UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL 2015 EXAMINATIONS
CSC418H1S: Computer Graphics

Duration: 3 hours

No aids allowed

There are 15 pages total (including this page)

Given name(s): _____

Family Name: _____

Student number: _____

Question	Marks
1	_____/21
2	_____/15
3	_____/12
4	_____/16
5	_____/20
6	_____/11
7	_____/16
8	_____/12
9	_____/20
10	_____/8
11	_____/4
Total	_____/155

1. [21 marks] **Curves and Surfaces**

   a) [4 marks] Derive the formula for a 3D parametric curve describing a spiral of radius 1 that circles the z-axis. Let the spiral begin at $(1,0,0)$, twist counter-clockwise about the z-axis, make one full revolution about the axis at $(1,0,2\pi)$, and end at $(1,0,6\pi)$.

   b) [1 mark] Adjust the curve so that the spiral can have an arbitrary radius r.

   c) [3 marks] Compute the tangent to the curve in part *b)*.

   d) [2 marks] Turn the curve into a surface by making it parameters of both t and r, for $r \in [0, 1]$. This surface is named a helicoid.

e) [4 marks] Come up with an implicit equation to describe the same helicoid surface. Hint: Consider the ratio of the x and y coordinates of the parametric surface.

f) [4 marks] Use the implicit form of the surface to compute the normal at any point on the surface. Hint: $\frac{d}{dx}\tan(x) = {}^{1}/_{\cos^2(x)}$

g) [3 marks] Show that the normal from part *f)* is perpendicular to the tangent from part *c)*.

2. [15 marks] **Transformation Chain for 3D Viewing**
A camera is located in world space at *(1,1,0)* and faces the direction *(-1,0,0)*, with an up vector of *(0,1,0)*. A triangle with object space coordinates *(0,0,0)*, *(0,1,0)*, and *(0,0,1)* is first uniformly scaled by 0.5 and then translated in *x* by -1. This triangle is then projected into image coordinates by the projection matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

a) [12 marks] Give the series of transformations, as a matrix product, that will transform each vertex of the triangle from homogeneous coordinates in object space into canonical view space.

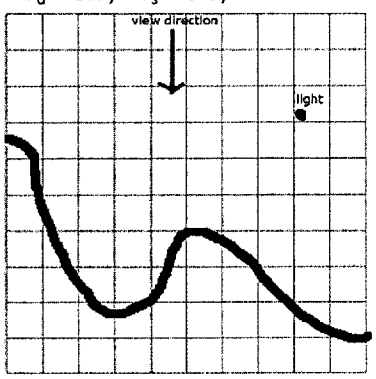
b) [3 marks] What are the image space coordinates and pseudo-depths of the triangle vertices?

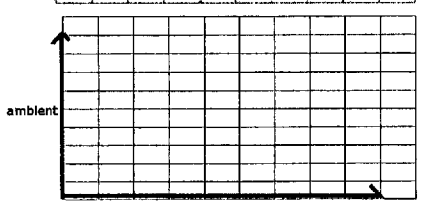
3. [12 marks] **Phong Lighting**

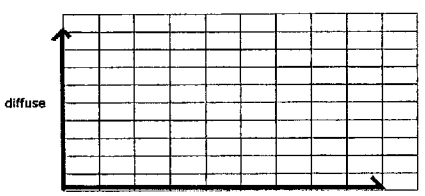
This question is about a Phong illumination model that includes shadows. For simplicity, assume a 1D surface in a 2D world, as shown below. The viewpoint is far above the scene looking straight down (as in orthographic projection) and the position of the light source is specified in the diagram. Sketch three graphs showing the ambient, diffuse, and specular components of the Phong model as applied to the scene, and properly account for shadows. In a fourth graph, show the total intensity that would be computed (the graphs should show the intensity as a function of *x*). For your convenience, two sets of graphs are provided. Use the first for a draft and the second for your final answer, drawn as accurately as you can. The parameters for this scene are:

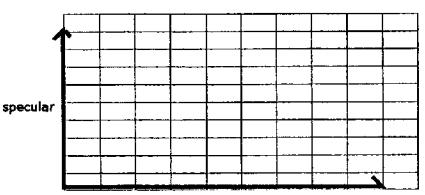
$I = r_a + r_d(\max(0, \vec{n} \cdot \vec{s})) + r_s(\max(0, \vec{v} \cdot \vec{r}))^n$ (when not in shadow)

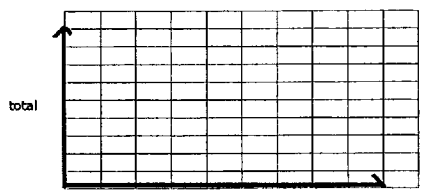
$r_a = 0.2, \quad r_d = 0.7, \quad r_s = 0.3, \quad n = 200$

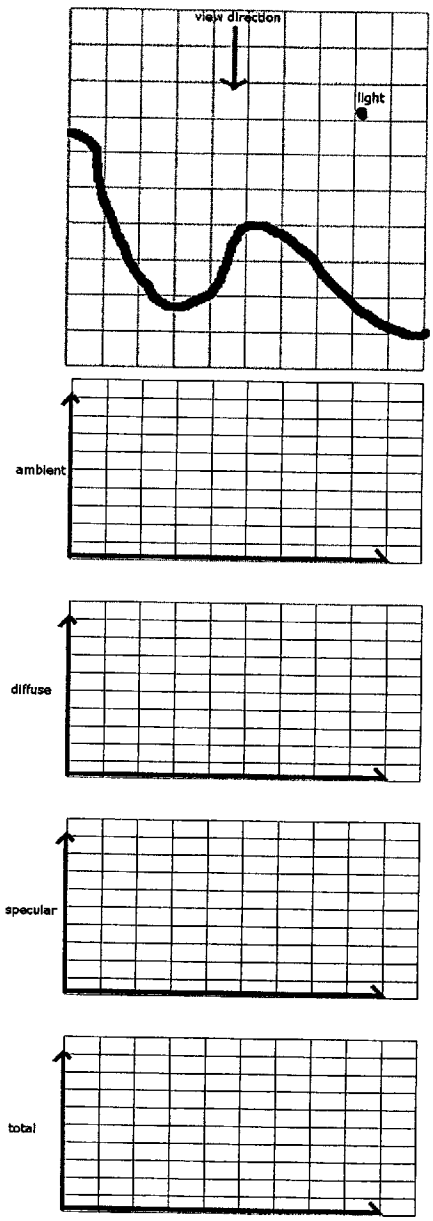




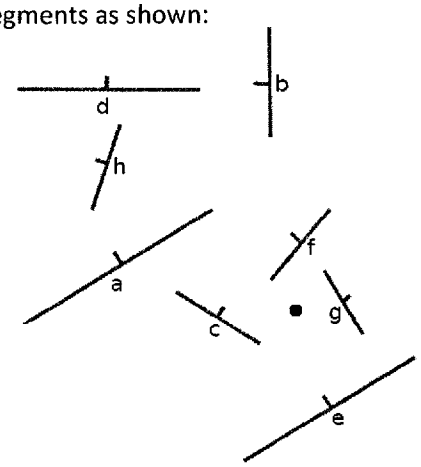







view direction
light
ambient
diffuse
specular
total

4. [16 marks] **BSP Trees**
Consider the following 2D scene with the camera point illustrated by the black dot and the outward normal of polygon segments as shown:



a) [8 marks] Draw the BSP tree for the scene by adding the segments in the labeled order, starting with a.

b) [8 marks] Describe how your tree would be traversed when rendering the scene from the specified camera location.

5. [20 marks] **Lighting and Shading**

a) [2 marks] How would you compute a normal vector for a vertex of a 3D triangulated polygonal mesh, assuming that the mesh is intended to approximate a smooth surface?

b) [8 marks] Using the Phong lighting model, give the mathematical expression for the light sent toward a camera center at location $\bar{c}$ from a surface point $\bar{p}$ with unit normal $\vec{n}$, given a point light source at location $\bar{l}$. Define any other variables needed by the model, and express all directions in terms of $\bar{p}$, $\vec{n}$, $\bar{c}$, and $\bar{l}$. Call the function $L(\bar{p}, \vec{n}, \bar{c}, \bar{l})$.

c) [2 marks] Let $\overline{p_0}$, $\overline{p_1}$, and $\overline{p_2}$ be the vertices of a triangle in a triangular mesh and $\overrightarrow{n_0}$, $\overrightarrow{n_1}$, and $\overrightarrow{n_2}$ be the normal associated to $\overline{p_0}$, $\overline{p_1}$, and $\overline{p_2}$, respectively. For any point $\bar{p}$ and its associated normal $\vec{n}$, $L(\bar{p}, \vec{n}, \bar{c}, \bar{l})$ is the color at the point p calculated using the Phong illumination model. Let $\bar{q}(a, b) = (1 - a - b) + p_0 + ap_1 + bp_2$ be a point in the triangle.

What are the valid ranges of the values a, b, and $a+b$ for $\bar{q}$ to lie in the triangle?

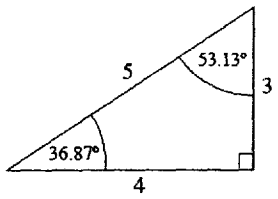
d) [2 marks] If Gouraud shading is applied, what is the colour at $\bar{q}$? (You may write this in terms of $L(\bar{p}, \vec{n}, \bar{c}, \bar{l})$.)

e) [2 marks] If Phong shading is applied, what is the colour at $\bar{q}$? (You may write this in terms of $L(\bar{p}, \vec{n}, \bar{c}, \bar{l})$.)

f) [4 marks] Explain briefly the difference between the two shading methods in terms of the accuracy and efficiency of shading.

6. [11 marks] **Refractions**

Suppose a laser is sitting 1m above the surface of a lake and is pointing down into the lake at an angle of 53.13 degrees with the water. If the lake is 20m deep, at what horizontal distance away from the laser's origin does the laser beam hit the lake's floor? Assume that the coefficient of air is 1 and the coefficient of water is 4/3. Your answer can contain trigonometric function calls and does not need to be simplified further.



7. [16 marks] **Ray-Surface Intersections**
Consider a hemispherical object: i.e. a sphere of unit radius that is cut by a plane through the center of the sphere (like half an orange). The surface of the object comprises two parts, the quadratic hemisphere and the planar base.

a) [6 marks] Express this object mathematically in implicit form using constraints.

b) [10 marks] Using this mathematical model, find the intersections (if any) of the object surface and a ray $\bar{r}(u) = \bar{p} + u\vec{d}$, assuming that the ray originates outside the object.

8. [12 marks] **Ray Tracing**

a) [3 marks] Basic (Whitted) ray tracing adds an extra term to the Phong lighting model. What is it, and what effect does it capture?

b) [5 marks] What is sub-surface scattering? Explain briefly why it cannot be represented as a BRDF.

c) [4 marks] Briefly describe what causes motion blur. Using words and/or pseudocode, explain how you could simulate motion blur using an OpenGL-like render (not a raytracer). The system does not need to run in real-time.

9. [20 marks] **Radiometry**

a) [4 marks] Give the equation for the radiance reflected from a surface point $\bar{p}$ with normal $\vec{n}$, in terms of the BRDF $\rho(\overrightarrow{d_e}, \overrightarrow{d_i})$ and incoming radiance $L(\overrightarrow{d_i}, \bar{p})$. You may parameterize directions in terms of spherical coordinates, e.g. $\vec{d}(\theta, \phi)$.

b) [8 marks] Given an expression for the irradiance at a surface point $\bar{p}$ with normal $\vec{n}$ due to an anisotropic point light source located at point $\bar{l}$.

c) [8 marks] Give an expression for the irradiance at surface point $\bar{p}$ with normal $\vec{n}$ due to an area light source defined by a polygon *P*.

10. [8 marks] **Cubic Curves**

Derive a cubic polynomial x(t) that satisfies the following constraints:

- $x(1) = 10$
- $x'(0) = 4$
- $x'(1) = 5$
- $x''(1) = -2$

11. [4 marks] **Animation**

a) [2 marks] Give two advantages of key-frame animation over physics-based animation.

b) [2 marks] Give two advantages of physics-based animation over key-frame animation.

END OF EXAM.

TOTAL PAGES = 15

TOTAL MARKS = 155