UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2013 EXAMINATIONS
CSC418H1F: Computer Graphics

Duration: 3 hours

No aids allowed

There are 15 pages total (including this page)

Given name(s): _____

Family Name: _____

Student number: _____

Question	Marks
1	_____/6
2	_____/22
3	_____/8
4	_____/10
5	_____/16
6	_____/20
7	_____/15
8	_____/12
9	_____/20
10	_____/8
11	_____/21
Total	_____/158

1. [6 marks] **Parametric Curves**

A comet is travelling towards Earth on a curve that can be expressed parametrically as follows:

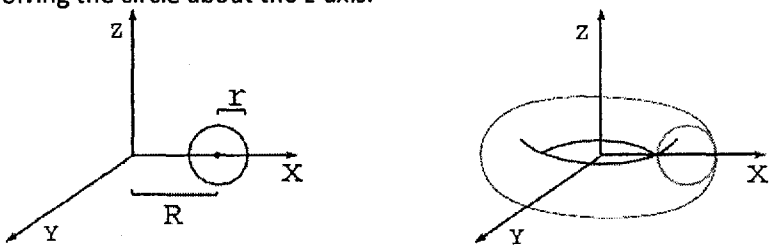
$$x(t) = t$$
$$y(t) = 4t^2 - 45$$

where $-\infty < t < \infty$ corresponds to the time.

Suppose the impact occurred at time *t=10*. What was the earth's position at the time of impact with the comet? With what velocity vector did the comet strike?

2. [22 marks] **Parametric and Implicit Surfaces**

As show in the figure below, a torus can be generated by first defining a circle in the xz-plane and then revolving the circle about the z-axis.



a) [8 marks] Derive a parametric equation for the torus, $p(\theta, \phi)$, that has major radius R and minor radius r, centered at the origin.

b) [4 marks] Derive two distinct tangent vectors for points on the surface as a function of (θ, ϕ), using the parametric form.

c) [4 marks] The same torus can be described with the following implicit equation:

$$f(x, y, z) = (x^2 + y^2 + z^2 + R^2 - r^2)^2 - 4R^2(x^2 + y^2) = 0$$

Derive an expression for the surface normal at a point p = (x, y, z), using the implicit form.

a) [6 marks] Show that the normal derived from the parametric form and the implicit form are equivalent are equivalent, assuming $\bar{q} = \bar{p}(\theta, \phi)$.

3. [8 marks] **Transformations** The transformations f and g are said to commute if and only if $f(g(p)) = g(f(p))$ for all p. For each of the cases below, prove whether or not f and g commute, where f and g are 3D affine transformations. You may prove non-commutativity by giving a counterexample.

   a) [4 marks] f and g are arbitrary rotations around the z-axis about the same point

   b) [4 marks] One is a rotation about the x-axis and one is a rotation about the y-axis

4. [10 marks] **Projections**

Let $p = (p_x, p_y, p_z)$ and $q = (q_x, q_y, q_z)$ be two endpoints of a line segment, defined in camera space coordinates. Let $m = (p + q)/2$ be the midpoint of a line segment. Using the perspective projection defined by the matrix

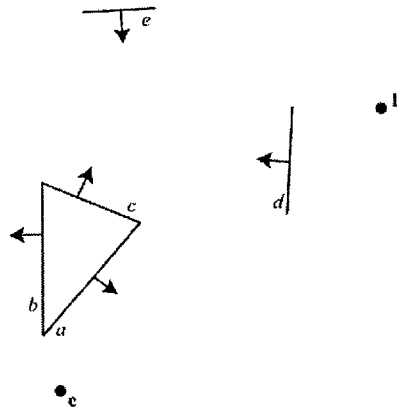
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & f & 0 \end{bmatrix}$$

with the camera direction facing the z-axis, let p', q', and m' be the perspective projections of p, q, and m, respectively.

a) [5 marks] Show whether or not $m' = (p' + q')/2$, in general.

b) [5 marks] If it is not true for all p and q, characterize the conditions under which it would be true.

5. [16 marks] **BSP Trees**
Consider the following 2D scene with a light source at point *l*, a camera at point *c*, and the outward normal of polygon segments as shown:



a) [8 marks] Draw the BSP tree for the scene by adding the segments in the labeled order, starting with a.

b) [8 marks] Suppose you want to compute, for every point on a segment, whether that point will be in shadow. Explain how to do this efficiently using the BSP tree in a).

6. [20 marks] **Lighting and Shading**

a) [2 marks] How would you compute a normal vector for a vertex of a 3D triangulated polygonal mesh, assuming that the mesh is intended to approximate a smooth surface?

b) [8 marks] Using the Phong lighting model, give the mathematical expression for the reflectance toward a camera eye (center of projection) at location $\bar{e}$ from a surface point $\bar{p}$ with unit normal $\vec{n}$, given a point light source at location $\bar{l}$. Define any other variables needed by the model, and express all directions in terms of $\bar{p}$, $\vec{n}$, $\bar{e}$, and $\bar{l}$. Call the function $L(\bar{p}, \vec{n}, \bar{e}, \bar{l})$.

c) [2 marks] Let $\overline{p_0}$, $\overline{p_1}$, and $\overline{p_2}$ be the vertices of a triangle in a triangular mesh and $\overrightarrow{n_0}$, $\overrightarrow{n_1}$, and $\overrightarrow{n_2}$ be the normal associated to $\overline{p_0}$, $\overline{p_1}$, and $\overline{p_2}$, respectively. For any point $\bar{p}$ and its associated normal $\vec{n}$, $L(\bar{p}, \vec{n}, \bar{e}, \bar{l})$ is the color at the point p calculated using the Phong illumination model. Let $\bar{q}(a, b) = (1 - a - b) + p_0 + ap_1 + bp_2$ be a point in the triangle.

What are the valid ranges of the values a, b, and $a+b$ for $\bar{q}$ to lie in the triangle?

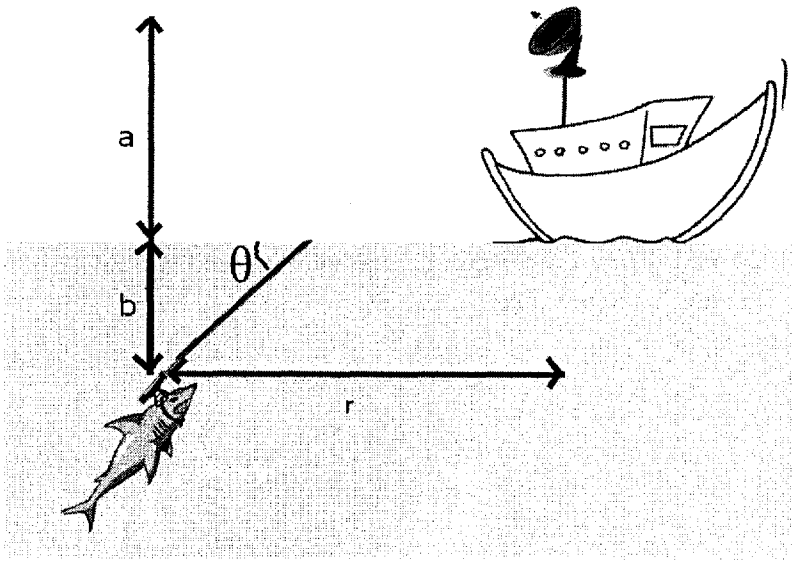
d) [2 marks] If Gouraud shading is applied, what is the colour at $\bar{q}$? (You may write this in terms of $L(\bar{p}, \vec{n}, \bar{e}, \bar{l})$.)

e) [2 marks] If Phong shading is applied, what is the colour at $\bar{q}$? (You may write this in terms of $L(\bar{p}, \vec{n}, \bar{e}, \bar{l})$.)

f) [4 marks] Explain briefly the difference between the two shading methods in terms of the accuracy and efficiency of shading.

7. [15 marks] **Ray Directions**

Suppose you are a shark with a laser beam mounted on your head, staying stationary *b* metres below the water. You are pointing the laser up to hit the center of tiny radar dish mounted on top of a boat. The center of the radar dish is *a* metres above the surface of the water and *r* metres to the right of you. At what angle θ should you aim the laser to hit the radar dish?



8. [12 marks] **Ray Tracing**

a) [3 marks] Basic (Whitted) ray tracing adds an extra term to the Phong lighting model. What is it, and what effect does it capture?

b) [5 marks] What is sub-surface scattering? Explain briefly why it cannot be represented as a BRDF.

c) [4 marks] In words, what is foreshortening, and how does it affect solid angle? How does it affect the amount of light that hits a surface?

9. [20 marks] **Radiometry**

a) [4 marks] Give the equation for the radiance reflected from a surface point $\bar{p}$ with normal $\vec{n}$, in terms of the BRDF $\rho(\overrightarrow{d_e}, \overrightarrow{d_i})$ and incoming radiance $L(\overrightarrow{d_i}, \bar{p})$. You may parameterize directions in terms of spherical coordinates, e.g. $\vec{d}(\theta, \phi)$.

b) [8 marks] Given an expression for the irradiance at a surface point $\bar{p}$ with normal $\vec{n}$ due to an anisotropic point light source located at point $\bar{l}$.

c) [8 marks] Give an expression for the irradiance at surface point $\bar{p}$ with normal $\vec{n}$ due to an area light source defined by a polygon *P*.

10. [8 marks] **Cubic Curves**

Derive a cubic polynomial x(t) that satisfies the following constraints:

- $x(1) = \frac{1}{2}$
- $x'(0) = 2$
- $x'(1) = 4$
- $x''(1) = 11$

11. [21 marks] **Miscellaneous**

a) [6 marks] Briefly describe what causes motion blur. Using words and/or pseudocode, explain how you could simulate motion blur using an OpenGL-like render (not a raytracer). The system does not need to run in real-time.

b) [6 marks] Explain what perspective-correct texture mapping is and why it is necessary.

c) [5 marks] How much extra memory is required, approximately, when using mipmaps for texture mapping? Assume that the image resolution of each mipmap level is half the width and height of the previous one.

d) [2 marks] Give two advantages of key-frame animation over physics-based animation.

e) [2 marks] Give two advantages of physics-based animation over key-frame animation.

END OF EXAM.

TOTAL PAGES = 15

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