UNIVERSITY OF TORONTO
Faculty of Arts and Science
DEC 2014 EXAMINATIONS

CSC 438H1F/2404H1F

Duration - 3 hours
No Aids Allowed

There are 8 questions worth a total of 80 marks.
Answer all questions on the question paper, using backs of pages for scratch work.
Check that your exam book has 10 pages (including this cover page).

PLEASE COMPLETE THIS SECTION:

Name ______________________________

(Please underline your family name.)

Student Number ______________________________

FOR USE IN MARKING:

1. __________/10
2. __________/10
3. __________/10
4. __________/10
5. __________/10
6. __________/10
7. __________/10
8. __________/10

Total:__________/80

[10] 1. Let f and g be unary function symbols, and let A be the formula $\forall x fgx = x$ and let B be the formula $\forall x gfx = x$. Prove that $A \not\models B$.

[10] 2. Let PRIMES be the set of prime numbers. Define A to be the set of numbers x such that the TM encoded by x halts on exactly those inputs that are prime numbers. Is A r.e.? Is A^c r.e.? Justify your answers. You may not use Rice's theorem.

[10] 3. Let A be the set of numbers x such that x encodes a TM with the property that it halts on at least one input. Is A recursive? Is A r.e.? Justify your answers. You may not use Rice's theorem.

[10] 4. Let f be a unary function (not necessarily total). Recall that $graph(f)$ is the relation $R_f(x, y) = (y = f(x))$. Prove that if $graph(f)$ is r.e. then f is recursive. (That is, show that if $graph(f)$ is r.e., then there is a TM that for every x, halts and outputs $f(x)$.)

[10] 5. A set Φ of $\mathcal{L}$ sentences is finitely axiomatizable if there is a finite set Γ of $\mathcal{L}$ sentences such that Φ and Γ have the same models. (Note that Γ is not necessarily a subset of Φ.) Prove that if $\Phi = \{A_1, A_2, \ldots\}$ and for all sufficiently large i, A_i is not a logical consequence of $\{A_1, \ldots, A_{i-1}\}$, then Φ is not finitely axiomatizable.

[10] 6. Let $\mathcal{L}$ be a first-order language with finitely many function and predicate symbols. Prove that the set of unsatisfiable $\mathcal{L}$-sentences is r.e. You may use Church's thesis, and the LK completeness theorem.

[10] 7. Recall that **RA** is a theory with 9 axioms P1, ... P9 over the language $\mathcal{L}_A$.
The **RA Representation Theorem** states that every r.e. relation is representable in **RA** by an $\exists\Delta_0$ formula. Use this theorem to prove that RA is undecidable. (That is, the set of numbers that encode the theorems of **RA** is an undecidable set.)

[10] 8. Let Σ be an axiomatizable theory over the vocabulary $\mathcal{L}_A$ of arithmetic such that every r.e. relation is representable in Σ by some $\exists\Delta_0$ formula. Show that there is a $\forall\Delta_0$ sentence A (one of the form $\forall y B$, where B is bounded) such that $\Sigma \nvdash A$ and $\Sigma \nvdash \neg A$.

(Extra sheet if you need more space.)