ALGORITHMS, FALL 2019, HOMEWORK 1

– This assignment is worth one unit.
– Remember to keep each of the three problems on a separate page, and to tag each problem in Gradescope.
– Due on Thursday, September 12, at noon.

1. Suppose that $f(n) = \Theta(g(n))$.

   (a) Must it be true that $\log f(n) = \Theta(\log g(n))$?

   (b) Must it be true that $2^{f(n)} = \Theta(2^{g(n)})$?

2. (a) Suppose you have a function of two variables, n and k. What would it mean, mathematically, to say that this function is $O(n + k)$?

   (b) Let $f(n) = O(n)$ and $g(n) = O(n)$. Let c be a positive constant.
   Prove or disprove that $f(n) + c \cdot g(k) = O(n + k)$.

3. Let $f(n) = \sum_{y=1}^{n} (n^6 \cdot y^{23})$.

   Find a simple $g(n)$ such that $f(n) = \Theta(g(n))$, by proving that $f(n) = O(g(n))$, and that $f(n) = \Omega(g(n))$.

   Don't use induction / substitution, or calculus, or any fancy formulas.
   Just exaggerate and simplify for big-O, then underestimate and simplify for Ω.