UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL 2013 EXAMINATIONS
CSC418H1S: Computer Graphics

Duration: 3 hours

No aids allowed

There are 16 pages total (including this page)

Given name(s): ______________________

Family Name: ______________________

Student number: ______________________

Question	Marks	
1	________	/22
2	________	/ 5
3	________	/14
4	________	/ 8
5	________	/10
6	________	/18
7	________	/31
8	________	/11
9	________	/23
10	________	/ 8
11	________	/10
Total	________	/160

1. [22 marks] **Implicit and Parametric Surface Representations**
   a) [5 marks] Give an implicit function of the form $\overline{f_1}(x, y, z) = 0$ for a cone whose apex (the tip of the cone) is at the point *(0, 0, 3)* and whose intersection with the *xy*-plane is the unit circle.

   b) [4 marks] Using the implicit function from a), derive an equation for the normal vector at the point *(x, y, z)*.

   c) [5 marks] Derive a parametric function of the form $\overline{f_2}(u, v)$ that describes the same surface as in part a). The curve $\overline{f_2}(u, 0)$ for $u \in [0, 1]$ should be for a unit circle in the *xy*-plane and $\overline{f_2}(u, 1)$ for $u \in [0, 1]$ should coincide with the apex of the cone.

d) [4 marks] Using the parametric function from c), derive equations for the tangent vectors at an arbitrary *(u, v)*.

e) [4 marks] Given a ray $\bar{r}(t) = \bar{p} + t\vec{d}$ for $t >= 0$, show how to compute its intersection(s) with the cone. It is sufficient to set up the equations, but you don't need to solve them.

2. [5 marks] **Parametric Curves**

   What types of 2D curves do the following parametric functions produce? Explain the difference between the two.

$$\bar{f}_1(u) = (\sin u\,, \cos u), \qquad 0 \le u \le 2\pi$$
$$\bar{f}_2(u) = \left(\sin \sqrt{u}\,, \cos \sqrt{u}\right), \qquad 0 \le u \le 4\pi^2$$

3. [14 marks] **Coordinate Systems**

   Suppose you are working with three coordinate systems: object, world, and camera space. The basic vectors of object space have world coordinates (1, 0, 0), (0, 0, -1), and (0, 1, 0) and the origin of object space has world coordinates (0, 0, 10). Camera space has basis vectors of $\left({}^{1}/_{\sqrt{3}},\ {}^{1}/_{\sqrt{3}},\ {}^{1}/_{\sqrt{3}}\right)$, $\left({}^{-1}/_{\sqrt{2}},\ 0,\ {}^{1}/_{\sqrt{2}}\right)$, and $\left({}^{1}/_{\sqrt{6}},\ {}^{-2}/_{\sqrt{6}},\ {}^{1}/_{\sqrt{6}}\right)$, and an origin of (5, 9, -4), also in world coordinates.

   a) [3 marks] Give the 4x4 matrix that transforms object to world coordinates.

b) [3 marks] Give the 4x4 matrix that transforms camera to world coordinates.

c) [4 marks] Give the 4x4 matrix that transforms world to camera coordinates.

d) [2 marks] Give the 4x4 matrix product that transforms object to camera coordinates. You do not need to multiply out the matrix product.

e) [2 marks] What are the camera coordinates of the object space's origin?

4. [8 marks] **Transformations**

a) [4 marks] Compute the transformation matrix for a rotation by $\pi/2$ radians about the x-axis followed by a rotation of $\pi/2$ radians about the y-axis. Leave the result as a product of two matrices.

b) [1 mark] Compute the transformation matrix for a rotation by $\pi/2$ radians about the y-axis followed by a rotation of $\pi/2$ radians about the x-axis. Leave the result as a product of two matrices.

c) [3 marks] Apply the rotations from a) to the point (1, 1, 1) and apply the rotations from b) to the point (1, 1, 1), to demonstrate that rotations are not commutative.

5. [10 marks] **Projections**

Consider two 3D lines passing through points

$$\overline{p_0} = \begin{bmatrix}1\\0\\2\end{bmatrix}, \qquad \overline{p_1} = \begin{bmatrix}0\\1\\2\end{bmatrix}$$

and

$$\overline{p_2} = \overline{p_0} + \begin{bmatrix}0\\0\\2\end{bmatrix}, \qquad \overline{p_3} = \overline{p_1} + \begin{bmatrix}0\\0\\2\end{bmatrix}$$

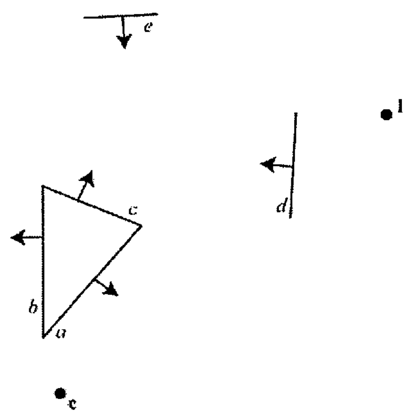
respectively. All points are expressed in camera space. Suppose the camera's project matrix is

$$M = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0.5 & 0\end{bmatrix}$$

Where do the lines' projections intersect on the image plane?

6. [18 marks] **BSP Trees**

   Consider the following 2D scene with a light source at point *l*, a camera at point *c*, and the outward normal of polygon segments as shown:



   a) [7 marks] Draw the BSP tree for the scene by adding the segments in the labeled order, starting with a.

   b) [5 marks] Suppose you want to compute, for every point on a segment, whether that point will be in shadow. Explain how to do this efficiently using the BSP tree in a).

c) [6 marks] Now suppose that segment *d* is a perfect mirror. Since segments may now be visible indirectly via their reflection in the mirror, your visibility computations must take mirror reflection into account. Would this change the BSP tree computed in a)? If your answer is yes, show the new BSP tree, otherwise explain how you would use the existing tree.

7. [31 marks] **Phong Lighting**

The Phong lighting model can be described using the following equation:

$$L(\vec{b}, \vec{n}, \vec{s}) = r_a I_a + r_d I_d max(0, \vec{n} \cdot \vec{s}) + r_s I_s max(0, \vec{r} \cdot \vec{b})^{\propto}$$

a) [3 marks] The right hand side of the equation is the sum of three components. Give the name of each component, in left-to-right order.

b) [4 marks] Draw a diagram illustrating the relationship of all the vectors, as well as the position of the eye and the light source.

c) [3 marks] Derive the formula for the angle of reflection, $\vec{r}$, given an incoming ray direction $\vec{v}$ and a surface normal $\vec{n}$.

d) [3 marks] Basic (Whitted) ray tracing adds another summand to the equation. What is it? What illumination effect does it capture?

e) [3 marks] Briefly explain how shadows are captured in basic ray tracing and how this may modify the equation above.

f) [5 marks] Does changing the $\propto$ parameter affect the total amount of reflected light, or just how the light is scattered? Explain.

g) [10 marks] Suppose we replace the third component in the equation with the following term that is a function of the surface normal $\vec{n}$, incidence direction $\vec{\imath}$, and outgoing direction $\vec{o}$:

$$e^{-\theta^2}$$

where

$$\theta = \text{angle}\left(\vec{n}, \frac{\vec{\imath}+\vec{o}}{2}\right) \qquad \text{(in radians)}$$

What appearance would this term simulate?

8. [11 marks] **Texture Mapping**

a) [6 marks] Give an intuitive explanation of how aliasing artifacts occur in texture mapping. You may use an example or draw a sketch. The sketch may show a one-dimensional function to illustrate the general problem. Describe the main idea of antialiasing techniques such as mipmapping.

b) [5 marks] How much extra memory is required, approximately, when using mipmaps for texture mapping? Assume that the image resolution of each mipmap level is half the width and half the height of the previous level. State any assumptions.

9. [23 marks] **Solid Angles and Radiometry**

a) [4 marks] A point light at position $\bar{p}$ is illuminating a point $\bar{q}$ on a surface. Explain why the strength of the light is proportional to

$$1/\|\bar{p}-\bar{q}\|^2$$

i.e. one over the squared distance between the light and the point on a surface?

b) [4 marks] In words, what is foreshortening, and how does it affect solid angle? How does it affect the amount of light that hits a surface?

c) [5 marks] State the definition of irradiance. Be as specific as possible, and specify the quantity's units of measurement.

d) [10 marks] For each of the following statements, indicate whether it is true or false. You will receive 2 marks for each correct answer and -2 for each incorrect one.

   i. The irradiance at A due to B depends on A's surface normal.

   ii. The irradiance at A due to B is equal to the radiance at B in the direction of A.

   iii. The irradiance at A due to B depends on the BRDF at B.

   iv. The irradiance at A due to B depends on the BRDF at A.

   v. The radiance from A to B depends on the distance between A and B.

10. [8 marks] **Beziers**

a) [4 marks] Give the equation for the Bezier curve $\bar{p}(t)$ defined by the control points $\overline{p_0}$, $\overline{p_1}$, $\overline{p_2}$, and $\overline{p_3}$.

b) [4 marks] Derive the tangent to the curve for an arbitrary point *t*.

11. [10 marks] **Catmull-Rom Splines**

Recall that Catmull-Rom splines are computed using the formula

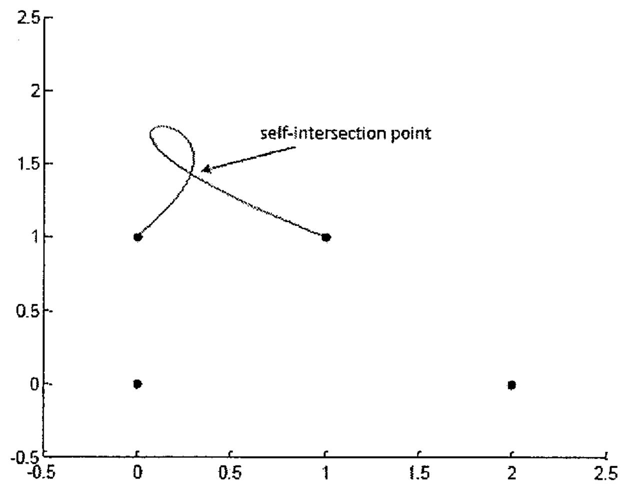
$$x(t) = \begin{bmatrix}1 & t & t^2 & t^3\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ -3 & 3 & -2 & -1\\ 2 & -2 & 1 & 1\end{bmatrix}\begin{bmatrix}x_j\\ x_{j+1}\\ \kappa(x_{j+1}-x_{j-1})\\ \kappa(x_{j+2}-x_j)\end{bmatrix}$$

$$= \begin{bmatrix}1 & t & t^2 & t^3\end{bmatrix}\begin{bmatrix}0 & 1 & 0 & 0\\ -\kappa & 0 & \kappa & 0\\ 2\kappa & \kappa-3 & 3-2\kappa & -\kappa\\ -\kappa & 2-\kappa & \kappa-2 & \kappa\end{bmatrix}\begin{bmatrix}x_{j-1}\\ x_j\\ x_{j+1}\\ x_{j+2}\end{bmatrix}$$

A similar formula applies to *y(t)*.

In the figure below, a segment of Catmull-Rom spline was computed using the parameters

$$\overline{p_1} = \begin{bmatrix}0\\0\end{bmatrix}, \overline{p_2} = \begin{bmatrix}0\\1\end{bmatrix}, \overline{p_3} = \begin{bmatrix}1\\1\end{bmatrix}, \overline{p_4} = \begin{bmatrix}2\\0\end{bmatrix}, \kappa = 3$$



As you can see, the spline is self-intersecting. Find the coordinates of the self-intersection point.
Hint: the following formulas could be useful

$$(a+b)(a-b) = a^2 - b^2$$
$$(a-b)(a^2+b^2+ab) = a^3 - b^3$$

extra space for the previous question)

END OF EXAM.

TOTAL PAGES = 16

TOTAL MARKS = 160