# ALGORITHMS, FALL 2019, HOMEWORK 7

– This assignment is worth 3 units.
– Due on Thursday, November 7, at noon.
– Points will be deducted if you include the problem statement in your submission. Sorry, it looks like it's the only way to enforce this. Just include your solution.

1. You are given an array $A$, containing $n$ real numbers without duplicates. For every pair of elements, we say that the pair is "in order" if the smaller value is found somewhere to the left of the larger one. For example, in the array $[13, 11, 14, 12]$ there are three pairs that are in order: $\{13, 14\}$, $\{11, 14\}$, $\{11, 12\}$. Show how to find the number of such pairs in $A$, in $O(n \log n)$ time.
   *You must use a technique learned in class this week.*

2. You are given two strings that each have length $n$. You must find their LCS; not only its length. You are allowed polynomial time to do this but you must only use linear space. Describe how to do this.

3. Let $S$ be a string with $n$ characters, indexed 1 to $n$. Let $X$ be a sorted array with $m$ integers within range 1 to $n-1$, inclusive. The integers in $X$ correspond to indices in $S$ after which we must insert a space. For example, if $S =$ ALGOISSOCOOL is to be split into 4 words of length 4,2,2,4 respectively, then $X = [4, 6, 8]$.

   Given $S$ and $X$, the spaces must be inserted one at a time, in any order. The order matters because it costs $k$ to insert a space anywhere on a substring of size $k$. When a space is inserted, we get two substrings that we eventually process disjointly. In the example above if you first split at position 6, the cost will be 12. Then on each side the cost of the next split will be 6. On the other hand if you were to first split at position 4, the cost would be 12 but then you would pay 8 for the next split, etc.

   (a) Derive a recursive formula that returns the minimum cost of splitting a string as defined above. Briefly explain why the formula is correct.

   (b) How many distinct subproblems might you need to solve during recursion?

   (c) How fast can you determine what the minimum cost of splitting a given string is? You may do whatever you like, algorithmically, but only polynomial time will receive credit.