$(\mathbf{A} - \mathbf{B})^T \mathbf{O} = \frac{(\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2)}{2} \quad (2.0.3)$$

Similarly,

$$(\mathbf{B} - \mathbf{C})^T \mathbf{O} = \frac{(\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2)}{2} \quad (2.0.4)$$

can be combined to form the matrix equation

$$\mathbf{N}^T \mathbf{O} = \mathbf{K} \quad (2.0.5)$$

$$\mathbf{O} = \mathbf{K} \mathbf{N}^{-T} \quad (2.0.6)$$

Where

$$\mathbf{N} = \begin{pmatrix} (\mathbf{A} - \mathbf{B}) & (\mathbf{B} - \mathbf{C}) \end{pmatrix} \quad (2.0.7)$$

$$\mathbf{K} = \frac{1}{2} \begin{pmatrix} \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 & \|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 \end{pmatrix} \quad (2.0.8)$$

by substituting A ,B and C in (2.0.7) and (2.0.8) we get

$$\mathbf{N} = \begin{pmatrix} -1 & 4 \\ -2 & 1 \end{pmatrix} \quad \mathbf{N}^{-T} = \begin{pmatrix} 1/7 & -4/7 \\ 2/7 & -1/7 \end{pmatrix} \quad (2.0.9)$$

$$\mathbf{K} = \frac{1}{2} \begin{pmatrix} (2 - 13) & (13 - 8) \end{pmatrix} \quad \mathbf{K} = \begin{pmatrix} -11/2 & 5/2 \end{pmatrix} \quad (2.0.10)$$

from (2.0.9) and (2.0.10) we get

$$\mathbf{O} = \begin{pmatrix} -1/14 & 39/14 \end{pmatrix} \quad (2.0.11)$$

Hence in vector form

$$\mathbf{O} = \begin{pmatrix} -1/14 \\ 39/14 \end{pmatrix}$$

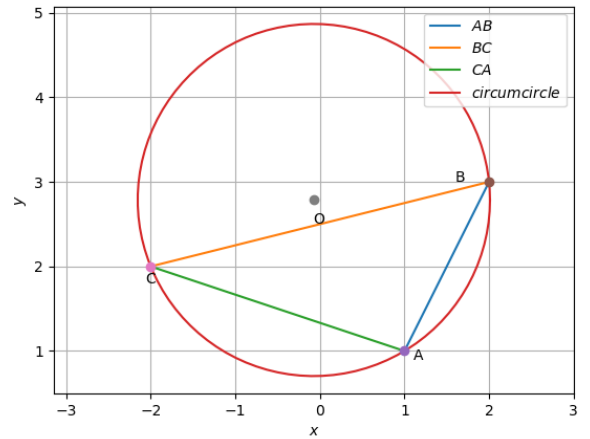


Fig. 0: Circumcircle with \mathbf{O} as center