Math 341 Homework 3

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1 Practice problems

1.1 Problem 1

Prove by induction: $n(n+7) \vdots 2$ for any $n \in \mathbb{N}$.

*Proof. Basis:*If $n = 1$, check: $1(1+7) \vdots 2$.
Inductive step: Assume the statement is true for $n = k$. Then $k(k+7) \vdots 2$. Need to show that it is true for $n = k+1 : (k+1)(k+8) \vdots 2$:

$$k^2 + 9k + 8 = k(k+7) + (2k+8) = k(k+7) + 2(k+4)$$

We know by assumption that $k(k+7) \vdots 2$, and by definition of divisibility, $2(k+4)$ is divisible by 2 for any $k \in \mathbb{N}$. Therefore, the statement is true for all $n \in \mathbb{N}$. QED

1.2 Problem 3

Prove by induction: $7^n + 3^n \vdots 2$ for all $n \in \mathbb{N}$.

Proof. Basis: If $n = 1$, check: $7^1 + 3^1 = 10 \vdots 2$.
Inductive step. Assume the statement holds for $n = k$, $7^k + 3^k \vdots 2$, or $7^k + 3^k = 2d$. We must show that the statement is true for $n = k+1$. So $7^{k+1} + 3^{k+1} \vdots 2$, or $7^{k+1} + 3^{k+1} = 2c$.

$$7^{k+1} + 3^{k+1} = 7 \cdot 7^k + 3 \cdot 3^k$$

Since $7^k + 3^k \vdots 2$, then $7^k + 3^k = 2d$, and $3^k = 2d - 7^k$. So:

$$= 7 \cdot 7^k + 3 \cdot (2d - 7^k) = 4 \cdot 7^k + 6d$$

We have 2 terms, $4 \cdot 7^k$ and $6d$. Since both are divisible by 2, the sum of them will always be divisible by 2, and since that equation is the same as $7^{k+1} + 3^{k+1}$, that means $7^{k+1} + 3^{k+1} \vdots 2$. Therefore, by induction, $7^n + 3^n \vdots 2$ for all $n \in \mathbb{N}$. QED

1.3 Problem 7

Prove: a number n is divisible by 5 iff its last digit is either 0 or 5.

Proof. ($\Longrightarrow$): All numbers n which end in 0 can be written as $n = 10k, k \in \mathbb{Z}$, since $10 \vdots 5$ always, $10k \vdots 5$, and therefore $n \vdots 5$ if n ends in 0. If a number n ends in 5 it can be expressed as $n = 10k + 5$, and since $10k + 5 = 5(k + 1)$, $10k + 5 \vdots 5$. Which means $n \vdots 5$ if n ends in 5.
($\Longleftarrow$): If n ends in any other number, then $n = 10k + a$, where $a \in S, S = \{1, 2, 3, 4, 6, 7, 8, 9\}$ since $10k \vdots 5$, we can remove it, so in essence, $n = a \pmod 5$. Since $a \neq 5l$ for $l \in \mathbb{Z}$, $10k + a \not\vdots 5$ and so $n \not\vdots 5$ if n ends in any a.
$\therefore n \vdots 5 \Longleftrightarrow n$ ends in 0 or 5. QED