Math 305 Midterm 2

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Problem 1

(a) Transition matrix:

	A	B	C	D	E	F	G
1		Tied - A serving	Tied - B serving	A ahead by 1 - A serving	B ahead by 1 - B serving	A wins	B wins
2	Tied - A serving	0	1-p	0	p	0	0
3	Tied - B serving	q	0	0	0	0	1-q
4	A ahead by 1 - A serving	0	1-p	0	p	0	0
5	B ahead by 1 - B serving	q	0	0	0	0	1-q
6	A wins	0	0	0	0	1	0
7	B wins	0	0	0	0	0	1

(b) Probability that the game will not be finished after four rallies is $0.0495 + 0.04095 = 0.09045$ or 9.04%.

(c) Transition matrix:

	A	B	C	D	E	F	G	H	I
1		State 1	State 2	State 3	State 4	State 5	State 6	State 7	State 8
2	State 1	0	q	0	p	0	0	0	0
3	State 2	p	0	0	0	q	0	0	0
4	State 3	0	0	0	p	0	q	0	0
5	State 4	0	0	0	0	q	p	0	0
6	State 5	0	p	q	0	0	0	0	0
7	State 6	0	0	0	0	p	0	0	q
8	State 7	0	0	0	0	0	0	1	0
9	State 8	0	0	0	0	0	0	0	1

(d) Probability that game will not be finished after three rallies is 2.8%

Problem 2

- Rank of page A: .134
- Rank of page B: .118
- Rank of page C: .162
- Rank of page D: .059
- Rank of page E: .142
- Rank of page F: .147
- Rank of page G: .121
- Rank of page H: .118

Problem 3

(a) The paper discusses the formation of cell patterns in epithelial tissues and challenges the notion that the hexagonal cell pattern observed in simple epithelia is a result of optimal cell packing. The authors propose a mathematical model based on a discrete Markov chain to demonstrate that the distribution of polygonal cell types in epithelia is a consequence of cell proliferation rather than cell packing.

(b)

$$f(t) = 2f(t-1)$$
$$e(t) = e(t-1) + 3f(t-1)$$
$$v(t) = v(t-1) + 2f(t-1)$$
$$s(t) = \frac{(e(t-1) + 3 * f(t-1))}{f(t-1)}$$

(c) $\lim_{t\to\infty} s(t) = 6$

(d) Transition matrix:

	A	B	C	D	E	F	G
1	Sides	4	5	6	7	8	9
2	4	0	1	0	0	0	0
3	5	0	0.5	0.5	0	0	0
4	6	0	0.25	0.5	0.25	0	0
5	7	0	0.125	0.375	0.375	0.125	0
6	8	0	0.0625	0.25	0.375	0.25	0.0625
7	9	0	0.03125	0.15625	0.3125	0.3125	0.15625

(e) Running this code gave the following results:

```
% Define the transition matrix
transition_matrix = [
    0        1       0        0       0       0;
    0        0.5     0.5      0       0       0;
    0        0.25    0.5      0.25    0       0;
    0        0.125   0.375    0.375   0.125   0;
    0        0.0625  0.25     0.375   0.25    0.0625;
    0        0.03125 0.15625  0.3125  0.3125  0.15625
];

% Perform power iteration to find the leading eigenvec
n = size(transition_matrix, 1);
x = ones(n, 1) / n;  % Initialize with a uniform distr
epsilon = 1e-8;  % Convergence criterion
max_iterations = 1000;

for i = 1:max_iterations
    prev_x = x;
    x = transition_matrix.' * x;
    x = x / sum(x);
    if max(abs(x - prev_x)) < epsilon
        break;
    end
end

% Print the resulting steady state distribution
disp('Steady State Distribution:');
disp(x);
```

```
Steady State Distribution:
         0
    0.2889
    0.4641
    0.2085
    0.0359
    0.0027
```

- 28.89% pentagons.
- 46.41% hexagons.

- 20.85% 7-gons.
- 3.59% 8-gons.
- 0.27% 9-gons.

This is almost exactly the results which the paper had.