

Student ID:	
Full Name:	

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# Midterm

          
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*University of Toronto*

*CSC418H  
Computer Graphics*

*Winter 2015*

Instructor: Luke Moore

*Tuesday, February 24, 2015  
Duration: 60 minutes*

*This is a closed book exam (no calculators, PDAs, laptops or other examination aids)*

*Note: It is a violation of the University's Code of Behaviour on Academic Matters to cheat on an examination. Cheating includes copying another student's work, possessing unauthorized examination aids, having someone else take your examination, or knowingly allowing your work to be copied.*

**Question 1:** Consider the 3D implicit surface defined by the following equation:

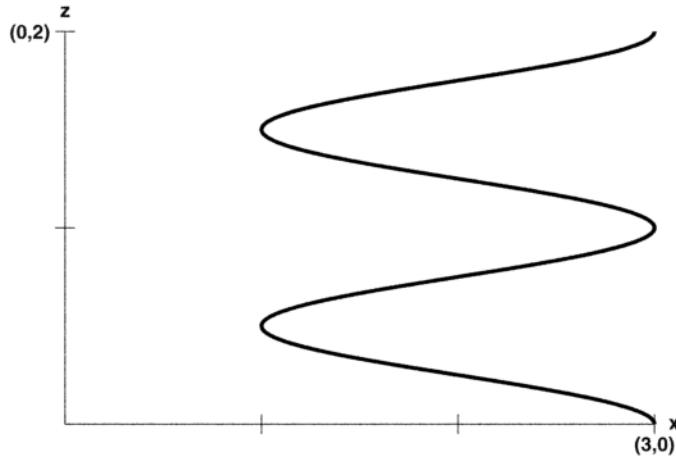
$$f(x, y, z) = 2x^2 + 3y^2 - xy + xz - 2yz + 2x + y - z + 9 = 0$$

a) [1 marks] Find the point on the surface at  $x=1$  and  $y=1$ .

b) [4 marks] Compute the surface normal at the point in a). You do not have to express it as a unit vector.

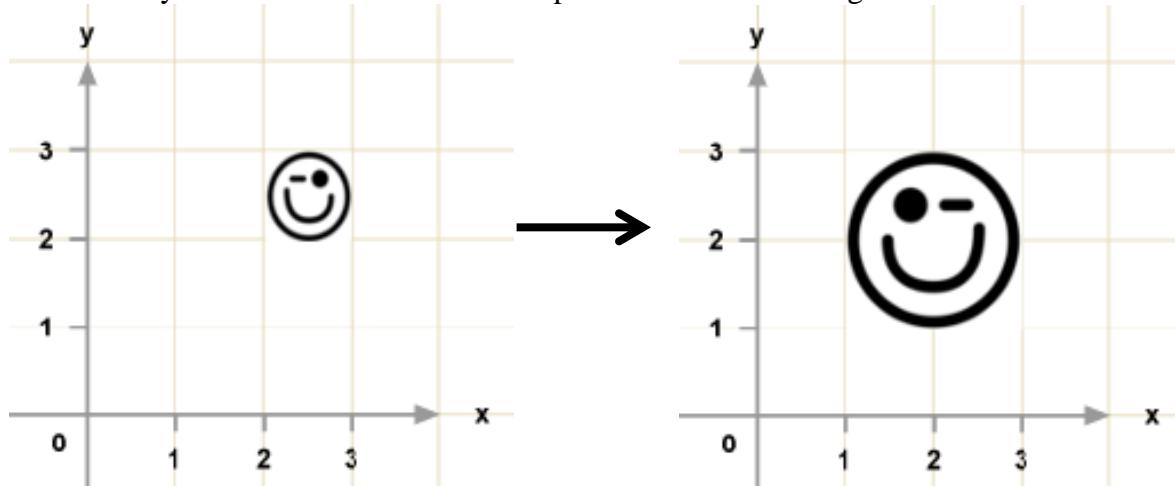
**Question 2:**

a) [4 marks] Give a parametric representation for the following 3D curve that lies on the  $x$ - $z$  plane. State the domain of the curve parameter.



- b) [4 marks] Give the parametric representation for the 3D surface formed by rotating the curve from a) about the  $z$  axis. State the domains of the two parameters.

**Question 3:** [8 marks] Find the  $3 \times 3$  homography matrix that transforms the 2D vertices of the smiley face as illustrated below. Express the result as a single matrix.



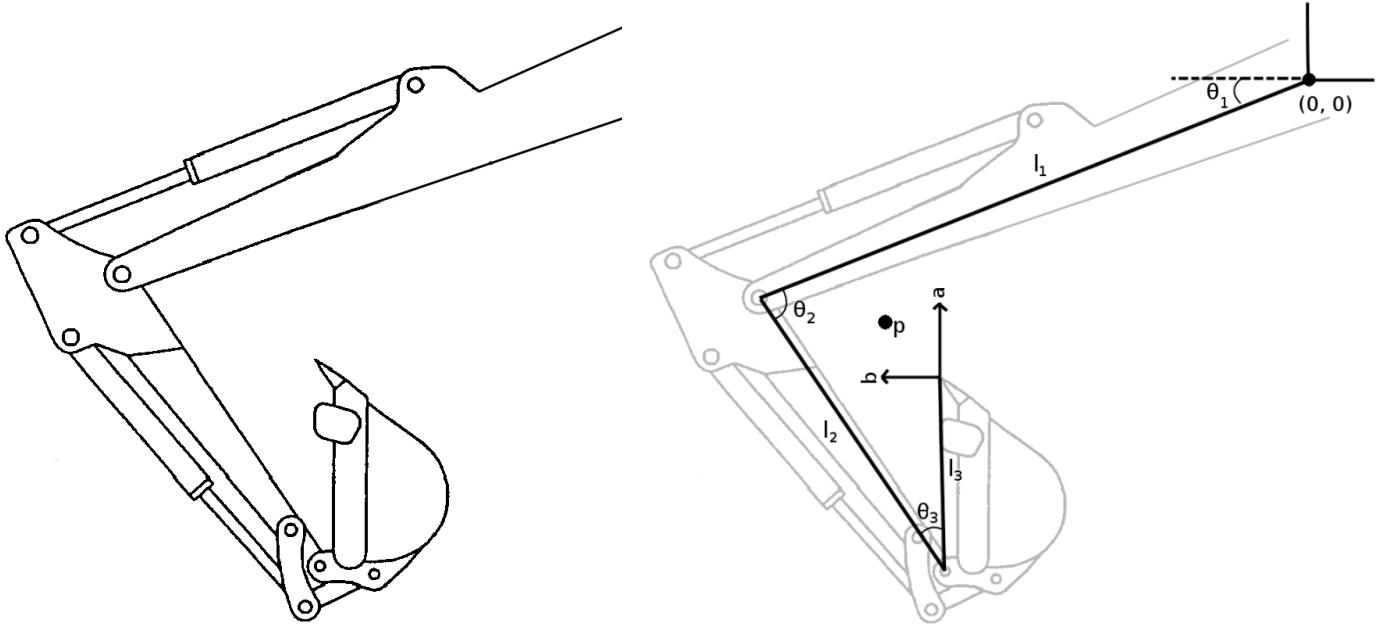
**Question 4:** Consider the following projection matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix}$$

- a) [2 marks] If  $f$  is 0.5, what are the 2D Cartesian coordinates of the 3D point (-12, 24, -3) after projecting it with the above matrix?
  - b) [3 marks] What is pseudo-depth and how does OpenGL use it?
  - c) [3 marks] What pseudo-depth values does the above projection matrix produce? Why isn't this matrix suitable for use with a z-buffer algorithm?
  - d) [4 marks] Modify the matrix so it produces a range of pseudo-depth values.

**Question 5:** [12 marks] Consider the following three-link, two-dimensional excavator arm determined by arm lengths  $l_1$ ,  $l_2$ , and  $l_3$  and angles  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ :

- Its base is affixed to the origin of the world coordinate system.
- Its first link has length  $l_1$  and can rotate about the angle  $\theta_1$  as in the diagram.
- Its second link has length  $l_2$  and can rotate about the angle  $\theta_2$ .
- Its third link has length  $l_3$  and can rotate about the angle  $\theta_3$ .



Suppose we attach a local coordinate system with basis vectors  $(\mathbf{a}, \mathbf{b})$  to the end of the bucket, as shown in the figure above. Suppose also that point  $p$  is expressed in coordinates with respect to this local coordinate system. Determine the transformation that maps local coordinates to world coordinates, and express it as a product of elementary transformation matrices. You do not need to perform any matrix multiplications.