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(1a) $P(X|\alpha) = \alpha \cdot (1-X)^{\alpha-1}$

$\text{Argmax}_{\alpha}\, P(x|\alpha) = \hat{\alpha}$

$\mathcal{L}(\alpha|X) = \prod_{i=1}^{n} P(X_i|\alpha)$ (likelihood function)

$\ell(\alpha|X) = \log \mathcal{L}(\alpha|X) = \sum_{i=1}^{n} \log P(x_i|\alpha)$

$= \sum_{i=1}^{n} \log(\alpha(1-x)^{\alpha-1}) = \sum_{i=1}^{n} [\log\alpha + (\alpha-1)\log(1-x_i)]$

$= n\log\alpha + (\alpha-1)\sum_{i=1}^{n} \log(1-x_i)$ taking the deriv. wrt α and solve

$= \frac{n}{\alpha} + \sum_{i=1}^{n} \log(1-x_i) = 0$

$\frac{n}{\alpha} = -\sum_{i=1}^{n} \log(1-x_i)$

$\alpha = -\frac{n}{\sum_{i=1}^{n} \log(1-x_i)}$

hence, MLE of α is

$$\frac{-n}{\sum_{i=1}^{n} \log(1-x_i)}$$ //

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(1b) $p(\alpha) = \underbrace{\lambda \alpha^{\lambda-1} e^{-\lambda\alpha}}_{\text{prior}}$

computing the posterior $p(\alpha|X)$ with Bayes Th

Bayes Theorem $\Rightarrow P(\alpha|X) = \dfrac{(P(X|\alpha) \cdot P(X)}{P(\alpha)}$

likelihood func.

prior

we found likelihood func in (1a) $\Rightarrow \prod_{i=1}^{n} P(X_i|\alpha)$

$\mathcal{L}(\alpha|X) = P(X|\alpha) = \alpha^n (1-x_1)^{\alpha-1} \cdot (1-x_2)^{\alpha-1} \cdot (1-x_3)^{\alpha-1} \cdots (1-x_n)^{\alpha-1}$

$P(X|\alpha) \cdot P(\alpha) = \alpha^n (1-x_1)^{\alpha-1} (1-x_2)^{\alpha-1} \cdots \lambda \alpha^{\lambda-1} e^{-\lambda\alpha}$

hard to maximize

taking log

$\log(P(X|\alpha)) = \log\left[\alpha^n (1-x_1)^{\alpha-1} (1-x_2)^{\alpha-1} \cdots (1-x_n)^{\alpha-1} \cdot \lambda \alpha^{\lambda-1} e^{-\lambda\alpha}\right]$

$= (n\lambda - 1)\log\alpha + \left(\sum_{i=1}^{n} \log(1-x_i) - \lambda\right)\alpha$

differentiate wrt α

$\dfrac{n\lambda - 1}{\alpha} + \left(\sum_{i=1}^{n} \log(1-x_i) - \lambda\right) = 0$ solve for α

$\alpha = \dfrac{(n\lambda - 1)}{-\sum_{i=1}^{n} \log(1-x_i) + \lambda}$ MAP estimation of α.