

# Assignment 3

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Find Python Codes from below link

[https://github.com/Sekharjala/Matrix-Matrix\\_2](https://github.com/Sekharjala/Matrix-Matrix_2)

and latex-tikz codes from

[https://github.com/Sekharjala/Matrix-Matrix\\_2](https://github.com/Sekharjala/Matrix-Matrix_2)

## 1 EXAMPLES I

### 1.1 Question 14

Prove that the point  $\begin{pmatrix} -1/14 \\ 39/14 \end{pmatrix}$  is the centre of the circle circumscribing the triangle whose angular points are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \quad (1.1.1) \quad \text{from (2.0.10) and (2.0.11) we get}$$

## 2 SOLUTION

let assume that circume centre of the triangle ABC is  $\mathbf{O}$

$$\|\mathbf{A} - \mathbf{O}\| = \|\mathbf{B} - \mathbf{O}\| = \|\mathbf{C} - \mathbf{O}\| \quad (2.0.1)$$

$$\|\mathbf{A} - \mathbf{O}\|^2 - \|\mathbf{B} - \mathbf{O}\|^2 = 0 \quad (2.0.2)$$

Which can be simplified as

$$(\mathbf{A} - \mathbf{B})^T \mathbf{O} = \frac{(\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2)}{2} \quad (2.0.3)$$

Similarly,

$$(\mathbf{B} - \mathbf{C})^T \mathbf{O} = \frac{(\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2)}{2} \quad (2.0.4)$$

can be combined to form the matrix equation

$$\mathbf{N}^T \mathbf{O} = \mathbf{K} \quad (2.0.5)$$

$$\mathbf{O} = \mathbf{K} \mathbf{N}^{-T} \quad (2.0.6)$$

Where

$$\mathbf{N} = \begin{pmatrix} (\mathbf{A} - \mathbf{B}) & (\mathbf{B} - \mathbf{C}) \end{pmatrix} \quad (2.0.7)$$

$$\mathbf{K} = \frac{1}{2} \begin{pmatrix} \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 & \|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 \end{pmatrix} \quad (2.0.8)$$

by substituting  $\mathbf{A}, \mathbf{B}$  and  $\mathbf{C}$  in (2.0.7) and (2.0.8) we get

$$\mathbf{N} = \begin{pmatrix} -1 & 4 \\ -2 & 1 \end{pmatrix} \quad (2.0.9)$$

$$\mathbf{N}^{-T} = \begin{pmatrix} 1/7 & -4/7 \\ 2/7 & -1/7 \end{pmatrix} \quad (2.0.10)$$

$$\mathbf{K} = \frac{1}{2} \begin{pmatrix} (2 - 13) & (13 - 8) \\ (-11/2 & 5/2) \end{pmatrix} \quad (2.0.11)$$

$$\mathbf{O} = \begin{pmatrix} -1/14 & 39/14 \end{pmatrix} \quad (2.0.12)$$

Hence in vector form

$$\mathbf{O} = \begin{pmatrix} -1/14 \\ 39/14 \end{pmatrix} \quad (2.0.13)$$

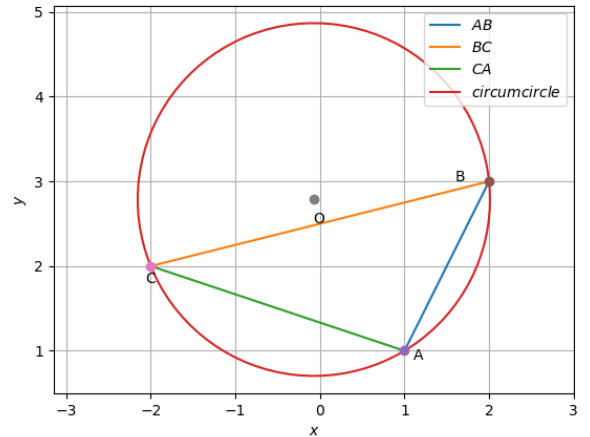


Fig. 0: Circumcircle with  $\mathbf{O}$  as center