Math 305 Homework 3

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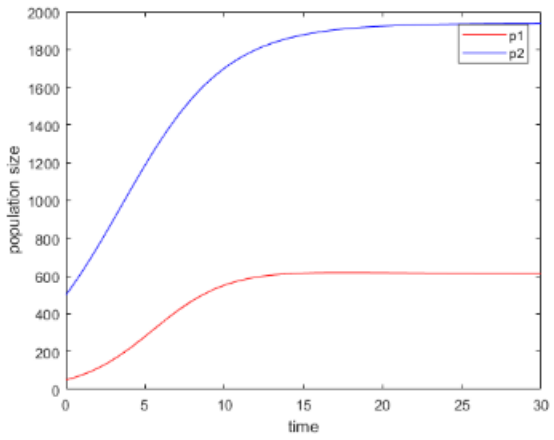
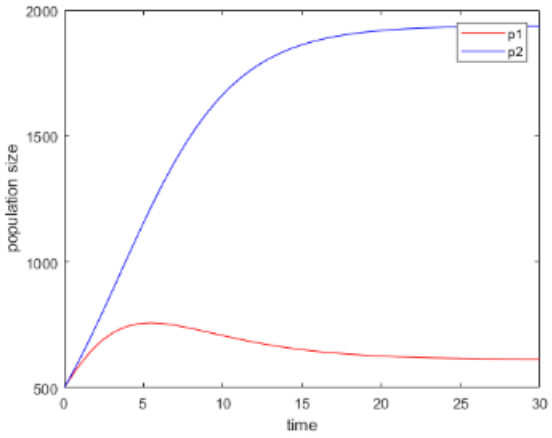
Section 3: Solving systems of differential equations

Exercise 3.1

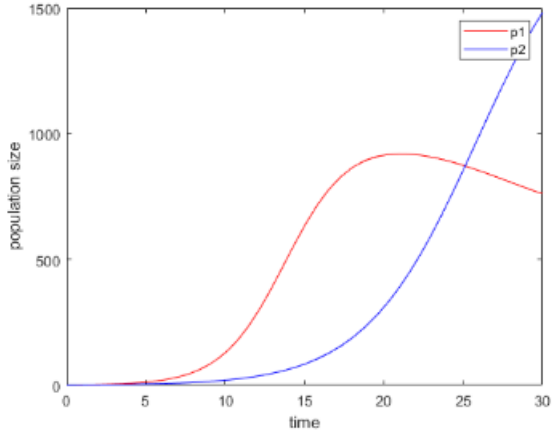
- $v_0 = (50, 500)$:



- $v_0 = (500, 50)$:



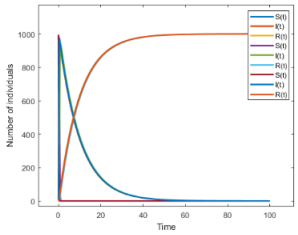
- $v_0 = (1, 1)$:



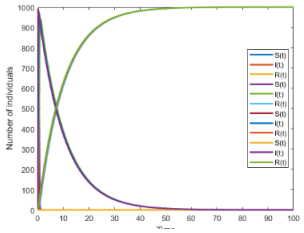
- The long-term behaviour of the populations do not change.
- P_1 and P_2 do not converge unless the first term of $v(0)$ is greater than the second. I.e, P_1 starts higher than P_2.

Exercise 3.2

```
clear;
% Initial conditions
S0 = 995;
I0 = 5;
R0 = 0;
y0 = [S0, I0, R0];

% Time vector
t = linspace(0, 100, 1000);

% Parameters
a = 0.5;
c = 0.05;
gamma = 0.1;

% Solve the SIR model
[t, sol] = ode45(@(t,y) SIR(t,y,a,c,gamma), t, y0);

% Plot the results
plot(t, sol(:, 1), 'LineWidth', 2, 'DisplayName', 'S(t)');
hold on
plot(t, sol(:, 2), 'LineWidth', 2, 'DisplayName', 'I(t)');
plot(t, sol(:, 3), 'LineWidth', 2, 'DisplayName', 'R(t)');
xlabel('Time');
ylabel('Number of individuals');
legend('Location', 'best');

% Define the SIR model
```




```
function dydt = SIR(t, y, a, c, gamma)
    S = y(1);
    I = y(2);

    % Parameters
    a = 0.8;
    c = 0.2;
    gamma = 0.1;

    % Solve the SIR model
    [t, sol] = ode45(@(t,y) SIR(t,y,a,c,gamma), t, y0);

    % Plot the results
    plot(t, sol(:, 1), 'LineWidth', 2, 'DisplayName', 'S(t)');
    hold on
    plot(t, sol(:, 2), 'LineWidth', 2, 'DisplayName', 'I(t)');
    plot(t, sol(:, 3), 'LineWidth', 2, 'DisplayName', 'R(t)');
    xlabel('Time');
    ylabel('Number of individuals');
```



The relationship between a, c, and γ to prevent an epidemic must be $ac < \gamma$. This can be achieved by increasing gamma, or decreasing a or c. Decreasing the rate of transmission (a) or the proportion of susceptible individuals who come into contact with infectious individuals (c), will reduce the chance of an epidemic. Increasing the recovery rate (γ) will also reduce the chance of an epidemic.

Problem 3

Diana is a 160-lb woman who consumes five standard glasses of wine in a short amount of time.

(a) Estimate her BAC. $BAC = (A \cdot 5.14/W \cdot r) - 0.015 \cdot H$, assuming she drank 5 5oz glasses in 1 hr:

$$BAC = (25 \cdot 5.14/160 \cdot 0.55) - 0.015 = 0.1185$$

(b) Use the Widmark model to estimate how long it will take before Diana is legally able to drive. Solving for H:

$$H = (A \cdot 5.14/W \cdot r - BAC)/0.015$$

Legal BAC is .08, so plugging in:

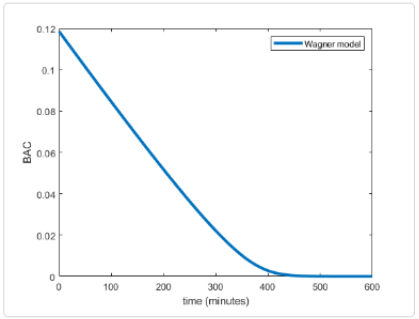
$$H = (25 \cdot 5.14/160 \cdot 0.55 - 0.08)/0.015 = 3.36 \text{ hours}$$

(c) Use the Wagner model simulation to estimate how long it will take before Diana is legally able to drive.

Wagner.mlx

```
clear all
close all
% wagner model with input and stopping
w=160; %weight in lb  140
g=2;   % gender - male 1, female 2
DT=1;  % number of hours of steady drinking
m=0;   % number of standard drinks in DT hours
tf=600; % time in minutes 300
wr(1)=.1185; % start with BAC .1185
d=bac_calc(w,g,m)/(DT*60);  % ingestion per minute
Vmax=0.0003693; Km=0.00858;
for i=1:tf
    %if i<=DT*60
      wr(i+1)=wr(i)-Vmax/(Km+wr(i))*wr(i)+d;
%   else
%       wr(i+1)=wr(i)-Vmax/(Km+wr(i))*wr(i);
%end

end
bac_steady_state=d*Km/(Vmax-d)
figure(1)
plot(0:tf, wr,'Linewidth',3)
```

```
bac_steady_state = 0
```



About 1 hour and 55 minutes, $t = 115$ minutes.