

Simulated Annealing

The Simulated Annealing algorithm (see the SA Algorithm section) generated is based on the stopping criteria introduced in [1]. Perturbation is performed by 1-Swap operator (see Perturb function under SA Algorithm section), while discrete representation is selected to keep the solutions. The parameters T, L, ip, r, and fp is set respectively to 10000000, 200, 0.7, 0.9, 0.01 respecting the findings derived from [1].

Pseudocode of Simulated Annealing algorithm is given as follows:

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1.    $X \leftarrow \text{InitialSolution}$ 
2.    $Z \leftarrow f(X)$ 
3.    $\text{Incumbent} \leftarrow Z$ 
4.   Initialize input parameters:  $T, L, ip, r, fp$ 
5.    $\text{NumberOfIterations} \leftarrow 0$ 
6.    $\text{terct} \leftarrow 0$ 
7.   while  $\text{terct} < 5$  :
8.        $j \leftarrow 0$ 
9.       do  $L$  times:
10.           $\text{NumberOfIterations} \leftarrow \text{NumberOfIterations} + 1$ 
11.           $X' \leftarrow \text{Perturb}(X)$  // remove an existing facility and add a new facility
12.           $Z' \leftarrow f(X')$ 
13.           $\Delta \leftarrow Z' - Z$ 
14.          if  $\Delta \leq 0$ :
15.               $\text{Incumbent} \leftarrow Z'$ 
16.               $X \leftarrow X'$ 
17.               $\text{BestSolution} \leftarrow X'$ 
18.               $\text{BestIteration} \leftarrow \text{NumberOfIterations}$ 
19.               $j \leftarrow j + 1$  //keep track number of accepted solution
20.          else  $\Delta < 0$ :
21.              select a random variable  $m \sim U(0,1)$ 
22.              if  $e^{-\frac{\Delta}{T}} > m$ :
23.                   $Z \leftarrow Z'$ 
24.                   $X \leftarrow X'$ 
25.                   $j \leftarrow j + 1$ 
26.              end if
27.          end if
28.      end do
29.      if  $\frac{j}{L} \leq fp$ :
30.           $\text{terct} = \text{terct} + 1$ 
31.      else:
32.           $\text{terct} = 0$ 
33.      end if
34.      if  $\frac{j}{L} > ip$ :
35.           $T = T/2$ 
36.      else:
37.           $T = r \cdot T$ 
38.      end if
```

39. **end while**
40. **Return** *Incumbent, BestSolution, BestIteration*

Variable Neighborhood Search

Within Variable Neighborhood Search Algorithm (see VNS function) discrete solution representation is preferred, while 1-swap, 2-swap, and 3-swap shaking moves are used as it is asked. For local search, 1-swap move operator is selected in order to discover the neighbors of the shake-d solution. The stopping criteria is met when no better solution is found within 20 iterations.

Pseudocode of Variable Neighborhood Search algorithm is given as follows:

1. $K \leftarrow \{1\text{-swap}, 2\text{-swap}, 3\text{-swap}\}$ //using 1-swap, 2-swap and 3-swap neighborhood structures set K is generated for shaking step
2. $X \leftarrow \text{InitialSolution}$
3. $Z \leftarrow f(X)$
4. $\text{Incumbent} \leftarrow Z$
5. $\text{NumberOfIterations} \leftarrow 0$
6. **Repeat:**
7. $\text{NumberOfIterations} \leftarrow \text{NumberOfIterations} + 1$
8. $k \leftarrow 0$
9. **while** $k \leq \text{length}(K)$:
10. $X' \leftarrow \text{random solution of } X \text{ using neighborhood structure } k$ //shaking step
11. Generate all neighbors of X' based on 1 – swap move //local search: randomly select one facility from X' and add new facility
12. $Z'' \leftarrow \text{minimum objective value within neighborhood } N(X')$
13. $X'' \leftarrow \text{solution corresponds to the minimum objective value}$
14. **if** $Z'' < \text{Incumbent}$:
15. $\text{Incumbent} \leftarrow Z''$
16. $X \leftarrow X''$
17. continue
18. **else:**
19. $k \leftarrow k + 1$
20. **end if**
21. **end while**
22. **if** $(\text{Incumbent} - Z'') < 0$ within last 20 neighbors:
23. **stop**
24. **Return** $X, \text{Incumbent}, \text{NumberOfIterations}$

[1] Liu, C., Kao, R., Wang, A., Solving Location-allocation Problems with Rectilinear Distances by Simulated Annealing, *J. Opl. Res. Soc.*, vol. 45, no. 11, 1994, pp. 1304-1315.