11601 hw4 ch6

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Question 6.1

You have 20 bottles of pills. 19 bottles have 1.0 gram pills, but one has pills of weight 1.1 grams.Given a scale that provides an exact measurement , how would you find the heavy bottle? You can only use the scale once.

Solution: Take one pill from Bottle #1, two pills from Bottle #2, three pills from Bottle #3, etc. Weigh these pills together. If all the pills were one gram each, then the scale would read 210 grams (1 + 2 + 3 + 4 + ... + 20 = 210) or (20 * 21)/2 = 210. Any weight over 210 must come from the heavier pills.

Then use this formula to find the bottle number. ((weight - 210) / 0.1). So for example, if the set of pills weighed 211.3 grams, then bottle #13 would have the 1.1g pills.

Question 6.2

You have a basketball hoop and someone says that you can play one of two games. Game 1: You get one shot to make the hoop. Game 2: You get three shots and you have to make two of three shots. If p is the probability of making a particular shot, for which values of p should you pick one game or the other?

Solution: Given: (1) The probability of winning Game 1 is p1 = p (2) The probability of winning Game 2 is p2 = $p^3 + 3 * (1 - p) * p * p = 3 * p^2 - 2 * p^3$.

The solution would be:

(1) If p1 is larger than p2, we should choose Game 1. (2) If p = 0 or 0.5 or 1, then p1 == p2, Game1 and Game2 have the same probability of winning. (3) If p is within the range of 0.5 and 1, that is p2 is larger than p1, we will choose Game 2.

Question 6.3

There is an 8*8 chessboard in which two diagonally opposite corners have been cut off. You are given 31 dominos, and a single domino can cover exactly two squares. Can you use the 31 dominos to cover the entire board? Prove your answer by providing an example or showing why it's impossible.

Solution:

Answer: We cannot cover the entire board by the 31 dominors.

Proof: The chessboard originally have 32 white squares and 32 black squares. If we cut off the two squares in diagonally opposite corners, there will be 30 white squares and 32 black squares, or 30 black squares and 32 white squares. Then assuming there are 30 black squares and 32 white squares. Given every dominor can cover exactly 1 black and 1 white square, and hence only if there are 31 black squares and 31 white squares, the dominors can cover the entire board. Therefore, we cannot cover the entire board by the 31 dominors.

Question 6.4

There are three ants on different vertices of a triangle. What is the probability of collision (between any two or all of them) if they start walking on the sides of the sides if the triangle? Assume that each ant randomly picks a direction, with either direction being equally likely to be chosen, and that they walk at the same speed. Similarly, and the probability of collision with n ants on an n-vertex polygon.

Solution:

Answer: There are two directions, the clockwise and the counter clockwise for every ant. Only if the three ants walk in the same direction, there will not be collisions. The probability of walking in the same direction is 2 * $(1/2)^3 = 1/4$.

Therefore, the probability of collision is 1 - 1/4 = 3/4.

Follow up: For the n-vertex polygon, every ant still only has two directions, the clockwise and the counter clockwise. Thus, the probability of walking in the same direction is 2 * $(1/2)^n = (1/2)^{n-1}$.

Therefore, the probability of collision is 1 - $(1/2)^{n-1}$.

Question 6.5

You have a five-quart jug, a three-quart jug, and an unlimited supply of water (but no measuring cups). How would you come up with exactly four quarts of water. Note that the jugs are oddly shaped, such that filling exactly "half" of the jug would be impossible.

Solution: Let the quarts of water in five-quart jug be quart5 and three-quart jug be quart3:

(1)Filld the 5-quart jug: quart5=5,quart3=0;
(2)Filled the 3-quart jug by water in 5-quart: quart5=2, quart3=3;
(3)Pulled out the water in 3-quart jug: quart5=2, quart3=0;
(4)Put the water in 5-quart jug to 3-quart jug: quart5=0, quart3=2;
(5)Filled the 5-quart jug: quart5=5; quart3=2;
(6)Filled the 3-quart jug by water in 5-quart: quart5=4, quart3=3;
Then we have the 4 quarts water.

Question 6.6

A bunch of people are living on an island, when a visitor comes with a strange order: all blue-eyed people must leave the island as soon as possible. There will be a flight out at 8:00pm every evening. Each person can see everyone else's eye color, but they do not know their own (nor is anyone allowed to tell them). Additionally, they do not know how many people have blue eyes, although they do know that at least one person does. How many days will it take the blue-eyed people to leave?

Solution:

Answer: If there is x people, it will take x days for leaving.

Proof: If there is only 1 blue-eyed person, then when somebody does not see any person with blue eyes, he can realize that he is the one in blue eyes and leave that night. If there are 2 blue-eyed people, then they can both see one person in blue eyes.Then they can realize that there is 1 or 2 people with blue eyes, one is the person they see, the other may be herself. If nobody leaves at the first night, then they can be sure that the other person also see a blue-eyed person, that is herself. Therefore, they both leave on the second night. For three people cases. If there are only two eyes in blue, they will leave on the second night. If not, there are three blue-eyed people Then we can see if there are x blue-eyed people, then they can all see x - 1 people in blue eyes. They need x - 1 nights to ensure if there is x - 1 blue-eyed people or herself also is blue-eyed.Then on the xth day, all blue-eyed people will leave out.

Question 6.9

There are 100 closed lockers in a hallway. A man begins by opening all 100 lockers. Next, he closes every second locker. Then on his thirs pass, he toggles every third locker (closes it if it is open or opens it if it is closed). This process continues for 100 passes, such that on each pass i, the man toggles every ith locker. After his 100th pass in the hallway, in which he toggles only locker No.100, how many locker are open?

Solution: Obviously if the locker n is in the mth turn and m is a factor of n, status will be changed. If some locker is still on after some times of operations, then it is operated by odd times. Given the factor number of this locker number is odd, it is a complete square number. Then we can search the complete square number from integer 1 to integer 100.

Therefore, we can see there are 10 lockers still open.