UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2012 EXAMINATIONS
CSC418H1F: Computer Graphics

Duration: 3 hours

No aids allowed

There are 17 pages total (including this page)

Given name(s): _____

Family Name: _____

Student number: _____

Question	Marks
1	_____/18
2	_____/10
3	_____/10
4	_____/16
5	_____/22
6	_____/10
7	_____/16
8	_____/15
9	_____/12
10	_____/10
11	_____/20
12	_____/8
13	_____/8
Total	_____/175

1. [18 marks] **Parametric and Implicit Surfaces**

An ellipsoid in 3D can be defined by the points (x, y, z) satisfying:

$$f(x, y, z) = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 - 1 = 0$$

It can also be defined parametrically as:

$$\bar{p}(\theta, \phi) = (a\cos\theta\sin\phi,\ b\sin\theta\sin\phi,\ c\cos\phi)$$

a) [4 marks] To show that these expressions define the same surface, show that every surface point produced by the parametric form satisfies the implicit equation of the surface.

b) [4 marks] Derive an expression for the surface normal at a point $\bar{q} = (x, y, z)$, using the implicit form.

c) [4 marks] Derive two distinct tangent vectors for points on the surface as a function of (θ, ϕ), using the parametric form.

d) [6 marks] Show that the tangent planes derived from the parametric form and the implicit form are equivalent are equivalent, assuming $\bar{q} = \bar{p}(\theta, \phi)$.

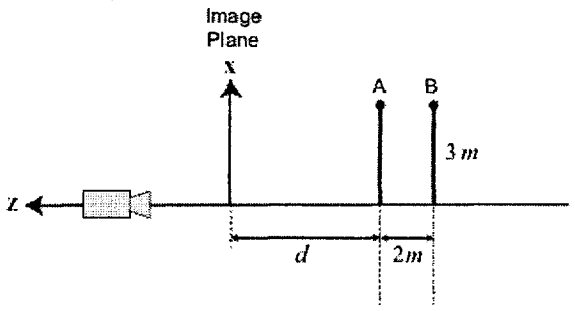
2. [10 marks] **Surfaces and Transformations**
Let $f(\bar{p}) = 0$ be the implicit equation of a surface. An affine transformation Q maps $\bar{p}$ to $Q(\bar{p}) = A\bar{p} + \vec{t}$, where A is an invertible 3 x 3 matrix and $\vec{t}$ is a translation vector.

a) [3 marks] Suppose we transform every point on the surface by Q. Give an implicit equation for this transformed surface.

b) [2 marks] Express the normal $\vec{n}$ to the original surface at point $\bar{p}$ in terms of f and $\bar{p}$.

c) [5 marks] Show that $((A^{-1})^{T})\vec{n}$ is normal to the transformed surface at point $Q(\bar{p})$.

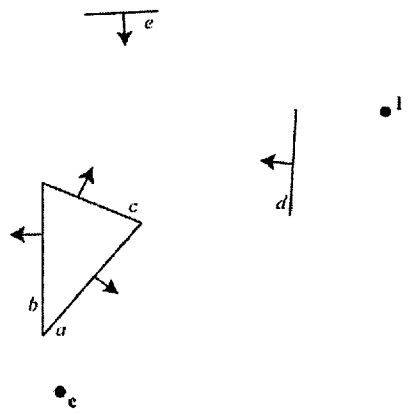
3. [10 marks] **Projections**
Consider a 2D scene consisting of two poles that are 3m high and 2m apart. Suppose that the scene is being viewed by a camera at a distance d away from the leftmost pole, as shown below:



a) [5 marks] Give the projected heights for each pole for a camera with focal length f.

b) [5 marks] How far should the camera be from Pole A so that the projected height of Pole A is within 5% of the projected height of Pole B?

4. [16 marks] **BSP Trees**
Consider the following 2D scene with a light source at point *l*, a camera at point *c*, and the outward normal of polygon segments as shown:



a) [8 marks] Draw the BSP tree for the scene by adding the segments in the labeled order, starting with a.

b) [8 marks] Suppose you want to compute, for every point on a segment, whether that point will be in shadow. Explain how to do this efficiently using the BSP tree in a).

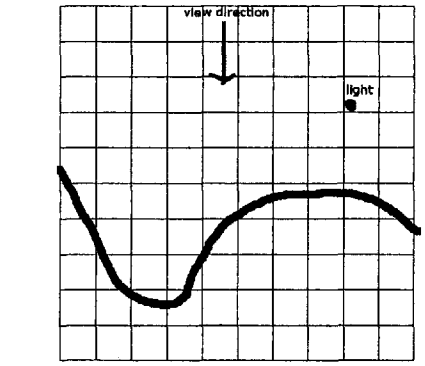
5. [22 marks] **Phong Lighting**

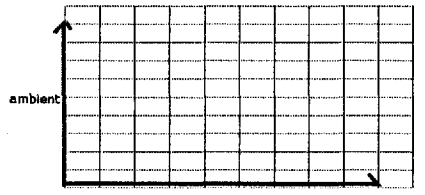
a) [12 marks] This question is about the Phong illumination model. For simplicity, assume a 1D surface in a 2D world, as shown below. The viewpoint is far above the scene looking straight down (as in orthographic projection) and the position of the light source is specified in the diagram. Sketch three graphs showing the ambient, diffuse, and specular components of the Phong model as applied to the scene. In a fourth graph, show the total intensity that would be computed (the graphs should show the intensity as a function of *x*). For your convenience, two sets of graphs are provided. Use the first for a draft and the second for your final answer, drawn as accurately as you can.

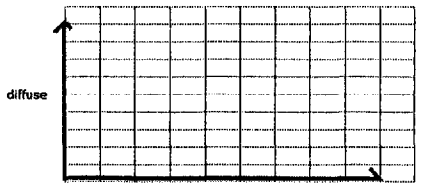
The parameters for this scene are:

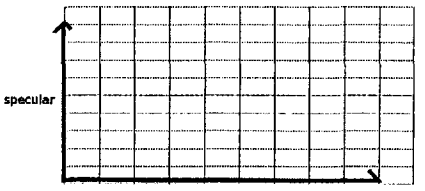
$I = r_a + r_d(\max(0, \vec{n} \cdot \vec{s})) + r_s(\max(0, \vec{v} \cdot \vec{r}))^n$

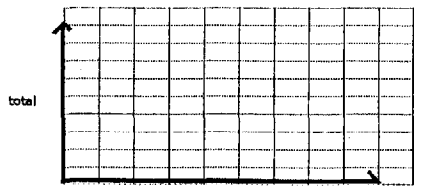
$r_a = 0.2, \quad r_d = 0.7, \quad r_s = 0.3, \quad n = 200$

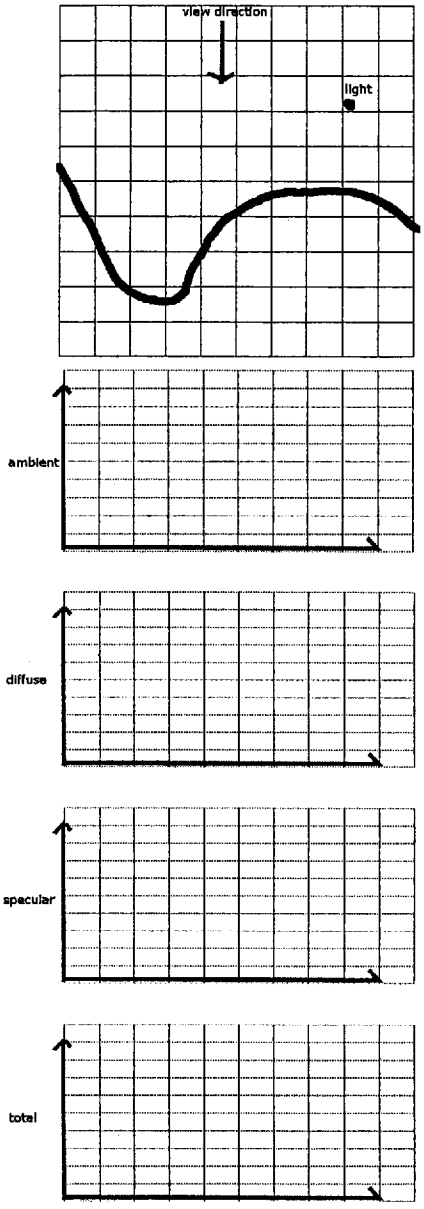











view direction
light
ambient
diffuse
specular
total

b) [10 marks] Suppose we replace the Phong reflection model with the following function the surface normal $\vec{n}$, incidence direction $\vec{\imath}$, and outgoing direction $\vec{o}$:

$$I(\vec{n}, \vec{\imath}, \vec{o}) = e^{-(\frac{10\theta}{\pi})^2}$$

where

$$\theta = \text{angle}(\vec{\imath}, \vec{o}) \qquad \text{(in radians)}$$

What appearance would function $I()$ simulate?

6. [10 marks] **Texture Mapping**

   a) [4 marks] Let $\bar{q}(\alpha, \beta)$ be a triangle with vertices p0, p1, and p2. Give a parametric equation for $q(\alpha, \beta)$ in terms of the vertices and the parameters (α, β), and specify bounds on α and β.

   b) [6 marks] Explain what perspective-correct texture mapping is and why it is necessary.

7. [16 marks] **Ray-Surface Intersections**

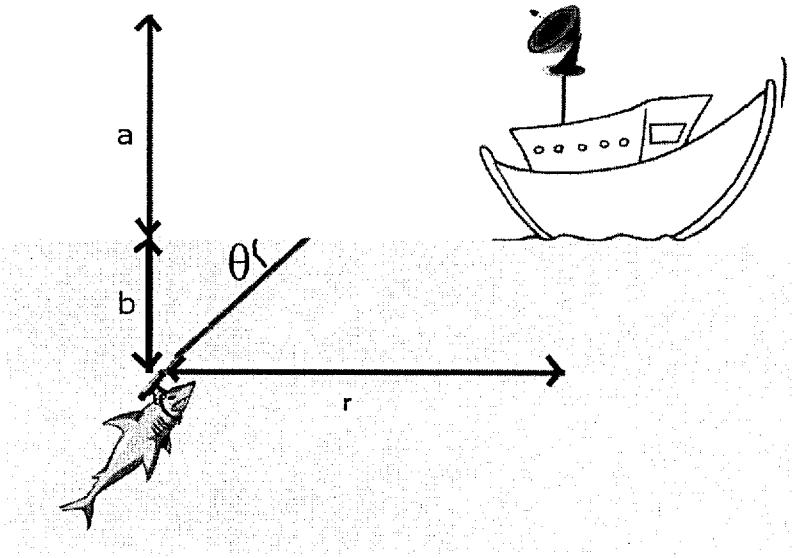
   Consider a hemispherical object: i.e. a sphere of unit radius that is cut by a plane through the center of the sphere (like half an orange). The surface of the object comprises two parts, the quadratic hemisphere and the planar base.

   a) [6 marks] Express this object mathematically in implicit form using constraints.

b) [10 marks] Using this mathematical model, find the intersections (if any) of the object surface and a ray $\vec{r}(u) = \vec{p} + u\vec{d}$, assuming that the ray originates outside the object.

8. [15 marks] **Ray Directions**

Suppose you are a shark with a laser beam mounted on your head, staying stationary *b* metres below the water. You are pointing the laser up to hit the center of tiny radar dish mounted on top of a boat. The center of the radar dish is *a* metres above the surface of the water and *r* metres to the right of you. At what angle θ should you aim the laser to hit the radar dish?



9. [12 marks] **Ray Tracing**

a) [3 marks] Basic (Whitted) ray tracing extends the local illumination model with another term. What illumination effect does it capture?

b) [4 marks] Describe two effects that cannot be rendered with Whitted ray tracing and briefly explain why.

c) [5 marks] What is sub-surface scattering? Explain briefly why it cannot be represented as a BRDF.

10. [10 marks] **Solid Angles**

a) [2 marks] In words, why is solid angle important for determining how much light hits an object from a point light source?

b) [4 marks] In words, what is foreshortening, and how does it affect solid angle? How does it affect the amount of light that hits a surface?

c) [4 marks] Give the equation for the radiance reflected from a surface point $\bar{p}$ with normal $\vec{n}$, in terms of the BRDF $\rho(\overrightarrow{d_e}, \overrightarrow{d_i})$ and incoming radiance $L(\overrightarrow{d_i}, \bar{p})$. You may parameterize directions in terms of spherical coordinates, e.g. $\vec{d}(\theta, \phi)$.

11. [20 marks] **Radiometry**

a) [10 marks] Given an expression for the irradiance at a surface point $\vec{p}$ with normal $\vec{n}$ due to an anisotropic point light source located at point $\vec{l}$.

b) [10 marks] Give an expression for the irradiance at surface point $\vec{p}$ with normal $\vec{n}$ due to an area light source defined by a polygon P.

12. [8 marks] **Beziers**

a) [4 marks] Give the equation for the Bezier curve $\bar{p}(t)$ defined by the control points $\overline{p_0}$, $\overline{p_1}$, $\overline{p_2}$, and $\overline{p_3}$.

b) [4 marks] Derive the tangent to the curve for an arbitrary point *t*.

13. [8 marks] **Cubic Curves**

Derive a cubic polynomial x(t) that satisfies the following constraints:

- $x(0) = 1$
- $x'(0) = 1$
- $x''(0) = 2$
- $x(1) = 5$

END OF EXAM.

TOTAL PAGES = 17

TOTAL MARKS = 175