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Problem Set 1

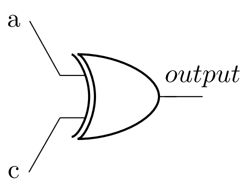
1. if x then y else $z \iff (\neg x \lor y) \land (x \lor z)$
   if y then $z \lor x$ else $z > x \iff (\neg y \lor z \lor x) \land (y \lor (z \geq x))$
   if z then $x \leq y$ else $x \land y \iff (\neg z \lor (x \leq y) \land (z \lor (x \land y)))$
   Proof by truth table:

x	y	z	if x then y else z	if y then $z \lor x$ else $z > x$	if z then $x \leq y$ else $x \land y$
F	F	F	F	F	F
F	F	T	T	T	T
F	T	F	F	F	F
F	T	T	T	T	T
T	F	F	F	F	F
T	F	T	F	F	F
T	T	F	T	T	T
T	T	T	T	T	T

   Clearly, those three expressions are equivalent.

   $\square$

2. First from the circuit we get: $(a \land \neg((b \land c) \lor (\neg b \land c))\ ) \lor \neg((a \land c) \lor (\neg a \land \neg c)) \lor (\neg a \land c)$
   $(b \land c) \lor (\neg b \land c) \iff c$, then we have $(a \land \neg c\ ) \lor \neg((a \land c) \lor (\neg a \land \neg c)) \lor (\neg a \land c)$
   $(a \land c) \lor (\neg a \land \neg c) \iff a = c$, then we have $(a \land \neg c\ ) \lor \neg(a = c) \lor (\neg a \land c)$
   $(a \land \neg c\ ) \lor (\neg a \land c) \iff a \oplus c$, also $\neg(a = c) \iff a \oplus c$; hence, we have $(a \oplus c) \lor (a \oplus c) \iff a \oplus c$



   $\square$

3. First we know set $\{\neg, \lor\}$ is complete, so in order to prove $\{\triangle\}$ we can use $\triangle$ the express $\neg$ and $\lor$.

   (a) $\neg a$, $\neg a \iff a \triangle a$, clearly if a is true the $a \triangle a$ is false and true when a is false.

   (b) $a \lor b$, we know that $a \lor b \iff \neg(\neg a \land \neg b) \iff (\neg a \triangle \neg b)$. Then, we have $a \lor b \iff ((a \triangle a) \triangle (b \triangle b))$.

(c) Hence, $\{\triangle\}$ is complete.

$\square$

4. Suppose that $\{\neq\}$ is complete. Then we know that using $\neq$ and two variables a, b we could generate all 16 numbers from 0 to 15 in binary, which in other words stands for all possible combination of F, T of length 4.
First, from $a \neq a$ we always get false.
Now, suppose we have two variables a and b and we can get a new combination $c : a \neq b$

a	b	$a \neq b$
0	0	0
0	1	1
1	0	1
1	1	0

now from $a \neq c$ we have:

a	c	$a \neq c$
0	0	0
0	1	1
1	1	0
1	0	1

clearly, that we can see b has the same sequence as $a \neq c$. Also, from $c \neq b$, we have:

b	c	$b \neq c$
0	0	0
1	1	0
0	1	1
1	0	1

Hence, a has the same sequence as $b \neq c$. Also, from $(a \neq F)$, $(b \neq F)$ and $(c \neq F)$, we have:

a	b	c	F	$a \neq F$	$b \neq F$	$c \neq F$
0	0	0	0	0	0	0
0	1	1	0	0	1	1

1	0	1	0	1	0	1
1	1	0	0	1	1	0

Clearly, no new string has been generated.

We now run out of strings and we only have 4 combinations $\{F, a, b, c\}$. Since, $\neq$ cannot generate all possible values between 0 and 15 in binary; therefore, it's incomplete.

$\square$

5. First we give the truth table

x_3	x_2	x_1	x_0	x	y	z	y_3	y_2	y_1	y_0	z_1	z_0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	1	0	1
0	0	1	0	2	2	1	0	0	1	0	0	1
0	0	1	1	3	3	1	0	0	1	1	0	1
0	1	0	0	4	2	2	0	0	1	0	1	0
0	1	0	1	5	5	1	0	1	0	1	0	1
0	1	1	0	6	3	2	0	0	1	1	1	0
0	1	1	1	7	7	1	0	1	1	1	0	1
1	0	0	0	8	4	2	0	1	0	0	1	0
1	0	0	1	9	3	3	0	0	1	1	1	1
1	0	1	0	10	5	2	0	1	0	1	1	0
1	0	1	1	11	11	1	1	0	1	1	0	1
1	1	0	0	12	4	3	0	1	0	0	1	1
1	1	0	1	13	13	1	1	1	0	1	0	1
1	1	1	0	14	7	2	0	1	1	1	1	0
1	1	1	1	15	5	3	0	1	0	1	1	1

Notice, from the truth table we could get,

$y_3 = x_3 \wedge x_0 \wedge (x_2 \neq x_1)$

$y_2 = (x_3 \wedge \bar{x}_0) \vee (x_2 \wedge x_0)$

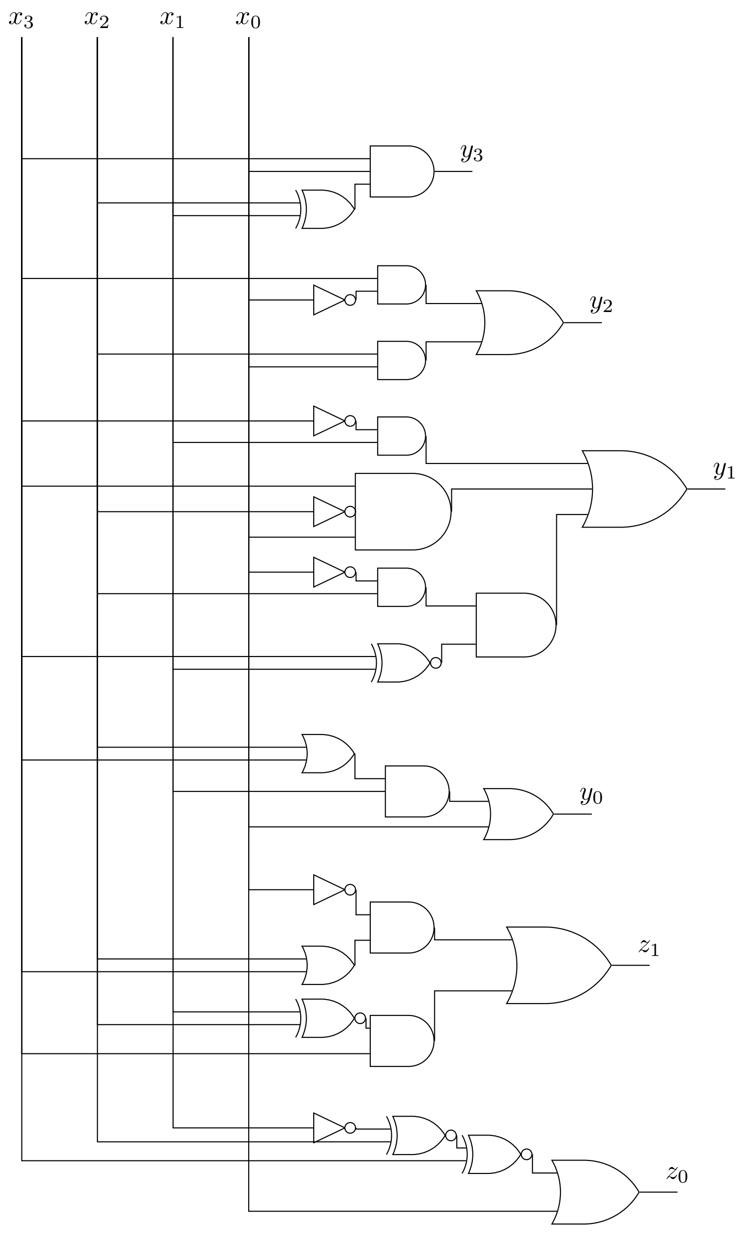
$y_1 = (\bar{x}_3 \wedge x_1) \vee (x_3 \wedge \bar{x}_2 \wedge x_0) \vee (\bar{x}_0 \wedge x_2 \wedge (x_3 = x_1))$

$$\begin{aligned} y_0 &= \neg((\bar{x}_1 \wedge x_0) \vee (\bar{x}_3 \wedge \bar{x}_2 \wedge x_1 \wedge \bar{x}_0)) \\ &= (x_1 \vee x_0) \wedge (x_3 \vee x_2 \vee \bar{x}_1 \vee x_0) \\ &= ((x_3 \vee x_2) \wedge x_1) \vee x_0 \end{aligned}$$

$z_1 = (\bar{x}_0 \wedge (x_2 \vee x_3)) \vee (x_3 \wedge (x_2 = x_1))$

$z_0 = x_0 \vee \bar{x}_0((\bar{x}_3 \wedge \bar{x}_2 \wedge x_1) \vee (x_3 \wedge x_2 \wedge \bar{x}_1))$

$$= ((x_3 = x_2 = \bar{x}_1) \wedge x_0) \vee x_0$$
$$= (x_3 = x_2 = \bar{x}_1) \vee x_0)$$



$\square$