Math 531 Homework 3

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1 Section 2.1

- Problem 1:
  (a) $f : \mathbb{R} \to \mathbb{R}; f(x) = x + 3$.
  Injective and surjective. This is a bijection
  (b) $f : \mathbb{C} \to \mathbb{C}; f(x) = x^2 + 2x + 1$.
  Surjective but not injective.
  (c) $f : \mathbb{Z}_n \to \mathbb{Z}_n; f([x]_n) = [mx + b]_n$, where $m, b \in \mathbb{Z}$.
  Neither injective nor surjective? (What if $m, b = 0$).
  (d) $f : \mathbb{R}^+ \to \mathbb{R}; f(x) = ln(x)$.
  Injective and surjective. This is a bijection.
- Problem 6: Let $S = \{1, 2, 3\}$ and $T = \{4, 5\}$.
  (a) How many functions are there from $S \to T$? From $T \to S$?
  There are $2^3 = 8$ from $S \to T$, and $3^2 = 9$ from $T \to S$.
  (b) How many of the functions from $S \to T$ are one-to-one? How many are onto?
  None of the functions are one-to-one. There are 6 functions that are onto.
  (c) How many of the functions from $T \to S$ are one-to-one? How many are onto?
  There are 6 functions from $T \to S$ that are one-to-one. None of the functions are onto.

- Problem 15: Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is one-to-one, the f is one-to-one, and that if $g \circ f$ is onto, then g is onto.

  Proof. Consider $A = \{x_1, x_2, x_3, ..., x_n\}$, $B = \{y_1, y_2, y_3, ..., y_n\}$ and $C = \{z_1, z_2, z_3, ..., z_n\}$. Since $g \circ f$ is one-to-one, QED

2 Section 2.2

- Problem 4: Let S be the set of all ordered pairs (m, n) of positive integers. For $(a_1, a_2) \in S$ and $(b_1, b_2) \in S$, define $(a_1, a_2) \sim (b_1, b_2)$ if $a_1 + b_2 = a_2 + b_1$. Show that $\sim$ is an equivalence relation.

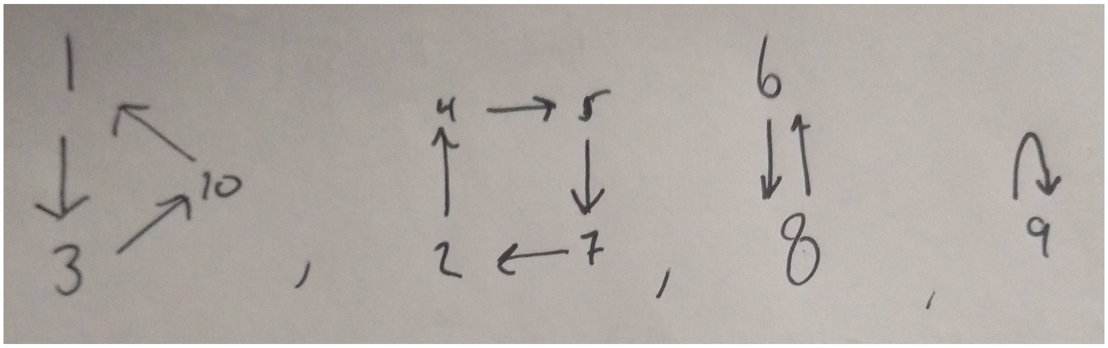
  Proof. Reflexivity: Check $(a, b) \sim (a, b) \implies a + b = a + b$.
  Symmetry: Check $(a, b) \sim (c, d)$ iff $(c, d) \sim (a, b)$. Then $a + b = c + d$ iff $c + d = a + b$. This is true.
  Transitivity: Check if $(a, b) \sim (c, d)$ and $(c, d) \sim (e, f)$, then $(a, b) \sim (e, f)$.
  $a + b = c + d$ and $c + d = e + f$, so we can replace $c + d$ in the second equation to get: $a + b = e + f$. Then $(a, b) \sim (e, f)$. Therefore this is an equivalence relation. QED

- Problem 9: Show that any circular relation is an equivalence relation.

  Proof. By definition, $R \subseteq S \times S$ is both symmetric and transitive. (IDK how to show reflexivity? I think it has to do with the fact that transitivity is stronger than reflexivity but idk) QED

3 Section 2.3

- Problem 3: $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 4 & 10 & 5 & 7 & 8 & 2 & 6 & 9 & 1 \end{pmatrix}$ As a product of disjoint cycles and as a product of transpositions, construct its associated diagram, and find its order.
  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 4 & 10 & 5 & 7 & 8 & 2 & 6 & 9 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 10 & 2 & 4 & 5 & 7 & 6 & 8 & 9 \\ 3 & 10 & 1 & 4 & 5 & 7 & 2 & 8 & 6 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 10 \\ 3 & 10 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 & 5 & 7 \\ 4 & 5 & 7 & 2 \end{pmatrix} \cdot \begin{pmatrix} 6 & 8 \\ 8 & 6 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ 9 \end{pmatrix}$



The order is 12.

- Problem 7: Not sure :/
- Problem 14: Prove that (a, b) cannot be written as a product of two cycles of length three.

  Proof. (a, b) is an odd permutation. A cycle of length 3 is an even permutation, and even times even can never be odd, therefore the product of two cycles of length 3 is even. QED