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Question 1

The recurrence relation of a recursive divide and conquer algorithm is given. Explain this recurrence, verbally, in terms of the size of each sub-problem, the cost of dividing the problem, and combining solutions.

$$T(n) = 3T(\tfrac{n}{4}) + 2n + n^3$$

Answer:

↳ Size of each subproblem ⇒ since input size is n, 3T(n/4) means that the problem is divided into 3 subproblems with n/4 size each. Thus size of each s.p. is n/4.

↳ cost of dividing the problem ⇒ 2n.

↳ cost of combining the solutions ⇒ n^3

Question 2

Find an asymptotically tight lower bound for the following recurrence by using the substitution method.

$$T(n) = T(\tfrac{n}{3}) + T(\tfrac{2n}{3}) + \Theta(n)$$

Answer: (1) making a guess : Since each recursive call's work proportional to n, let's assume that $T(n)$ is bounded below by $cn\log n$ for some constant $c>0$ In other words, we claim $T(n) = \Omega(n\log n)$

(2) Inductive Step: We need to prove that $T(n) \geq cn.\log n$ for all sufficiently large n We have the recurrence rela.

$T(n) = T(n/3) + T(2n/3) + \Theta(n)$

Assuming the lower bound $T(n) \geq cn\log n$, by inductive hypothesis :

$T(n) \geq \left(\frac{cn}{3}\log\frac{n}{3}\right) + \left(\frac{2cn}{3}.\log\frac{2n}{3}\right) + \Theta(n)$

By simplifying :

$T(n) \geq cn.\log n - cn\log 3 + \frac{2cn}{3}\log 2 - \frac{2cn}{3}\log 3 + \Theta(n)$

Dropping the lower-order terms

$T(n) \geq cn\log n + \Theta(n)$

Thus, we prove that the tight lower bound for $T(n)$:

$T(n) = \Omega(n\log n)$

Question 3

For the following recurrences, either solve it by using the master method or show that it cannot be solved with the master method.

(a) $T(n) = T(\frac{n}{2}) + \Theta(1)$

$a \geq 1$, $b \geq 1$, $f(n)$ is asymp. positive. } applicable.

Answer: With Master Theorem, we can solve the recurreces with the structure:

$T(n) = a.T(n/b) + f(n) \Rightarrow a=1, b=2, f(n)=\Theta(1)$

Analysis of the case:

$\log_b a = \log_2 1 = 0$
$f(n) = \Theta(1) = \Theta(n^0)$. } Since $f(n) = \Theta(n^{\log_b a})$ Case 2 applies.

Therefore, the solution to the recurrece relation:

$T(n) = \Theta(n^{\log_b a}.\log n) \Rightarrow T(n) = \Theta(n^0.\log n)$
$= \Theta(\log n)$//

(b) $T(n) = 3T(\frac{n}{4}) + n\lg n$

$3 \to a$, $4 \to b$, $n\lg n \to f(n)$

$a \geq 1$, $b \geq 1$, $f(n)$ is asym. positive } applicable

Answer:

$n^{\log_b a} = n^{\log_4 3} \Rightarrow \log_4 3 < 1$, we need to compare $f(n)$

necessary checks

$f(n) = n.\lg n = \Omega(n^{\log_4 3+\epsilon})$

$a.f(n/b) \leq cf(n) \Rightarrow 3.\frac{n}{4}.\log\frac{n}{4} \leq c.n.\log n$
$\hookrightarrow c = 3/4 \Rightarrow c<1$

Case 3:

$T(n) = \Theta(f(n))$
$= \Theta(n\log n)$//