**Worth:** 10% **Due:** Before 10:00pm on **Thursday 19 November 2015**.

**Your submission must be a PDF file named a3.pdf and it must be handed-in using the MarkUs system. You are to create the PDF file using a document preparation system (*e.g.* Word or LaTeX). Only hand in the PDF file - do not hand in other files (*e.g.* a .doc file or a .tex file). (If you expect to have a difficult time with the typesetting, speak with Tom.)**

**For questions where you are asked to write a program, you must include your program and any output in your submission.**

**See the Syllabus for the course policy on late assignment submissions.**

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1. To find a root of the equation

$$f(x) = x^2 - 3x + 2 = 0,$$

we can consider fixed-point problems involving the following different functions :

$$\begin{aligned} g_1(x) &= (x^2 + 2)/3, \\ g_2(x) &= \sqrt{3x - 2}, \\ g_3(x) &= 3 - 2/x, \\ g_4(x) &= (x^2 - 2)/(2x - 3). \end{aligned}$$

   (a) Analyze the convergence properties of each of the corresponding fixed-point iteration schemes for the root $x = 2$ by considering $|g_i'(2)|$.

   (b) Confirm your analysis by implementing each of the schemes and verifying its convergence (or lack thereof) and approximate convergence rate.

2. (a) Show that the iterative method

$$x_{k+1} = \frac{x_{k-1} f(x_k) - x_k f(x_{k-1})}{f(x_k) - f(x_{k-1})}$$

   is mathematically equivalent to the secant method for solving a scalar nonlinear equation $f(x) = 0$.

   (b) When implemented in finite-precision floating-point arithmetic, what advantages or disadvantages does the formula given in part (a) have compared with the formula for the secant method given in lecture

$$x_{k+1} = x_k - f(x_k) \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} \quad ?$$

3. Using `matlab` implement the bisection, Newton and secant methods for solving nonlinear equations in one dimension, and test your implementations by finding at least one root for each of the following equations. What termination criteria should you use? What convergence rate is achieved in each case? Compare your solution results with those produced by `matlab`'s `fzero` function.

   (a) $x^3 - 2x - 5 = 0$,

   (b) $e^{-x} = x$,

   (c) $x \sin(x) = 1$,

   (d) $x^3 - 3x^2 + 3x - 1 = 0$.

4. Consider the function $f(x) = (1 - 3/(4x))^{1/3}$.

   (a) By hand, determine the root of $f(x)$.

   (b) Apply Newton's Method to the problem of computing a root of $f(x)$, with each of $x_0 = 0$, $x_0 = 1$ and three randomly chosen $x_0$ in the interval $(0, 1)$. Simplify the Newton iteration expression to eliminate division by $x$ before performing the computations. Compute at most 50 iterations. For each of the random starting points, plot the first 50 iterates. Use the **matlab** function `rand` to generate random initial guesses and the function `plot` to generate plots.

   (c) Comment on your results.

If you have questions or encounter any difficulties, post a description of the issue on the Piazza course forum.