Assignment 3

Rajasekhar Jala

Find Python Codes from below link

https://github.com/Sekharjala/Matrix−Matrix_2

and latex-tikz codes from

https://github.com/Sekharjala/Matrix−Matrix_2

1 Examples I

1.1 Question 14

Prove that the point $\begin{pmatrix} -1/14 \\ 39/14 \end{pmatrix}$ is the centre of the circle circumscribing the triangle whose angular points are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{1.1.1}$$

2 Solution

let assume that circumecentre of the triangle ABC is $\mathbf{O}$

$$\|\mathbf{A} - \mathbf{O}\| = \|\mathbf{B} - \mathbf{O}\| = \|\mathbf{C} - \mathbf{O}\| \tag{2.0.1}$$

$$\|\mathbf{A} - \mathbf{O}\|^2 - \|\mathbf{B} - \mathbf{O}\|^2 = 0 \tag{2.0.2}$$

Which can be simplified as

$$\left(\mathbf{A} - \mathbf{B}\right)^T \mathbf{O} = \frac{(\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2)}{2} \tag{2.0.3}$$

Similarly,

$$\left(\mathbf{B} - \mathbf{C}\right)^T \mathbf{O} = \frac{(\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2)}{2} \tag{2.0.4}$$

can be combined to form the matrix equation

$$\mathbf{N}^T \mathbf{O} = \mathbf{K} \tag{2.0.5}$$

$$\mathbf{O} = \mathbf{K}\mathbf{N}^{-T} \tag{2.0.6}$$

Where

$$\mathbf{N} = \left((\mathbf{A} - \mathbf{B}) \quad (\mathbf{B} - \mathbf{C})\right) \tag{2.0.7}$$

$$\mathbf{K} = \frac{1}{2}\left(\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad \|\mathbf{B}\|^2 - \|\mathbf{C}\|^2\right) \tag{2.0.8}$$

by substituting $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ in (2.0.7) and (2.0.8)
we get

$$\mathbf{N} = \begin{pmatrix} -1 & 4 \\ -2 & 1 \end{pmatrix} \tag{2.0.9}$$

$$\mathbf{N}^{-T} = \begin{pmatrix} 1/7 & -4/7 \\ 2/7 & -1/7 \end{pmatrix} \tag{2.0.10}$$

$$\mathbf{K} = \frac{1}{2}\left((2-13) \quad (13-8)\right)$$

$$\mathbf{K} = \begin{pmatrix} -11/2 & 5/2 \end{pmatrix} \tag{2.0.11}$$

from (2.0.10) and (2.0.11) we get

$$\mathbf{O} = \begin{pmatrix} -1/14 & 39/14 \end{pmatrix} \tag{2.0.12}$$

Hence in vector form

$$\mathbf{O} = \begin{pmatrix} -1/14 \\ 39/14 \end{pmatrix} \tag{2.0.13}$$

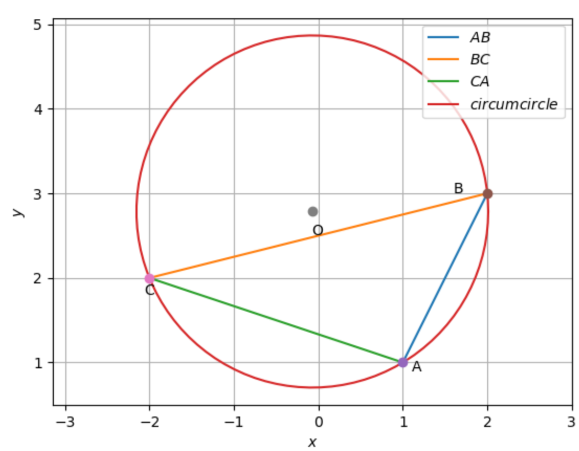

Fig. 0: Circumcircle with $\mathbf{O}$ as center