ECON 590
Final Project

Investment in new Technology

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Research Question

How does the substitutability between AI capital and labor influence a firm's optimal AI investment, the value of increasing AI, and the evolution of AI capital?

In simpler terms:

how easily AI capital can replace labor affects...

- a firm's investment choices
- net output generated from AI
- AI capital accumulation over time.

Our project is figuring out what those effects are👊

Problem Setup (model)

4 Investment in new technology

In this project we are going to solve a representative agent model about the accumulation of automation capital in a model with a constant elasticity of substitution (CES) production function. We want to compare how the model solutions vary with different values for the elasticity of substitution between labor and capital in the production function.

Consider the following production technology

$$Y_t = \left[\frac{1}{2}L^{\frac{\sigma-1}{\sigma}} + \frac{1}{2}A_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{26}$$

where $L = 1$ denotes labor input and A_t denotes AI-capital, such as data centers. With $\sigma > 1$ AI capital is a substitute for labor. Consider a firm that chooses how much to invest into AI technology:

$$V(A_t) = \max_{I_t \geq 0} \log(Y_t - I_t) + \beta V(A_{t+1}) \tag{27}$$

subject to:

$$Y_t = \left[\frac{1}{2}L^{\frac{\sigma-1}{\sigma}} + \frac{1}{2}A_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{28}$$

$$A_{t+1} = A_t(1-\delta_A) + I_t. \tag{29}$$

For parameters choose $\beta = 0.9$ and $\delta_A = 0.1$.

- Solve and simulate the model for $\sigma = 1.05$, $\sigma = 1.2$, and $\sigma = 1.3$
- Interpret the differences

Breakdown:

- The firm produces **output** using **labor (L)** and **AI capital (A)**, like data centers,
- **The firm *can* invest in AI each period**, which grows its AI capital for the future
- Some AI can replace labor, and how easily it can do that is measured by **sigma (σ)**

*** Variation of Ramseys continuous model!

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Firm's goal:

- Decide **how much to invest in AI each period** to maximize net output over time
- **Balance current output with future growth** from more AI capital

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Key parts of the model:

- **Production function**
  - Output today from AI and Labor
- **Value function**
  - The firm's utility from today and the future based on today's decision
- **Policy function**
  - AI capital we will have tomorrow

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Parameters:

- Yt = total output
- L = labor (fixed at 1)
- At = AI capital today
- At+1 = AI capital tomorrow
- It = Investment today
- β = discount factor
- δ = depreciation
- σ = elasticity of substitution

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what we REALLY care about

Key parameter: elasticity of substitution, σ

- Sigma (σ) is the substitutability between labor and capital, appearing **only in production function**

$$Y_t = \left[\frac{1}{2}L^{\frac{\sigma-1}{\sigma}} + \frac{1}{2}A_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

- Why is that important? Because **changing sigma, changes output** and the firm's goal is to maximize output

- **Low σ** → AI and labor are complements → investing in AI gives less bang for our buck

- **High σ** → AI and labor are good substitutes → investing in AI is more valuable

  - **HELPFUL NOTE**
    Since labor is fixed at 1, differences in sigma highlight how AI capital alone contributes to output and investment decisions

Solution method: value function iteration

- So how do we solve this firm's problem with varying sigmas? 🤔
- We solve the dynamic model (similar to the Ramsey model) with continuous capital, using cubic spline interpolation to approximate the value function

Solution method: value function iteration

1. Define parameters, production function and utility function

2. get_update
   - Calculates the updated value and the optimal policy for a given level of AI capital

3. update_V
   - Computes updated value and policy functions for an AI capital grid, utilizing the get_update method

4. converge_bellman
   - Updates value and policy function using update_V method until convergence

Solution method: value function iteration

4. simulation()
    - Computes capital, output, investment and consumption over T periods

5. solve_model_for_sigma()
    - Solves for value function, policy function, simulations and steady states given sigma value

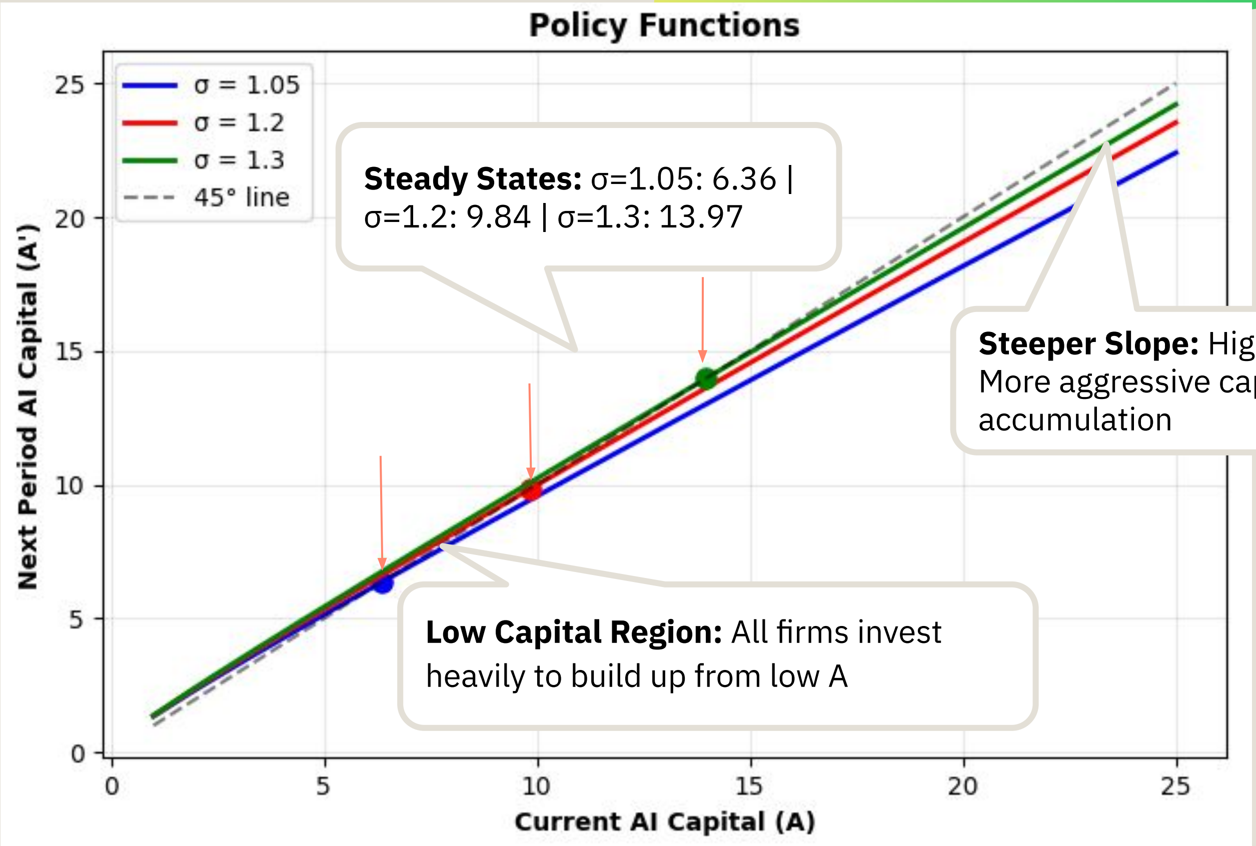
With the information obtained from step 5, we were able to visualize and analyze our results.

Results (Convergence & Steady States)

Convergence Progress				Steady State Comparison		
σ	Iter 0	Iter 150	Status	Ass	Yss	Iss
1.05	3.29e+00	5.46e-08	Success	6.36	2.57	0.64
1.20	3.32e+00	8.01e-08	Success	9.84	3.50	0.98
1.30	3.34e+00	1.03e-07	Success	13.97	4.55	1.40
All models converged in ~150 iterations				↑ +120%	↑ +77%	↑ +119%

Key Finding: Higher elasticity of substitution (σ) increases steady state outcomes.
A 24% increase in σ from 1.05 to 1.3 more than doubles capital and investment, with 77% higher output.

Results (Policy Functions)

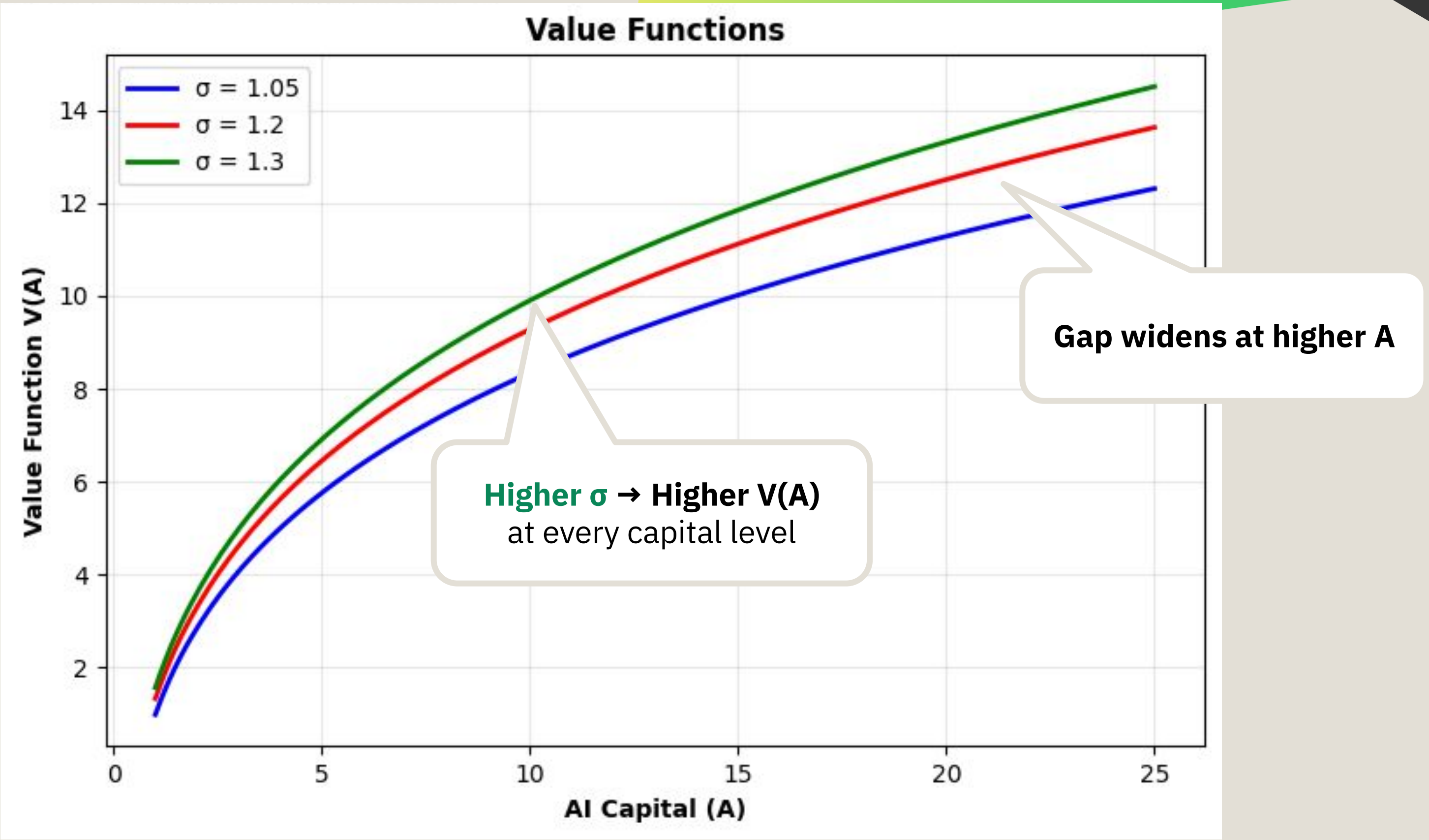


Steady States: σ=1.05: 6.36 | σ=1.2: 9.84 | σ=1.3: 13.97

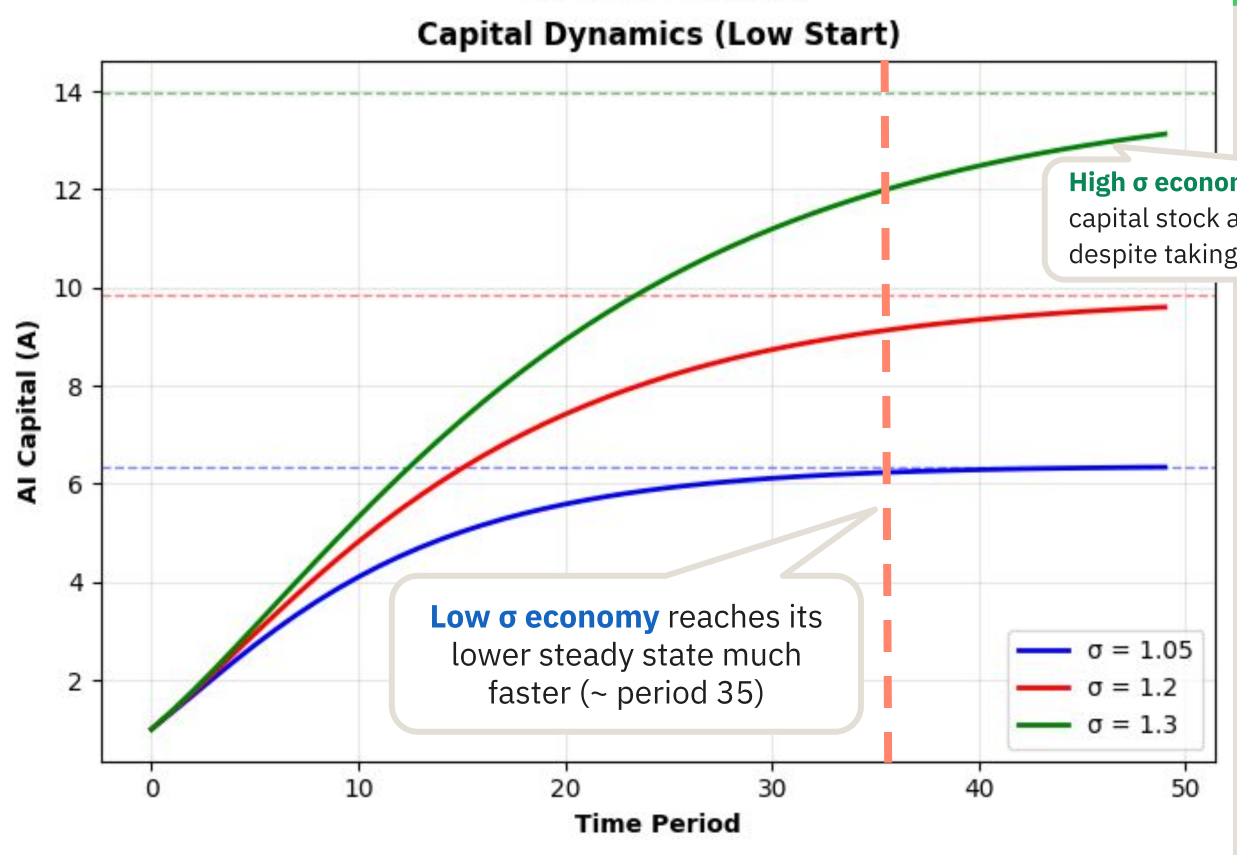
Steeper Slope: Higher σ → More aggressive capital accumulation

Low Capital Region: All firms invest heavily to build up from low A

Results (Value Functions)



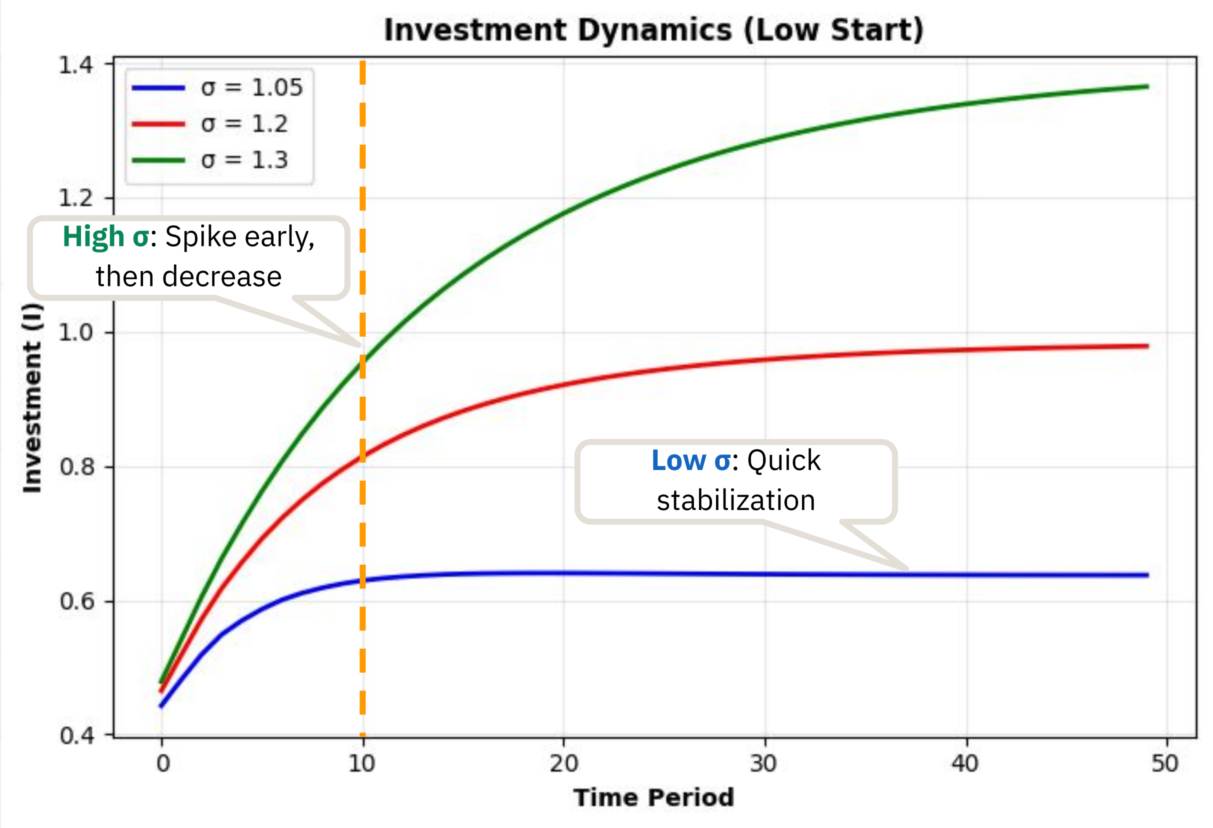
Results (Capital Dynamics)



High σ economy builds a much larger capital stock and output over time, despite taking longer to converge

Low σ economy reaches its lower steady state much faster (~ period 35)

Results (Investment Dynamics)



Economic Interpretation

Finding 1: Scale & Magnitude

Higher σ → **AI capital is a better substitute** for the fixed labor input

→ Investing in AI yields a **higher marginal product** and is **more valuable** to the firm

→ Firm chooses to **invest more**, resulting in **higher long-run levels of AI capital, output, and investment**

Finding 2: Speed vs. Magnitude Tradeoff

High σ → takes **longer** (> period 50) to build a much **larger capital stock**

Low σ → reaches its **lower steady state faster** (~ period 35)

Finding 3: Investment Front-Loading

High σ → **Aggressive, Front-Loaded Investment**→ Investment **spikes early** to quickly build capital, **then decreases** as the steady state approaches

Low σ → **Gradual Adoption** → Investment shows a **smooth buildup and quick stabilization**

Economic Interpretation (Real-Life)

If σ is HIGH (highly substitutable):

⚠️ Adoption: Firms pursue **massive capital buildup via a front-loaded investment boom**

⚠️ Labor Market Impact: This signals **a large structural transformation** that could lead to rapid worker displacement

⚠️ Policy Needs: **Aggressive intervention** (e.g., large-scale social safety nets, urgent retraining programs) **is needed to mitigate disruption**

If σ is LOW (less substitutable):

✓ Adoption: Firms **adopt AI at a gradual pace**

✓ Labor Market Impact: Modest capital accumulation **allows more time for worker adjustment and retraining**

✓ Policy Needs: **Standard economic policies** are likely sufficient

Insight 1: The need for policy intervention is **directly proportional** to the degree of substitutability **σ.**
Insight 2: Policymakers and industry leaders must prioritize **measuring the true σ** in specific sectors to anticipate the scale and speed of AI-driven change.

Limitations

Fixed Labor (L=1) 👥

- Model: Labor fixed
- Reality: Workers exit labor force
- Impact: Understates labor market effects

No Uncertainty 🎲

- Model: Perfect foresight
- Reality: Tech shocks, recessions
- Impact: Overstates ability to plan

No Human Capital 🎓

- Model: Labor quality fixed
- Reality: Workers can retrain
- Impact: Understates adaptation

Partial Equilibrium ⚖️

- Model: Wages exogenous
- Reality: Wages respond to AI
- Impact: Misses feedback effects

Extensions (Future Research)

Measure True σ

Conduct empirical work using firm data to accurately estimate the elasticity of substitution

Human Capital

Include worker adaptation/retraining for more optimistic long-run scenarios.

General Equilibrium

Introduce endogenous wages to capture feedback between AI, investment, and the labor market

Heterogeneous Firms

Model different types of firms to analyze distributional effects (who wins/loses)

Conclusions

Results

As the substitutability between AI capital and labor increases

- All firms invest heavily in AI capital and higher levels maintain stronger accumulation
- The value of owning more AI capital increases at every level of AI
- Economics with higher substitutability accumulate dramatically more AI capital and achieve much higher output over time

Thank you!

We hope you understood this early in the morning <3