CS2109S Midterms
AY24/25 SEM 1
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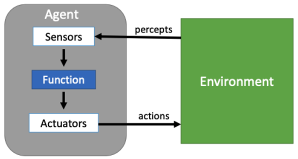
1 Introduction to AI

1.1 Intelligent Agents



- **PEAS**
  1. Performance Measure: Best for whom, what are we optimizing, what information is available, any unintended effects, what are the costs
  2. Environment: Refer to Environment section
  3. Actuators: Allow intelligent agent to take actions or affect its environment
  4. Sensors: Allow intelligent agent to perceive information about its environment
- **Agent Function:** Maps from percept histories to actions, refer to Agent Function section

1.2 Task Environment

1. **Fully Observable VS Partially Observable**
   - Fully Observable: Agent has complete & accurate info about env state at all times (Eg. Chess)
   - Partially Observable: Agent has access to incomplete, uncertain or noisy info about env state (Eg. Self-driving cars)
2. **Deterministic VS Stochastic VS Strategic**
   - Deterministic: Next env state is completely determined by current state & agent action | Outcome is fully predictable (Eg. Sudoku)
   - Stochastic: Next env state is not completely determined by current state & agent action | Outcome is uncertain (Eg. Self-driving car)
   - Strategic: Env is deterministic, but outcomes depend on other agents' actions, requiring agent to consider strategies & behaviors of others (Eg. Chess)
3. **Episodic VS Sequential**
   - Episodic: Agent actions are divided into discrete periods, each episode is independent of one another, agent makes decisions based on current episode (Eg. Classification task)
   - Sequential: Agent actions are inter-dependent, each action affects future states & decisions, agent considers sequence of actions over time (Eg. Chess)
4. **Static VS Dynamic**
   - Static: Env state does not change while agent is deliberating
   - Dynamic: Env state changes over time even when agent is deliberating
   - Semi-dynamic: Env state does not change, but agent's performance score does
5. **Discrete VS Continuous**
   - Discrete: Finite # of distinct, clearly defined percepts & actions
   - Continuous: Infinite # of percepts & actions
6. **Single Agent VS Multi Agent**
   - Single Agent: Agent operating by itself in an env
   - Multi Agent: Multiple agents in an env

1.3 Agent Structures

Note: Agent is completely specified by Agent Function mapping percept sequences to actions

1. **Simple Reflex Agent:** Operates based on a set of predefined rules or conditions $\longrightarrow$ Reacts to current state of env with a corresponding action $\longrightarrow$ Does not have memory of past states or actions & does not consider future consequences
2. **Model-based Reflex Agent:** Extends simple reflex agent by maintaining internal model of world $\longrightarrow$ Allows agent to keep track of current env state & handle situations where env state is partially observable or changes over time
3. **Goal-based Agent:** Operates with specific goals in mind $\longrightarrow$ Selects actions based on ability to achieve these goals $\longrightarrow$ Considers future & plans its actions to achieve desired end state | Uses goal representation & perform search and planning
4. **Utility-based Agent:** Extends goal based agent by considering not just whether goals are achieved, but how well they are achieved $\longrightarrow$ Assigns utility value to different states & chooses actions that maximize overall utility
5. **Learning Agent:** Improves performance over time by learning from its experiences | Can be reflex, model, goal & utility based

- **Exploitation:** Maximize expected utility according to current knowledge about world
- **Exploration:** Trying to learn more about the world

2 Solving Problems by Searching

2.1 Designing an Agent

- **Assumptions:** Goal-based agent | Env is fully observable, deterministic, static, discrete
- **Problem-solving Agent:** Agent that plans ahead (considers a seq. of actions that form a path to a goal state), undertakes SEARCH process
- **Steps:**
  1. Goal Formulation: *(What do we want?)*
  2. Problem Formulation: *(How the world works?)* $\longrightarrow$ States (state space), Initial State(initial state of agent), Goal State/Test (goal state of agent), Actions (things that agent can do in a given state), Transition Model (specifies outcome of an action to a given state & how it leads to new states), Action Cost Function (cost of performing an action)
  3. Search: *(How to achieve it?)* $\longrightarrow$ Path (seq. of actions), Solution (path to a goal)
  4. Execute
- **Representation Invariant:** A condition that must be true over all valid concrete representations of a class

2.2 Search Algorithms (Introduction)

- **Search Algorithm:** Takes in search problem (input), returns solution/failure (output) | Defined by Order of Expansion (FRONTIER)
- **Evaluation Criteria:**
  1. Time Complexity: # of nodes generated/expanded
  2. Space Complexity: Max # of nodes in memory
  3. Completeness: Does it return solution if it exists?
  4. Optimality: Does it always find least cost solution?

Tree Search:

```
create frontier

insert initial state to frontier
while frontier is not empty:
    state = frontier.pop()
    if state is goal: return solution

    for action in actions(state):
        next state = transition(state, action)
        frontier.add(next state)
return failure
```

Graph Search:

```
create frontier
create visited
insert initial state to frontier
while frontier is not empty:
    state = frontier.pop()
    if state is goal: return solution
    if state in visited: continue
    visited.add(state)
    for action in actions(state):
        next state = transition(state, action)
        frontier.add(next state)
return failure
```

- **Checking of Goal State:**
  - New state is checked for goal state before new states are PUSHED to frontier $\longrightarrow$ Expand less states, may skip states with less cost
  - State is checked for goal state after state is POPPED from frontier $\longrightarrow$ Expand more states, will not skip states with less cost

2.3 Search Algorithms (Uninformed Search)

- **Key Idea:** Search Algo is given no clue about how close a state is to the goal | Can be Tree or Graph Search
- **BFS:** Queue Frontier | Time Complexity: $O(b^d) = 1 + b + b^2 + \ldots + b^d$, where b is branching factor, d is depth of optimal solution | Space Complexity: $O(b^d)$ when expanded until last child in worst case | Completeness: Complete if b is finite | Optimality: Optimal if step cost is same everywhere
- **UCS:** Priority Queue (path cost) Frontier, where path cost == cost from root to a state | Time Complexity: $O(b^{C^*/\epsilon})$, where C^* is cost of optimal solution, ϵ is minimum edge cost $\rightarrow C^*/\epsilon$ is est. depth of optimal solution in worst case | Completeness: Complete if $\epsilon > 0$ and C^* is finite (if $\epsilon = 0$, zero cost cycle may occur) | Optimality: Optimal if $\epsilon > 0$
  Note: BFS is special case of UCS where step cost == 1 for every edge
- **DFS:** Stack Frontier | Time Complexity: $O(b^m)$ where b is branching factor, m is max depth | Space Complexity: $O(bm)$ as only 1 path is expanded at one time | Completeness: Not complete (when depth is infinite or can go back or forth) | Optimality: Not optimal (there can be paths with less cost not explored yet)
- **DLS (Depth Limited Search):** Limit the search depth to l where $l <= m$, backtrack once depth limit is reached | Time Complexity: $O(b^l)$ | Space Complexity: $O(bl)$ | Completeness: Not complete when soln lies deeper l | Optimality: Not optimal when soln lies deeper than l
  Note: We dk the depth of solution, which is a downside
- **IDS (Iterative Deepening Search):** Do DLS with max depth of $0, \ldots, \infty$ $\rightarrow$ return soln if found, otherwise increase depth | Time Complexity: $O(b^d), Overhead = (n_{IDS} - n_{DLS})/n_{DLS}$ | Space Complexity: $O(bd)$ | Completeness: Complete | Optimality: Optimal if step cost is same everywhere
  Note: IDS is not always faster than DFS $\longrightarrow$ Consider state space s.t. each state has only single successor & goal node is at depth n $\longrightarrow$ IDS will run in $O(n^2)$, DFS will run in $O(n)$
- **Backward Search:** Search from goal
- **Bidirectional Search:** Combine forward search & backward search, stop when 2 searches meet | Time Complexity: $2 * O(b^{d/2}) < O(b^d)$

2.4 Search Algorithms (Informed Search)

- **Key Idea:** Search Algo has a clue on how close a state is to the goal
- **Best First Search:** Priority Queue ($f(n)$) Frontier, where $f(n)$ estimates the goodness of a state (Node with lowest $f(n)$ is selected first to be expanded) | $f(n)$ can be purely heuristic (estimated cost from n to goal) or a combi of path cost & heuristic
- **Greedy Best First Search:** Priority Queue ($f(n) = h(n)$) Frontier, where $h(n)$ is heuristic function that est. cost from n to goal (Expands node that seems closest to goal according to $h(n)$ without considering path cost so far) | Time Complexity: $O(b^m)$ | Space Complexity: $O(b^m)$ | Completeness: Not complete since GBFS might keep expanding nodes based on $h(n)$ without ever finding goal | Optimality: Not optimal since GBFS selects nodes based on $h(n)$ without considering path cost
- A^* **Search:** Priority Queue ($f(n) = g(n) + h(n)$) where $g(n)$ is cost so far to reach n | Time Complexity: $O(b^m)$ | Space Complexity: $O(b^m)$ | Completeness: Complete | Optimality: Optimal
  - If $h(n)$ is admissible $\rightarrow A^*$ using Tree search is optimal
  - If $h(n)$ is consistent $\rightarrow A^*$ using Graph search is optimal
  - *Note: UCS is special case of A^* search where $h(n) = 0$*

2.5 Heuristics

- Estimate cost from node n to goal
- **Admissible Heuristics:** For every node n, $h(n) \le h^*(n)$, where $h^*(n)$ is true cost to reach goal state from n (Never over-estimate)
- **Consistent Heuristics:** For every node n, every successor n' generated by action a, $h(n) \le c(n, a, n') + h(n')$ and $h(G) = 0$ (Proof $h(n) - h(n') \le c(n, a, n')$)
  Note: If $h(n)$ is consistent, $f(n') \ge f(n) \rightarrow f(n)$ is non-decreasing along any path $\longrightarrow$ Nodes are expanded in order of increasing f cost
- **Dominance:** If $h_2(n) \ge h_1(n)$ for all $n \rightarrow h_2$ dominates h_1 | If h_2 is admissible $\rightarrow h_2$ is better for search
- **Creating Admissible Heuristics:**
  - Problem with fewer restrictions on actions is called a relaxed problem
  - Cost of an optimal soln to a relaxed problem is an admissible h for original problem

3 Local Search & Adversarial Search

3.1 Local Search

- **Assumptions:** Agent is a Goal/Utility-based agent, Env has a very large state space
- **Informed & Uninformed Search VS Local Search**
  1. IUS: Low to moderate state space | Optimal or no soln | Search path is usually the soln
  2. LS: Very large state space | Good enuf soln is preferable rather than no soln | State is the soln (don't care about search path)
- **Local Search Overview:**
  - Basic Idea: Start somewhere in state space, move towards a better spot
  - Problem Formulation: States(state space), Initial State(initial state of agent), Goal test (optional, coz we actually dk the goal state, rely on eval function instead), Successor Function (possible states from a state), Evaluation Function (Output value/goodness of a state)
- **Hill Climbing Algorithm**

```
current = initial state

while True:
    neighbor = a highest-valued successor of current
    if value(neighbor) <= value(current):
        return current
    current = neighbor
```

- Known as Greedy Local Search (pick best amongst neighbors, repeat)
- Best Soln: State space where eval. function has a max value (global max)
- Disadvantages: Cannot reach global max if it enters local max, plateau | Sensitive to choice of initial state, poor initial state may result in poor final state (Can overcome with random restarts, walks)
- **Simulated Annealing**

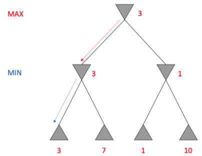
```
current = initial state
T = a large positive value
while T > 0:
    next = a randomly selected successor of current
    if value(next) > value(current): current = next
    else with probability P(current, next , T): current = next
    decrease T
return current
```

- P(current, next, T) = $e^{(value(next) - value(current))/T}$
- More exploration of bad states is allowed when T is high, more exploitation is done when T is low $\rightarrow$ basically choosing worse successor may lead to a better max
- Theorem: If T decreases slowly enough, SA will find global optimum with high probability

3.2 Adversarial Search

- **Assumptions:** Agent is Utility-based | Env is a game (game cannot be single player, partially observable, stochastic, but must be fully observable, deterministic, discrete, terminal states exist, 2 players, zero-sum, turn taking)
- **Minimax Algorithm:**

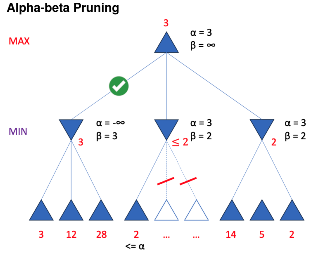
```
def minimax(state):
    v = max_value(state)
    return action in successors(state) with value v

def max_value(state):
    if is_terminal(state): return utility(state)
    v = -∞
    for action, next_state in successors(state):
        v = max(v, min_value(next_state))
    return v

def min_value(state):
    if is_terminal(state): return utility(state)
    v = ∞
    for action, next_state in successors(state):
        v = min(v, max_value(next_state))
    return v
```



Assumes opponent play **optimally**: trying to **minimize** player's value

- Intuition: MAX wins when utility is high, MIN wins when utility is low | Assign utility values to all terminal states & start tracing from terminal states $\longrightarrow$ Eventually, all states will have utility values, starting player can choose a state that will max/min his utility
- Analysis: Completeness: Complete if tree is finite | Time Complexity: $O(b^m)$ | Space Complexity: $O(bm)$ depth first exploration | Optimality: Optimal against optimal opponent

- **Alpha-beta Pruning**

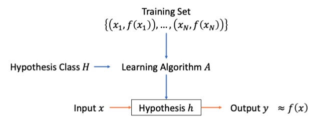


- Definitions: α is best explored option to the root for MAX player (Highest value for MAX) | β is best explored option along path to the root for MIN player (Lowest value for MIN)
- Procedure: 1. Assign $\alpha = -\infty$, $\beta = \infty$ for root 2. Propagate values down to the terminal node 3. Update α value at MAX node, β value at MIN node 4. Propagate values up 5. Prune branches of nodes where $\alpha \ge \beta$
- **Minimax with Cutoff**
  - Instead of calling $is\_terminal$, call is_cutoff which returns TRUE if (1): State is terminal or (2): Cut-off is reached
  - Instead of using $utility$, call $eval$ which is an eval. function that returns (1): Utility for terminal states or (2): Heuristic value for non-terminal states

4 Introduction to ML & Decision Trees

4.1 Introduction to ML

- **Definitions:** Computer program is said to learn from experience E w.r.t. some class of tasks T & performance measure P, if its performance at tasks in T, as measured by P, improves with experience E
- **Types of Feedback:**
  1. Supervised Learning: Involves training a model on a labeled dataset, where input data is paired with correct output $\longrightarrow$ Model learns to map inputs to outputs based on this labeled data, allowing it to make predictions on new data
     - Regression: Predict continuous input
     - Classification: Predict discrete input
  2. Unsupervised Learning: Deals with dataset that do not have labeled outputs $\longrightarrow$ Goal is to identify patterns & structures within data
  3. Reinforcement Learning: Agent learns to make decisions by interacting with an environment $\longrightarrow$ Agent receives feedback in the form of rewards or penalties based on its actions $\longrightarrow$ Learns optimal behaviors over time
- **Formal Definitions:**



4.2 Performance Measure

- **Regression:**

For a set of N examples $\{(x_1, y_1), \ldots, (x_N, y_N)\}$ we can compute the average (mean) squared error as follows.

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2$$

Where $\hat{y}_i = h(x_i)$ and $y_i = f(x_i)$.

For a set of N examples $\{(x_1, y_1), \ldots, (x_N, y_N)\}$ we can compute the average (mean) absolute error as follows.

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|\hat{y}_i - y_i|$$

Where $\hat{y}_i = h(x_i)$ and $y_i = f(x_i)$.

- **Classification:**

Classification is correct when the prediction $\hat{y} = y$ (true label).

For a set of N examples $\{(x_1, y_1), \ldots, (x_N, y_N)\}$ we can compute the average **correctness** (accuracy) as follows.

$$Accuracy = \frac{1}{N}\sum_{i=1}^{N}\mathbb{1}_{\hat{y}_i = y_i}$$

Where $\hat{y}_i = h(x_i)$ and $y_i = f(x_i)$.

Predicted Label \ Actual Label	Cancer	Benign
Cancer	2 True Positive	1 False Positive
Benign	3 False Negative	4 True Negative

- **Accuracy:** $\frac{TP+TN}{TP+FN+FP+TN}$
- **Precision:** $\frac{TP}{TP+FP}$ (How many selected items are relevant, maximise if FP is costly)
- **Recall:** $\frac{TP}{TP+FN}$ (How many relevant items are selected, maximise if FN is dangerous)
- **F1 Score:** $\frac{2}{1/precision + 1/recall}$

4.3 Decision Trees

- **Traits of Decision Trees:**
  - Decision Trees can express any function of input attributes
  - Consistent Decision Tree for any training set, but probably will not generalize to new examples
  - # of distinct decision trees with n boolean attributes = 2^{2^n}
- **Decision Tree Learning Algorithm**

```
def DTL(examples, attributes, default):
    if examples is empty: return default
    if examples have the same classification:
        return classification
    if attributes is empty:
        return mode(examples)
    best = choose_attribute(attributes, examples)
    tree = a new decision tree with root best
    for each value v_i of best:
        examples_i = {rows in examples with best = v_i}
        subtree = DTL(examples_i, attributes − best, mode(examples))
        add a branch to tree with label v_i and subtree subtree
```

- mode: Category with the highest number
- choose attribute: Chooses attribute with the highest information gain
- **Choosing an attribute:**
  - Ideally select an attribute that splits examples into "all positive" or "all negative"
  - Entropy (Measure of randomness): $I(P(v_1), \ldots, P(v_n)) = -\sum_{i=1}^{n} P(v_i) log_2 P(v_i)$, where for data set containing p positive & n negative examples, $I(\frac{p}{p+n}, \frac{n}{p+n}) = -\frac{p}{p+n} log_2 \frac{p}{p+n} - \frac{n}{p+n} log_2 \frac{n}{p+n}$
    Note: I(1,0) = I(0,1) = 0, I(0.5, 0.5) = 1
  - Information Gain (Entropy of curr. node - Total Entropy of children nodes): $IG(A) = I(\frac{p}{p+n}, \frac{n}{p+n}) - remainder(A)$
    remainder(A) = $\sum_{i=1}^{v} \frac{p_i + n_i}{p+n} I(\frac{p_i}{p_i+n_i}, \frac{n_i}{p_i+n_i})$, where examples are split into v subsets by attribute A
- **Dealing with continuous valued attributes:** Define a discrete valued input attribute to partition values into discrete set of intervals
- **Dealing with missing values:** Assign most common value of attribute, assign probability to each value and sample, drop attribute, drop rows
- **Overfitting:** Decision Tree is perfect on training data, but worse on test data
- **Occam Razor:** Prefer short/simple hypothesis (long/complex hypothesis that fits data may be coincidence)
- **Pruning:** Prevents nodes from being split even when it fails to cleanly separate examples (Min samples leaf: Merge until leaf node is above min. samples number | Max depth: Merge until leaf nodes are at depth less than max depth)

5 Linear Regression

5.1 Linear Regression Basics

- **Purpose:** Given $h_w(x) = w_0 + w_1 x + \ldots$, find a w that fits the data well
- **Mean Squared Error (Loss Function):**
  $J_{MSE}(w) = \frac{1}{2m}\sum_{i=1}^{m}(h_w(x^{(i)}) - y^{(i)})^2$
  1. m is the number of training examples
  2. $h_w(x^{(i)}) == \hat{y}^{(i)}$
  3. $y^{(i)}$ is actual y value for the ith training example
- **Partial Derivative:**
  $\frac{\partial J_{MSE}(w)}{\partial w_1} = \frac{1}{m}\sum_{i=1}^{m}(w_1 x^{(i)} - y^{(i)})x^{(i)}$ (when we differentiate $J_{MSE}(w)$ wrt. particular weight, i.e. w_1)

5.2 Gradient Descent

- **Gradient Definition:** $\begin{bmatrix} \frac{\partial J(w)}{\partial w_0} \\ \frac{\partial J(w)}{\partial w_1} \\ \ldots \end{bmatrix}$, where each expression is a partial derivative wrt. different weights
- **Gradient Descent Algorithm:**
  1. Start at some w
  2. Pick a nearby w that reduces $J(w)$:
     $w_j \leftarrow w_j - \gamma \frac{\partial J(w_0, w_1, \ldots)}{\partial w_j}$
  3. Repeat until minimum is reached
- **Gradient Descent with 2 parameters:** Always store gradient in intermediate variable, then conduct gradient descent on each weight separately $\longrightarrow$ Do not conduct gradient descent on all weights concurrently (or else a new weight will be updating another weight unknowingly)
- **Theorem:** $J_{MSE}(w)$ is convex for linear regression $\longrightarrow$ 1 minimum, global minimum only

5.3 Gradient Descent Variants

1. **Batch Gradient Descent:** All training examples involved
2. **Mini-batch Gradient Descent:** Subset of training examples at a time; Cheaper (faster); Random, may escape local minima for non-convex function
3. **Stochastic Gradient Descent:** 1 random data point at a time (Order which we use the samples should be randomly decided); Cheapest (fastest); More random, may escape local minima for non-convex function

5.4 Extension of Linear Regression

- **Features of different scales:** A feature with smaller magnitude will take much smaller steps each update than another feature with larger magnitude $\longrightarrow$ Converging becomes slower
  1. Solution 1: Have different learning rates for each weight
  2. Solution 2: Conduct mean normalisation $x_j \leftarrow \frac{x_j - \mu_j}{\sigma_j}$, where σ_j is the standard deviation of the feature across all training examples & μ_j is the mean of the feature across all training examples
- **Non-linear relationship:** Use polynomial regression for non-linear relationship (transform features) — Terms that are raised to a power more than 1 might need to be scaled as they can become too big — Max degree of polynomial needed to fit any set of n points is $n - 1$ (otherwise will overfit)

5.5 Normal Equation

- **Normal Equation Procedure:**

$$X = \begin{bmatrix} 1 & x_1^{(1)} & & x_n^{(1)} \\ 1 & x_1^{(2)} & \dots & x_n^{(2)} \\ 1 & \vdots & & \vdots \\ 1 & x_1^{(m)} & & x_n^{(m)} \end{bmatrix} \quad w = \begin{bmatrix} w_0 \\ w_1 \\ \dots \\ w_n \end{bmatrix} \quad Y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \dots \\ y^{(m)} \end{bmatrix}$$

Bias

$$h_w(X) = Xw$$

$$\text{Set } \frac{\partial J_{MSE}(w)}{\partial w} = 0$$

A bunch of math...

$$2X^T Xw - 2X^T Y = 0$$
$$2X^T Xw = 2X^T Y$$
$$X^T Xw = X^T Y$$
$$\boxed{w = (X^T X)^{-1} X^T Y}$$

Assume invertible

$X^T X$ becomes non-invertible when it is singular, which occurs when its rows (repeated or identical data points) or columns (redundant or highly correlated features) are linearly dependent

- **Gradient Descent VS Normal Equation:**

	Gradient Descent	Normal Equation
Need to choose γ	Yes	No
Iteration(s)	Many	None
Large number of features n?	No problem	Slow, $(X^T X)^{-1} \rightarrow O(n^3)$
Feature scaling?	May be necessary	Not necessary
Constraints	-	$X^T X$ needs to be invertible

6 Logistic Regression

6.1 Logistic Regression Basics

- Used for classification problems
- **Logistic Regression (1D)**
  - Logistic Function: $\sigma(z) = \frac{1}{1+e^{-z}}$, where $z = wx$
  - Output of $\sigma(z)$ aka. $h_w(x)$ is in [0,1] and treated as a probability $\longrightarrow$ If $\sigma(z) > \alpha$, then label as 1
- **Logistic Regression (2D):** Decision boundary is a line of intersection between prediction boundary plane and plane containing all the prediction points $\longrightarrow$ Decision boundary is perpendicular to w

6.2 Measuring Fit

- **Why MSE is bad for Logistic Regression:** J_{MSE} for logistic regression would not work well as it is non-linear $\longrightarrow$ non-convex $\longrightarrow$ multiple local minima
- **Cross Entropy for C classes:** $CE(y, \hat{y}) = \sum_{i=1}^{C} -y_i log(\hat{y}_i)$ (Measures the average number of bits required to identify an event from 1 probability distribution $\longrightarrow$ measures the difference between discovered probability distribution of a classification model & predicted values)
- **Binary Cross Entropy for 2 classes:** $BCE(y, \hat{y}) = -ylog(\hat{y}) - (1 - y)log(1 - \hat{y})$
  1. Given $y = 1$ & value of $\hat{y}$ is high, model will be rewarded for making a correct prediction
  2. Given $y = 1$ & value of $\hat{y}$ is low, model will be penalised for making a wrong prediction
- **BCE Loss Function:** $J_{BCE}(w) = \frac{1}{m} \sum_{i=1}^{m} BCE(y^{(i)}, h_w(x^{(i)}))$, which is convex for logistic regression $\longrightarrow$ Can find global minimum during gradient descent

6.3 Gradient Descent

- **Partial Derivative:**

$$\frac{\partial J_{BCE}(w)}{\partial w_0} = \frac{1}{m} \sum_{i=1}^{m} \left(h_w(x^{(i)}) - y^{(i)} \right)$$
$$\frac{\partial J_{BCE}(w)}{\partial w_1} = \frac{1}{m} \sum_{i=1}^{m} \left(h_w(x^{(i)}) - y^{(i)} \right) x_1^{(i)}$$