Math 341 Homework 2

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1 Practice problems

1.1 Problem 2

Prove: if $(a-b) \vdots c$, then $a \vdots c$ iff $b \vdots c$.

Proof. To prove iff, we must first prove "If A, then B" (forwards), then prove"if B, then A" (backwards).
($\Longrightarrow$): Assume that $a \vdots c$ is true, that is, $a = cn$. $(a-b) \vdots c$ implies that $(a-b) = cl, l \in \mathbb{Z}$. Rearranging for b we get $b = a - cl = cn - cl = c \cdot (n-l) \therefore$ if $a \vdots c, b \vdots c$.
($\Longleftarrow$): Assume that $b \vdots c$ is true, that is, $b = cm$. We know from above that $(a-b) = cl$. Rearranging for a we get $a = cl + b = cl + cm = c \cdot (l+m) \therefore$ if $b \vdots c, a \vdots c$. QED

1.2 Problem 3

Prove: If $a \vdots c$, then for any $b, (ab) \vdots c$.

Proof. $a \vdots c$ implies that $\exists n \in \mathbb{Z}$ s.t. $a = cn$. Thus, $a \cdot b = b(cn)$, and by the commutative property, $b(cn) = c(bn)$. Let $(bn) = m$, so $a \cdot b = cm, m \in \mathbb{Z}$, therefore $a \cdot b$ is divisible by c. QED

1.3 Problem 4

Prove: if $a \vdots b$ and $b \vdots c$, then $a \vdots c$.

Proof. $a \vdots b \implies a = bn, n \in \mathbb{Z}$. Also, $b \vdots c \implies b = cm$. Solving the first equation for b: $b = \frac{a}{n}$. Plugging it into the second: $\frac{a}{n} = cm$. So $a = cmn = c \cdot (mn)$. Therefore $a \vdots c$. QED