Due: Friday, October 2, beginning of tutorial

NOTE: Each problem set counts 15% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offence and will be dealt with accordingly.

1. Suppose that we allow $\oplus$ (exclusive OR) as a primitive connective in building propositional formulas. Give the appropriate left and right introduction rules for $\oplus$. The rules should be in the same style as the introduction rules for $\wedge, \vee, \neg$ (the formulas in the bottom sequent are constructed from formulas in the top sequent by sometimes adding $\oplus$). Make sure your rules satisfy the Sequent Soundness Principle (page 11 of the Notes) and the Inversion Principle (page 13).

2. Consider the algorithm on page 9 of the Notes which, given a set S of propositional clauses either outputs a satisfying assignment for S, or outputs a resolution refutation of S. Show that if every clause in S has at most two literals, then the procedure halts in time polynomial in the size of the input S.

3. Do Exercise 14, page 17 of the Notes. (This involves using propositional formulas to express colourability of graphs.)

4. Do Exercise 9, page 25 in the Notes: Give a predicate calculus sentence A with vocabulary consisting of $=$ and a binary predicate symbol R such that for all positive integers n, A has a model whose universe has n elements if and only if n is even. Justify your answer. (*Hint*: think of R as a pairing relation.)

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Practice Exercises (Do not hand in):

- Exercise 6 page 12: Find PK proofs.

- Exercise 11 page 16: Prove the equivalence of the three formulations of the Propositional Compactness Theorem.

- Exercise 1, page 19 in the Notes: Prove the unique readability of terms.