

Örnek 9.34.

$$A = \begin{bmatrix} 2 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 5 \end{bmatrix}$$

Matrisinin karakteristik değerleri ve karşı gelen karakteristik vektörlerini bulunuz

$$\det \left(\begin{bmatrix} 2 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right) = \det \left(\begin{bmatrix} 2-\lambda & -2 & -1 \\ -2 & 2-\lambda & 1 \\ -1 & 1 & 5-\lambda \end{bmatrix} \right)$$

$$\begin{bmatrix} 2-\lambda & -2 & -1 \\ -2 & 2-\lambda & 1 \\ -1 & 1 & 5-\lambda \end{bmatrix} \Rightarrow [(2-\lambda)(2-\lambda)(5-\lambda)+4)-((2-\lambda)+(2-\lambda)4(5-\lambda))] \\ [(\lambda^2-4\lambda+4)(5-\lambda)+4)-(24-6\lambda)] \\ 5\lambda^2-20\lambda+20-\lambda^3+4\lambda^2=\lambda(\lambda-3)(\lambda-6)=0$$

$$\Rightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = 3 \\ \lambda_3 = 6 \end{cases}$$

$$\lambda = 0 \text{ için } \vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$$

$$A\vec{\alpha} = 0, \vec{\alpha} = 0$$

$$\begin{bmatrix} 2 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 5 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} 2\alpha_1 - 2\alpha_2 - \alpha_3 = 0 \\ -2\alpha_1 + 2\alpha_2 + \alpha_3 = 0 \\ -\alpha_3 + \alpha_2 + 5\alpha_3 = 0 \end{cases}$$

$$\text{rank } A = 2$$

$$n - r = 3 - 2 = 1 \text{ adet bilinmeyeni keyfi seçeriz. } \alpha_2 = k \text{ dersek}$$

$$2\alpha_1 - \alpha_3 = 2k$$

$$-\alpha_1 + 5\alpha_3 = -k$$

$$2\alpha_1 - \alpha_3 = 2k$$

$$-2\alpha_1 + 10\alpha_3 = -2k$$

$$\alpha_3 = 0$$

$$2\alpha_1 - 2k = 0$$

$$\alpha_1 = k \Rightarrow \vec{\alpha} = (k, k, 0)$$

$$\lambda = 3 \text{ için } \vec{\beta} = (\beta_1, \beta_2, \beta_3)$$

$$\begin{bmatrix} 2 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 5 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 3\beta_1 \\ 3\beta_2 \\ 3\beta_3 \end{bmatrix}$$

$$2\beta_1 - 2\beta_2 - \beta_3 = 3\beta_1$$

$$-2\beta_1 + 2\beta_2 + \beta_3 = 3\beta_2$$

$$-\beta_1 + \beta_2 + 5\beta_3 = 3\beta_3$$

$$2\beta_2 + \beta_3 = -\beta_1$$

$$-2\beta_1 + \beta_3 = \beta_2$$

$$\beta_2 - \beta_1 = -2\beta_3$$

$$-\beta_1 - 2\beta_2 - \beta_3 = 0$$

$$-2\beta_1 - \beta_2 + \beta_3 = 0$$

$$-\beta_1 + \beta_2 + 2\beta_3 = 0$$

$$\beta = \begin{bmatrix} -1 & -2 & -1 \\ -2 & -1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

$\det \beta = 0$ olduğundan $\text{rank } \beta$ ya bakılır..

$$2\beta_2 + \beta_3 = -\beta_1$$

$$-2\beta_1 + \beta_3 = \beta_2$$

$$\beta_2 - \beta_1 = -2\beta_3$$

$$2\beta_2 + \beta_3 = -\beta_1$$

$$2\beta_1 - \beta_3 = -\beta_2$$

$$2\beta_2 + 2\beta_1 = -\beta_1 - \beta_2$$

$$\vec{\beta} = (k, -k, k)$$

$$3\beta_2 = -3\beta_1$$

$$\beta_2 = -\beta_1$$

$$\beta_1 = k \quad \beta_2 = -k$$

$$\beta_3 = k \Rightarrow \vec{\beta} = (k, -k, k)$$

$$\lambda = 6 \text{ için } \vec{\gamma} = (\gamma_1, \gamma_2, \gamma_3)$$

$$A\gamma = \lambda \cdot \gamma$$

$$\begin{bmatrix} 2 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 5 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} = \begin{bmatrix} 6\gamma_1 \\ 6\gamma_2 \\ 6\gamma_3 \end{bmatrix} \Rightarrow \begin{cases} 2\gamma_1 - 2\gamma_2 - \gamma_3 = 6\gamma_1 \\ -2\gamma_1 + 2\gamma_2 + \gamma_3 = 6\gamma_2 \\ -\gamma_1 + \gamma_2 + 5\gamma_3 = 6\gamma_3 \end{cases} \Rightarrow \begin{cases} -4\gamma_1 - 2\gamma_2 - \gamma_3 = 0 \\ -2\gamma_1 - 4\gamma_2 + \gamma_3 = 0 \\ -\gamma_1 + \gamma_2 - \gamma_3 = 0 \end{cases}$$

$$\gamma = \begin{bmatrix} -4 & -2 & -1 \\ -2 & -4 & 1 \\ -1 & 1 & -1 \end{bmatrix} \Rightarrow \det \gamma = \begin{vmatrix} -4 & -2 & -1 \\ -2 & -4 & 1 \\ -1 & 1 & -1 \end{vmatrix} = (-16 + 2 + 2) - (-4 - 4 - 4) = 0$$

$$\text{rank} A = 2, \quad \gamma_3 = k \text{ olduğundan}$$

$$-4\gamma_1 - 2\gamma_2 - k = 0 \quad -4\gamma_1 - 2\gamma_2 - k = 0$$

$$-2\gamma_1 - 4\gamma_2 + k = 0 \Rightarrow -2\gamma_1 - 4\gamma_2 = -k$$

$$-\gamma_1 + \gamma_2 - k = 0 \quad + \quad 2\gamma_1 - 2\gamma_2 = -2k$$

$$-6\gamma_2 = -3k$$

$$-2\gamma_2 = k \quad \gamma_2 = -\frac{k}{2} \quad \gamma_3 = k$$

$$-4\gamma_1 + 2\frac{k}{2} - k = 0, \quad \gamma_1 = 0 \Rightarrow \vec{\gamma} = \left(0, -\frac{k}{2}, k\right)$$