A New Formula for Secondary Emission Yield

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Abstract—**Mathematical expressions for the secondary emission yield as a function of impact voltage and direction are required by computer programs for ray tracing, and are intended to follow secondary trajectories also. It is shown that the accuracy requirements are quite severe for lower voltage impacts, up to about three times V_{max} (where V_{max} is the voltage at which the normal secondary emission ratio reaches its maximum value δ_{max}), but are less stringent above this range. A new formula is proposed for the low-voltage region, which agrees with experimental data better than the Lye and Dekker formula. For the voltage region above 4 V_{max}, the Lye and Dekker formula is retained, with a smooth transition centered between 3 and 4 V_{max}. The directional effect is modeled by adjusting the values of δ_{max} and V_{max} before applying either formula, rather than applying either Bruining's or Muller's correction factor afterward. This results in an automatic modeling of the known absence of any appreciable directional effect below about half V_{max}. Some experimental verification of the formula used to modify δ_{max} is presented.**

I. Introduction—Requirement

COMPUTER programs for tracing rays in electron tubes are now often required to continue to trace secondary electrons resulting from primary impacts with interior surfaces of the tube. Unless the program is merely tracing rays in the absence of space-charge forces, the current carried by each secondary ray must be known, which implies that the program must include formulas for evaluating the secondary emission ratio δ at each impact. The relation between δ and impact velocity (expressed in volts) has the generic form shown in Fig. 1. The *shape* of this curve is somewhat universal, but the parameters shown on the figure vary over a considerable range: δ_{max} can range from about 0.3 for textured carbon, through values between 1.3 and 2 for most metals, to higher than 10 for some oxides. V_{max} varies between about 200 and 1000 V. V_1 and V_2, known as the first and second crossovers, are important in applications, because they determine whether the net current is into or out of the surface, and they define a range in which multipactor can occur with normal impact. They are also important experimentally, since they can be evaluated by a null measurement—the net current is zero at these points.

Fig. 1 also shows a curve representing the impact of the primary at a 60-degree angle of incidence: it will be seen that the two curves are essentially coincident below V_1, implying that there is practically no directional effect in this low-voltage region. There are significant increases in both V_{max} and δ_{max} for the oblique impact, while the V_2 value has moved far to the right, well beyond the limit of the diagram. This last feature is of particular importance in considering cross-field multipactors, since it allows them to operate at much higher power levels than normal-impact discharges. This curve is fairly representative of the directional effect for a dull surface. For a polished surface there is a greater increase in V_{max} and in δ at the higher voltages, while for a very rough surface, such as textured carbon [1], there is practically no directional effect at any voltage.

The accuracy requirements for the formulas to be used depend on the application: two common cases are the cold-cathode cross-field amplifier (CFA), and the depressed collector of a TWT or klystron. In the first of these cases, secondary emission is a desirable phenomenon, on which the buildup of the beam depends; it is always a struggle to get the impedance levels high enough to cause impacts on the cathode at even a few hundred volts so that the region of interest for these tubes is the left-hand half of Fig. 1. But the beam builds up by a succession of impacts—20 or 30 in typical designs—so that any error will grow exponentially: if the error is 5 percent at each impact, then after 20 impacts the accumulated error is $1.05^{20} = 265$ percent. Thus, on the left half of Fig. 1, we need the highest accuracy we can get, of both data and mathematical representation. However, in the beam buildup region the impacts are all near the cusps of cycloidal loops, so that they are substantially normal to the cathode, and there is no appreciable directional effect here. It is only when the space-charge density is above about 10 percent of the Brillouin value that oblique impacts begin to appear, and this is in the last few wavelengths of the tube.

The depressed collector application, on the other hand, utilizes mainly the right-hand half of Fig. 1 and extending on out to much higher voltages where the generic curve continues to decrease smoothly. In this application, secondary emission is regarded as an undesirable (but not entirely avoidable) phenomenon. Efforts are made to *reduce* the impact velocities as far as possible, to save the most power, but even with good design, impact voltages of more than 3 V_{max} are typical. Accuracy, while always desirable, is no longer critical for two reasons. First, at most two or three generations of secondaries will be traced, so there is no serious compounding of error. Second, if the design is at all satisfactory, only a small minority of primary electrons will give rise to secondaries that can escape from the immediate vicinity of their cre-

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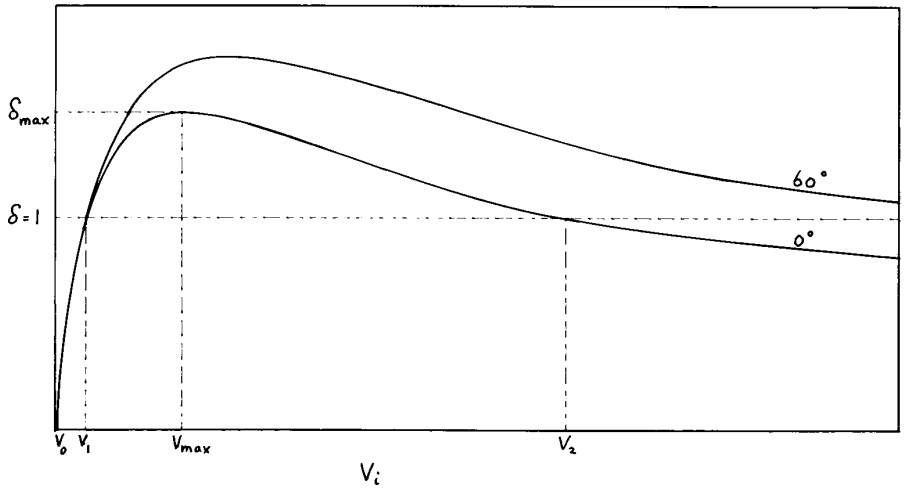

Fig. 1. Generic curves of δ versus impact voltage V_i, for normal and 60 degree incidence. The curves were computed from the formulas given in the text using $V_{max} = 600$ V and $\delta_{max} = 1.5$. The horizontal scale covers 0 to 4000 V, and the vertical scale 0 to 2.0.

ation so that the total contribution of secondaries to the space-charge forces will be minor compared to that of the primaries. However, this minority consists mainly of rays making grazing impacts on the edges of electrodes so that the directional effect is maximized; these secondaries will nearly always be accelerated to a less depressed electrode so that their effect on collector efficiency is disproportionate but still not dominant. We conclude that, in the right-hand half of Fig. 1 and beyond, we can be content with a lower standard of accuracy but need a formula that is easy to evaluate and that will not "blow up" if it is applied at very high voltages. A negative power of the impact voltage appears suitable.

II. Prior Art

The prior art can be considered to be represented by three items:

1) Formulas given by Bruining and by Muller [2, p. 76] for the directional effect; both of these formulas express the δ for oblique impact in terms of the normal δ calculated for the same impact voltage, and the present writer formerly used this approach. But this does not reproduce either the known absence of a directional effect at low voltages or the increase of V_{max} with angle of incidence. After reading [5] and [6], to which his attention was drawn by a reviewer, the writer realized that it was far more satisfactory to correct the values of V_{max} and δ_{max} for the impact angle *first*, and then calculate the value of δ from these corrected values. If the impact is at an angle θ to the normal, then empirical correction formulas for V_{max} and δ_{max} are

$$V_{max\theta} = V_{max0}(1 + k_s\theta^2/\pi) \tag{1}$$

and

$$\delta_{max\theta} = \delta_{max0}(1 + k_s\theta^2/2\pi) \tag{2}$$

where k_s is a "smoothness factor" for the surface, in the range from 0 for textured carbon to 1.5, possibly 2, for a polished or crystalline surface; in the absence of specific data, a typical dull surface should be assigned $k_s = 1$.

2) A curve of normalized δ versus impact voltage, obtained by Gibbons and published by Beck and Ahmed [3] and others. No formula is given, but the curve represents the average of data taken by several earlier workers on many materials (each data set being normalized to its own V_{max} and δ_{max} so that they can be meaningfully combined). The clustering of the data points around Gibbons' curve justifies the adoption of the "universal curve shape" theory for practical purposes, though it is clearly not a high precision characteristic.

3) A formula published by Lye and Dekker in 1957 [4] which is (with a slight change of notation)

$$\frac{\delta}{\delta_{max}} = \frac{1}{g_n(z_m)} \cdot g_n\left(z_m \frac{V_i}{V_{max}}\right) \tag{3}$$

where

$$g_n(z) = \frac{1 - \exp(-z^{n+1})}{z^n} \tag{4}$$

and z_m is the value of z for which $g_n(z)$ is a maximum; n is an adjustable parameter for which Lye and Dekker adopted the value of 0.35, which gave a fairly good fit to experimental data in the low-voltage region. They suggested that a higher value of n would be better for high-voltage impacts and considered values as high as unity, but they decided to stick with $n = 0.35$ because the low-voltage region was more important to them. A review by the present writer of more recent data showed a large scatter of n values (from 0.16 to 0.58) but no compelling reason to depart from 0.35 as a default value, to be used unless there are reliable measurements on the material in use. When $n = 0.35$, we find $z_m = 1.84$, $g_n(z_m) = 0.725$,

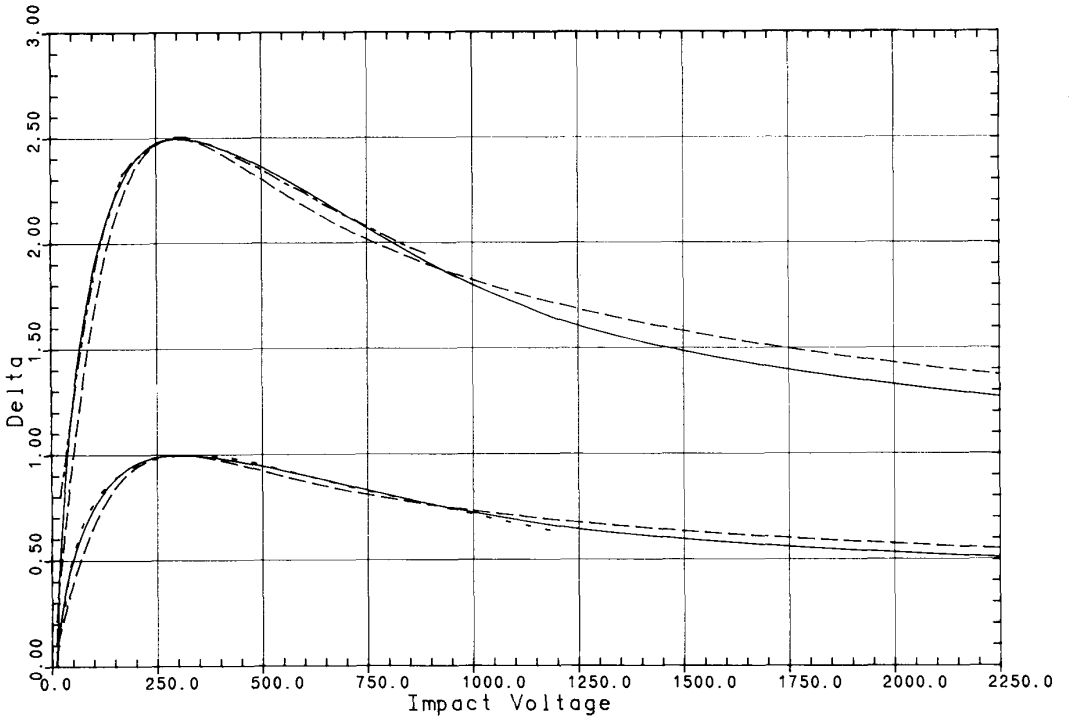

Fig. 2. Comparison of computed yield curves of Vaughan (solid line), and Lye and Dekker (dashed line) with experimental data of Gibbons (short dashes) and of Ritz (dash-dot).

and (1) reduces to

$$\frac{\delta}{\delta_{max}} = 1.379 \frac{1 - \exp\left\{-(1.844\ V_i/V_{max})^{1.35}\right\}}{(1.844\ V_i/V_{max})^{0.35}}. \quad (5)$$

In the high-voltage region, above about 3 V_{max}, the numerator of (5) becomes essentially unity, so that the formula becomes

$$\frac{\delta}{\delta_{max}} = 1.113/(V_i/V_{max})^{0.35}. \quad (6)$$

III. New Empirical Formula

A new formula is proposed for the low-voltage region:

$$\frac{\delta}{\delta_{max}} = (ve^{1-v})^k \quad (7)$$

where

$$v = \frac{V_i - V_0}{V_{max} - V_0}. \quad (8)$$

V_0 and V_{max} are defined in Fig. 1. The value of V_0 is taken as 12.5 V. V_i is the impact voltage, and

$$k = k_1 = 0.62 \quad \text{for} \quad v < 1$$
$$k = k_2 = 0.25 \quad \text{for} \quad v > 1. \quad (9)$$

It is quite acceptable for k to be discontinuous at $v = 1$ since the form of (7) makes it insensitive to k when v is close to unity; the function is still continuous with a continuous differential. If a smooth variation of k is desired, it can be obtained by putting

$$k = \frac{k_1 + k_2}{2} - \frac{k_1 - k_2}{\pi} \arctan(\pi \ln v). \quad (10)$$

These values of k_1 and k_2 were determined to give the best fit to the Gibbons' curve; as shown in Fig. 2 (lower curves), the fit is not just at 3 points, but is quite good all along the line and is distinctly better than the Lye and Dekker curve in this region.

If V_1 and V_2 are known from accurate measurements, then corresponding values of k_1 and k_2 could be determined from

$$k_{1,2} = \ln \delta_{max}/(v_{1,2} - \ln v_{1,2} - 1) \quad (11)$$

where $v_{1,2}$ correspond to $V_{1,2}$ by (8). It is implied, of course, that $\delta_{max} > 1$, otherwise V_1 and V_2 do not exist. The converse problem of finding V_1 and V_2 when k_1 and k_2 are given is slightly more difficult, since (11) cannot be solved explicitly for v.

Convenient iterative procedures are

for v_1

$$v_1 = 0.26/(\delta_{max}\sqrt{\delta_{max} - 1}) \quad \text{(first approx.)}$$

then

$$v_1 = v_1 + 0.25\, v_1^{1.5}\left\{v_1 - \ln v_1 - 1 - (\ln \delta_{max})/k_1\right\} \quad (12)$$

repeated twice;

for v_2

$$v_2 = 2\delta_{max} \quad \text{(first approx.)}$$

then

$$v_2 = v_2 - 2\left\{v_2 - \ln v_2 - 1 - (\ln \delta_{max})/k_2\right\} \quad (13)$$

repeated twice.

The upper curves in Fig. 2 compare (3) and (7) with a set of data kindly supplied by Dr. V. L. Ritz of the Naval Research Laboratory. This is a single set of values taken with a well-calibrated computer-controlled test set on a BeO surface of the kind used in cold-cathode CFA's [7]. It will be seen that (7) is a very good fit using the same k_1 and k_2 values that were determined by the Gibbons curve. This simultaneous fit to two independent sets of data gives some confidence that both the functional form of (7) and the values of k_1 and k_2 are well chosen for this impact voltage region.

In the high-voltage region, however, (7) will decrease nearly as e^{-v}, which is too strong; in this region, (6) is a better choice, though one should be ready to change the exponent from $n = 0.35$ if specific data for the surface being used becomes available.

Expressions (6) and (7) intersect at a point given, to a sufficient degree of accuracy (since the intersection is at a small angle), by

$$v_3 = \frac{1}{k_2} - 0.25. \quad (14)$$

For most purposes, one could therefore use (7) when $v < v_3$ and (6) when $v > v_3$, ignoring the discontinuity of slope in the curve that then occurs at v_3.

One can in fact smooth out the slope discontinuity by using the same mathematical artifice that we have already used in (10): this equation is simply a device that asymptotes to k_1 on the left and to k_2 on the right, with a smooth transition centered at $v = 1$. If δ_1 and δ_2 were values given by (7) and (6), respectively, then a combined formula would be

$$\delta = \frac{\delta_1 + \delta_2}{2} - \frac{\delta_1 - \delta_2}{\pi} \arctan\left\{\pi \ln (v/v_3)\right\}. \quad (15)$$

This is, in fact, the expression that was used to draw the "generic" curves of Fig. 1. However, this approach tends to obscure the functional forms of (5) and (7), and it may be more important to keep them unobscured, as this is the point where further advances could be made. The rather elegant form of (7), which is at present a purely empirical formula, gives the author hope that a theoretician may feel impelled to develop a physical theory from which (7) could be logically derived.

TABLE I

Theta (deg.)	$\delta_{max\theta}$ (M&W)	δ_{max0} Muller	Bruining	Eq. (2), $k_s = 1$	$k_s = 1.5$
50	1.51	.970	1.02	1.35	1.28
78	1.90	.395	.795	1.47	1.32

The value π inside the arctan function in (10) or (15) controls the rapidity of the transition from one asymptotic value to the other; lower values give a long, soft transition, higher ones a sharper transition, with 30 approaching a hard step function. The excuse for using π rather than, say, 3 is simply that it gives the formula a more elegant look. The π in the denominator, on the other hand, is a true trigonometric value, which *must* be π for the two asymptotic values to be approached correctly.

Equation (2), and also the Muller and Bruining formulas, were tested against the data of Mayer and Weiss [8], a recent source of apparently reliable data at large angles of incidence (50° and 78°). The material was nickel, specifically the (110) face. The test method was simply to invert the formulas so as to calculate δ_{max0} from the given oblique data and check the results for consistency with each other and with more widely available normal impact data. The results are shown in Table I. The text-book value for δ_{max0} for nickel is 1.3, though Dudley [9] gives values of 1.35 and 1.47 for well-outgassed specimens. Clearly (2), used in the manner described, is a much closer and more self-consistent representation than either Bruining's or Muller's formula; the assignment of $k_s = 1.5$ for a prepared crystalline surface seems justifiable.

If (2) is accepted as valid, then the observation that there is essentially no directional effect at voltages below about half V_{max} is sufficient to validate (1) also since it is just the 2:1 ratio of the coefficients of θ^2 in (1) and (2) that eliminates the sensitivity to θ at these low voltages.

A source of error that has not yet been considered is the proportion of reflected primaries; these include both elastically reflected electrons and those inelastically reflected that return with velocities above 50 V (an arbitrary limit set by convention). In ray-tracing programs, these will have entirely different subsequent trajectories from true slow secondaries. A discussion by McKay [2, pp. 83–87] indicates that reflected primaries may account for 20 to 50 percent of the total emission in the voltage range in which we are interested for microwave tubes. The data on which that discussion was based goes back to the 1920's, so it may not be very accurate. There is undoubtedly a case for investigation of this aspect using modern equipment and actual tube materials.

Acknowledgment

The author is grateful to a reviewer who drew his attention to [5] and [6]. The data in these references confirm the validity of (7) in the low-voltage region; but, it was reading these papers that made the author realize that his prior handling of the directional effect by applying either

Bruining's or Muller's factor *after* calculating δ gave unrealistic results, and that the directional effect must be dealt with first instead of last.

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