Math 341 Homework 13

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1 Practice problems

1.1 Problem 13.1

Find all $\phi(n)$ for $n \leq 12$.

n	1	2	3	4	5	6	7	8	9	10	11	12
$\phi(n)$	1	1	2	2	4	2	6	4	6	4	10	4

1.2 Problem 13.2

Let p be prime, find $\phi(p)$.

Proof. By definition, because p is prime, there are no numbers smaller than p which evenly divide it. Therefore, since all numbers $\{1, 2, \ldots, p-1\}$ are relatively prime to p, $\phi(p) = p - 1$. QED

1.3 Problem 13.6

Prove that $\phi(n)$ is the number of invertible elements in $\mathbb{Z}_n$.

Proof. By definition, an element $x \in \mathbb{Z}_n$ is invertible iff $\gcd(x, n) = 1$, (they are relatively prime). Conveniently, the totient function at n, $\phi(n)$, counts how many numbers smaller than n are relatively prime to n. Therefore $\phi(n)$ is the number of invertible elements in $\mathbb{Z}_n$. QED