

UNIVERSITY OF TORONTO  
Faculty of Arts and Science

DECEMBER 2014 EXAMINATIONS  
CSC418H1F: Computer Graphics

Duration: 3 hours

No aids allowed

There are 17 pages total (including this page)

Given name(s): \_\_\_\_\_

Family Name: \_\_\_\_\_

Student number: \_\_\_\_\_

Question	Marks
1	_____/21
2	_____/21
3	_____/10
4	_____/6
5	_____/10
6	_____/16
7	_____/17
8	_____/15
9	_____/13
10	_____/10
11	_____/20
12	_____/8
13	_____/8
Total	_____/175

1. [21 marks] **Implicit Surfaces and Curves on Surfaces**

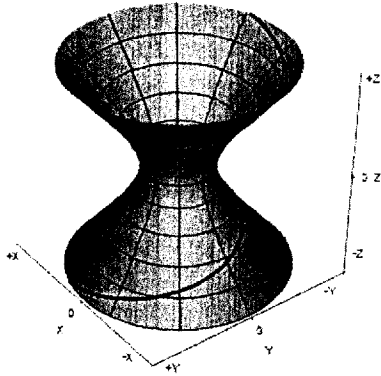
a) [2 marks] Give the implicit equation in the form  $f(x, y) = 0$  for a 2D circle with radius  $r$ , centered at the origin.

b) [2 marks] Now consider the same implicit equation from part a) in 3D instead of 2D. What does it represent in 3D?

c) [4 marks] Consider an infinite set of circles in 3D, where each circle lies in a plane parallel to the  $x$ - $y$  plane. In addition, each circle's radius changes according to the  $z$  value of the plane it lies in, and the radius is given by  $\sqrt{z^2 + 1}$ . Give an implicit equation for this surface in the form  $g(x, y, z) = 0$ .

d) [2 marks] Given a point on the surface in part c), give a formula for the normal vector at that point. You do not have to convert the normal vector into a unit vector.

- e) [5 marks] Give the parametric equation of a 3D curve  $h(t)$  that spirals around this surface as illustrated below. Assume that the points  $(1, 0, 0)$  and  $(\sqrt{4\pi^2 + 1}, 0, 2\pi)$  lie on the curve. Let  $-\infty < t < \infty$ .



- f) [3 marks] Compute the tangent vector of the curve from part e). You do not have to compute a unit vector.
- g) [3 marks] Show that the tangent vector from part f) is perpendicular to the normal from part d).

2. [21 marks] **Parametric Curves and Surfaces**

Let  $f(a)$  be a 3D parametric curve defined as

$$f(a) = (\sin(6\pi a), 0, a) \text{ where } 0 \leq a \leq 1$$

- a) [3 marks] Let  $\vec{t}$  be the unit tangent vector of  $f(a)$  at an arbitrary value of  $a$ . Compute  $\vec{t}$ .

- b) [2 marks] For the subsequent parts of this question, let the x, y, and z components of  $f'(a) / \|f'(a)\|$  be named  $\alpha$ ,  $\gamma$ , and  $\beta$ .

Let  $\vec{u}$  be the vector  $(0, 1, 0)$  and let it represent the “up” direction. Compute the cross product of  $\vec{t}$  with  $\vec{u}$  and name the result  $\vec{v}$ .

- c) [3 marks] Form a coordinate frame using  $\vec{t}$ ,  $\vec{u}$ , and  $\vec{v}$  as the basis vectors and  $f(a)$  as the location of the origin, and call the resulting coordinate space “curve space”. Give the transformation matrix M that transforms coordinates from curve space into world coordinate space.
- d) [1 marks] Note that  $\vec{t}$ ,  $\vec{u}$ , and  $\vec{v}$  are all unit vectors. Show that  $\vec{t}$ ,  $\vec{u}$ , and  $\vec{v}$  are orthogonal to one another.
- e) [4 marks] Form the 3x3 matrix  $A = [\vec{t} \ \vec{u} \ \vec{v}]$  by choosing  $\vec{t}$ ,  $\vec{u}$ , and  $\vec{v}$  to be the columns of the matrix. Show that if  $\vec{t}$ ,  $\vec{u}$ , and  $\vec{v}$  are unit vectors that are orthogonal to one another, the inverse A is the transpose of A:  $A^T = \begin{bmatrix} \vec{t}^T \\ \vec{u}^T \\ \vec{v}^T \end{bmatrix}$ . Compute  $A^{-1}A$ .

- f) [3 marks] Give the transformation matrix  $M^{-1}$  that transforms from world space coordinates into curve space coordinates. The construction of this matrix will be similar to constructing the world to camera transform out of the camera to world transform.

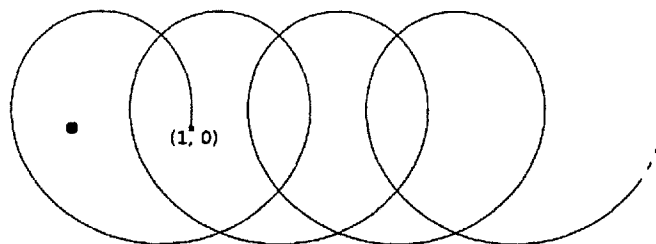
- g) [5 marks] Suppose an ellipse curve in 3D is defined parametrically by

$$g(b) = (\cos(2\pi b), 2 \sin(2\pi b), 0) \text{ where } 0 \leq b \leq 1$$

Give the parametric surface  $h(a, b)$ , where  $0 \leq a, b \leq 1$ , formed by sweeping the curve  $g(b)$  along the curve  $f(a)$ . Do this by transforming the world space coordinates of  $g(b)$  into the curve space coordinates of  $f(a)$  for arbitrary values of  $a$  and  $b$ .

3. [10 marks] **Parametric Curves and Projections**

- a) [3 marks] Give a parametric equation  $f(t)$ , for  $t \geq 0$ , that generates the following 2D curve. Note that the origin is indicated by a dot and the curve starts at  $(1, 0)$ . Allow the curve to continue infinitely to the right.

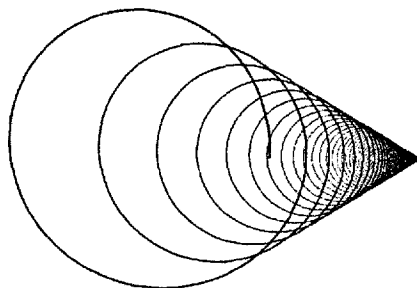


- b) [1 mark] Turn this curve into the 3D curve  $g(t)$  that has the same  $x$  and  $y$  values but whose  $z$  value is given by  $z(t) = -t$ . Give the formula for  $g(t)$ .

- c) [4 marks] Now consider the perspective projection matrix with focal length of 2

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/2 & 0 \end{bmatrix}$$

Give the 2D parametric curve  $h(t)$  that corresponds to the projection of  $g(t)$  onto the image plane.



- d) [2 marks] As  $t$  increases to infinity, what 2D point does  $h(t)$  convert to on the image plane?

4. [6 marks] **3D Transformations**

- a) [3 marks] Give the transformation matrix for a rotation in 3D about the x-axis by an arbitrary angle  $\theta$ .
- b) [3 marks] Give the transformation matrix for a translation in 3D by an arbitrary vector  $(t_x, t_y, t_z)$ .

5. [10 marks] **Transforming Normals**

Let  $f(\vec{p}) = 0$  be the implicit equation of a surface. An affine transformation  $Q$  maps  $\vec{p}$  to  $Q(\vec{p}) = A\vec{p} + \vec{t}$ , where  $A$  is an invertible  $3 \times 3$  matrix and  $\vec{t}$  is a translation vector.

- a) [3 marks] Suppose we transform every point on the surface by  $Q$ . Give an implicit equation for this transformed surface.

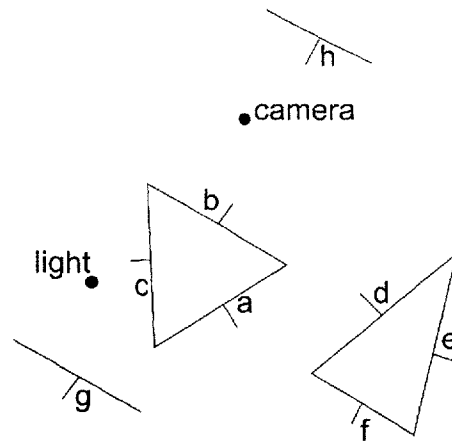
b) [2 marks] Express the normal  $\vec{n}$  to the original surface at point  $\vec{p}$  in terms of  $f$  and  $\vec{p}$ .

c) [5 marks] Show that  $((A^{-1})^T)\vec{n}$  is normal to the transformed surface at point  $Q(\vec{p})$ .



6. [16 marks] **BSP Trees**

Consider the following 2D scene with a light source, camera at point, and the outward normal of polygon segments as shown:



- a) [8 marks] Draw the BSP tree for the scene by adding the segments in the labeled order, starting with *a*.

- b) [8 marks] Suppose you want to compute, for every point on a segment, whether that point will be in shadow. Explain how to do this efficiently using the BSP tree in a).

7. [17 marks] **Phong Lighting**

a) [6 marks] For each of the three main terms in the Phong lighting model, briefly describe what properties of visual appearance they generate.

b) [8 marks] Using the Phong lighting model, give the mathematical expression for the reflectance toward a camera eye (center of projection) at location  $\vec{e}$  from a surface point  $\vec{p}$  with unit normal  $\vec{n}$ , given a point light source at location  $\vec{l}$ . Define any other variables needed by the model, and express all directions in terms of  $\vec{p}$ ,  $\vec{n}$ ,  $\vec{e}$ , and  $\vec{l}$ . Call the function  $L(\vec{p}, \vec{n}, \vec{e}, \vec{l})$ .

c) [3 marks] Explain the difference between Phong shading and Gouraud shading.

8. [15 marks] **Raytracing**

- a) [3 marks] Basic (Whitted) ray tracing adds an extra term to the Phong lighting model. What is it, and what effect does it capture?

- a) [6 marks] Describe three effects other than motion blur that cannot be rendered with Whitted ray tracing and briefly explain why.

- b) [3 marks] Briefly describe what causes motion blur and how you can implement it in a distributed raytracer.

- c) [3 marks] Describe how you could simulate motion blur using an OpenGL-like renderer (not a raytracer). The system does not need to run in real-time.

9. [13 marks] **Miscellaneous**

- a) [2 marks] How would you compute a normal vector for a vertex on a 3D triangulated polygonal mesh, assuming that the mesh is intended to approximate a smooth surface?
  
  
  
  
  
  
  
  
  
  
- b) [3 marks] Explain what perspective-correct texture mapping is and why it is necessary.
  
  
  
  
  
  
  
  
  
  
- c) [3 marks] The graphics pipeline performs clipping against the view volume in homogenous coordinates. Explain how this can be done without explicitly converting to Euclidean coordinates.

d) [2 marks] Describe the difference between forward kinematics and inverse kinematics.

e) [3 marks] When doing animation, why is it preferable to use an interpolation scheme that exhibits local control over one that exhibits global control?

10. [10 marks] **Solid Angles**

a) [2 marks] In words, why is solid angle important for determining how much light hits an object from a point light source?

b) [4 marks] In words, what is foreshortening, and how does it affect solid angle? How does it affect the amount of light that hits a surface?

- c) [4 marks] Give the equation for the radiance reflected from a surface point  $\bar{p}$  with normal  $\vec{n}$ , in terms of the BRDF  $\rho(\vec{d}_e, \vec{d}_i)$  and incoming radiance  $L(\vec{d}_i, \bar{p})$ . You may parameterize directions in terms of spherical coordinates, e.g.  $\vec{d}(\theta, \phi)$ .

11. [20 marks] **Radiometry**

- a) [10 marks] Given an expression for the irradiance at a surface point  $\vec{p}$  with normal  $\vec{n}$  due to an anisotropic point light source located at point  $\vec{l}$ .

- b) [10 marks] Give an expression for the irradiance at surface point  $\vec{p}$  with normal  $\vec{n}$  due to an area light source defined by a polygon  $P$ .

12. [8 marks] **Beziers**

- a) [4 marks] Give the equation for the Bezier curve  $\vec{p}(t)$  defined by the control points  $\vec{p}_0$ ,  $\vec{p}_1$ ,  $\vec{p}_2$ , and  $\vec{p}_3$ .

- b) [4 marks] Derive the tangent to the curve for an arbitrary point  $t$ .



**13. [8 marks] Cubic Curves**

Derive a cubic polynomial  $x(t)$  that satisfies the following constraints:

- $x(1) = 1$
- $x'(0) = -3$
- $x'(1) = 4$
- $x''(0) = 4$

**END OF EXAM.**

**TOTAL PAGES = 17**

**TOTAL MARKS = 175**