Math 341 Homework 6

Theo Koss

September 2020

1 Practice problems

1.1 Problem 1

Find all prime numbers p such that $p + 1$ is prime.
$p = 2$.

Proof.

Theorem 1. *For any odd numbers n, a, $n + a$ is even. Recall the definition of an odd number is some number $n = 2k + 1, k \in \mathbb{Z}$, or $a = 2k_1 + 1, k_1 \in \mathbb{Z}$. An even number $m = 2l, l \in \mathbb{Z}$. So $n + a = 2k + 2k_1 + 2 = 2(k + k_1 + 1)$, and since $(k + k_1 + 1) \in \mathbb{Z}$, $n + a$ is even.*

Theorem 2. *Any positive even integer $n > 2$ is composite, since $n = 2k$, for some $k > 1 \in \mathbb{N}$, therefore 2 divides n, and since $2 \neq 1$ and $2 \neq n$, by definition n is composite.*

Corollary 2.1. *All prime numbers $p \neq 2$ are odd.*

There are 2 cases for this problem:

1. Case 1: $p = 2$, if $p = 2$, $p + 1 = 3$ is prime. So this case is a solution.

2. Case 2: p is a prime number greater than 2. Thus, by Theorem 1, $p + 1 = n$, where n is some some positive even integer > 2. And by Theorem 2, any positive even integer greater than 2 is composite, thus every prime number greater than 2 does not work.

QED

1.2 Problem 5

Prove that for any $n \in \mathbb{N}$, n and $n+1$ are relatively prime.

Proof.

Remark. *Two numbers* $a, b \in \mathbb{N}$ *are relatively prime if* $\gcd(a, b) = 1$*.*

Also recall the Euclidean Algorithm, by definition 5.1 using Euclidean Alg. on $(n+1, n)$, we achieve:

$$n + 1 = n \cdot 1 + 1$$

$$n = 1 \cdot n + 0$$

The Euclidean Algorithm is over, and it states that $\gcd(n+1, n) = 1$, therefore $n+1, n$ are relatively prime. QED

1.3 Problem 9

True or false: for any $n \in \mathbb{N}, n^2 + n + 41$ is prime. False.

Proof. Counterexample: Let $n = 40$, $40^2 + 40 + 41 = 1681$, and $1681 = 41 \cdot 41$ $\therefore$ by definition, since $41 \in \mathbb{N}$, the number is composite and the proposition is false. QED