Proton Density Fat Fractio in MRI

This is the multi-point fat-water separation with phase error correction and R2* using combined complex and magnitude fitting.

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Complex Fitting

First, the projection onto dipole field (PDF) using multi-echo data algorithm was implemented to calculate the initial phase and field map. To save the fitting time, this code utilized the weighted linear least-square (WLS) to obtain the phase map. Due to the phase calculated from the complex MR signal by using the angle function is generally wrapped into the range of $[-\pi, \pi]$ radian, it will produce the wrapped phase.

Second, to estimate the true phase from the wrapped phase, this code implemented the region growing algorithm to unwrap the phase. Since the SNR rely mostly on the amplitude of the signal, this code chose the point (x,y) which has the maximum value of amplitude to become the seed. The eight nearest neighbors of the seed pixel are sequentially compared to the seed to obtain the phase difference $\Delta\emptyset_{pkx,pky} = \emptyset_{seed} - \emptyset_{pkx,pky}$. If the phase difference $\Delta\emptyset_{pkx,pky} > -\pi,$ add the pixel phase to the unwrapped map; otherwise, if the phase difference $\Delta\emptyset_{pkx,pky} < -\pi,$ $2\ \pi$ will be added to the pixel phase and then put it to the wrapped map. When the eight nearest neighbors are already unwrapped, the seed will move to another seed to repeat the step mentioned earlier until all the point in the phase map are unwrapped.

Third, this code utilized the Auto-Regression on Linear Operations (ARLO) algorithm to do the R2* fitting. Before running the ARLO algorithm, the MR signal will be fitted by the unwrapped field map to correct the phase error. After fitting the R2*, we can get the initial estimated water and fat complex value to calculate the estimated complex signal *shat* and compare it with the frequency fitted MR signal *s*. If the error $|s - shat|$ isn't less than the tolerance, *shat* will replace *s* and repeat the ARLO. Otherwise, if the error $|s - shat|$ is less than the tolerance or the iteration is larger than max iteration, the ARLO loop will be immediately terminated and calculate the water and fat complex signal.

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Magnitude Fitting

This code implemented the simple iterative linear fitting algorithm to obtain water and fat signal. First, the results of fat and water from the complex-based fitting are used as the initial guess for the magnitude-based fitting. S_{magn}=$W^2 + F^2 \cdot |c_i|^2$+2$\cdot$ $Re\{c_i\} \cdot W \cdot F$, where $c_i = \sum_{p=1}^{P} \alpha_p \, e^{j2\pi\Delta f_p TE_i}$. Since the function of magnitude-based fitting is $argmin_{W,F}\left[\sum_{n=1}^{Nechos}|S| - |\,S_{magn}|\right]$. Second, to estimate of the error terms of water and fat, this code define the matrix B as

$$B=\begin{bmatrix} 2\cdot W + 2\cdot Re\{c_1\}\cdot F & 2\cdot F\cdot |c_1|^2 + 2\cdot Re\{c_1\}\cdot W \\ 2\cdot W + 2\cdot Re\{c_2\}\cdot F & 2\cdot F\cdot |c_2|^2 + 2\cdot Re\{c_2\}\cdot W \\ 2\cdot W + 2\cdot Re\{c_3\}\cdot F & 2\cdot F\cdot |c_3|^2 + 2\cdot Re\{c_3\}\cdot W \\ 2\cdot W + 2\cdot Re\{c_4\}\cdot F & 2\cdot F\cdot |c_4|^2 + 2\cdot Re\{c_4\}\cdot W \\ 2\cdot W + 2\cdot Re\{c_5\}\cdot F & 2\cdot F\cdot |c_5|^2 + 2\cdot Re\{c_5\}\cdot W \\ 2\cdot W + 2\cdot Re\{c_6\}\cdot F & 2\cdot F\cdot |c_6|^2 + 2\cdot Re\{c_6\}\cdot W \end{bmatrix}.$$

The estimate of the error $\begin{bmatrix}\Delta W \\ \Delta F\end{bmatrix} = (B^T \cdot B^{-1}) \cdot B^T \cdot [|s| - |S_{magn}|$. After obtaining the estimated water and fat errors $(\Delta W, \Delta F)$, this code utilized the error to correct the water and fat signal and update the current estimates W=W+ ΔW; F=F+ΔF. Finally, repeat the first step until the iteration is larger than max iteration.

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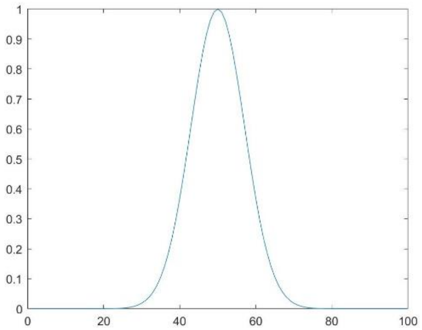
Combined Complex Fitting and Magnitude Fitting

The complex-based fitting provides relatively superior noise performance; however, it will created more phase error at low- and high- fat fraction. The magnitude-based fitting is more accurate in the presence of phase errors, but it has poor noise performance around 50%. This code combined the complex-based fitting and magnitude-based fitting to improve the accuracy of the fat fraction image.

$$W(x,y)=\text{weight} \cdot Wc(x,y)+(1-\text{weight}) \cdot Wm(x,y)$$

$$F(x,y)=\text{weight} \cdot Fc(x,y)+(1-\text{weight}) \cdot Fm(x,y)$$

The weight is determined from the fat-fraction calculated from the first step of complex-based fitting algorithm. In this code, the simple exponential function $e^{\frac{-(ff_C-50)^2}{100}}$ is implemented. As shown in figure [1], If the p point's fat-fraction is closer to 0 or 100, the weight approach to 0; therefore, the water and fat images at p point are built more by the results from the magnitude-based fitting algorithm. On the contrary, if the p point's fat-fraction is closer to 50, the weight will approach to 1. The results of water and fat images from the complex-based fitting algorithm will impact the final water and fat images.



Figure[1]

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Fat-Fraction calculation

The fat-fraction function is ff=$\frac{100 \cdot |f|}{|f|+|w|}$. To correct the noise in the fat-fraction image, if the pixel's ff is more than 50, the ff will be recalculated by implementing the function ff=$\frac{100 \cdot |f|}{|f+w|}$. And if the pixel's ff is less than or equal to 50, the ff will be recalculated by using the function ff= $100-\frac{100 \cdot |w|}{|f+w|}$.