Reliability of Mechatronics System

1.1 Reliability: Quality of a product for a given time is to be known as Reliability

1.2 Reliability Engineering objectives:

- Reduce/ prevent frequency of failures
- Identification of root cause for failures
- Deal with failures which are not yet corrected.
- Estimating reliability of new products i.e analyzing

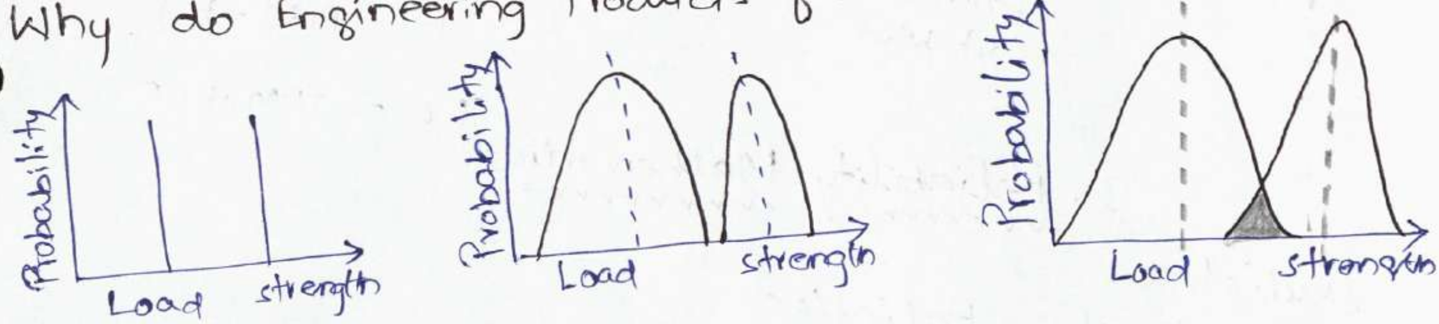
1.3 Why to learn Reliability Engineering?

- To maintain the product quality with expected life time by reducing variability or omitting the uncertainity & risks

Aspects that influence uncertainity

- new technology / development
- human error, verification process
- demand, customer
- safety
- management attention, legal

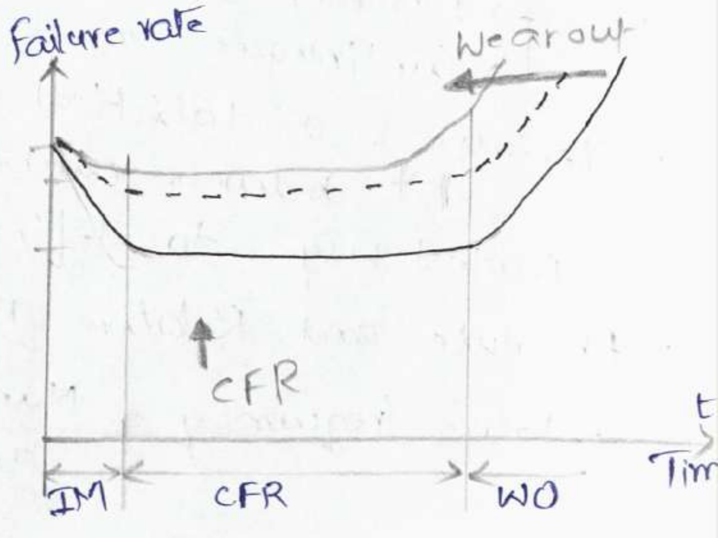
1.4 Why do Engineering Products fail?



1.5 Bathtub - Curve:

IM - ~~Initial~~ Infant Mortality
CFR - Constant Failure Rate
WO - Wear out

Figure shows Product failure rate trend as technology scales. CFR increases with wear-out failures occuring earlier than expected.



1.6 Quantifing Reliability :

Reliability is Quantified in two ways.

1) MTBF - Mean Time Between failures

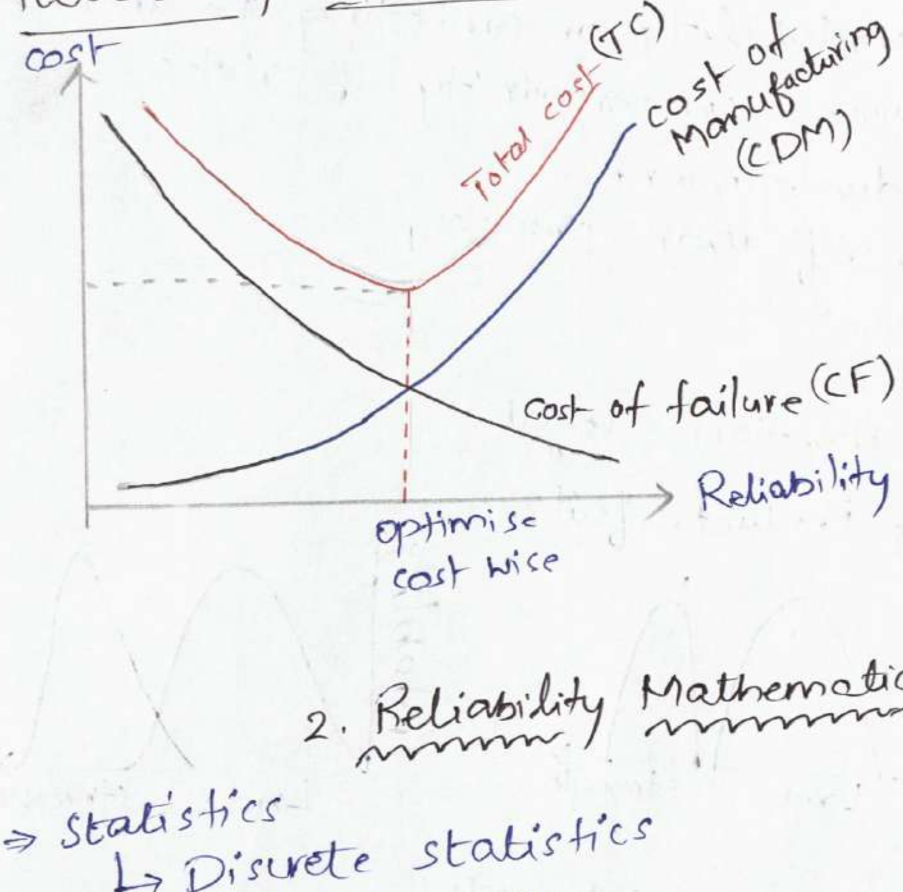
- It implies a ~~reliab~~ repairable item with a constant failure rate

2) MTTF = Mean Time to failure

i.e mean time between the start of an operation and the failure time.

$$\therefore \text{Availability} = \frac{MTBF}{MTBF + MTTR}$$

1.7 Reliability Economics :-



CDM = fixed cost + cost * R^2

CF = fixed cost + cost * F

where,

R = Reliability

F = Failure

F = n*(1-R)

TC = CDM + CF

2. Reliability Mathematics

⇒ Statistics

- → Discrete statistics
- → Continuous "

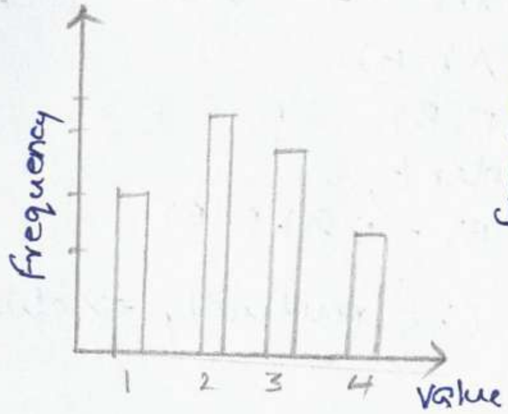
1) Variation : a statistical measurement of the spread between numbers in the dataset

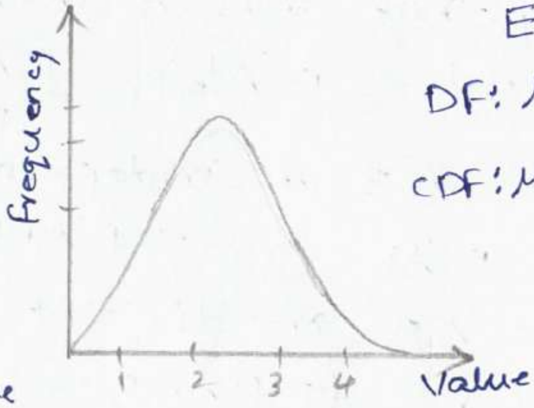
Easiest way - to define of an error bar and interval

2) Absolute and Relative frequency histograms:

$$\text{Relative frequency} = \frac{\text{Number of occurances}}{\text{Total no. of occurances}}$$

3.) Discrete Distribution Vs continuous Distribution





Expected values,

DF: $\mu = \sum_{i=1}^{n} x_i P_x$

CDF: $\mu = \int_{-\infty}^{\infty} x_i f(x) dx$

4) Probability:-

$$\text{Probability}, P = \frac{\text{Event}}{\text{sample set}} = \frac{\text{Desired outcome}}{\text{Total possible outcomes}}$$

Axioms:-

i) $0 \leq P(A) \leq 1$, for all events 'A' in a system 'A'.
 i.e existence of non-negativity

ii) 2nd Axiom of Kolmorgov is $\sum_{i=1}^{n} P_i = 1$

iii) σ-Additivity : If $A_1, A_2 \dots A_n$ be mutually exclusive events, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

$\because A \cup A^c = S$ and $A \cap A^c = \phi$

$$P(A \cup A^c) = P(A) + P(A^c) = P(S) = 1$$

$$P(A^c) = 1 - P(A)$$

$\therefore$ Sum Rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

□ If $A_1, A_2 \dots A_n$ be independent events, then

$$P\left(\bigcap_{i=1}^{n} A_i\right) = \prod_{i=1}^{n} P(A_i)$$

□ Generalization for any random events $A_1, A_2 \dots A_n$

Equation of poincare and sylvester

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{K=1}^{n} (-1)^{K-1} \sum_{1 \leq i_1 \leq i_2 \leq \dots i_K} P(A_{i_1} \cap A_{i_2} \cap \dots A_{i_K})$$

Eg: $P\left(\bigcup_{i=1}^{2} A_i\right) = \sum_{i=1}^{2} P(A_i) - \sum_{1 \leq i \leq j \leq 2} P(A_i \cap A_j)$

$$P\left(\bigcup_{i=1}^{3} A_i\right) = \sum_{i=1}^{3} P(A_i) - \sum_{1 \leq i \leq j \leq 2} P(A_i \cap A_j) + \sum_{1 \leq i \leq j \leq K \leq 3} (P\, A_i \cap A_j \cap A_K)$$

Conditional Probability:

Probability that event A occurs if event B has already occured, $P(A/B) = \frac{P(A \cap B)}{P(B)}$

- If events A, B are independent,
  $P(A/B) = P(A)$ $\quad \because P(A \cap B) = P(A) \cdot P(B)$
- If events A, B are disjoint, (i.e mutually exclusive)
  $P(A/B) = 0$ $\quad [\because P\, A \cap B = 0]$
- If event $B \in A$, then $P(A/B) = 1$

Proposition of Total Probability:-

If $A_1, A_2, \ldots A_n$ are random events with $\bigcup_{i=1} A_i = \Omega$ (sample space) and $A_i \cap A_j = \phi$ for $i \neq j$ and $P(A_i) > 0$

then $P(B) = P(A_1) P(B/A_1) + P(A_2) P(B/A_2) + \ldots\ldots$

$$= \sum_{i=1}^{n} P(A_i) \cdot P(B|A_i)$$

Bayes Theorem:-

Equation for the probability of Hypothesis.

with $\bigcup_{i=1} A_i = \Omega$ & $A_1, A_2, \ldots A_n$ be random events
$A_i \cap A_j = \phi$ and $P(A_i), P(B) > 0$

then $$P(A_K/B) = \frac{P(B/A_K) \cdot P(A_K)}{P(B)}$$

$$P(A_K/B) = \frac{P(B|A_K) \cdot P(A_K)}{\sum_{i=1}^{n} P(B/A_i) \cdot P(A_i)}$$

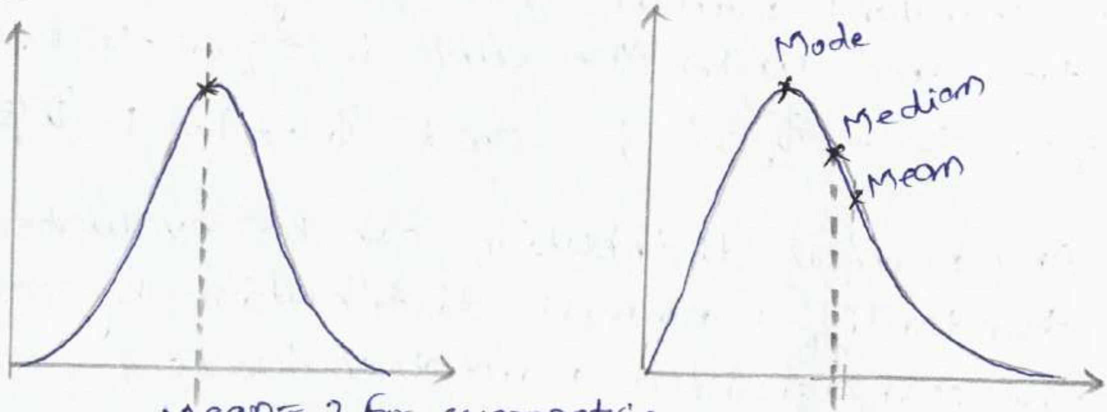
Bayesian Statistics

Previous knowledge of an event $P_0(X)$ i.e a priori knowledge $\quad \cup \quad$ additional knowledge for the event x through an experimentally

$\downarrow$

Improved knowledge $P_1(X) = P(X/y_i)$ for the event x i.e, posterior knowledge.

Measuring / Describing the central tendancy of cdf (Cumulative Distribution function)



Mean = Median = Mode } for symmetric distribution, all statistical values are equal

	Discrete function (probability mass function) $P(a)=P(X=a)$	Continuous Distribution function (probability density function) $F(a)=P(X\le a)=\int_{-\infty}^{a} f(x)dx$
Expectation Value (mean / avg. value)	$\mu = \sum_{i=1}^{n} x_i P(X=i)$	$\mu = \int_{-\infty}^{\infty} x\cdot f(x)\,dx$
standard Deviation	$\sigma = \sqrt{\sum_{i=1}^{n} P_i (x_i-\mu)^2}$	$\sigma = \sqrt{\int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx}$

- If $f(x)$ is a continuous distribution function, $F(x)$ is the cumulative distribution function

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

Probability is $P(x_1 < x < x_2) = \int_{x_1}^{x_2} f(x)\,dx$

Total probability is, $P(-\infty < x < \infty) = \int_{-\infty}^{\infty} f(x)\,dx = 1$

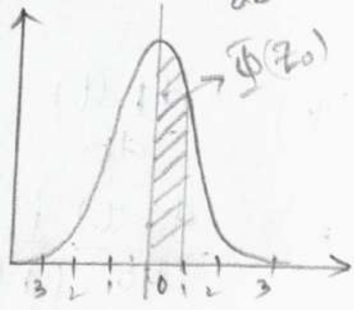
Normal or Gaussian Distribution:-

$$N(x) = f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma}\cdot \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

standard normal distribution is given by

$(\mu=0, \sigma=1)$ $\quad f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$

$$P(X \le z) = \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left(-\frac{x^2}{2}\right) dx$$



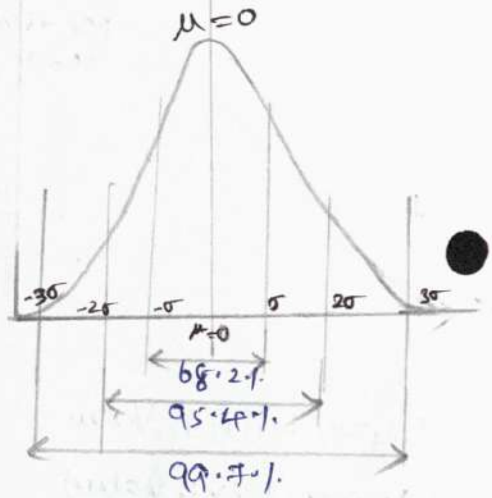
$$\Phi_0(z) = \int_0^z \frac{1}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2}\right)$$

Since standard normal curve is symmetric about x-axis and total area under the curve is equal to 1.

$\therefore \Phi(z) = \frac{1}{2} + \Phi_0(z)$; and $\Phi(-z) = 1 - \Phi(z)$

<u>Note</u>: Any normal distribution can be evaluated from standardized normal distribution by calculating the standardized normal variante 'z',

$$z = \frac{x-\mu}{\sigma}$$



<u>Eg</u>: Calculate the probability for ±1 SD, ±2SD, ±3 SD ?

$P(-1 < x < 1)$ with $\mu = 0, \sigma = 1$

$= P\left(\frac{-1-0}{1} < \frac{x-\mu}{\sigma} < \frac{-1-0}{1}\right)$

$= P(-1 < z < 1) = \Phi(1) - \Phi(-1)$

$= 2\Phi(1) - 1$

$= 68.26\%$

$P(-2 < x < 2) = P\left(\frac{-2-0}{1} < z < \frac{+2-0}{1}\right) = \Phi(2) - \Phi(-2)$

$= 2\Phi(2) - 1$

$= 95.45\%$

$P(-3 < x < 3) = 2\Phi(3) - 1 = 99.72\%$

	Probability of Failure $F(t)$	Survival Reliability function $R(t) = 1 - F(t)$	Failure density function $f(t)$	Hazard rate / Failure rate $h(t) = f(t)/R(t)$
$F(t)$	-	$1 - R(t)$	$\int_0^t f(t) \cdot dt$	$1 - \exp\left[\int_0^t -h(t) \cdot dt\right]$
$R(t)$	$1 - F(t)$	-	$\int_t^\infty f(t) \cdot dt$	$\exp\left[\int_0^t -h(t) dt\right]$
$f(t)$	$\frac{dF(t)}{dt}$	$-\frac{dR(t)}{dt}$	-	$\exp\left[\int_0^t -h(t) dt\right]$
$h(t)$	$\frac{dF(t)}{dt} \cdot \frac{1}{1-F(t)}$	$-\frac{dR(t)}{dt} \cdot \frac{1}{R(t)}$	$\frac{f(t)}{\int_t^\infty f(t) dt}$	-

Lognormal distribution :-

lognormal distribution,

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\cdot\sigma x}\cdot \exp\left[\frac{-(\ln x-\mu)^2}{2\sigma^2}\right], & x \geq 0 \\ 0, & x \leq 0 \end{cases}$$

Expectation value, $E = \left[\mu + \frac{\sigma^2}{2}\right]$

Standard deviation, $SD = \sqrt{\exp(2\mu+2\sigma^2) - \exp(2\mu+\sigma^2)}$

Exponential distribution :-

$$f(t) = \begin{cases} \lambda\cdot e^{-\lambda t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

, is the probability density function of random variable 't' is said to be exponential random variable.

$F(t) = 1 - e^{-\lambda t}$

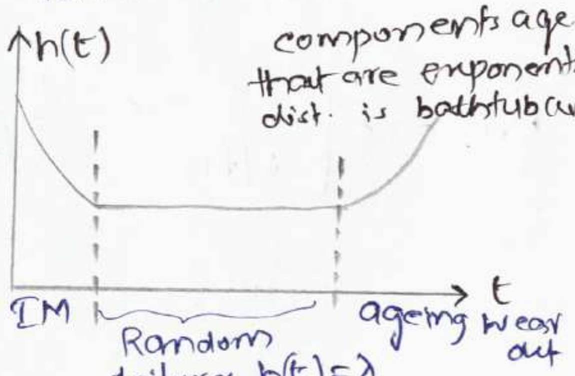
$F(t)$ - cumulative distribution function.

$R(t) = 1 - F(t) = e^{-\lambda t}$

$h(t) = f(t)/R(t) = \lambda$

Expectation value, $E(t) = \lambda$

components age that are exponential dist. is bathtub curve



Weibull Distribution :-

Weibull probability (density) distribution,

$$f(t) = \begin{cases} \alpha\beta t^{\beta-1}\cdot \exp\left[-\alpha t^{\beta}\right], & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Weibull cumulative distribution

$$F(t) = \begin{cases} 1 - \exp\left(-\left[t/\eta\right]^{\beta}\right), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

where, $\beta > 0$ is called shapeparameter

$\eta \geq 0$ is scale parameter / characterstic lifetime

$\eta = \alpha^{-1/\beta} \Rightarrow \eta^{-\beta} = \alpha$

$$F(t) = 1 - \exp(-\alpha t^{\beta})$$

Reliability function, $R(t) = \begin{cases} \exp(-\alpha t^{\beta}), & t \geq 0 \\ 1, & t < 0 \end{cases}$

Hazard function, $h(t) = \frac{f(t)}{R(t)} = \alpha \beta t^{\beta-1}$

If $\beta = 1 \Rightarrow h(t) = \alpha$, is a constant (i.e, exp. dist.)
→ random failures

$\beta < 1 \Rightarrow h(t)$ decreases (early failures), burn in

$\beta > 1 \Rightarrow h(t)$ increases monotonically

$\beta > 2 \Rightarrow h(t)$ increases progressively

$\beta > 3 \Rightarrow$ Weibull distribution is replaced by Gaussian distribution

P.%. Confidence Interval :-

P.%. confidence interval expresses as a probability that the expectation value lies with an interval of $\pm \varepsilon$ around the mean value : $[\bar{x} - \varepsilon, \bar{x} + \varepsilon]$

$$P.\%. = P\left[\frac{-\varepsilon}{\sigma/\sqrt{n}} \leq \frac{\mu - \bar{x}}{\sigma/\sqrt{n}} \leq \frac{\varepsilon}{\sigma/\sqrt{n}}\right]$$