

UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL 2014 EXAMINATIONS
CSC418H1S: Computer Graphics

Duration: 3 hours

No aids allowed

There are 16 pages total (including this page)

Given name(s): _____

Family Name: _____

Student number: _____

Question	Marks
1	_____/9
2	_____/10
3	_____/14
4	_____/10
5	_____/14
6	_____/16
7	_____/20
8	_____/12
9	_____/12
10	_____/23
11	_____/8
12	_____/17
Total	_____/165

1. [9 marks] **Parametric Curves**

a) [4 marks] A Crunodal cubic is described as:

$$x(t) = t^2 - 1$$

$$y(t) = t(t^2 - 1)$$

where $-\infty < t < \infty$.

Give a formula for the tangent of the curve at an arbitrary value of t .

b) [5 marks] This curve intersects itself. Give the coordinates and all tangents of the curve at the intersection point.

2. [10 marks] **Parametric Curve Reparameterizations**

a) [4 marks] Suppose

$f(t): R \rightarrow R^3$ is a parametric curve

$s(t): R \rightarrow R$ is a differentiable function

then $g(t) = f(s(t))$ is a parametric curve called a reparameterization of $f(t)$ and it traces out the same curve as $f(t)$.

Show that $\frac{g'(t)}{\|g'(t)\|} = \pm \frac{f'(t)}{\|f'(t)\|}$

b) [6 marks] Given the following formula for $f(t)$:

$$f(t) = \begin{pmatrix} a(t+1)^2 \cos((t+1)^2) \\ a(t+1)^2 \sin((t+1)^2) \\ t^2 + 2t \end{pmatrix}$$

Choose a function $s(t)$ such that $g'(t)$ is easier to derive than $f'(t)$, where $g(t) = f(s(t))$. Give formulas for each of $s(t)$, $g(t)$, and $g'(t)$.

3. [14 marks] **Parametric and Implicit Surfaces**

- a) [4 marks] For non-zero constants a and b , an elliptic cylinder surface can be described parametrically by

$$f(u, v) = (x(u, v), y(u, v), z(u, v))$$

where

$$x(u, v) = a \cos(u) \sin(v)$$

$$y(u, v) = b \sin(u) \sin(v)$$

$$z(u, v) = v$$

for $u \in [0, 2\pi]$, $v \in [-\infty, \infty]$.

Compute two distinct tangent vectors for an arbitrary parametric location (u, v) .

- b) [3 marks] Using the two tangent vectors computed in *a*), give a formula for the normal vector at an arbitrary parametric location (u, v) .

- c) [4 marks] Consider the yz -plane. This surface can be described parametrically as

$$g(u, v) = (0, u, v) \quad \text{for } u, v \in [-\infty, \infty]$$

This same surface can be described in implicit form as $x=0$. Note that $y(u, v) = u$ for any value of u , and since the plane can take on an arbitrary value of y as long as $x=0$ is satisfied, there is no y term in the implicit form. Similarly there is no z term in the implicit form.

Knowing the above, take the squares of the parametric forms of the x , y , and z functions for the elliptic cylinder and use them derive an implicit function describing the elliptic cylinder.

- d) [3 marks] Using the implicit form, give the formula for the normal at a given point (x, y, z) on the surface. Show that this vector is in the same direction, or possibly the opposite direction, of the vector computed in b).

4. [10 marks] **Transformations**

- a) [3 marks] Given the following 2D affine transformation, and a triangle with an area of 2 units squared, what is the triangle's area after it is transformed?

$$\begin{bmatrix} 2 & 1 & 4 \\ -3 & 5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

- b) [3 marks] Can affine transformations be represented with a formula where points can be represented in Cartesian coordinates instead of homogeneous coordinates? What is the main advantage of representing affine transformations in the same matrix form as general homographies?

- c) [4 marks] Show that the inverse of a 3x3 matrix $M = [\vec{u} \ \vec{v} \ \vec{w}]$ is the transpose of M , where \vec{u} , \vec{v} , and \vec{w} are unit vectors in 3D that are orthogonal to one another.

5. [14 marks] **Coordinate Systems**

Suppose you are working with three coordinate systems: object, world, and camera space. The basic vectors of object space have world coordinates $(1, 0, 0)$, $(0, 0, -1)$, and $(0, 1, 0)$ and the origin of object space has world coordinates $(0, 0, 10)$. Camera space has basis vectors of $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, and $\left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$, and an origin of $(5, 9, -4)$, also in world coordinates.

a) [3 marks] Give the 4x4 matrix that transforms object to world coordinates.

b) [3 marks] Give the 4x4 matrix that transforms camera to world coordinates.

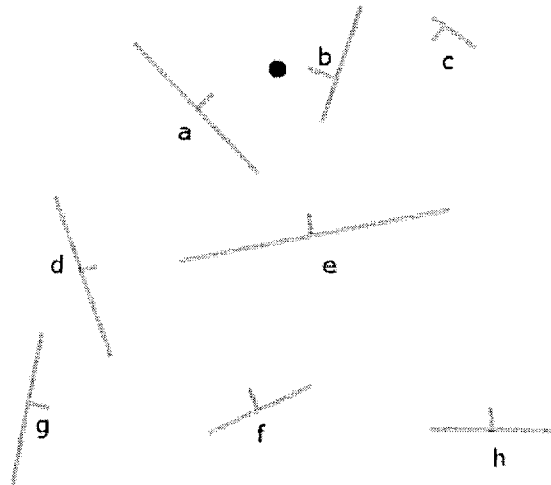
c) [4 marks] Give the 4x4 matrix that transforms world to camera coordinates.

d) [2 marks] Give the 4x4 matrix product that transforms object to camera coordinates. You do not need to multiply out the matrix product.

e) [2 marks] What are the camera coordinates of the object space's origin?

6. [16 marks] **BSP Trees**

Consider the following 2D scene with the camera at the point illustrated by the black dot and the outward normal of polygon segments as shown:



- a) [8 marks] Draw the BSP tree for the scene by adding the segments in the labeled order, starting with a.
- b) [8 marks] Describe how your tree will be traversed when rendering the scene from the location specified.

7. [20 marks] **Lighting and Shading**

- a) [6 marks] Give the mathematical expression for the Phong lighting model function for one light source, $L(\vec{b}, \vec{n}, \vec{s})$, where \vec{b} is the unit vector toward the eye, \vec{n} is the unit normal, and \vec{s} is the unit vector toward the light.
- b) [3 marks] In the diffuse term of the Phong lighting model, the angle between the normal vector and the direction to the light can be larger than 90 degrees. Draw a diagram of this case, and explain how the lighting model handles this case.
- c) [5 marks] To make the specular highlights on a material larger, how should you modify the specular reflection exponent of the material? Explain in detail, and include the specular reflection direction in your explanation.
- d) [3 marks] What is the purpose of the ambient light term in the Phong lighting model?

- e) [3 marks] What is the difference between Phong shading and Gouraud shading? What is the effect on specular highlights?

8. [12 marks] **Basic (Whitted) Ray Tracing**

Consider a basic (Whitted) ray tracing model with a global illumination term. Suppose \bar{p}_1 is the point of intersection, \bar{b}_1 is the unit vector toward the camera, \bar{n}_1 is the unit normal vector, and \bar{s}_1 is the unit vector toward the light source. Suppose also there are no surfaces needing refraction. Let $(\bar{p}_2, \bar{b}_2, \bar{n}_2, \bar{s}_2)$ correspond to the intersection of a ray spawned from \bar{p}_1 , and let $(\bar{p}_3, \bar{b}_3, \bar{n}_3, \bar{s}_3)$ be the result of spawning a third ray from \bar{p}_2 . No rays are spawned from \bar{p}_3 .

Let the material properties of the surface at \bar{p}_i be given by r_{di} for the material's diffuse colour, r_{ai} for the material's ambient colour, r_{si} for the material's specular color, and $r_{\alpha i}$ for the material's specular exponent. Let $L(\bar{b}_i, \bar{n}_i, \bar{s}_i)$ be the value of the Phong lighting model when computed at point \bar{p}_i .

- a) [6 marks] Give the formula for the lighting model at point \bar{p}_1 , accounting for secondary (global) illumination, where none of \bar{p}_1 , \bar{p}_2 , or \bar{p}_3 are in shadow.

- b) [4 marks] What value is used in place of $L(\bar{b}_i, \bar{n}_i, \bar{s}_i)$ when the point \bar{p}_i is in shadow? Explain why.

- c) [2 marks] In a basic (Whitted) raytracing model on surfaces without refraction, why are rays spawned in the perfect specular reflection direction?

9. [12 marks] **Refractions**

Suppose a laser is sitting 10m above the surface of a lake and is pointing down into the lake at an angle of 45 degrees. If the lake is 100m deep, at what horizontal distance away from the laser's origin does the laser beam hit the lake's floor? Assume that the coefficient of air is 1 and the coefficient of water is $\frac{4}{3}$. Your answer can contain trigonometric function calls and does not need to be simplified further.

10. [23 marks] **Solid Angles and Radiometry**

- a) [4 marks] A point light at position \vec{p} is illuminating a point \vec{q} on a surface. Explain why the strength of the light is proportional to

$$1/\|\vec{p} - \vec{q}\|^2$$

i.e. one over the squared distance between the light and the point on a surface?

- b) [4 marks] In words, what is foreshortening, and how does it affect solid angle? How does it affect the amount of light that hits a surface?

- c) [5 marks] State the definition of irradiance. Be as specific as possible, and specify the quantity's units of measurement.

- d) [10 marks] For each of the following statements, indicate whether it is true or false. You will receive 2 marks for each correct answer and -2 for each incorrect one.
- a. The irradiance at A due to B depends on B's surface normal.
 - b. The irradiance at A due to B is equal to the radiance at B in the direction of A.
 - c. The irradiance at A due to B does not depend on the BRDF at B.
 - d. The irradiance at A due to B does not depend on the BRDF at A.
 - e. The radiance from A to B depends on the distance between A and B.

11. [8 marks] **Beziers**

- a) [4 marks] Give the equation for the Bezier curve $\bar{p}(t)$ defined by the control points \bar{p}_0 , \bar{p}_1 , \bar{p}_2 , and \bar{p}_3 .

- b) [4 marks] Derive the tangent to the curve for an arbitrary point t .

12. [17 marks] **Miscellaneous**

- [4 marks] What is importance sampling in distribution ray tracing? Explain how the values produced from a BRDF can be used to perform importance sampling?
- [2 marks] Why is it possible to compute the intersection of two lines in 2D using a cross product, but you cannot compute the intersection of two lines in 3D using a cross product?
- [4 marks] Given the eye position \vec{e} , a point a polygon \vec{p} , and the polygon's normal \vec{n} , give a test to determine if the polygon should not be displayed because of backface culling.

- d) [2 marks] Why cannot a BRDF be used to model the appearance of surfaces with subsurface scattering?
- e) [5 marks] When performing texture mapping, why can you not simply interpolate texture coordinates of a triangle in the 2D projected space?

END OF EXAM.

TOTAL PAGES = 16

TOTAL MARKS = 165