20296 Advanced Microeconomics
Group Project

Problem 1

Sem Manna - 3087964 Giovanna Milone - 3062147 Friedrich Schaper - 3074167

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1 Question 1

Document the fact described above about the difference between hours worked in the US vs Europe in recent years, if needed even across different European countries.

To document the difference in hours worked we consider two main indicators: *annual hours worked* and *average weekly hours worked*. Our full sample comprises the United States of America (USA) and 9 other high-income Western European Countries in the OECD: Austria (AUT), Belgium (BEL), Germany (DEU), Spain (ESP), France (FRA), Great Britain (GBR), Italy (ITA), and the Netherlands (NLD). Our full sample, sourced from the OECD Statistics Web Browser App, includes annual observations from 2010 to 2021. Because the European sovereign debt crisis crippled most Eurozone economies in the early 2010s and given how Governments responded heterogeneously to the Covid-induced crisis, our main sample is restricted to the 5 years from 2014-2019. This is done to ensure the representatives of the data to the underlying labour-market conditions and labour regulations during ordinary times, as well as to improve the comparability of the data across Countries.

1.1 Annual hours worked

We start by considering average annual hours actually worked per worker, an aggregate measure computed as the total number of hours worked over the year divided by the average number of people in employment. Total hours worked are computed considering both part- and full-time workers. Yet, because the measure is sourced by the OECD from the National Accounts of each Country, sources differ and comparisons should be taken with a grain of salt. Nevertheless, this measure provides preliminary evidence of the differences in working hours between the US and European Countries, summarized in Table [1] over the 2014-2019 period.

Table 1: Cross-country differences in the Average Annual number of hours worked per worker by employment category

Average Annual Hours Actually Worked per Worker

	Total Employment		Dependent Employment	
	Avg ('14-'19)	% Diff to USA	Avg ('14-'19)	% Diff to USA
AUT	1,503	-16%	1,427	-20%
BEL	1,578	-11%	1,438	-19%
DEU	1,392	-22%	1,332	-25%
ESP	1,693	-5%	1,613	-10%
EUR AVG	1,546	-13%	1,460	-18%
FRA	1,517	-15%	1,423	-20%
GBR	1,536	-14%	1,509	-16%
ITA	1,717	-4%	1,577	-12%
NLD	1,433	-19%	1,364	-24%
USA	1,780	-	1,786	-

Note: EUR AVG refers to the unweighted average for the selected European Countries. "Diff to USA" stands for the per cent difference from the US annual hours actually worked (computed as a share of the US average between 2014 and 2019). Source: OECD Statistics for the USA and selected Western European Countries, available here .

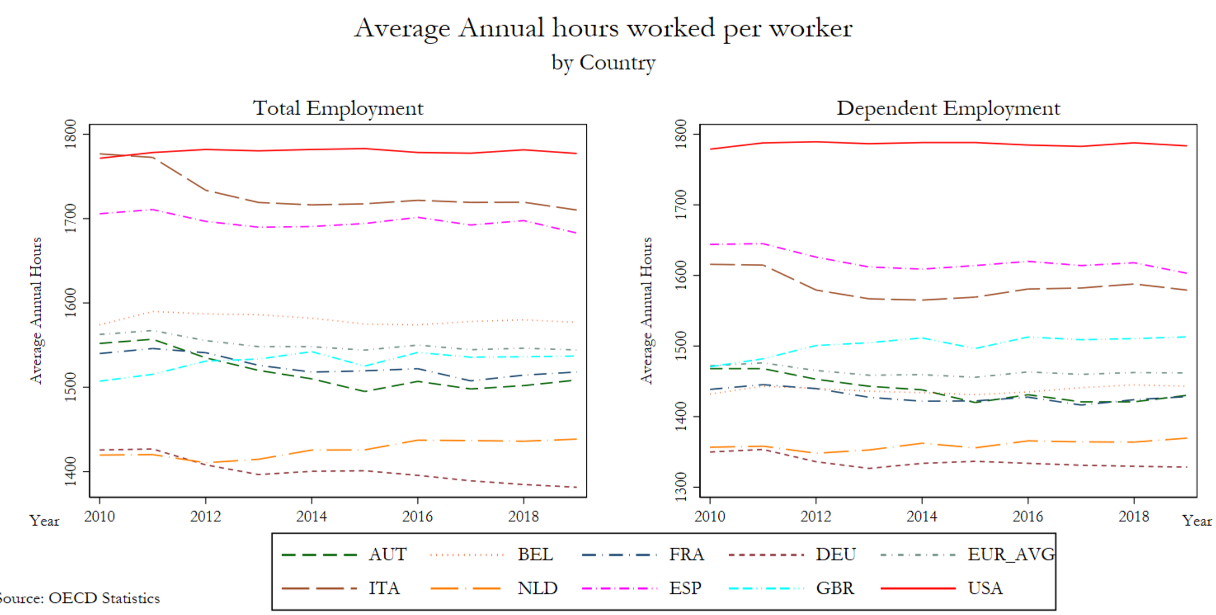
American workers top the ranking of annual hours worked for both dependent and independent employment. Between 2014 and 2019 American workers spent, every year, an average of 1,780 hours on the job − 388 hours more than their German counterparts. This difference is consistent also across all other major European countries, as summarized by the percentage difference in average annual hours worked by European vs US workers under the heading "Diff w.r.t. USA" of Table [1]. Even in absolute terms, this translates into huge gaps: Americans worked 263 hours more than the French, 244 more than the British, and 87 and 63 more than, respectively, Italians and Spaniards. On average, between 2014 and 2010, European workers in our main sample worked around 13% less than Americans did.

Those differences are even starker if we restrict the comparison only to those employed in dependent work. Compared to Europeans, the average American employee works an average of 454 hours more than Germans, 362 hours more than the French, 277 more than the British, 208 more than the Italians and 173 more than the Spanish. On average, Europeans under dependent job contracts worked a staggering 18% less than American employees.

A possible explanation for the widening gap between Europe and the US when restricting the analysis to dependent employment is that dependent labour is more heavily regulated in Europe than in the US, while self-employed workers enjoy more freedom everywhere. The qualitative comparison between columns 2 and 4 in Table [1] could serve as suggestive evidence of this underlying systematic difference.

It is also worth noting how the average annual hours worked by all workers remained fairly stable in the 5 years from 2014 to 2019. If we extend our analysis to include the years from 2010, while the annual hours worked by US workers remain stable, we see a qualitative but sizeable decline across most European countries, as summarized in Figure [1]. The UK, for which worked hours rose throughout our sample, is a notable exception. This possibly hints at differences in the cultural determinants of labour supply, which are more similar across the two Anglo-Saxon countries.

Figure 1: Average Weekly Hours Worked per Worker on the Main Full-Time Job, by Country



As anticipated, because the OECD figures for annual hours worked are sourced by National Accounts employing different methodologies, they are best meant for comparing time trends rather than levels across Countries. Figure [1] represents those trends for our main sample. From it, we can also note how, rather than being a peculiarity of the '14-'19 period, this gap in hours worked by American and European workers is sizable and consistent across the entirety of the last decade. As outlined by the solid red line, Americans consistently worked more hours in both dependent and total employment.

Because this measure is not fit for cross-country comparisons, we move to a qualitative analysis of the average weekly hours worked. Yet, it must be noted that a comparison of yearly hours can more comprehensively capture the overall differences in working conditions, accounting for differences in paid leave and holidays which are not reflected in the contractual weekly hours.

1.2 Weekly hours worked

Narrowing our analysis to differences in the average weekly hours worked on the main job helps unveil a potential mechanism for the observed differences in the annual hours worked. The weekly data is available only for contracts concerning dependent employment, which is in line with our main interest in addressing differences stemming from regulatory and policy conditions, which tend to impose stricter restrictions (through regulation or union bargaining) on dependent rather than independent employment. Table [2] summarizes the differences in the average weekly hours worked by European and American workers on their main full- or part-time job, as well as on the total declared employment.

Table 2: Cross-country differences in the average weekly hours worked per employment category

Average Weekly Hours Worked on the Main Job per Worker in Dependent Employment (all Genders and Ages)

	Full Time		Part Time		Total Declared Employment	
	Avg ('14-'19)	Diff w.r.t USA	Avg ('14-'19)	Diff w.r.t USA	Avg ('14-'19)	Diff w.r.t USA
AUT	40.32	-3%	17.63	-4%	35.69	-8%
BEL	38.77	-7%	20.34	10%	35.40	-8%
DEU	39.64	-5%	17.12	-7%	34.76	-10%
ESP	39.71	-4%	16.38	-11%	34.52	-11%
EUR AVG	39.51	-5%	17.86	-3%	36.47	-6%
FRA	38.90	-6%	19.44	5%	36.17	-6%
GBR	41.94	1%	18.27	-1%	36.67	-5%
ITA	39.38	-5%	20.02	9%	35.56	-8%
NLD	37.39	-10%	16.38	-11%	29.26	-24%
USA	41.54	-	18.43	-	38.63	-

Note: EUR AVG refers to the unweighted average for the selected European Countries. "Diff to USA" stands for the per cent difference from the US annual hours actually worked (computed as a share of the US average between 2014 and 2019). Source: OECD Statistics for the USA and selected Western European Countries, available here.

Notably, Americans in full-time employment work more than everyone else in the sample but the British. Compared to the average American full-time employee, the average French works 2.64 fewer hours per week, a 6% decrease from the US average. This difference is still significant, at 2.16 hours, for Italians, Germans (1.90), and Spaniards (1.83). Every week, the average European works 5% fewer hours than its American counterpart.

When looking at part-time work the picture looks rather different, with employees in multiple countries working *more* hours compared to American workers. On average, Europeans still work 3% fewer hours than Americans do. Yet, because our metric only considers hours worked on the **main job**, those differences may hide underlying discrepancies in the number of jobs held per person in the USA as compared to Europe. If Americans tend to take up multiple part-time jobs while most Europeans tend to only work one job, this metric may misrepresent the true difference in the total amount of work supplied by part-time employees in Europe and the US.

An important takeaway from Table [2] is that the largest (percentual) difference is in the Total Declared Employment category, hinting instead at the fact that, overall, the labour supply of the average American is still well above that of European workers, who work on average 6% less.

Yet, weekly differences should also not be taken at face value as they hide cross-country heterogeneity in the annual leave time (paid or unpaid) that workers are allowed to take. Because Europeans have more paid leave (Ray et al. (2013)), even comparable differences in the weekly hours could lead to significant yearly gaps.

Notably, as shown in Figure [2], those differences are consistent over time even when extending our analysis to the 2010-2019 time span.

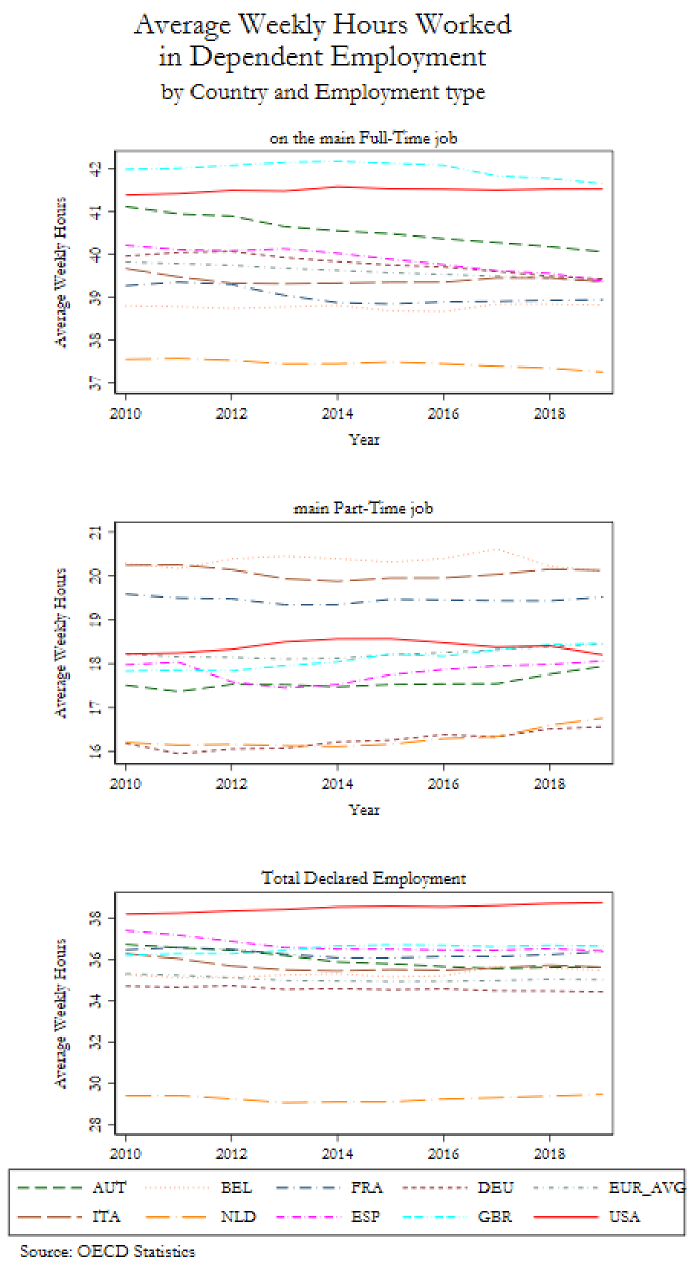


Figure 2: Average Weekly Hours Worked, by Job Type and Country

2 Question 2

One explanation that has been put forward - for example, by Ed Prescott in 2004 ('Why do Americans work so much more than Europeans?') - is higher taxes. Summarize the main differences in the income tax between US and Europe (or a few European countries). Write down a model of labour supply and derive formally the conditions under which the European tax system will lead to fewer hours worked than in the US. [Hint: start with the simplest case of differences only in the average tax rates.]

2.1 Empirics

To Summarize the main differences in the income tax between US and Europe we consider the OECD's Taxing Wages Dataset. This dataset provides information on income taxes borne by workers in all the OECD member states. It reports multiple measures accounting also for social security contributions, family benefits received (cash transfers) and social security contributions and payroll taxes paid by employers. Because we care about studying how income taxation affects the labour supply, we will simply focus on the *average income tax rate as a percentage of gross wage earnings*. This metric is not representative of the resulting disposable income or overall welfare conditions of workers, as it masks the underlying cross-country heterogeneity in income compensation and welfare schemes present. Yet, it should limit the possibility of confounding our results and should be broadly representative of the tax burden faced by the workers in a Country. Moreover, because in the US many welfare measures come in the form of tax deductions (rather than public services), we are likely to underestimate the true tax differential faced by most individuals in Europe compared to the US.

All the measures in the OECD database are recorded for 8 distinct household categories distinguished by marital status, number of earners, number of children, and earnings levels (expressed as a percentage of the average wage). In Table [3] we report the *average annual tax rate* for 3 selected household types, broadly representative of the heterogeneity in income tax schemes across household conditions. Note that, for a more comprehensive understanding of the impact of taxation on labour supply, one should also consider the marginal tax rate. We have this data in the OECD database, but we chose to focus on the average tax rate in coherence with our model where there is a constant tax rate (see Section 2.2).

Strikingly, the difference between US and European workers varies a lot by family type. Most of this seems to stem from reliance on tax deductions to support single-parent families: a single-earner household with two children and earnings at 67% of the Country average in Europe faces an income tax rate of 3.80% of gross income, while in the US they would be receiving a subsidy of almost 4% of their annual income. Yet this relationship changes when we look at the same worker without children: the average tax rate spikes both in the US and Europe, but on average, workers in the selected European countries pay a tax rate 4 percentage points lower, an almost 20% decrease from the US average of 14.79%. Looking at a more populous and perhaps representative category of households we get a result more in line with the results from Prescott (2004). Married couples with two earners, one at the national average and one at 67% of the national average, with two children face higher taxes in all selected European countries but Germany and Spain. Considering the unweighted average, in this case, partly driven by Belgian workers, Europeans pay on average 17% higher taxes than Americans do.

Table 3: Cross-country differences in the average income tax rates per employment household type

Average Income Tax Rate as a % of Gross Wage Earnings

	Two-Earner Married Couple one at 100% of average earning, the other at 67%. With two children		Single Person at 67% of average earnings. With two children		Single Person at 67% of average earnings. With no child	
	Avg ('14-'19)	Diff w.r.t USA	Avg ('14-'19)	Diff w.r.t USA	Avg ('14-'19)	Diff w.r.t USA
AUT	12.08	4%	5.28	232%	9.70	-34%
BEL	22.47	94%	14.18	455%	19.69	33%
DEU	10.97	-6%	-2.38	40%	14.14	-4%
ESP	11.19	-4%	-3.23	19%	10.64	-28%
EUR AVG	13.53	17%	3.80	195%	12.03	-19%
FRA	11.69	1%	8.37	310%	11.94	-19%
GBR	12.90	11%	-1.21	70%	11.15	-25%
ITA	15.17	31%	5.06	227%	12.72	-14%
NLD	11.80	2%	4.33	208%	6.29	-57%
USA	11.61	-	-3.99	-	14.79	-

Note: EUR AVG refers to the unweighted average for the selected European Countries. "Diff to USA" stands for the per cent difference from the US average income tax rate (computed as a share of the US average between 2014 and 2019). Source: OECD Statistics for the USA and selected Western European Countries, available here.

2.2 Model

2.2.1 Baseline without taxes

Let us introduce a simple labour supply model: we have a representative household. For simplicity, let us assume that each household corresponds to a representative agent/consumer/worker so that the two terms will be used interchangeably throughout the exercise. The representative household maximises lifetime utility, whilst facing a trade-off between consumption C_t, which it likes, and labour N_t, which it dislikes but for which it receives a nominal wage W_t. We assume that there is no saving in our economy, so the agent solves a static problem each period t and has no intertemporal savings-versus-current consumption decisions to make.

Let us assume that the agent has a Constant Relative Risk Aversion (CRRA) utility:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{1}$$

where $\sigma > 0$ measures both the degree of risk aversion of agents (and makes the problem concave) and the relative strength of the income vis-à-vis the substitution effect governing the intratemporal optimality condition of the household, which - as is shown below - determines the optimal labour supply decision. The parameter φ is the so-called *Frisch elasticity of labour supply*, which captures the elasticity of hours worked to the wage rate, given a constant marginal utility of wealth.

The problem of lifetime utility maximisation is equivalent to one where the agent maximises utility in each period t. The problem can be thus stated as follows:

$$\max_{\{C_t, N_t\}} \quad \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{2}$$

$$\text{s.t.} \quad P_t C_t \leq W_t\, N_t$$

From this we get the following Lagrangian

$$\mathcal{L} = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t \left[W_t N_t - C_t P_t \right]$$

yielding the following first-order conditions (FOCs henceforth):

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t}: &\quad C_t^{-\sigma} - \lambda_t P_t = 0 \iff C_t^{-\sigma} = \lambda_t P_t \\
\frac{\partial \mathcal{L}}{\partial N_t}: &\quad -N_t^{\varphi} + \lambda_t W_t = 0 \iff N_t^{\varphi} = \lambda_t W_t
\end{aligned}$$

which we can combine to obtain

$$\frac{C_t^{-\sigma}}{P_t} = \frac{N_t^{\varphi}}{W_t}$$

or, alternatively

$$\frac{N_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{3}$$

That is, the agent equates his real wage to his marginal rate of substitution between leisure and consumption.

Furthermore, since $\lambda = \frac{C_t^{-\sigma}}{P_t} > 0$, by the complementary slackness condition on the constraint, we conclude that the budget constraint must be binding. Intuitively, this result makes sense from an economic perspective: if there is no possibility to save and utility is strictly increasing in consumption, at the optimum the household consumes the entirety of its labour income (expressed in real terms). If it were not doing so, it could improve its utility by consuming an additional $\varepsilon > 0$, arbitrarily small, and strictly improve its objective function (utility), contradicting the fact that the previous allocation was assumed to be the optimal one. Hence, at the optimum, we must have $P_t C_t = W_t N_t$.

2.2.2 Introducing taxes

Let us now introduce taxes to the model. In particular, we introduce a positive tax rate τ paid on wage income. The tax rate is constant over time and we assume that agents do not receive any utility in return for their taxes − say, from Government spending on public goods funded through tax revenues. An example of this could be a feudal system, where individuals are obliged to pay tributes to their lord in the form of taxation. In a more modern context, we can assume that all tax revenue is used for military expenditure, which does not enter households' utility function, neither directly nor indirectly and is thus disregarded entirely in their maximisation problem.

We take τ to be the single tax rate in the economy, measured as a constant percentage of income. Adding a positive tax rate, the consumer now solves the following problem:

$$\max_{\{C_t, N_t,\}} \quad \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{4}$$

$$\text{s.t.} \quad P_t C_t = (1-\tau) W_t \, N_t$$

From this we get the following Lagrangian

$$\mathcal{L} = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t \left[(1-\tau)W_t N_t - C_t P_t\right]$$

and the associated first-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t}: &\quad C_t^{-\sigma} - \lambda_t P_t = 0 \iff C_t^{-\sigma} = \lambda_t P_t \\
\frac{\partial \mathcal{L}}{\partial N_t}: &\quad -N_t^{\varphi} + \lambda_t (1-\tau) W_t = 0 \iff N_t^{\varphi} = \lambda_t (1-\tau) W_t
\end{aligned}$$

which we can combine into

$$\frac{C_t^{-\sigma}}{P_t} = \frac{N_t^{\varphi}}{(1-\tau)W_t}$$

or, alternatively

$$\frac{N_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t}(1-\tau) \tag{5}$$

This result shows how, at optimum, the agent equates her marginal rate of substitution to her real, after-tax, wage, that is, her wage in terms of consumption. This is just as in the baseline model − but now the agent also takes into account the reduction in her real disposable income due to the taxes (which reduces their net wage available for consumption spending).

By rearranging the optimality condition in Equation (5) we observe that labour supply is a decreasing function of the tax rate. We can therefore hypothesize that all else equal, workers in Countries with a higher tax rate should work less. Formally:

$$\begin{aligned}
N_t^{\varphi} &= C_t^{-\sigma}(1-\tau) W_t P_t^{-1} \\
\iff N_t &= \left[C_t^{-\sigma}(1-\tau) W_t P_t^{-1}\right]^{\frac{1}{\varphi}}
\end{aligned}$$

We can then assess the partial equilibrium effect. Taking consumption as fixed, we can see how the effect of a marginal increase in the tax rate on hours worked is negative:

$$\frac{\partial N_t}{\partial \tau}\Big|_{C_t = \overline{C}} = -\frac{1}{\varphi}\left[\overline{C}^{-\sigma}(1-\tau) W_t P_t^{-1}\right]^{\frac{1-\varphi}{\varphi}} \cdot \overline{C}^{-\sigma} W_t P_t^{-1} < 0 \tag{6}$$

as prices, consumption and wages are all positive.

We can also compute the elasticity of labour supply to the tax rate as:

$$\varepsilon_{N,\tau} = \underbrace{\frac{\partial N_t}{\partial \tau}}_{<0} \cdot \underbrace{\frac{\tau}{N_t}}_{>0} < 0 \tag{7}$$

where the partial effect of taxes on hours worked is negative − as computed above − and the second factor is always positive, so that the overall response is **always** negative.

Then, in this stylized model, the partial equilibrium effect of a positive marginal tax rate corresponds to a decrease in hours worked for any given level of the real wage with respect to the baseline with no tax. Intuitively, a tax rate acts as a wage markdown, reducing the returns to labour and so disincentivizing labour − graphically, this would correspond to an inward shift of the upward-sloping labour supply curve. Another way to interpret the result is that the opportunity cost of labour vis-à-vis leisure decreases as a result of the tax rate. Hence, by a standard substitution effect, households substitute away from labour as leisure becomes cheaper.

Taking this result into account, we justify the observed empirical cleavage in working hours in the US vs. Europe. In particular, assuming that for the bulk of workers it holds that $\tau_{EU} > \tau_{US}(> 0)$, our model would predict that the higher tax rate in most European Countries will result in a smaller per-household supply of working hours. As reported in Table [3] for the tax differential and in Table [2] for weekly hours worked, this prediction is consistent with the stylized facts observed across Countries.

3 Question 3

Some authors have claimed that individual-level estimates of labour supply suggest that the European-U.S. differences cannot be the result of higher taxes. Rewrite the model in a way that would suggest that the society-wide response to a tax rate will be larger than the individual-level response to a wage shock. Suggest some empirical implications of this model.

3.1 Aggregating the economy: Supply and Demand

To see why the overall societal effect of taxation might be greater than the individual effect we need to aggregate the economy. In the answer to the previous question, when analyzing the individual response to a tax rate, we found that a positive tax rate has a negative impact on the individual supply of hours of work. Since all workers are homogeneous in this economy, on aggregate, the labour supply decision of households will be symmetric in equilibrium. In particular, if all workers equally reduce their labour supply, aggregate supply will go down as well, and each worker will have a lower disposable income due to the tax she now pays and her reduced working hours. In turn, this contraction in aggregate demand will contribute to a reduction in output. Thus, in our simplified framework a higher tax rate can also reduce aggregate supply.

A secondary effect of this reduction in disposable income for every worker will be that aggregate demand for goods also goes down in response to the drop in spending power by the workers. Since there is less demand for goods, firms will hence also have less demand for labour and overall working hours are reduced even further. Therefore, a second-round effect is a reduction in aggregate demand and as a consequence, a further reduction in working hours follows. This way, individuals reduce their working hours in response to high taxes, causing a drop in their disposable income and consumption. The second round effects at the aggregate level further exacerbate the individual effects. Overall, output and working hours drop even more than if we only accounted for the individual response.

We formulate the household problem as we did in Question 2.2. Then, we can interpret the result also from an aggregate perspective, given the symmetric equilibrium, and denote individual consumption, when aggregated across all individuals in society, as aggregate demand. Note that for aggregate demand we abstract from agents' investment decisions as there is no saving nor intratemporal asset trade in this economy. This follows from our assumption that all agents are homogeneous. As a result, all agents would either want to save or borrow equally, hence the credit market would clear only when bonds are in *zero net supply*. We also abstract from imports and exports by assuming we are in a closed economy. Lastly, we once again disregard government spending in our demand function by assuming that the government spends tax revenues on goods outside of our model, such as national defence. One could, however, include government spending. In that case, the question of the size of the multiplier on government spending arises. As long as the multiplier is smaller or equal to one, our results would still hold. This assumption seems consistent with known results from empirical literature (Barro (1981), Christiano and Eichenbaum (1992)).

Concerning the aggregate supply, we need to consider the behaviour of the firms. To see this analytically, let us introduce a representative firm that maximizes profits by producing a good, using labour and technology. We assume that all firms are perfectly competitive in this economy.

The firm solves:

$$\max_{Y_t, N_t} \quad P_t Y_t - W_t N_t \tag{8}$$

$$\text{s.t.} \quad Y_t = A_t N_t^{1-\alpha}$$

For simplicity, let us use a constant return to scale (CRS) production function, and simplify the production function by assuming $\alpha = 0$. Then, the output becomes a linear function of labour hours:

$$Y_t = A_t N_t$$

At optimum, the firm equates its marginal cost to its price, as standard in the perfect competition model. Formally, the problem yields the following Lagrangian:

$$\mathcal{L} = P_t Y_t - W_t N_t + \lambda_t \left[A_t N_t - Y_t \right]$$

with the associated FOCs:

$$\frac{\partial \mathcal{L}}{\partial Y_t} : \quad P_t - \lambda_t = \iff P_t = \lambda_t$$
$$\frac{\partial \mathcal{L}}{\partial N_t} : \quad -W_t + \lambda A_t \iff W_t = \lambda A_t$$

which can be then combined to derive the firm's optimality condition,

$$P_t = \frac{W_t}{A_t}$$

where the left-hand side of the expression is the price, determined so as to clear the goods market, while the right-hand side is the marginal cost of production. Thus, at optimum, a representative firm produces up to when their price equals their marginal (labour) cost since we assume all firms to be perfectly competitive $-$ with zero profits in equilibrium.

We can rearrange it to get the following expression:

$$\frac{W_t}{P_t} = A_t \tag{9}$$

which determines the aggregate labour demand. The interpretation is the following: firms demand labour to the point where the real wage (left-hand-side of the equality) equals the marginal product of labour MPN_t (on the right-hand-side). From a graphical point of view, this implies that labour demand by firms is a downward sloping in the $W - N$ plane.

Equilibrium. An equilibrium allocation is described by allocation $\{N_t, C_t\}_{t=0}^{\infty}$ for given prices $\{W_t, P_t\}_{t=0}^{\infty}$, and a stochastic process for technology $\{A_t\}_{t=0}^{\infty}$, such that all agents in the economy are optimising and markets $-$ both the goods and the labour market $-$ clear.

Since our model is static and there is no saving, we study a partial static equilibrium at generic t.

In equilibrium goods markets clear, that is:

$$Y_t = C_t = A_t N_t$$

where the second equality follows from the production function of the firm.
Furthermore, by combining the first-order conditions of the representative firm (in Equation (9)) with the ones from the representative worker (in Equation(5)) we get:

$$A_t = \frac{W_t}{P_t} = N_t^{\varphi} C_t^{\sigma} (1-\tau)^{-1}$$

Let us further simplify our model to focus only on labour by assuming that technology $A_t = 1$. Labour supply can be expressed as follows:

$$N_t = \left[(1-\tau) C_t^{-\sigma}\right]^{\frac{1}{\varphi}} \tag{10}$$

Then, recall the partial equilibrium effect as derived in Section (2.2), Equation (6) is:

$$\frac{\partial N_t}{\partial \tau}\Big|_{C_t=\overline{C}} = -\frac{1}{\varphi}\left[\overline{C}^{-\sigma}(1-\tau)\right]^{\frac{1-\varphi}{\varphi}} \cdot \overline{C}^{-\sigma} < 0 \tag{11}$$

which has now been simplified using the optimality condition of the firm combined with our assumption on technology, that is, $W_t/P_t = A_t = 1$.

From market-clearing in both the goods and the labour market, we have $N_t = Y_t = C_t$. Then, the optimality condition on households' labour supply can be rewritten as:

$$\begin{aligned} &\frac{Y_t^{\varphi}}{Y_t^{-\sigma}} = (1-\tau) \iff \\ &\iff Y_t^{\varphi+\sigma} = (1-\tau) \iff \\ &\iff Y_t = (1-\tau)^{\frac{1}{\sigma+\varphi}} \end{aligned}$$

From this we obtain that the aggregate level of output depends negatively on the tax rate:

$$\begin{aligned} \frac{\partial Y_t}{\partial \tau} &= -\frac{1}{\sigma+\varphi}(1-\tau)^{\frac{1}{\sigma+\varphi}-1} = \\ &= -\frac{1}{\sigma+\varphi}(1-\tau)^{\frac{1-\sigma-\varphi}{\sigma+\varphi}} \end{aligned}$$

In order to assess the effect of the tax rate on hours worked, we can exploit the identity $Y_t = N_t$ and the fact that the effect can be decomposed as follows:

$$\frac{\partial N_t}{\partial \tau} = \underbrace{\frac{\partial N_t}{\partial Y_t}}_{=1} \cdot \frac{\partial Y_t}{\partial \tau} \tag{12}$$

from which we have that:

$$\frac{\partial N_t}{\partial \tau} = \frac{\partial Y_t}{\partial \tau} = -\frac{1}{\sigma+\varphi}(1-\tau)^{\frac{1-\sigma-\varphi}{\sigma+\varphi}} < 0 \tag{13}$$

We now need to assess that the general equilibrium effect of taxes on labour is larger − in absolute terms − than the partial effect counterpart.

This is the case when the aggregate response, in Equation (13) is larger than the individual one, in Equation (11). Hence, when the following condition is satisfied:

$$
\begin{aligned}
\frac{\partial N_t}{\partial \tau} > \frac{\partial N_t}{\partial \tau}\Big|_{C_t=\overline{C}} \iff & \\
& \iff -\frac{1}{\sigma+\varphi}(1-\tau)^{\frac{1-\sigma-\varphi}{\sigma+\varphi}} > -\frac{1}{\varphi}\left[\overline{C}^{-\sigma}(1-\tau)\right]^{\frac{1-\varphi}{\varphi}} \cdot \overline{C}^{-\sigma} \\
& \iff (1-\tau)^{\frac{1-\sigma-\varphi}{\sigma+\varphi}} < \frac{\sigma+\varphi}{\varphi}(1-\tau)^{\frac{1-\varphi}{\varphi}} \cdot \overline{C}^{\frac{-\sigma(1-\varphi)}{\varphi}-\sigma} \\
& \iff (1-\tau)^{\frac{1-\sigma-\varphi}{\sigma+\varphi}+\frac{\varphi}{1-\varphi}} < \frac{\sigma+\varphi}{\varphi}\overline{C}^{-\frac{\sigma}{\varphi}} \\
& \iff (1-\tau)^{\overbrace{\frac{(1-\sigma-\varphi)(1-\varphi)+\varphi(\sigma+\varphi)}{(\sigma+\varphi)(1-\varphi)}}^{:=T}} < \frac{\sigma+\varphi}{\varphi}\overline{C}^{-\frac{\sigma}{\varphi}} \\
& \iff (1-\tau) < \left[\frac{\sigma+\varphi}{\varphi}\overline{C}^{-\frac{\sigma}{\varphi}}\right]^{\frac{1}{T}} \\
& \iff \tau > 1-\left[\frac{\sigma+\varphi}{\varphi}\overline{C}^{-\frac{\sigma}{\varphi}}\right]^{\frac{1}{T}}
\end{aligned}
$$

And total output is obtained by imposing market clearing in the goods market.

$$Y_t = C_t = \left[(1-\tau)N_t^{-\varphi}\right]^{\frac{1}{\sigma}}$$

In equilibrium, a lower labour supply will translate into a decrease in consumption and output, i.e., a drop in aggregate demand. Firms will adjust by producing less, thus demanding less labour and total labour hours worked will thus decrease, giving rise to feedback from output to labour supply that goes beyond the labour supply and production decisions at the level of the household and of the firm, respectively. Thus, we can see that the society-wide response to a higher tax rate will be larger than the individual response to it.

We can see this analytically by noting that, because of this effect, aggregate output also negatively depends on τ. Thus, on aggregate, we can see that the sensitivity of labour with respect to taxes can be decomposed into two parts, the direct reduction in response to the higher tax rate, and the indirect effect through the elasticity of output to the tax rate. Figure [4] summarises this logic.

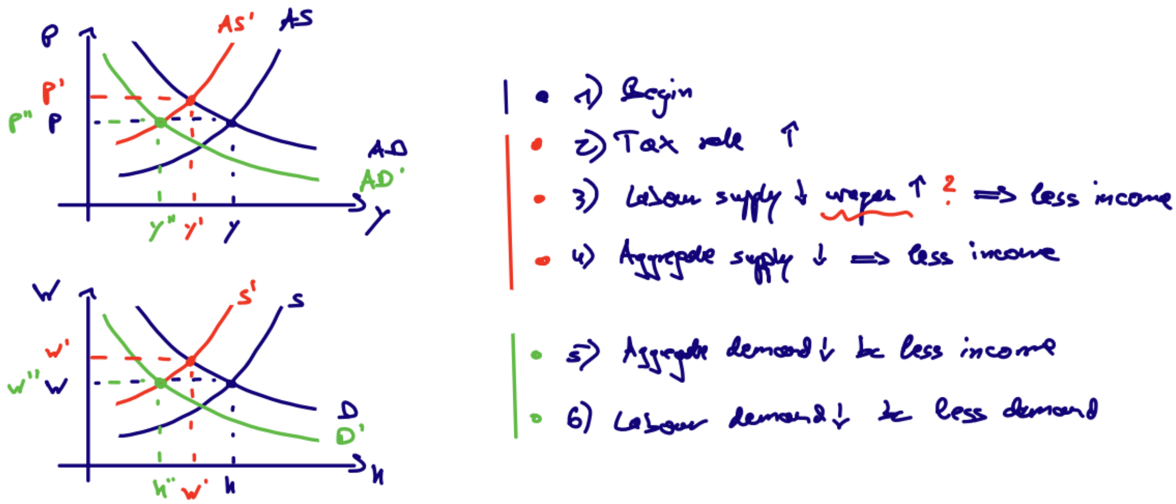


Figure 3: Aggregate demand and supply

3.2 Empirical implications

The main empirical prediction of this simplified model is that the introduction of a constant tax rate on wages − or, more generally, an increase in payroll taxes − unambiguously decreases the labour supply and, through it, total output in a country. Of course, this prediction would become less reliable once we account for heterogeneous workers and allow for income effects to dominate substitution effects for some workers. An interesting insight from our model is also that part of the aggregate decline in the equilibrium level of total hours worked passes through firms and a decline in the aggregate demand. A relevant empirical implication is then that wages stabilize through a suppression in firms' labour demand as the economy enters a depression.

Another possible empirical implication suggested by the model is that a tax-induced increase in the cost of labour inputs for firms may create an incentive to substitute human labour with technology and robots. This is because payroll taxes make labour relatively more expensive than capital. This thesis has found at least partial empirical validation: higher taxes on labour seem to increase reliance on automation as compared to low-skilled labour (Acemoglu et al. 2020). This is especially relevant in light of the recent wave of improvements in Artificial Intelligence technology, led by Natural Language Processing models that can both complement and substitute high-skilled labour.

4 Question 4

Another explanation that has been put forward is labour market regulation limiting the number of hours that Europeans can work. The impact of such regulation is obvious, but the welfare effects are more subtle. Can you derive the welfare effects of such labour legislation on workers and firms? How would your answer change if workers are heterogeneous?

4.1 Labour market regulation − Homogeneous case

4.1.1 Efficiency benchmark

In order to analyse any effects on welfare, we first define an efficiency benchmark. To this end, we formulate the social planner's problem, in which a planner maximises social welfare subject to a resource constraint and a production/technology constraint:

$$\max_{C_t, N_t} \quad U(C_t, N_t)$$
$$\text{s.t.} \quad \begin{cases} Y_t \leq A_t N_t & \text{Technology constraint } (\lambda) \\ C_t \leq Y_t & \text{Resource constraint } (\mu) \end{cases}$$

where the utility function is CRRA, as assumed in the previous points. Furthermore, note that we are able to write the problem as the maximisation of the utility of a representative agent as we do not assume any form of heterogeneity at this stage and thus the household problem is symmetric. Since the problem is concave and all constraints are linear, we can assume that utility is monotonically increasing in consumption, which implies that the equilibrium allocation will be non-wasteful, with $C_t = Y_t$, allowing us to can combine the two constraints.

Then, we can write the Lagrangian of the problem as follows:

$$\mathcal{L} = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda(A_t N_t - C_t)$$

Taking FOCs, we obtain:

$$\frac{\partial \mathcal{L}}{\partial C_t}: \quad C_t^{-\sigma} - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial N_t}: \quad -N_t^{\varphi} + \lambda A_t = 0$$

Combining the two FOCs, and noting that $\lambda > 0$ by non-negativity of the multipliers, we have:

$$\frac{N_t^{\varphi}}{C_t^{-\sigma}} = A_t$$

and, by complementary slackness, $A_t N_t = C_t$. Then, we can combine the constraint with the intratemporal optimality condition for labour-consumption allocation to obtain the efficient level of output and employment:

$$\frac{N_t^{\varphi}}{(A_t N_t)^{-\sigma}} = A_t \iff$$
$$\iff N_t^{\varphi+\sigma} = A_t^{1-\sigma} \iff$$
$$\iff N_t^{FB} = A_t^{\frac{1-\sigma}{\varphi+\sigma}}$$

where N_t^{FB} are the hours worked level in the first best (planner's solution).
Then, using $Y_t = A_t N_t$, the efficient level of output is

$$Y_t^{FB} = A_t N_t^{FB} = A_t^{\frac{1-\sigma}{\varphi+\sigma}+1} = A_t^{\frac{1+\varphi}{\varphi+\sigma}}$$

4.1.2 Competitive equilibrium with regulation constraints

Next, let us introduce the problem of a representative agent that maximises aggregate welfare subject to a regulation constraint on hours worked. In particular, the limit on workable hours is set at a level $\overline{N}$. Because all agents are homogeneous, we can still solve the aggregate problem by maximising individual-level utility over the continuum of households/agents, subject to the regulation constraint and the standard budget constraint faced by the household:

$$\begin{aligned} \max_{C_t, N_t} \quad & U(C_t, N_t) \\ \text{s.t.} \quad & \begin{cases} P_t C_t \leq W_t N_t & (\lambda) \\ N_t \leq \overline{N} & (\gamma) \end{cases} \end{aligned}$$

Then, we can set the Lagrangian as follows:

$$\mathcal{L} = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda(W_t N_t - P_t C_t) + \gamma(\overline{N} - N_t)$$

Taking FOCs, we obtain:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_t} : & \quad C_t^{-\sigma} - \lambda P_t = 0 \\ \frac{\partial \mathcal{L}}{\partial N_t} : & \quad -N_t^{\varphi} + \lambda W_t - \gamma = 0 \end{aligned}$$

From the first FOC, we have that $\lambda = \frac{C_t^{\sigma}}{P_t} > 0$, which, by the CSC implies that the budget constraint is binding at optimum. Combining the two FOCs we obtain the following:

$$-N_t^{\varphi} + C_t^{-\sigma} \frac{W_t}{P_t} - \gamma = 0$$

By complementary slackness, we have that the regulation constraint is binding whenever $\gamma > 0$, which means:

$$\begin{aligned} \gamma = C_t^{-\sigma} \frac{W_t}{P_t} - N_t^{\varphi} > 0 \\ \iff (A_t N_t)^{-\sigma} W_t - N_t^{\varphi} P_t > 0 \\ \iff N_t^{SB} < A_t^{\frac{1-\sigma}{1+\varphi}} = N_t^{FB} \end{aligned}$$

where N_t^{SB} denotes the employment level for the union problem, that we derived by using the standard optimality condition for the firm (see Section [3], Equation (9)).

This implies that, whenever the regulation constraint binds, we have that $\overline{N} = N_t^{SB} < N_t^{FB}$. By contrast, when $\gamma = 0$, the regulation constraint is slack, and we can disregard it when solving the problem. Not surprisingly, when the regulation constraint is not binding, under the underlying assumption on perfect competition and the utility function, we are able to achieve exactly the efficient outcome for employment, and hence, output and consumption. In all other cases, the welfare effects are negative.

To add further to this intuition, the result makes even more sense by adopting a purely mathematical standpoint. If agents were optimizing absent the constraint, the introduction of a new constraint can only reduce their choice set, and utility can therefore at most be unchanged when compared to the unconstrained case.

4.2 Labour market regulation − Heterogeneous case

Let us introduce now a dimension of workers' heterogeneity. In particular, suppose there is a fraction $(1-\theta)$ of workers that work full time, whereas the remaining fraction θ of workers are part-time workers, i.e., they only work a fraction $\alpha \in (0,1)$ hours compared to the full-time workers:

$$N_{P,t} = \alpha N_{F,t}$$

where $N_{P,t}$ denotes hours worked by the part-time worker, whereas $N_{F,t}$ denotes hours worked by the full-time worker.
Next, we proceed by setting the problem of a social planner, maximizing the agent's utility subject to a series of constraints.

4.2.1 Efficiency benchmark with Heterogeneous agents

Let us first consider the problem of a social planner that is maximising social welfare. This corresponds to maximising *aggregate* utility, which in this case can be expressed as a weighted average of individual utilities, weighted by the numerosity of the two types in the population. As for the fully homogeneous case, the planner does not face prices and wages but only cares about allocation. Thus, the problem of the planner can be written as follows:

$$\max_{\{C_t^i, N_t^i\}_{i\in\{F,P\}}} \quad \theta U_P(C_t^P, N_t^P) + (1-\theta)U_F(C_t^F, N_t^F)$$
$$\text{s.t.} \quad \begin{cases} C_t \leq Y_t & (\gamma) \\ Y_t \leq A_t N_t & (\delta) \end{cases}$$

where $\theta C_t^P + (1-\theta)C_t^F = C_t$ and $\theta N_t^P + (1-\theta)N_t^F = N_t$. Thus, the planner maximises utility subject to a resource constraint and the production function, as before. We can combine the two constraints and rewrite the problem as:

$$\max_{\{C_t^i, N_t^i\}_{i\in\{F,P\}}} \quad \theta U_P(C_t^P, N_t^P) + (1-\theta)U_F(C_t^F, N_t^F)$$
$$\text{s.t.} \quad C_t \leq A_t N_t \quad (\gamma)$$

The Lagrangian can be set as follows:

$$\mathcal{L} = \theta U_P(C_t^P, N_t^P) + (1-\theta)U_F(C_t^F, N_t^F) + \gamma \left[A_t(\theta N_t^P + (1-\theta)N_t^F) - \theta C_t^P - (1-\theta)C_t^F \right]$$

The FOCs can be derived as follows:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t^P} &= \theta (C_t^P)^{-\sigma} - \gamma\theta = 0 \iff (C_t^P)^{-\sigma} = \gamma \\
\frac{\partial \mathcal{L}}{\partial C_t^F} &= (1-\theta)(C_t^F)^{-\sigma} - \gamma(1-\theta) = 0 \iff (C_t^F)^{-\sigma} = \gamma \\
\frac{\partial \mathcal{L}}{\partial N_t^P} &= -\theta (N_t^P)^{\varphi} + \gamma\theta A_t = 0 \iff (N_t^P)^{\varphi} = \gamma A_t \\
\frac{\partial \mathcal{L}}{\partial N_t^F} &= -(1-\theta)(N_t^F)^{\varphi} + \gamma(1-\theta) A_t = 0 \iff (N_t^F)^{\varphi} = \gamma A_t
\end{aligned}
$$

We can then combine the FOCs to obtain efficient labour supply conditions:

$$
\begin{aligned}
\frac{(N_t^P)^{\varphi}}{(C_t^P)^{-\sigma}} &= A_t \\
\frac{(N_t^F)^{\varphi}}{(C_t^F)^{-\sigma}} &= A_t
\end{aligned}
$$

Then, knowing that $N_t^P = \alpha N_t^F$ and $N_t = \theta N_t^P + (1-\theta) N_t^F$ we can write:

$$
\begin{aligned}
N_t^P &= \frac{N_t}{\theta} - \left(\frac{1-\theta}{\theta} N_t^F \right) \\
\iff \alpha N_t^F &= \frac{N_t}{\theta} - \left(\frac{1-\theta}{\theta} N_t^F \right) \\
\iff N_t^F &= \frac{N_t}{1-\theta+\theta\alpha}
\end{aligned}
$$

from which we have also:

$$
\iff N_t^P = \frac{\alpha N_t}{1-\theta+\theta\alpha}
$$

Then, we can retrieve consumption as a function of labour from the optimality conditions. For $i \in \{F, P\}$

$$
C_t^i = \left(\frac{(N_t^i)^{\varphi}}{A_t} \right)^{-\frac{1}{\sigma}} = (N_t^i)^{-\frac{\varphi}{\sigma}} A_t^{\frac{1}{\sigma}}
$$

Next, we can retrieve the optimal level of consumption - and output, by market clearing:

$$
\begin{aligned}
Y_t &= C_t \\
&= \theta C_t^P + (1-\theta) C_t^F \\
&= \theta (N_t^P)^{-\frac{\varphi}{\sigma}} A_t^{\frac{1}{\sigma}} + (1-\theta)(N_t^F)^{-\frac{\varphi}{\sigma}} A_t^{\frac{1}{\sigma}} \\
&= \theta \left(\frac{\alpha N_t}{1-\theta+\theta\alpha} \right)^{-\frac{\varphi}{\sigma}} A_t^{\frac{1}{\sigma}} + (1-\theta) \left(\frac{N_t}{1-\theta+\theta\alpha} \right)^{-\frac{\varphi}{\sigma}} A_t^{\frac{1}{\sigma}}
\end{aligned}
$$

Furthermore, in equilibrium, we have $Y_t = A_t N_t$, hence $N_t = Y_t / A_t$

$$Y_t = \theta\left(\frac{\alpha(Y_t/A_t)}{1-\theta+\theta\alpha}\right)^{-\frac{\varphi}{\sigma}} A_t^{\frac{1}{\sigma}} + (1-\theta)\left(\frac{Y_t/A_t}{1-\theta+\theta\alpha}\right)^{-\frac{\varphi}{\sigma}} A_t^{\frac{1}{\sigma}}$$

The expression can be further simplified as

$$Y_t^* = A_t^{\frac{1+\varphi}{\sigma+\varphi}}\left[\frac{1-\theta+\theta\alpha^{-\frac{\varphi}{\sigma}}}{(1-\theta+\theta\alpha)^{-\frac{\varphi}{\sigma}}}\right]^{\frac{\sigma}{\sigma+\varphi}} = A_t^{\frac{1+\varphi}{\sigma+\varphi}}\left[\frac{1+\theta(\alpha^{-\frac{\varphi}{\sigma}}-1)}{(1+\theta(\alpha-1))^{-\frac{\varphi}{\sigma}}}\right]^{\frac{\sigma}{\sigma+\varphi}}$$

where both the numerator and the denominator in the square brackets are positive since, for $\alpha \in (0,1)$:

$$1+\theta(\alpha^{-\frac{\varphi}{\sigma}}-1) > 0$$
$$1+\theta(\alpha-1) > 0$$

Then, since both expressions are positive, also their ratio will be positive.

Notice that the first factor is the efficient level of output with homogeneous agents - we have defined it as Y_t^{FB}. The second factor - the one in $[\cdot]$ - is a distortionary term that depends on underlying parameters. We can see that the output deviates from the homogeneous benchmark computed above. In particular, it is lower than the homogeneous efficiency benchmark, since we can show that the second factor is smaller than 1.

$$\begin{aligned}
\left[\frac{1-\theta+\theta\alpha^{-\frac{\varphi}{\sigma}}}{(1-\theta+\theta\alpha)^{-\frac{\varphi}{\sigma}}}\right]^{\frac{\sigma}{\sigma+\varphi}} &= \left[\frac{(1-\theta)\alpha^{\frac{\varphi}{\sigma}}+\theta}{(1-\theta+\theta\alpha)^{\frac{\varphi}{\sigma}}}\right]^{\frac{\sigma}{\sigma+\varphi}} \\
&= \left[\frac{(1-\theta)\alpha^{\frac{\varphi}{\sigma}}/\theta+1}{(1-\theta)/\theta+\alpha^{\frac{\varphi}{\sigma}}}\right]^{\frac{\sigma}{\sigma+\varphi}} \\
&= \left[\frac{(1-\theta)\alpha^{\frac{\varphi}{\sigma}}/\theta+\theta/\theta}{(1-\theta)/\theta+\alpha^{\frac{\varphi}{\sigma}}}\right]^{\frac{\sigma}{\sigma+\varphi}} \\
&= \left[\frac{(1-\theta)\alpha^{\frac{\varphi}{\sigma}}/\theta+1}{\alpha^{\frac{\varphi}{\sigma}}+(1-\alpha)^{\frac{\varphi}{\sigma}}\theta/\alpha^{\frac{\varphi}{\sigma}}}\right]^{\frac{\sigma}{\sigma+\varphi}} \\
&= \left[\frac{\alpha^{\frac{\varphi}{\sigma}}}{\alpha^{\frac{\varphi}{\sigma}}+(1-\alpha)^{\frac{\varphi}{\sigma}}\theta}\right]^{\frac{\sigma}{\sigma+\varphi}} \cdot \left[\frac{(1-\theta)\alpha^{\frac{\varphi}{\sigma}}+1}{\alpha^{\frac{\varphi}{\sigma}}}\right]^{\frac{\sigma}{\sigma+\varphi}}
\end{aligned}$$

The first term in the last expression is less than 1 because $\theta \in (0,1)$, $\alpha \in (0,1)$, and $\varphi > 0$, so $(1-\alpha)^{\frac{\varphi}{\sigma}} > 0$, which implies that $\alpha^{\frac{\varphi}{\sigma}} + (1-\alpha)^{\frac{\varphi}{\sigma}}\theta > \alpha^{\frac{\varphi}{\sigma}}$. The second term is less than 1 because $(1-\theta)\alpha^{\frac{\varphi}{\sigma}} + 1 \leq \alpha^{\frac{\varphi}{\sigma}} + 1 < \alpha^{\frac{\varphi}{\sigma}}$. Therefore, the whole expression is less than 1, which implies also that

$$\widetilde{Y}_t^* = A_t^{\frac{1+\varphi}{\sigma+\varphi}}\left[\frac{1-\theta+\theta\alpha^{-\frac{\varphi}{\sigma}}}{(1-\theta+\theta\alpha)^{-\frac{\varphi}{\sigma}}}\right]^{\frac{\sigma}{\sigma+\varphi}} < A_t^{\frac{1+\varphi}{\sigma+\varphi}} = Y_t^{FB}$$

where Y_t^{FB} is the efficient level of output of the social planner's problem when all agents are homogeneous and work full time. This is trivial, knowing that whenever at least one agent works part-time, that is, works fewer hours than a full-time worker, she will produce less than in the first best, hence aggregate output -which we take as a measure of welfare - will fall compared to the homogeneous benchmark.

4.2.2 Heterogeneous labour supply − C.E. with regulation constraints

Now, let us analyse a competitive equilibrium outcome where agents face the regulation constraint. Then, from a social welfare perspective, the problem can be written as follows:

$$\max_{\{C_t^i, N_t^i\}_{i\in\{F,P\}}} \quad \theta U_P(C_t^P, N_t^P) + (1-\theta)U_F(C_t^F, N_t^F)$$

$$\text{s.t.} \quad \begin{cases} P_t C_t^P \leq W_t N_t^P & (\lambda_P) \\ P_t C_t^F \leq W_t N_t^F & (\lambda_F) \\ N_t^P \leq \overline{N} & (\mu^P) \\ N_t^F \leq \overline{N} & (\mu^F) \\ N_t^P = \alpha N_t^F & (\xi) \end{cases}$$

Since we have two types of agents, social welfare is determined by a sum of individual utilities weighted by the numerosity of the two types.
The Lagrangian can be written as follows:

$$\mathcal{L} = \theta U_P(C_t^P, N_t^P) + (1-\theta)U_F(C_t^F, N_t^F) + \lambda_P[W_t N_t^P - P_t C_t^P] + \lambda_F[W_t N_t^F - P_t C_t^F] + \\ + \mu_P[\overline{N} - N_t^P] + \mu_F[\overline{N} - N_t^F]$$

where we assume the usual CRRA functional form for $U_i(C_t^i, N_t^i)$. Next, let us take the FOCs:

$$\begin{aligned} C_t^P: &\quad \theta(C_t^P)^{-\sigma} - \lambda_P P_t = 0 \\ C_t^F: &\quad (1-\theta)(C_t^F)^{-\sigma} - \lambda_F P_t = 0 \\ N_t^P: &\quad -\theta(N_t^P)^{\varphi} + \lambda_P W_t - \mu_P = 0 \\ N_t^F: &\quad -(1-\theta)(N_t^F)^{\varphi} + \lambda_F W_t - \mu_F = 0 \end{aligned}$$

We can rearrange the first two FOCs to obtain

$$\lambda_P = \frac{\theta(C_t^P)^{-\sigma}}{P_t} > 0$$
$$\lambda_F = \frac{(1-\theta)(C_t^F)^{-\sigma}}{P_t} > 0$$

Then, $\lambda_P > 0$ and $\lambda_F > 0$ imply, by complementary slackness, that both budget constraints are always binding.
By replacing the multipliers in the next two FOCs with their respective expressions, we obtain:

$$-\theta(N_t^P)^{\varphi} + W_t \frac{\theta(C_t^P)^{-\sigma}}{P_t} - \mu_P = 0$$
$$-(1-\theta) = (N_t^F)^{\varphi} + W_t \frac{(1-\theta)(C_t^F)^{-\sigma}}{P_t} - \mu_F = 0$$

which can be then rearranged as follows:

$$\theta(N_t^P)^{\varphi} + \mu_P = \frac{W_t}{P_t}\theta(C_t^P)^{-\sigma}$$
$$(1-\theta)(N_t^F)^{\varphi} + \mu_F = \frac{W_t}{P_t}(1-\theta)(C_t^F)^{-\sigma}$$

Multipliers Let us analyse the signs of the multipliers associated with the regulation constraints, namely μ_P and μ_F. Since $N_{t,P} < N_{t,F}$ by construction, when $\mu^F = 0$ – hence the constraint on hours worked does not bind for full-time workers, it must be the case that it does not bind for the part-time worker as well. Then, by complementary slackness, $\mu^P = \mu^F = 0$ and the expressions above simplify to:

$$\frac{(N_t^P)^\varphi}{(C_t^P)^{-\sigma}} = \frac{W_t}{P_t}$$
$$\frac{(N_t^F)^\varphi}{(C_t^F)^{-\sigma}} = \frac{W_t}{P_t}$$

Then, knowing that both budget constraints are binding, we can rewrite the two expressions as follows:

$$N_t^P = \left(\frac{W_t}{P_t}\right)^{\frac{1-\sigma}{\sigma+\varphi}}$$
$$N_t^F = \frac{1}{\alpha}N_t^P = \frac{1}{\alpha}\left(\frac{W_t}{P_t}\right)^{\frac{1-\sigma}{\sigma+\varphi}} > N_t^P$$

Let us replace the expressions to retrieve the associated consumption values when the regulatory constraints are not binding.

$$C_t^P = \left(\frac{W_t}{P_t}\right)^{\frac{1+\varphi}{\sigma+\varphi}}$$
$$C_t^F = \frac{1}{\alpha}\left(\frac{W_t}{P_t}\right)^{\frac{1+\varphi}{\sigma+\varphi}} > C_t^P$$

which means that, when neither type faces the regulation constraint, the full-time worker works more and consumes more (from a higher income). In particular, both types will choose an allocation equivalent to the heterogeneous efficiency benchmark.

Now, let us assume that the regulation constraint binds only for the full-time worker, which means $N_t^P < \overline{N} = N_t^F$ and $\mu_F > 0 = \mu_P$. The optimality conditions derived above read:

$$(N_t^P)^\varphi = \frac{W_t}{P_t}(C_t^P)^{-\sigma}$$
$$(1-\theta)(\overline{N})^\varphi + \mu_F = \frac{W_t}{P_t}(1-\theta)(\overline{C})^{-\sigma}$$

Then, we can substitute $N_t^F = \overline{N}$, with $C_t^F = \overline{C}$ being the consumption value associated with this level of employment. By contrast, the intratemporal optimality condition for the part-time worker is the same as the one analysed in the previous case.

$$\frac{(N_t^P)^\varphi}{(C_t^P)^{-\sigma}} = \frac{W_t}{P_t}$$
$$(1-\theta)(\overline{N})^\varphi + \mu_F = \frac{W_t}{P_t}(1-\theta)(\overline{C})^{-\sigma}$$

Let us focus on the second expression. We have $\mu_F > 0$ if and only if:

$$\frac{W_t}{P_t}(1-\theta)(\overline{C})^{-\sigma} - (1-\theta)(\overline{N})^{\varphi} > 0 \iff$$
$$\iff \frac{W_t}{P_t}\overline{C}^{-\sigma} - \overline{N}^{\varphi} > 0 \iff \overline{N}^{\varphi} < \frac{W_t}{P_t}\overline{C}^{-\sigma}$$

Then, using the fact that the budget constraint is still binding, we can use it as we did above to retrieve the actual values of $\overline{N}$ and $\overline{C}$ and obtain that

$$\overline{N} < \left(\frac{W_t}{P_t}\right)^{\frac{1-\sigma}{\sigma+\varphi}} = A_t^{\frac{1-\sigma}{\sigma+\varphi}}$$

where the last equality comes from the standard optimality condition of the firm, equivalent to the ones derived in previous problems (for reference, see Question [3]). This result implies that in this scenario the full-time workers would work fewer hours so that they produce and consume less compared to a scenario where the constraint is not binding.

Since the labour supply of full-time workers has decreased, whereas that of part-time workers remains unchanged, in the aggregate, labour supply will drop, and so will output (together with consumption, by market clearing). Thus, the regulation constraint on working hours creates an inefficiency compared to the efficiency benchmark under heterogeneity in labour supply.
The aggregate level of output associated with the level of labour supply $\overline{N}$ can be computed as

$$\overline{Y} = \overline{C} < \left(\frac{W_t}{P_t}\right)^{\frac{1+\varphi}{\sigma+\varphi}} = A_t^{\frac{1+\varphi}{\sigma+\varphi}}$$

where the second equality comes from the problem of the firm, which is unchanged. The condition can be then expressed as

$$\overline{Y} = \overline{C} < A_t^{\frac{1+\varphi}{\sigma+\varphi}} = Y_t^{FB}$$

that is, the level of output is lower compared to the efficient level of output derived in the social planner's problem.

Finally, let us consider the case where both constraints are binding, that is, when $\mu_P > 0$ and $\mu_F > 0$, so that $\overline{N} = N_t^P = N_t^F$. However, since we assumed that $N_t^P = \alpha N_t^F < N_t^F$, this cannot be the case.

To conclude, we have seen that whenever the regulation is binding, it binds only for the full-time workers, which consume and work at a suboptimal level, compared to the case where they maximise utility subject only to the standard budget constraint which corresponds instead to the efficient allocation derived in the social planner's problem. Thus, we can conclude that, even in a framework where the constraint is binding only for a portion of the workers, the effect on social welfare is negative.

5 Question 5

Yet a third explanation is the presence of unions in the economy and in particular how unions respond to sectoral shifts in the economy. Show the impact of sectoral shifts (i.e. one part of the economy gets more productive and the other less so) on hours worked in a competitive economy. Argue, preferably by means of a formal model, how this response will change in an unionised economy. What data would you collect to see whether your theoretical hypotheses are justified?

5.1 A two-sectors economy

Consider a two-sector economy where each sector has a representative firm, allowing us to talk about firms and sectors interchangeably. Then, let 1 and 2 denote the two representative firms in this economy. Assume that both firms have the same production function, which is a function of labour and technology, however, let the technology process, A_t^i, $i = 1, 2$ be sector-specific.

Let us assume that both firms are price-takers, for instance, because imports from abroad can freely enter the market. Instead, assume workers are not free to move abroad and can only switch between the two companies (or sectors). This implies that prices are exogenous while wages are not. In fact, labour markets inside the economy are competitive: the two representative firms compete for sales and workers and set wages according to their productivity.
The labour factor is ex-ante homogeneous, and workers can move costlessly from one sector to another, without any implications for human capital accumulation and sector-specific skills.
The problem of a representative firm in sector $i = 1, 2$ can be stated as follows:

$$\max_{N_t^i, Y_t^i} \quad P_t Y_t^i - W_t N_t^i \tag{14}$$

$$\text{s.t.} \quad Y_t^i = A_t^i \ln N_t^i$$

Where the production function is concave, displaying decreasing marginal returns to scale and decreasing marginal returns to labour.
We abstract from any consideration for capital by assuming it to be fixed in the short-to-medium term.

Suppose that, initially, both firms have the same productivity, that is, $A_t^1 = A_t^2$. Hence, both have the same following FOCs and pay the same wage. For both firms, the Lagrangian can be written as:

$$\mathcal{L} = P_t Y_t^i - W_t^i N_t^i + \lambda_t \left[A_t^i \ln N_t^i - Y_t^i \right]$$

and the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial Y_t^i} : \qquad P_t = \lambda_t^i$$

$$\frac{\partial \mathcal{L}}{\partial N_t^i} : \qquad W_t^i = \lambda_t^i A_t^i \frac{1}{N_t^i}$$

which can be combined to get rid of the (positive) multiplier.

$$\frac{W_t^i}{P_t} = \frac{A_t^i}{N_t^i} = MPN_i \quad i = 1, 2 \tag{15}$$

that is, each firm equates its (real) marginal labour cost to its marginal product.

From Equation (15) we notice how the labour demand function is a positive function of productivity and a negative function of the real wage. Let us also recall the labour supply function derived from a standard household's utility maximisation problem (See Question 2) and note that the agents supply labour as a positive function of the hourly wage.

Now assume there is a sectoral shift following a positive productivity shock for sector 1 $-$ or, equivalently, a negative one for sector 2, given that what matters is how the *relative* productivity between the two firms changes.

As a consequence, firm 1 now has a higher marginal product $A_t^{'1} > A_t^1 = A_t^2$, and - ceteris paribus - will make positive profits, therefore it will demand more labour in order to produce more, which causes real wages in firm 1 to go up, $\frac{W_t}{P_t}$. Note that prices remain constant since they are determined exogenously. Analytically:

$$\frac{A_t^{'1}}{N_t^{'1}} = \frac{W_t^{1'}}{P_t} > \frac{W_t^1}{P_t} = \frac{A_t^1}{N_t^1}$$

and so $N_t^{'1} > N_t^1 = N_t^2$ and/or $W_t^{'1} > W_t^1 = W_t^2$. Note that the difference in wages is temporal. Graphically, we move from A to B as shown in Figure [4].

As a result, workers at firm 2 will want to switch to firm 1 to take advantage of the higher wages. But since the labour market is competitive, the wage will readjust, and go down as long as there is an excess supply of workers from firm 2 that want to move to firm 1, up until both firms will again pay the same wage. In Figure 5, we graphically represented the partial equilibrium dynamics of wage adjustments caused by the technology shock in Sector 1. A shock to A_t^1 shifts labour demand upward and real wage is higher for any level of labour supply, and equilibrium moves from point A to point B. As a result, the workers' response causes an upward shift in the labour supply, moving the equilibrium from B to C. Equilibrium in C features increased labour supply and the wage prevailing in equilibrium is exactly the original wage level, that is $W_t1'' = W_t^1$. Note, that after the labour market has swung back into equilibrium, wages at both firms will be the same, and firm 1 will demand more labour and produce more sectoral output.
In particular, we have that

$$N_t^{1'} = \frac{W_t^{1''}}{P_t} A_t^{1'} > \frac{W_t^{2''}}{P_t} A_t^2 = N_t^2$$

and, as a consequence,

$$Y_t^1 = A_t^{1'} \ln N_t^{1'} > A_t^2 \ln N_t^2 = Y_t^2$$

since output is an increasing function of both technology and output for both sectors.

5.2 Introducing unions

Let us now introduce unions into our economy. In our settings, we make the following assumption about the role of unions in labour markets, as ensuring that wages, once increased, are not

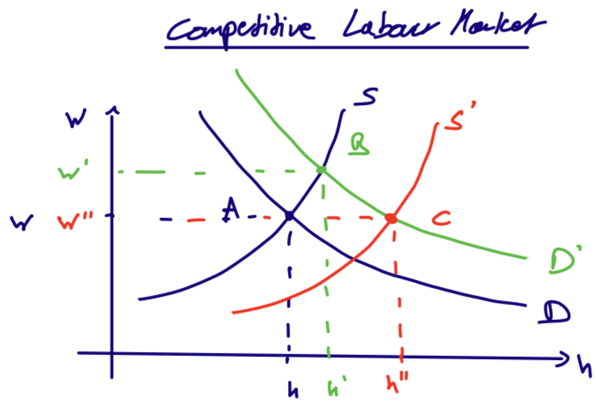


Figure 4: Labour Supply and Demand

allowed to go down again. In our model, through collective bargaining, unions immediately renegotiate wages once they are up, and ensure that they cannot go down again.

The problem faced by the firms is the same as before, except for wages now being downwardly rigid. For $i = 1, 2$:

$$\max_{N_t^i, Y_t^i} P_t Y_t^i - W_t^i N_t^i$$

$$\text{s.t.} \quad \begin{cases} Y_t^i = A_t^i \ln N_t^i \\ W_t \geq W_{t-1} \end{cases}$$

yielding the following Lagrangian

$$P_t Y_t^i - W_t^i N_t^i + \lambda_t \left[A_t lnN_t - Y_t \right] + \phi_t \left[W_t - W_{t-1} \right]$$

and the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial Y_t^i} : P_t = \lambda_t^i$$
$$\frac{\partial \mathcal{L}}{\partial N_t^i} : W_t^i = \lambda_t^i A_t^i \frac{1}{N_t^i}$$
$$\frac{\partial \mathcal{L}}{\partial W_t} : N_t^i = \phi_t$$

which can be rearranged to

$$\frac{W_t^i}{P_t} = \frac{A_t^i}{N_t}$$

as before, that is, each firm equalizes the marginal product of labour to the marginal cost of labour, the real wage. From complementary slackness, we get that

$$W_t = W_{t-1}$$

so it is optimal for the firm to keep the wages constant since it knows that once the wages have risen they can never come down again.

Note that this condition might be in conflict with the other first-order condition, that is, in times of productivity changes it would be optimal to adjust the wage. However, since the firm wants to keep wages constant, and prices are exogenous, the only way to respond for the firm is to increase labour.

$$\frac{\overline{W}}{P_t} = \frac{A_t}{N_t}$$

Yet in a competitive market, at the given wage, it will not attract labour. Thus, at the unionized firm, the wages and working hours are constant.

As a result, whilst we observe an increase (and subsequent drop) in wages, and an increase in working hours in the non-unionized firm in response to a sectoral shift, we do not observe any changes in labour in the unionized firm. This can be seen as an explanation of why Americans work more than Europeans. Graphically, we see that the unionized firm will remain in A, despite the sectoral shift, whilst the un-unionized firm moves first up to B, then to C.
Furthermore, since it cannot reap any gains from increased productivity, the unionized firm is not trying to innovate.
Note, however, that if the sectoral shift occurs "out of the blue" and the higher-productivity technology is available to all firms, even the unionized firm can expand output, by adopting the new technology, only after the labour market has first shifted to B, then to C, thanks to the un-unionized firm. At that point, for the given wage, even the unionized firm can utilize more labour. In that case, unions delay the increase in labour with respect to the un-unionized firms.

5.3 Empirical strategy

To study how sectoral shifts in an unionised economy may impact the total amount of hours worked in a competitive economy we first need to identify exogenous, sector-specific productivity shocks. Technological shocks exogenous to single firms or industries but that are specific to one sector may be a good candidate for such productivity shocks. Using exogenous (common for everyone) technological shocks to proxy for (exogenous) sector-specific spikes in total factor productivity (TFP) may allow us to study how total employment changes differently in economies displaying different degrees of unionization (more unionized vs less unionized economies). Assuming that in sectors with unionized workers, an increase in TFP leads to a rise in wages that cannot be adjusted downwards in response to the increase in labour supply (as in our model), we would expect to see that in countries (or firms) with higher union coverage, total employment in the affected sector does not increase, whereas the shift in labour supply should be larger in less unionized economies.

One simple approach to answering this question is to see how the recent and sharp developments in precision agriculture technologies in the farming sector differently impacted farming employment in unionised vs non-unionised countries. To this end, we would first need to collect data on union coverage across countries. This data is widely available (albeit not always fully comparable across countries due to methodological caveats). One important and highly regarded source for this data is the International Labour Organization ILOSTAT database, which includes industry-specific union density rates. Following the analysis in Questions [1] and [2], we could instead focus on OECD Countries by OECD trade union membership and collective bargaining data. Then, we would need to find data on the adoption of precision agriculture technologies, which dramatically increase land yield and farming productivity. This data is more subtle and hard to find but could be retrieved by looking at sales data from the main firms supplying farming drones and

GPS technologies. Alternatively, for the USA there is granular data from the Food and Agriculture Organization of the United States (FAO) which releases yearly statistical data which includes measures on technological inputs which could proxy for precision agriculture technologies adoption. More relevantly for a cross-country analysis, we could rely on data and existing analysis from the International Society of Precision Agriculture (ISPA), an NGO which carries out research on the topic.

Yet, note that, because union coverage is not exogenous, a more sophisticated identification strategy may be better suited. One could, for instance, employ a triple Difference-in-Differences estimation between firms in affected sectors vs. not affected sectors operating in countries (or industries) with high vs. low union coverage. The first difference would clean for baseline differences in firms affected by the shock vs those not affected by the shock, while the second for baseline differences in countries (or, alternatively, industries) with high vs low union coverage. Then, comparing this difference pre- vs. post- the exogenous productivity shock should provide a credible identification of the causal effect of productivity shocks in an unionized economy.

6 Question 6

Finally, one can think of a fourth explanation based on the distinction between men's and women's labour supply and suggests the labour supply decision of the couple is affected by the possibility (and likelihood) of divorce and the consequences of it. For example, in an economy with unstable marriages, partners might decide to invest in their ability to generate income when single. How would you set up a model capturing this aspect? What kind of data would you collect to check the relevance of this mechanism?

Let us consider, for the sake of simplicity, a two-periods model, $t \in \{0, 1\}$ with a one-sector economy where total wage W and working hours are uniformly set by a union for all workers in period 1. In time $t = 1$ workers are heterogeneous along an observable parameter, $\theta \in \Theta \subseteq \mathbb{R}_+$, which defines one's productivity (as perceived in the eyes of the employers) and is determined by worker's seniority. Given how we define it, θ is an **inverse measure of seniority** as it is a decreasing function of how one has worked in the previous period ($t = 0$ in our two-periods case).

Then, at $t = 0$ a worker can choose how much time to work, accounting for the fact doing so decreases their θ in the subsequent period. Since everyone earns (and works) the same, firms want to hire high-productivity (seniority) workers 'first'. Then, let p_{θ_i} denote the probability that a worker with seniority θ_i is employed at time $t = 1$. We will define this object more rigorously later on, but it is a decreasing function of θ_i: the higher θ_i, the lower the seniority of agent i and so the less likely that she will be employed in the next period. The desire to increase one's seniority following an increase in the divorce rates will drive our result that in an economy with unstable marriages agents invest in their ability to generate income.

6.1 Marriage as an insurance policy

Now assume that at $t = 0$ all workers are married to one another. If both they and their spouse are still employed at $t = 1$, workers will get to consume their wages. In this economy marriages also work as insurance policies: in case they lose their job, workers' consumption does not drop to zero but, if their spouse is employed, they get to consume I $(< W)$, which call their "marital insurance" [1]. Accordingly, if they are employed but their spouse is not, they pay I to their spouse. Unfortunately, due to exogenous reasons, some marriages are cut short and workers divorce. For how much two workers may love each other at time $t = 0$, let $P^D \in [0, 1]$ be the ex-ante probability that their marriage comes to grief in $t = 1$. Accordingly, let $P^M = 1 - P^D$ be the probability that the workers are still married.

Finally, let the expected utility of agent i with spouse j in $t = 1$ be given by:

$$\begin{aligned} u_i(W; \theta_i, \theta_j) =& p_{\theta_i} \cdot \left[W + P^M \cdot (1 - p_{\theta_j})(-I) \right] + (1 - p_{\theta_i}) \cdot \left[P^M \cdot p_{\theta_j} I \right] = \\ =& p_{\theta_i} W + P^M I \left[-p_{\theta_i}(1 - p_{\theta_j}) + p_{\theta_j}(1 - p_{\theta_i}) \right] = \\ =& p_{\theta_i} W + P^M I \left[p_{\theta_j} - p_{\theta_i} \right] \end{aligned} \tag{16}$$

Now notice: the sign of $\left[p_{\theta_j} - p_{\theta_i} \right]$ defines the sign of what we call the "expected net monetary value of marriage". This captures the fact that if your spouse has a higher probability of being

[1] This notion is not inherently a novel concept. See, among others, Hess (2004), Kotlikoff and Spivak (1981).

employed than you do, then marriage is a sound financial decision for you. Also, observe that the magnitude of the marriage premium (or cost) rises with an increase in the absolute value of P^M. Conversely, the lower P^M, the more unstable marriages are, and so insurance benefits of marriage for high-θ agents decrease. In turn then, following an increase in the risk of divorce, these agents have greater motivation to improve their own θ to minimize the risk of unemployment and single, and so of getting a monetary payoff of 0.

To see this point more rigorously, let's define the probability of being employed, p_{θ_i}. Assume that labour demand in $t = 1$ takes the following stochastic form, defined over the space of seniority parameters Θ:

$$L(W) = \Phi(W) + \varepsilon, \qquad \varepsilon \sim F(\cdot), \quad \Phi'(W) < 0$$

Where ε is a random shock to labour demand which introduces risk into the analysis.

Then, recall that more skilled workers are strictly preferred over less skilled ones and so the firm's preferences are strictly monotonic over worker's seniority θ_i − also note again that, under our parametrization, the higher the seniority, the **lower** the θ. We can then assume that everyone wants to work at $t = 1$ and that workers are employed if the following holds:

$$L(W) > \theta_i \tag{17}$$

So if demand is large enough that the highest θ employed by the firms is larger than one's own θ_i, agent i is employed. In turn, then, ex-ante agent i can expect to be employed with probability:

$$\begin{aligned} p_{\theta_i} &= \Pr\left[L(W) > \theta_i\right] \\ &= \Pr\left[\Phi(W) + \varepsilon > \theta_i\right] \\ &= \Pr\left[\varepsilon > \theta_i - \Phi(W)\right] \\ &= 1 - \Pr\left[\varepsilon < \theta_i - \Phi(W)\right] \\ &= 1 - F\left[\theta_i - \Phi(W)\right] \end{aligned}$$

which, plugged into Equation (16) yields the following:

$$\begin{aligned} u_i(W; \theta_i, \theta_j) =& p_{\theta_i} W + P^M I \left[p_{\theta_j} - p_{\theta_i}\right] = \\ &= \left[1 - F\left[\theta_i - \Phi(W)\right]\right] W + P^M I \left\{\left[1 - F\left[\theta_j - \Phi(W)\right]\right] - \left[1 - F\left[\theta_i - \Phi(W)\right]\right]\right\} = \\ &= \left[1 - F\left[\theta_i - \Phi(W)\right]\right] W + P^M I \left\{F\left[\theta_i - \Phi(W)\right] - F\left[\theta_j - \Phi(W)\right]\right\} \end{aligned} \tag{18}$$

where the "expected net monetary value of marriage" is represented by I times the term enclosed in the curly brackets $\{\cdot\}$. This term is strictly increasing in one's own seniority, θ_i and decreasing in that of one's partner, θ_j. More relevantly for what we want to model, notice that, if one is less "senior" than their spouse, that is, if $\theta_i > \theta_j$, then their probability of being unemployed ($F\left[\theta_i - \Phi(W)\right]$) is larger than that of their spouse, so the net value of marriage is positive and consequently their utility is strictly increasing in the probability of remaining married in $t = 1$, denoted by P^M.

Now consider the following problem at $t = 0$, when workers choose their labour supply whilst accounting for the fact that their working decision at time $t = 0$ determines their seniority at time $t = 1$. Assume that everyone has the same initial seniority, normalized to 0 and that everyone can work as much as they wish at $t = 0$ – for instance, because the government offers an infinite supply of traineeships in the public sector to all youngsters, at wage $w_0 \ll W$, allowing youngsters to choose how many hours of work to supply. For simplicity, we will let $w_0 = 0$.

To model the extent to which discriminating gender norms and perceptions negatively impact female employment, let us assume that women's labour supply convert to seniority at a lower rate than man's labour supply does. Remember that in this model **the lower** the θ_i, the **higher** the seniority, and so the probability of being unemployed. Then, let the seniority be defined as

$$\theta_i := \frac{\alpha_{G_i}}{N_i^0}, \qquad G_i \in \{M, F\}$$

where N_i^0 are i's hours of work supplied at $t = 0$ and $G_i \in \{M, F\}$ is a binary variable which indicates i's gender. To model labour discrimination, we assume male seniority to be given by $\frac{\alpha_{G_M}}{N_i^0}$, while female seniority is given by $\frac{\alpha_{G_F}}{N_i^0}$ with $\alpha_{G_F} > \alpha_{G_M}$. Then, to attain the same θ_i of a man working N_i^0 hours in time $t = 0$, a woman has to work $\frac{\alpha_{G_F}}{\alpha_{G_M}} \cdot N_i^0 > N_i^0$ hours.

Finally, assume that everyone dislikes working at $t = 0$: the dis-utility of working N_i^0 hours is given by: $\frac{(N_i^0)^{1+\varphi}}{1+\varphi}$, with $\varphi > 0$. Then, i's period $t = 0$ utility is given by:

$$u_i^0(w_0, N_i^0) = w_0 \cdot N_i^0 - \frac{(N_i^0)^{1+\varphi}}{1+\varphi} \tag{19}$$

The optimization at $t = 0$ reads:

$$\max_{N_i^0} \quad u_i^0\left(N_i^0; \alpha_{G_i}\right) + \beta \cdot u_i^1\left(\theta_i(N_i^0); \theta_j\right)$$
$$\text{s.t.} \quad \theta_i = \frac{\alpha_{G_i}}{N_i^0}$$

where $\beta \in (0, 1)$ is an intertemporal discount factor.

Then, plugging in Equation (19) and Equation (18) we get the following:

$$\max_{N_i^0} \quad w_0 \cdot N_i^0 - \frac{(N_i^0)^{1+\varphi}}{1+\varphi} + \beta \cdot [1 - F[\theta_i - \Phi(W)]]\, W + P^M I \left\{ F[\theta_i - \Phi(W)] - F\left[\theta_j - \Phi(W)\right] \right\} \tag{20}$$
$$\text{s.t.} \quad \theta_i = \frac{\alpha_{G_i}}{N_i^0}$$

Now recall that men have uniformly higher returns from working than women do since, by assumption, they experience a larger increase in seniority and thus decrease in the probability of being unemployed for the same number of hours worked at $t = 0$. Then note that, since everything else is equal, at optimum we must have $\theta_M \geq \theta_F$, where M denotes a generic male and F a generic female. Then, focusing only on heterosexual couples we have that for men the number in $\left\{F\left[\theta_j - \Phi(W)\right] - F[\theta_i - \Phi(W)]\right\}$ is always negative, while for women is always positive. This times I, is exactly the "expected net monetary value of marriage" discussed earlier in Equation (18). Then, following a drop in the probability that a marriage is long-lasting, P^M, women's labour supply should increase in order to compensate for the drop in their expected utility coming from the decrease in the probability of being insured in case of unemployment.

To see this point more formally, let us assume for the sake of brevity that the stochastic part of labour demand is uniformly distributed: $\varepsilon \sim \mathcal{U}[0, \delta]$. Then, the probability of being employed is:

$$F[\theta_i - \Phi(W)] = \begin{cases} 0 & \text{if } \theta_i - \Phi(W) \leq 0, \\ \dfrac{\theta_i - \Phi(W)}{\delta} & \text{if } 0 < \theta_i - \Phi(W) < \delta, \\ 1 & \text{if } \theta_i - \Phi(W) \geq \delta. \end{cases}$$

Assuming internality to study the interesting case, we can rewrite the optimization in (20) as:

$$\begin{aligned} \max_{N_i^0} \quad & w_0 \cdot N_i^0 - \frac{(N_i^0)^{1+\varphi}}{1+\varphi} + \left[1 - \frac{\theta_i - \Phi(W)}{\delta}\right] W + P^M I \left\{\frac{\theta_i - \Phi(W)}{\delta} - \frac{\theta_j - \Phi(W)}{\delta}\right\} = \\ = \max_{N_i^0} \quad & w_0 \cdot N_i^0 - \frac{(N_i^0)^{1+\varphi}}{1+\varphi} + \left[1 - \frac{\theta_i - \Phi(W)}{\delta}\right] W + P^M I \left\{\frac{\theta_i - \theta_j}{\delta}\right\} \\ \text{s.t.} \quad & \theta_i = \frac{\alpha_{G_i}}{N_i^0} \end{aligned}$$

Plugging-in θ_i:

$$\max_{N_i^0} \quad w_0 \cdot N_i^0 - \frac{(N_i^0)^{1+\varphi}}{1+\varphi} + \left[1 - \frac{\frac{\alpha_{G_i}}{N_i^0} - \Phi(W)}{\delta}\right] W + P^M I \left\{\frac{\frac{\alpha_{G_i}}{N_i^0} - \theta_j}{\delta}\right\}$$

$\text{FOC}_{N_i^0}$ of the free maximization:

$$\begin{aligned} w_0 - (N_i^0)^\varphi + \frac{\alpha_{G_i}}{(N_i^0)^2}\frac{W}{\delta} - P^M \frac{I}{\delta}\frac{\alpha_{G_i}}{(N_i^0)^2} = 0 \iff & (N_i^0)^2 w_0 - (N_i^0)^{\varphi+2} + \frac{\alpha_{G_i}}{\delta}(W - P^M I) = 0 \\ \iff & (N_i^0)^2(w_0 - (N_i^0)^\varphi) = \frac{\alpha_{G_i}}{\delta}(P^M I - W) \end{aligned}$$

Now, noting that $w_0 = 0$ we get:

$$(N_i^0)^{\varphi+2} = \frac{\alpha_{G_i}}{\delta}(W - P^M I) \iff (N_i^0)^* = \left[\frac{\alpha_{G_i}}{\delta}(W - P^M I)\right]^{\frac{1}{\varphi+2}}$$

Now, because by construction $I < W$ and $P^M < 1 \Longrightarrow W - P^M I > 0$ and therefore $N_i^{0^*}$ is *decreasing* in P^M, the probability that a marriage is long-lasting and therefore *increasing* in $P^D = 1 - P^M$, the divorce rate.

More in general, the behind this result can be interpreted as follows: if the divorce rate rises, women, who are ex-ante more likely to be unemployed than men are (for the same level of hours worked at $t = 0$), and therefore likely to benefit from the "marriage insurance" I in case of unemployment, are now less likely to get a monetary payoff of 0 in the $t = 1$. To avoid this, they compensate for the loss stemming from the decrease in the probability of being insured by increasing their supply of labour at $t = 0$ in order to boost their seniority and decrease the probability of needing insurance, to begin with.

To conclude, in an unequal society where women need to work harder to achieve the same level of seniority as men, an expected increase in the divorce rate at $t = 1$ leads to an increase in the female labour supply at $t = 0$. That is, to increase their investment in their ability to generate more income when single. The model also shows that the larger the discrimination against women, the lower their labour supply (and implicitly, reliance on the marriage contract and men for insurance). With some hard graft, we could extend this to multiple periods and build an intertemporal model where the same results hold for actual labour across multiple periods as agents optimize intertemporally.

We also note that this result needs not to be exclusive to women, even in a society plagued by labour discrimination: if rather than letting utility equal to expected wage in time $t = 1$ we considered a concave function of this expectation, the induced risk aversion may lead everyone to increase their investment in labour following an increase in the marriage rate. However, we neglected the firms' optimization, assuming that everyone with a large enough θ is hired regardless of their number under a very simplified labour demand function. However, it must be noted how simplified our setting is. For instance, if firms were to only hire a share of workers (rather than all those who are productive enough) and workers are strategic and forward-looking, then the equilibrium response to a decline in marriage rate could be different to the one modelled here, as workers compete against each other. Plausibly, if women's increased investment in seniority reduces men's likelihood of being employed, they will respond by investing more in their own seniority. Since, due to the labour discrimination modelled, men have uniformly higher returns to investing time in increasing their seniority, they would therefore outcompete women, possibly leading to a separating equilibrium where women do supply any work at $t = 0$.

6.2 Empirical strategy

Empirically validating the role of marriage instability in training and labour supply choices can be difficult. The first complication stems from the possibility of a reverse causality bias: if the female labour force participation rate (LFP) increases, the "insurance value" of marriages declines, reducing women's expected cost of a divorce and thus increasing the divorce rate (positing the presence of unhappy marriages endured only for their insurance valence). A second complication stems from the inability to compare cross-country (or even cross-regional) values of married women's LFP and divorce rates. The presence of omitted variables, such as cultural differences or different local labour markets would likely bias the resulting estimates or, even if studied in a panel data framework to control for time-constant unobservable factors, introduce complications in their interpretability and external validity.

Moreover, even the empirical validation of a causal link from the probability of divorcing to the female labour force participation may be due to a different channel from the one assumed in the model. For instance, Fernández and Wong (2014) exclude the hypothesis that women increase their labour supply to increase their labour market experience. Instead, they assume that an increase in the divorce risk results in changes in preferences over intra-household consumption decisions which led women to increase their own income.

Therefore, our model is best studied in a quasi-experimental setting to identify quasi-random (exogenous) variations in the divorce rate to see how they affect the female LFP. One interesting example would be to study the effect of the very *introduction* of the possibility of divorcing $-$ so of a sharp drop in P^M from 1 to some value below it. One example of this would be the introduction of the 1970 Fortuna-Baslini law, which first introduced divorce in the Italian legal code.

Since the law was approved at end-year of 1970, a simple strategy to empirically assess the impact of this sharp (and arguably exogenous) increase in the probability of divorcing would be to compare the LFP or married women in 1971 with their LFP right before the introduction of the law, in 1970, in a Diff-in-Diff framework where the control group are unmarried Italian women. Yet, our control group may also be affected by treatment if unmarried women expect to get married and are thus to rely on the same marriage insurance as married women do. Nevertheless, this could serve as preliminary evidence and is supported by the recent studying the impact of no-fault divorce laws on the labour supply of women with vs without children (Genadek et al. 2007).

To validate our hypothesis, we would need data on the labour force participation rate of Italian women around 1970. To do so, we would have to consult the historical archives of the Italian National Statistical Institutes, only available upon request. Luckily, very detailed historical data on the female LFP in Italy, dating back to the 1860s, has been recently surveyed and recorded (see Mancini (2018) and Mancini (2019)). To further validate the hypothesis that the increase in married women's LFP is attributable to the perceived increase in the probability of divorce, we would need to find some proxy for this measure in survey data from the time.

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Appendix

A An alternative model with unions (Question 5.2)

Let us consider a different model to analyse the effects of sectoral shocks in an economy with labour unions.
Let us introduce nominal downward wage rigidity imposed by unions as follows:

$$(W_t^i - \overline{W})(N_t^i - \overline{N}) > 0 \tag{A.1}$$

where the strict inequality crucially captures nominal downward wage rigidity. In particular, the nominal wage must be no smaller than a threshold $\overline{W}$ − which could be interpreted as a minimum wage − set by the union, and when $N_t^i > \overline{N}$, wages are set to the lower bound $W_t = \overline{W}$, whereas if workers experience an increase in wages, it must be that $N_t = \overline{N}$ (which, with some abuse of terminology, is equivalent of having full capacity in labour).

The problem faced by the firms is the same as before, except for wages now being downwardly rigid. For $i = 1, 2$:

$$\max_{N_t^i, Y_t^i} \; P_t Y_t^i - W_t^i N_t^i$$

s.t.

$$Y_t^i = A_t^i \ln N_t^i$$

Yielding the Lagrangian:

$$\mathcal{L} = P_t Y_t^i - W_t^i N_t^i + \lambda_t \left[A_t^i \ln N_t^i - Y_t^i \right]$$

and the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial Y_t^i} : P_t = \lambda_t^i$$
$$\frac{\partial \mathcal{L}}{\partial N_t^i} : W_t^i = \lambda_t^i A_t^i \frac{1}{N_t^i}$$

which can be rearranged to

$$\frac{W_t^i}{P_t} = \frac{A_t^i}{N_t}$$

This is the same optimality condition we have obtained above. Let us now explore what is the effect of introducing unions and downward wage rigidity to the economy.
First of all, let us assume first the case where $W_t^i = \overline{W}$ and $N_t^i > \overline{N}$.
Then, the optimality condition for both firms becomes

$$\frac{\overline{W}}{P_t} = \frac{A_t^i}{N_t^i}$$

Since prices are also exogenous, the real wage is exactly pinned down, and cannot adjust in equilibrium.

When a productivity shock is introduced in sector 1, the right-hand side of the optimality condition increases, from $\frac{A_t^1}{N_t^1}$ to $\frac{A_t^{\prime 1}}{N_t^1}$. For a given real wage, at optimum labour demand in sector 1 must decrease. Furthermore, since there has not been a shift in wage, there will be no incentive for mobility across the two sectors. The result is a drop in labour supply in the aggregate, since there is no spillover effect on sector 2 and $N_t^{1'} < N_t^1$. Hence, in this economy sector 1 would experience labour rationing, as demand for labour cannot accomodate the excess supply from sector 2's workers who wish to reallocate.
Since unions have a more widespread and stronger in European labour markets compared to their US counterpart, one could conclude, based on the underlying assumptions, that a lower labour supply in Europe is in line with a more pervasive role of unions.

By contrast, and for the sake of completeness, suppose we have a case where $W_t^i > \overline{W}$ and $N_t^i = \overline{N}$. The optimality condition of firm i becomes:

$$\frac{W_t^i}{P_t} = \frac{A_t^i}{\overline{N}}$$

An increase in firm 1's productivity will thus result in an increase in the nominal wage $W_t^{\prime 1} > W_t^1$ for any level of $\overline{N}$. This will trigger the same mechanism analysed in a unionless economy: workers from sector 2 will want to move to sector 1, being attracted by the higher wage. However, when wages increase, labour supply is fixed in sector 1, so that it cannot accomodate the excess demand from workers that would like to shift from sector 2 to sector 1. As a result, the wage will not readjust to a common level, but is permanently affected by a sectoral shift in technology, driving a wedge in labour earnings between the two sectors even when workers are ex-ante homogeneous.