1. Transformation

   (a) We know that an affine transformation has the form

   $$\begin{bmatrix} A & t \\ 0\ 0 & 1 \end{bmatrix}$$

   Now suppose the affine transformation is

   $$\begin{bmatrix} a & b & x \\ c & d & y \\ 0 & 0 & 1 \end{bmatrix}$$

   Then we plug in those points and get 6 equation:

   $$2a + 3b + x = 8$$
   $$2c + 3d + y = -4$$
   $$a + 2b + x = 2$$
   $$c + 2d + y = 0$$
   $$3a - b + x = 10$$
   $$3c - d + y = 8$$

   After solving this linear equation we have:

   $$\begin{bmatrix} 5.2 & 0.8 & -4.8 \\ -0.8 & -3.2 & 7.2 \\ 0 & 0 & 1 \end{bmatrix}$$

   (b) There are 8 variables for a 2D-Homography; hence, we need 8 points which means 4 maps to 4.
   There are 3 variables for a 2D-rigid transform; hence, we need 4 points which means 2 maps to 2.

   (c) Both centroid and ortho-center are preserved under affine transformation due the the similarly triangle.
   centroid:

   Suppose after an affine transformation a triangle ΔABC maps to $\Delta A'B'C'$. And the centroid O maps to O'.

   Now, connect AO and $A'O'$. Suppose line AO intersects BC at D and $A'O'$ intersects $B'C'$ at D'. Clearly triangle ΔADC and $\Delta A'D'C'$ are similarly triangles; hence D' is also the mid-point of $B'C'$.

   similarly, we can get O' is the centroid of $\Delta A'B'C'$.

   ortho-center:

   Suppose after an affine transformation a triangle ΔABC maps to $\Delta A'B'C'$. And the ortho-center O maps to O'.

   Now, connect AO and $A'O'$. Suppose line AO intersects BC at D and $A'O'$ intersects $B'C'$ at D'. Clearly triangle ΔADC and $\Delta A'D'C'$ are similarly triangles; hence $A'D'$ is also the perpendicular to $B'C'$.

similarly, we can get O' is the ortho-center of $\Delta A'B'C'$.

2. Viewing and Projection

   (a) The function of a real camera would be a complicated function about light, distance etc. The focal length of the lens is the distance between the lens and the image sensor when the subject is in focus. Smaller focal length would give a broader image view.
   Aperture is an opening through which light travels size of the aperture has a direct impact on the depth of field. A smaller aperture will bring all foreground and background objects in focus, while a large aperture will isolate the foreground from the background by making the foreground objects sharp and the background blurry.

   (b) camera position $e(2,1,3)$, look at point $p(-1,2,1)$, and up vector $\vec{r}(0,1,0)$. Then we have,

$$\vec{s} = \frac{p-e}{\|p-e\|} = (\frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}})$$

$$\vec{u} = \frac{\vec{r}\times\vec{s}}{\|\vec{r}\times\vec{s}\|} = (\frac{-2}{\sqrt{13}}, 0, \frac{3}{\sqrt{13}})$$

$$\vec{v} = \frac{\vec{r}\times\vec{u}}{\|\vec{r}\times\vec{u}\|} = (\frac{3}{\sqrt{182}}, \frac{13}{\sqrt{182}}, \frac{2}{\sqrt{182}})$$

   Then we have,

$$M_{cw} = \begin{bmatrix} \frac{-2}{\sqrt{13}} & \frac{3}{\sqrt{182}} & \frac{-3}{\sqrt{14}} & 2 \\ 0 & \frac{13}{\sqrt{182}} & \frac{1}{\sqrt{14}} & 1 \\ \frac{3}{\sqrt{13}} & \frac{2}{\sqrt{182}} & \frac{-2}{\sqrt{14}} & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   Then

$$M_{wc} = M_{cw}^{-1} = \begin{bmatrix} \frac{-2}{\sqrt{13}} & 0 & \frac{3}{\sqrt{13}} & \frac{5}{\sqrt{13}} \\ \frac{3}{\sqrt{182}} & \frac{13}{\sqrt{182}} & \frac{2}{\sqrt{182}} & \frac{22}{\sqrt{182}} \\ \frac{-3}{\sqrt{14}} & \frac{1}{\sqrt{14}} & \frac{-2}{\sqrt{14}} & \frac{1}{\sqrt{14}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (c) From perspective projection we could get,

$$(\frac{dp_x}{p_z}, \frac{dp_y}{p_z})$$

   (d) Suppose the line function is $l(\lambda) = (a+\lambda b_x, d+\lambda b_y, c+\lambda b_z)$.
   Then the 2D-point of any point point on this line would be,

$$(\frac{d(a+\lambda b_x)}{c+\lambda b_z}, \frac{d(d+\lambda b_y)}{c+\lambda b_z})$$

   As λ goes to infinite we get $(\frac{db_x}{b_z}, \frac{db_y}{b_z})$, which means the points converge to $(\frac{db_x}{b_z}, \frac{db_y}{b_z})$.

3. Surfaces

(a) At any point $p = (x, y, z)$, normal is $(\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz})$. Then we have,

$$\frac{df}{dx} = 2x - \frac{2Rx}{\sqrt{x^2 + y^2}}$$

$$\frac{df}{dy} = 2y - \frac{2Ry}{\sqrt{x^2 + y^2}}$$

$$\frac{df}{dz} = 2z$$

(b) Suppose the function for the tangent plane at point p is $t(q)$ where q is the any point on this tangent plane, then we have,

$$t(q) = (q - p)normal_p = 0, \text{ where } normal_p stands\ for\ the\ normal\ vector\ at\ the\ point\ p$$

(c) Substitute the parametric curve $q(\lambda) = (R\cos\lambda, R\sin\lambda, r)$ into the surface function and we have,

$$(R - \sqrt{(R\cos\lambda)^2 + (R\cos\lambda)^2})^2 + r^2 - r^2$$

$$= (R - \sqrt{R^2})^2 + 0$$

$$= 0$$

Hence, parametric curve is on the surface.

(d) the tangent vector for $q(\lambda)$ is $(\frac{dx}{d\lambda}, \frac{dx}{d\lambda}, z)$; therefore we have the tangent vector,

$$(-R\sin\lambda, R\cos\lambda, r)$$

for any point on curve $q(\lambda)$.

(e) First we know that the normal vector for any point at plane is,

$$(2R - \frac{2R^2(\cos(\lambda))}{R}, 2R - \frac{2R^2(\sin(\lambda))}{R}, 2r)$$

which is,

$$(0, 0, 2r)$$

And from b we know that for any point q, if its tangent vector (x, y, z) lie on the tangent plane, we will have,

$$0(x - R\cos\lambda) + 0(y - R\sin\lambda) + 2r(z - r) = 0$$

which is,

$$rz - z^2 = 0$$

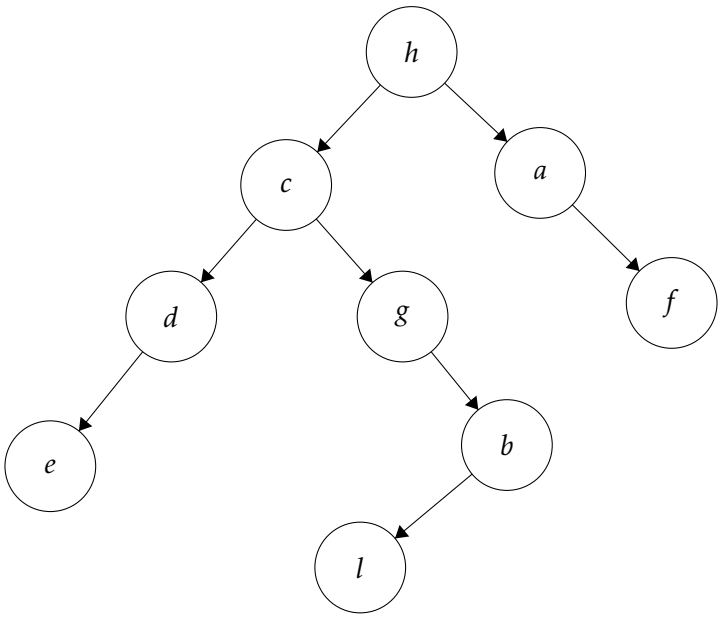
Now from (d) we have that the tangent vector for any point at the curve $p(\lambda)$ is $(-R\sin\lambda, R\cos\lambda, r)$. Then we substitute this into $rz - z^2 = 0$, and we have,

$$r^2 - r^2 = 0$$

Hence, the this tangent vector of $q(\lambda)$ does lie the the tangent plane.

4. (a) It is possible to exclude things, for instance it is possible to exclude d, d might be blocked by c and it is pointing aways from the general camera view.

(b)



(c) We based on the rule that if *eye* is outside a face *i*

- draw everything inside *i*
- draw *i*
- draw everything out side *i*

if *eye* is inside a face *i*

- draw everything outside *i*
- draw *i*
- draw everything inside *i*

Hence, we would get, [*a*, *f* , *h*, *d*, *e*, *c*, *b*, *l*, *g*,]