A Model of Political Competition with Citizen-Candidates

Osborne and Slivinski (1996)

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Outline

Outline

Introduction

- Novel spatial model of electoral competition
- Used to study the electoral outcomes under different majority rules setting
  - **Plurality rule**: the winner of the election is the candidate who obtains the most votes
  - **Runoff system**: if no candidate obtains a majority in the first-round election, a second round is held between the two most voted candidates
- Main novelty: notion of `citizen-candidate`

Definition: Citizen-candidate

- Each citizen in the population chooses **whether** to run for election or not
- The winner of the election implements her favourite policy

Introduction

- The Model focuses on two main questions:
  - How does the **number of candidates** at equilibrium differ between the two systems?
  - How does the **dispersion between different positions** change under the two systems?

Preview of Results

- The number of candidates depends negatively on the cost of running for office, c; and positively on the benefit of winning elections, b
- Two-candidate elections are more likely under plurality
  - Result in line with Duverger's Law: runoff elections favour multi-partism
- Maximum dispersion of candidates' position is smaller under runoff
  - Equilibria with many candidates in the same position are possible under runoff but not plurality
- There exist equilibria in which losing candidates always runs

Outline

Model Assumptions - Set Up

- Continuum of citizens
- Each citizen has *single-peaked* preferences over the set of policy positions, assumed to be the interval $[0, 1] \subset \mathbb{R}$
- F is the distribution of citizen's ideal points over $[0, 1]$, it is continuous and with unique median m
- No cost of voting, <u>no abstention</u>
- The 'ideological' payoff of each citizen i depends on the distance between her ideal point, x_i and the ideal point of the winner, x^*:

$$-|x^* - x_i|$$

Model Assumption - Rules of the Game

- Each citizen can choose either to enter the electoral competition (E), or not (N)
- A citizen who chooses (E) is referred to as *candidate*, and incurs a (utility) cost $c > 0$ to run for office
- The benefit of winning the elections is $b > 0$
- A candidate can only propose her preferred/ideal policy, and citizens rationally anticipate that a winning candidate will implement her preferred policy – thus computing the expected payoff on this
- Voting is ***sincere***:
    - A candidate whose position x_j is occupied by k candidates (including herself), attracts $1/k$ of the votes of the citizens whose ideal points are closer to x_j than to any other candidate.
    - No strategic voting

Model Assumptions - Payoffs

- Assume that the ideal position of the winner is x^*
- If a citizen i decides not to enter the competition (N) and her ideal position is x_i, then her payoff is:

$$-|x^* - x_i|$$

- If a citizen instead enters the competition, then her payoff is:

$$\begin{cases} b - c & \text{if she wins outright} \\ -|x^* - x_i| - c & \text{if she loses outright} \end{cases}$$

- If no one runs, everyone gets $-\infty$

Model Assumptions - Timing of the Game

- **Stage 1**: all citizens simultaneously make a choice of entering or not the electoral competition
  - Candidates are assumed to perfectly anticipate how citizens will vote for any given set of candidates
- **Stage 2**: after choosing between E (becoming a candidate) and N (not entering the competition), citizens cast a vote
  - Important: citizens are assumed to know each candidate's true favourite policy (complete information)
  - Everybody votes, no abstention
- **Stage 3**: the winning candidate is elected and implements her favourite policy when in office
  - Citizens rationally anticipate this

Model Assumptions - Solution Concept

- The model is solved through Pure Strategy Nash Equilibrium

PSNE in this game

An equilibrium is a set of candidates such that, given perfect anticipation of voting behaviour:

- Every citizen who is a candidate is better off being in the race given who else is in the race
- Every citizen who is not a candidate is better off not being in the race

Outline

Some elections are **won by acclamation!**

Proposition 1 – One-candidate equilibrium

There exists a one-candidate equilibrium $\iff b \leq 2c$.
Moreover:

1. if $c \leq b \leq 2c$, then the candidate's ideal position is m
2. if $b \leq c$, then it may be *any* position $x \in [m \pm \frac{(c-b)}{2}]$

▸ See Proof

Sketch of the Proof:

- Start by noting that, to ensure that no other citizen with the same ideal position as the candidate wants to run, the costs of running must outweigh the expected benefits: $\frac{1}{2}b \leq c \Rightarrow b \leq 2c$

Proposition 1

Proof of (1): if $c \leq b \leq 2c$, then the candidate's ideal position is m.

Let $b \leq 2c$, then there is an equilibrium where a single citizen with ideal position m runs since:

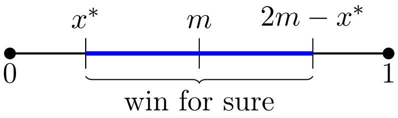
- any other entrant i with $x_i \neq m$ surely loses
- if the candidate at m withdraws, she gets $-\infty$

If there is one candidate at m, another citizen with ideal point m can enter and win with probability $1/2$, getting $\frac{1}{2}b - c$
$\Rightarrow b \leq 2c$ to have an eq.

Proposition 1

Intuition (2): if $b \leq c$, then it may be *any* position $x \in [m \pm \frac{(c-b)}{2}]$.

Note: for $x^* \neq m$, any citizen i with ideal point $x_i \in [x^*, 2m - x^*]$ wins for sure if she enters, getting $b - c$ instead of $-|x^* - x_i|$.



Then, to have an eq. it must be that *all* such citizens do not want to run: $-|x^* - x_i| \geq b - c$ for all $x_i \in [x^*, 2m - x^*]$ $\quad (\Rightarrow b \leq c)$

$$\implies -|x^* - 2m + x^*| \geq b - c \iff 2|x^* - m| \leq c - b$$

$$\iff |x^* - m| \leq \frac{(c-b)}{2} \iff x^* \in [m \pm \frac{(c-b)}{2}] \qquad \blacksquare$$

Proposition 1

Note: *Proposition 1* holds for both plurality and runoff elections

Q: Can you think of a real-life example of One-candidate equilibrium?

- High cost...
- ...Low benefits of winning

Proposition 1

Note: *Proposition 1* holds for both plurality and runoff elections

Q: Can you think of a real-life example of One-candidate equilibrium?

- High cost...
- ...Low benefits of winning

  $\Longrightarrow$ Class representative elections!

Outline

Candidates Running on the Same Platform

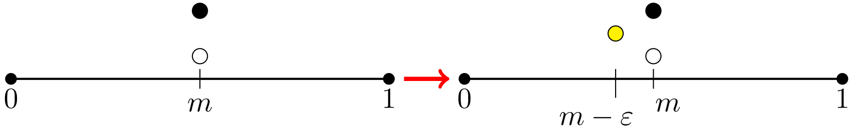
Imagine now an equilibrium where $k \geq 2$ candidates run at the median

- Under **plurality**, there cannot be two or more candidates: if there were, a citizen with ideal position nearby could enter and win.
  - c.f. Cox [1987]: no convergent equilibria under plurality for $k > 2$, with citizen-candidates this does not hold even with $k = 2$ $\Rightarrow$ no Downsian convergence at the median voter
- Under **runoff** elections instead this is possible: the entrant would surely lose the second round against a candidate at the median $\Rightarrow$ More convergence (less dispersion) under runoff!

Clustering at the median

Consider a 2-candidate equilibrium where both cluster at m

Under **Plurality** this is not possible: there are **winning entrants**



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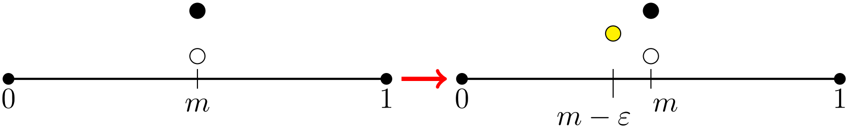
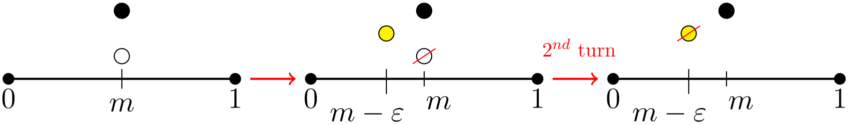


Under **Runoff** this can hold since **entrants lose the second round**



The candidate at the median always wins the second round!

Proposition 2

This result is more general: for an appropriate set of parameters (b, c), runoff elections can support any number of candidates clustered at the median.

Proposition 2 – Single-cluster equilibria under runoff

For any $k \geq 2$ there exists a k-candidate equilibrium in which the ideal position of every candidate is $m \iff kc \leq b \leq (k+1)c$

See Proof

Outline

Two-candidate Equilibria: Duverger's Law

Let i, j, be two candidates with different ideal positions x_i, x_j.

To have a two-candidate equilibrium it must hold that:

1. No other citizen wants to enter the race
2. i and j want to run one against the other

We show that the set of parameter values for which this result holds under a runoff system is a subset of those under which it holds in plurality elections: the model is consistent with **Duverger's Law**

Moreover, the two candidates will be on **opposite sides of the median** voter's ideal point.

To simplify the analysis, we further assume that **preferences are uniformly distributed** over $[0, 1]$, but results extend to any density F continuous with unique median m

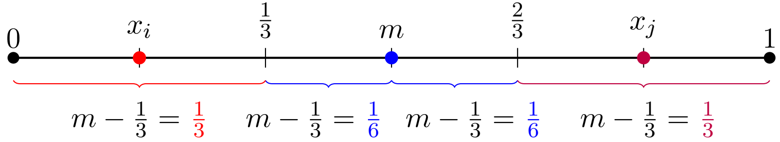
Two-Candidate Equilibria Under Plurality Rule

Suppose that $x_i = m - \varepsilon$, $x_j = m + \varepsilon$ so that each gets half of the votes

- if a candidate with ideal position m enters the race, i's share of votes is still the same as j's: $F[m - \varepsilon] = 1 - F[m + \varepsilon]$,
- and m gets a share of votes of: $\frac{1}{2}(m + \varepsilon - (m - \varepsilon)) = \varepsilon$.

Now note: x_i and x_j cannot be **too dispersed**, because if $x_i \leq m - \frac{1}{3} \Rightarrow \varepsilon \geq \frac{1}{3}$, then m can enter and win for sure:

$\varepsilon = \frac{1}{3}$



Proposition 3

But x_i and x_j cannot be **too similar** either because otherwise either candidate may prefer to give up b, save c and let the other candidate implement their preferred policy for sure. That is:

$$\frac{b}{2} - c > -|x_i - x_j| \iff 2\varepsilon > c - \frac{b}{2} \iff \varepsilon > \frac{1}{2}(c - \frac{b}{2})$$

Proposition 3 – Two-Candidate Equilibria Under Plurality

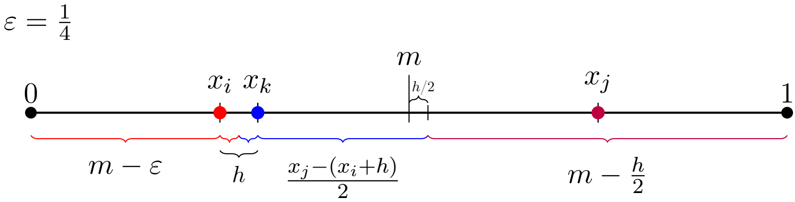
- Two-candidate equilibria exist only if $\frac{1}{3} > \varepsilon > \frac{1}{2}(c - \frac{b}{2})$
- In *any* two-candidate equilibrium, the candidate's ideal positions are $m - \varepsilon$ and $m + \varepsilon$

Two-Candidate Equilibria Under Runoff

- As per plurality, x_i and x_j cannot be **too similar**, or either candidate may prefer to give up b, save c and let the other candidate win. As before then: $\varepsilon > \frac{1}{2}(c - \frac{b}{2})$.
- Also under runoff elections x_i and x_j cannot be **too dispersed**, but now any citizen with ideal policy in the interval $(m - \varepsilon, m + \varepsilon)$ who receives more votes than **at least one** of the two candidates can enter the race, get to the second run and surely win (since she's closer to the median) $\Longrightarrow$ max dispersion must be even smaller.

Two-Candidate Equilibria Under Runoff

Wlog, consider a citizen k marginally closer to m than x_i:



Then, for the equilibrium to exist, k must be unable to advance to the runoff, that is, k's share of votes must be smaller than i's:

$$\frac{h}{2} + \frac{x_j - x_i - h}{2} < x_i + \frac{h}{2} \iff \frac{x_j - x_i}{2} - \frac{h}{2} < x_i \textit{ letting } h \to 0 :$$

$$\frac{2\varepsilon}{2} < m - \varepsilon \iff \varepsilon < \frac{1}{4} \quad (< \frac{1}{3} \textit{ c.f. plurality})$$

Two-Candidate Equilibria

Proposition 4 – Two-Candidate Equilibria Under Runoff

- Two-candidate equilibria exist only if $\frac{1}{4} > \varepsilon > \frac{1}{2}(c - \frac{b}{2})$
- In *any* two-candidate equilibrium, the candidate's ideal positions are $m - \varepsilon$ and $m + \varepsilon$

Therefore:

- Dispersion will be smaller under Runoff elections
- The values of (b, c) for which two-candidate equilibria are possible under runoff elections are a subset of those for which it exists under plurality

$\Longrightarrow$ Two-candidate equilibria are *more likely* under plurality: This is in line with the first hypothesis of **Duverger's law**

Outline

Three-Candidate Equilibria with a Sure-loser

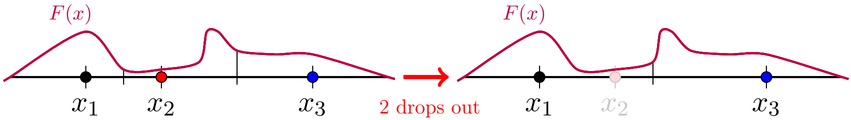
Under **plurality**, there exist equilibria where one candidate surely loses, but runs anyway to change the identity of the winner:

- the two other candidates get the same share of votes,
- the sure-loser must prefer the resulting equal-probability lottery between the other two over the sure victory of one of them if she withdraws.
- This result never holds under runoff: a sure-loser does not affect who gets to the runoff and thus the ultimate winner.

Note: the distribution of voters' preferences **cannot be symmetric**, otherwise withdrawal by the sure-loser would result in a certain victory by the candidate she likes the most!

Three-Candidate Equilibria with a Sure-loser

Consider 3 candidates and an asymmetric distribution of preferences:



Under **plurality**, if the candidate at x_2 drops out, the one at x_3 wins!
Under **runoff**, candidate 2 never affects the ultimate winner.

Q: Why can't the preference distribution be symmetric?

Three-Candidate Equilibria with a Sure-loser

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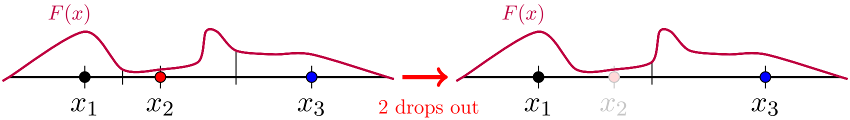

Under **plurality**, if the candidate at x_2 drops out, the one at x_3 wins! Under **runoff**, candidate 2 never affects the ultimate winner.

Q: Why can't the preference distribution be symmetric?

A: If it were, 2's withdrawal would result in the certain victory of candidate 1, since 1 is tying with 3 and is closer to 2 than 3 is. Then, since candidate 2 prefers x_1 over x_3 $\Rightarrow$ not running would be a dominant strategy for 2!

Three-Candidate Equilibria with Competitive Candidates

For 3 competitive candidates, the following characterizations hold.

- **Plurality**: candidates' positions are **not all the same**.
  - Two candidates may share the same positions with the third one running on a different position;
  - or all positions can be different.
  - Each candidate obtains $\frac{1}{3}$ of the votes.

HOWEVER, the necessary conditions set forth by the authors are **not sufficient**. Recall that if F is symmetric there is no equilibrium in which one candidate surely loses.

- If preferences are **symmetric and single-peaked** neither a sure loser, nor two candidates sharing the same position are feasible in equilibrium.

  $\Rightarrow$ All positions **must be different** to have an equilibrium!

- **Runoff**: the necessary condition is also sufficient, three-candidate equilibria exist for any distribution F if $3c \leq b \leq 4c$.
  - These are converging equilibria, where all candidates share the same position, m.
  - Equilibria are less dispersed than under plurality! This results holds also for $k = 4$. See more
- **HOWEVER,** for some values of our parameters b, c there are three-candidate equilibria under plurality but not under runoff!

Electoral rules' effect on equilibria's likelihood is ambiguous!

- Notice that Runoff does not require any specific distributional assumption...
- ...but is more restrictive than Plurality on parameters' values!

Outline

Conclusions

Divide Duverger's Law into two statements:

1. a two-candidate election is more likely under plurality rule than under a runoff system;
2. an election with k candidates, for any $k > 2$, is more likely under a runoff system than under plurality rule.

The model predicts (1) in the strongest possible sense and predicts (2) for k equal to 3 or 4 in a weaker sense.

In particular, three- and four-candidate equilibria:

- exist under a **runoff** system **for any distribution** F, for appropriate values of parameters b and c,
- do not exist under **plurality** system **for some distributions** F, for any parameter values.

Conclusions

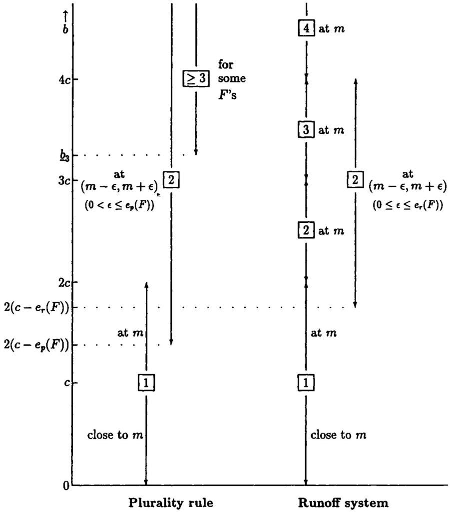

Figure 1: N° of Candidates in Possible Equilibria, as a function of b, c

Conclusions

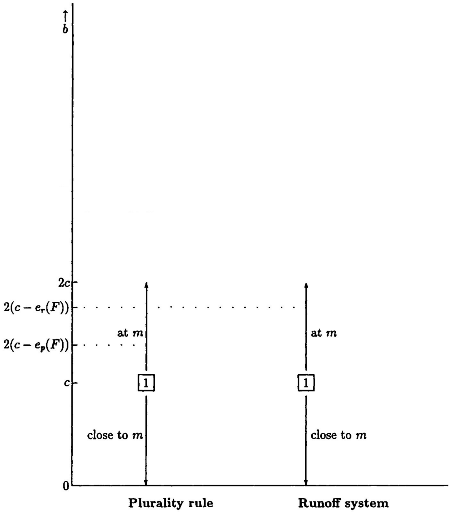

Figure 2: Equilibria characterization for $k = 1$, as a function of b, c

Conclusions

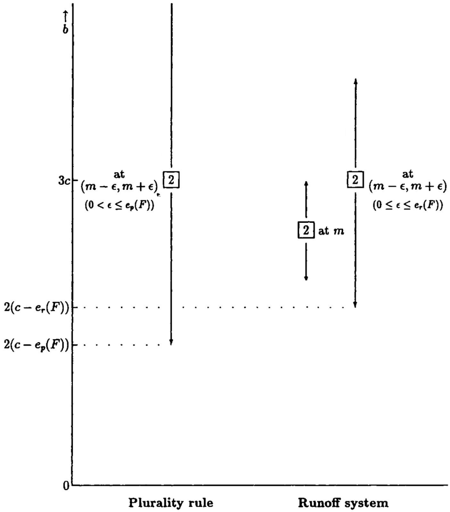

Figure 3: Equilibria characterization for $k = 2$, as a function of b, c

Conclusions

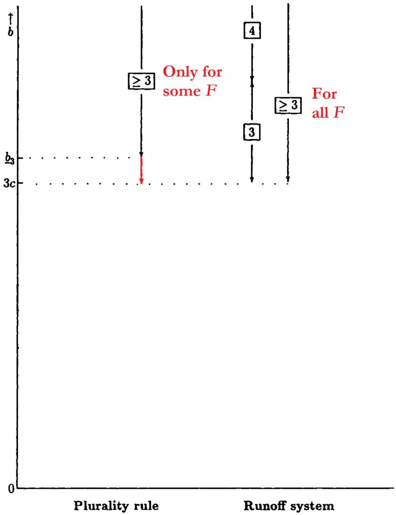

Figure 4: Equilibria characterization for $k \geq 3$, as a function of b, c

Relation with Previous Work

- This paper departs from **Hotelling (1929)** in two respects:
  1. the set of candidates is endogenous;
  2. candidates are (also) policy-motivated, rather than just office-motivated.
- **Palfrey (1984)** studies a three-candidate model in which the third candidates chooses to enter after observing the two other candidates. The third candidate loses in equilibrium.
- **Besley and Coate (1997)** develop a notion citizen-candidate applied to a model of strategic voting behaviour. They find that there are never more than 2 candidates in plurality in equilibrium.

Relation with Previous Work

- Duverger's Law (**Duverger, 1954** via Riker, 1982):
  1. Two-candidate election is more likely under plurality than under a runoff system
  2. An election with $k > 2$ candidates is more likely under a runoff system than under plurality
- **Palfrey (1989)** and **Feddersen (1992)** both predict that under plurality two candidates get (almost) all the votes, but that there might be more than two candidates.

Limitations and Open Questions (I)

- Everything showed holds also if we consider a separate pool of candidates with preferences drawn from the same distribution as the voters
  - This seems unrealistic: citizens who can/decide to run are a selected sub-sample of the voting population.
- If citizens can run (and credibly commit) to policies which are not their favourite ones the results no longer hold (agency problems).
- With strategic voting, some results no longer hold: there are never more than 2 candidates in plurality in equilibrium [Besley and Coate (1997a)].

Limitations and Open Questions (II)

- No analysis of the efficiency-properties of Citizen-candidates? [Basley and Coate (1997b)]
- Lack of pre-existing electoral candidates makes it hard to extend the model to include political parties [Persson and Tabellini 2002]
- No characterization of equilibria for k arbitrarily large
- Multiple equilibria make it hard to use the model to make testable predictions
- **No empirics!**

Outline

Bibliography

- Besley, T., Coate, S. (1997). An Economic Model of Representative Democracy. The Quarterly Journal of Economics, 112(1), 85–114.
- Cox, G. W. (1987). Electoral Equilibrium under Alternative Voting Institutions. American Journal of Political Science, 31(1), 82.
- Cox, G. W. (1997). Making Votes Count: Strategic Coordination in the World's Electoral Systems (1st ed.). Cambridge University Press.
- Duverger, M. (1954). Political Parties, Their Organization and Activity in the Modern State. Methuen.
- Feddersen, T. J. (1992). A Voting Model Implying Duverger's Law and Positive Turnout. American Journal of Political Science, 36(4), 938.

Bibliography

- Hotelling, H. (1929). Stability in Competition. The Economic Journal, 39(153), 41.
- Osborne, M. J., Slivinski, A. (1996). A Model of Political Competition with Citizen-Candidates. The Quarterly Journal of Economics, 111(1), 65–96.
- Palfrey, T. R. (1984). Spatial Equilibrium with Entry. The Review of Economic Studies, 51(1), 139.
- Palfrey, Thomas R. (1989) A mathematical proof of Duverger's Law. In: Models of Strategic Choice in Politics. University of Michigan Press, Ann Arbor, pp. 69-91.
- Persson, T., Tabellini, G. (2002). Political Economics and Public Finance Handbook of Public Economics (Chapter 24, Vol. 3, pp. 1549–1659).
- Riker, W. H. (1982). The Two-Party System and Duverger's Law: An Essay on the History of Political Science. The American Political Science Review, 76(4), 753–766.

Appendix: Proof of Proposition 1

To ensure that no other citizens with the same ideal position wants to enter it must hold that $\frac{1}{2}b \leq c$.
Now, notice that if $\frac{1}{2}b \leq c$ there is an equilibrium where a single citizen with ideal position m enters since:

- any other entrant surely loses
- if the candidate at m withdraws she gets $-\infty$

Instead, if there is a single candidate with position $x^* \neq m$, then any citizen i with ideal point $x_i \in [x^*, 2m - x^*]$ wins for sure if enters, getting $b - c$ instead of $-|x^* - x_i|$.

- Then, $-|x^* - x_i| \geq b - c \ \forall x_i \in [x^*, 2m - x^*]$ is a necessary condition for the existence of this equilibrium.
- Moreover, it implies $b \leq c$ and $|m - x^*| \leq \frac{(c-b)}{2}$
- and is also a sufficient since a citizen with $x_i \notin [x^*, 2m - x^*]$ wins with probability $\frac{1}{2}$ if enters, while the candidate gets $-\infty$ from withdrawing ∎

◂ Return to Proposition 1

Appendix: Proof of Proposition 2

Suppose that there are k candidates with common ideal position m.

For this to be an equilibrium it must hold that no other citizen wants to enter the race:

- Consider a citizen i with ideal position $x_i \neq m$. Then for $k \geq 2$:
  - if the candidate is too far from m, she fails to enter the runoff
  - if she does gets to the runoff, she always lose against a candidate running on m
    $\Longrightarrow$ for i it is optimal not to run

For this to be an equilibrium, it must also hold that no other citizen with ideal position m wants to run, then:

- $\frac{1}{(k+1)} \cdot b \leq c$

Finally, it must be optimal for the k candidates to run:

- $\frac{1}{k} \cdot b \geq c$ ■

◂ Return to Proposition 2

Appendix: Four-Candidate Equilibria

With $k = 4$, the following results hold.

- **Plurality:** there are four different equilibrium configurations depending on candidates' positions.
  - There can be a surely losing candidate,
  - or each candidate receives $\frac{1}{4}$ of votes.
  - **No more than two positions can be exactly the same!**
- **Runoff:** equilibria are more agglomerated!
  - As before, there is a converging equilibrium on the median,
  - or, there can be two clusters of candidates around the median, which was **impossible under plurality!**
  - The same characterization holds for any $k \geq 4$ **even**.

Appendix: Four-Candidate Equilibria

Under a generic F there is only one configuration resulting in an equilibrium under both Plurality and Runoff elections for $k = 4$. It is one where:

- all candidates' positions are different,
- two extreme candidates and one of the middle candidates obtain the same number of votes in the first round,
- the remaining candidate obtains fewer votes.

Meaning that:

- under **Plurality** the two "extremists" and one of the "centrists" tie. Each one of them wins the election with probability $\frac{1}{3}$.
- Under **Runoff** two of the same three candidates move with equal probability to the second round to determine the winner.
- In both cases there is a **sure loser!**

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