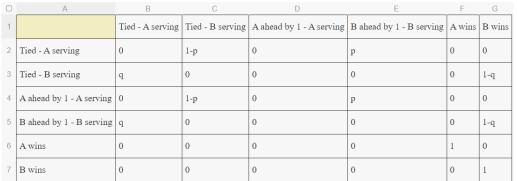
Math 305 Midterm 2

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Problem 1

(a) Transition matrix:



- (b) Probability that the game will not be finished after four rallies is 0.0495+0.04095=0.09045 or 9.04%.
- (c) Transition matrix:

	Α	В	С	D	E	F	G	Н	- 1
1		State 1	State 2	State 3	State 4	State 5	State 6	State 7	State 8
2	State 1	0	q	0	p	0	0	0	0
3	State 2	p	0	0	0	q	0	0	0
4	State 3	0	0	0	p	0	q	0	0
5	State 4	0	0	0	0	q	p	0	0
6	State 5	0	p	q	0	0	0	0	0
7	State 6	0	0	0	0	p	0	0	q
8	State 7	0	0	0	0	0	0	1	0
9	State 8	0	0	0	0	0	0	0	1

(d) Probability that game will not be finished after three rallies is 2.8%

Problem 2

- Rank of page A: .134
- Rank of page B: .118
- Rank of page C: .162
- Rank of page D: .059
- Rank of page E: .142
- Rank of page F: .147
- Rank of page G: .121
- Rank of page H: .118

Problem 3

(a) The paper discusses the formation of cell patterns in epithelial tissues and challenges the notion that the hexagonal cell pattern observed in simple epithelia is a result of optimal cell packing. The authors propose a mathematical model based on a discrete Markov chain to demonstrate that the distribution of polygonal cell types in epithelia is a consequence of cell proliferation rather than cell packing.

(b)

$$f(t) = 2f(t-1)$$

$$e(t) = e(t-1) + 3f(t-1)$$

$$v(t) = v(t-1) + 2f(t-1)$$

$$s(t) = \frac{(e(t-1) + 3 * f(t-1))}{f(t-1)}$$

(c) $\lim_{t\to\infty} s(t) = 6$

(d) Transition matrix:

	Α	В	С	D	Е	F	G
1	Sides	4	5	6	7	8	9
2	4	0	1	0	0	0	0
3	5	0	0.5	0.5	0	0	0
4	6	0	0.25	0.5	0.25	0	0
5	7	0	0.125	0.375	0.375	0.125	0
6	8	0	0.0625	0.25	0.375	0.25	0.0625
7	9	0	0.03125	0.15625	0.3125	0.3125	0.15625

(e) Running this code gave the following results:

```
% Define the transition matrix
transition_matrix = [
   0
            1
                                             0;
   0
            0.5
                 0.5
                             0
                                     0
                                             0;
   0
            0.25
                  0.5
                             0.25
                                     0
                                             0;
            0.125 0.375
                             0.375 0.125
   0
                                             0;
            0.0625 0.25
                             0.375
                                    0.25
                                             0.0625;
            0.03125 0.15625 0.3125 0.3125 0.15625
];
% Perform power iteration to find the leading eigenvector
n = size(transition_matrix, 1);
x = ones(n, 1) / n; % Initialize with a uniform distr
epsilon = 1e-8; % Convergence criterion
max_iterations = 1000;
for i = 1:max_iterations
   prev x = x;
   x = transition_matrix.' * x;
   x = x / sum(x);
   if max(abs(x - prev_x)) < epsilon
       break;
   end
end
% Print the resulting steady state distribution
disp('Steady State Distribution:');
disp(x);
```

Steady State Distribution:

```
0
0.2889
0.4641
0.2085
0.0359
0.0027
```

- \bullet 28.89% pentagons.
- $\bullet~46.41\%$ hexagons.

- 20.85% 7-gons.
- 3.59% 8-gons.
- 0.27% 9-gons.

This is almost exactly the results which the paper had.