Last Name ______________________ First Name & Initial ______________________

Student No. [redacted]

NO AIDS ALLOWED. Answer ALL questions on test paper. Use backs of sheets for scratch work.

Total Marks: 40

[5] 1. Give a specific formula A such that

$$\exists x A \not\models A(c/x)$$

where c is a constant symbol not occurring in A. Prove your answer, based on the definition of $\models$.

[10] 2. Recall that **TA** (True Arithmetic) is the set of all sentences A in the language $\mathcal{L}_A = [0, s, +, \cdot; =]$ of arithmetic such that A is true in the standard model $\underline{\mathbb{N}}$. Suppose that $A(x)$ is a formula of $\mathcal{L}_A$ whose only free variable is x, such that $A(s^n 0)$ is in **TA** for arbitrarily large $n \in \mathbb{N}$. Show that the infinite set of sentences

$$\mathbf{TA} \cup \{A(c), c \neq 0, c \neq s0, c \neq ss0, \cdots\}$$

is satisfiable, where c is a new constant.

[15] 3. Give an **LK**-Φ proof showing that $A \models B$, where

$A =_{syn} \forall x \exists y Pxy$
$B =_{syn} \exists x \exists y \exists z (Pxy \wedge Pyz)$

You do not need to indicate weakenings and exchanges.
You may use abbreviations for formulas (including A, B) in your proof.

Hint: You will need two instances of A in your proof. The first part of your proof will not have quantifiers.

[10] 4. Give an LK proof of the sequent

$$s0 \neq ss0 \rightarrow \forall x(x \neq s0 \vee x \neq ss0)$$

(Here $t \neq u$ stands for $\neg t = u$.)

You may leave out weakenings and exchanges.
Start by giving the specific instances of the **LK** equality axioms that you need in your proof.

Here are the **LK** equality axioms:

EL1: $\rightarrow t = t$
EL2: $t = u \rightarrow u = t$
EL3: $t = u, u = v \rightarrow t = v$
EL4: $t_1 = u_1, ..., t_n = u_n \rightarrow ft_1...t_n = fu_1...u_n$, for each f in $\mathcal{L}$
EL5: $t_1 = u_1, ..., t_n = u_n, Pt_1...t_n \rightarrow Pu_1...u_n$, for each P in $\mathcal{L}$