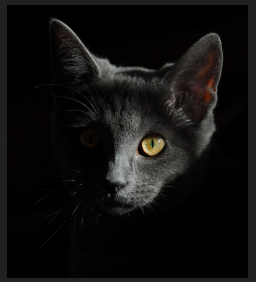
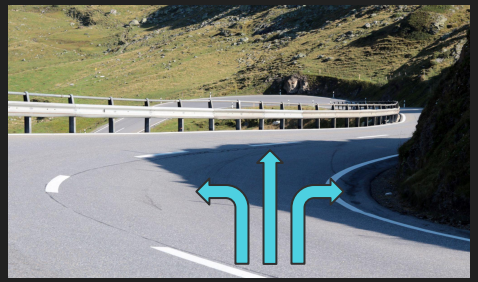
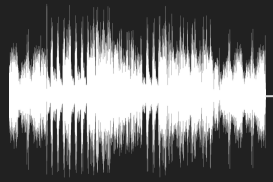
Lecture 10
Reinforcement learning - DQN



Dog or cat?



Left, straight or right?



/ˌgʊd ˈmɔː.nɪŋ/

Speech to phonemes

Yellow paperclip

Given a sentence create an image

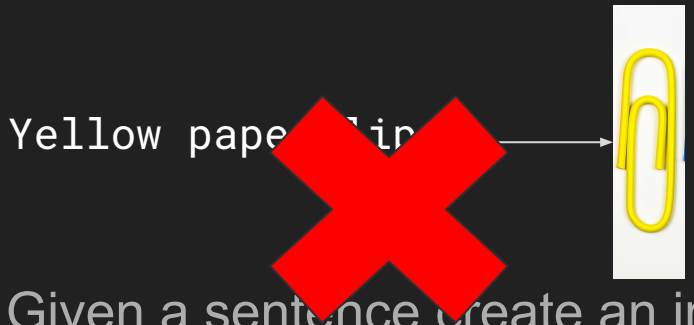
Supervised learning:

- Unique output (correct answer aka label) for each input.
- Input samples are independent.

Current output influences future inputs.

Left, straight or right?

Non-unique output.



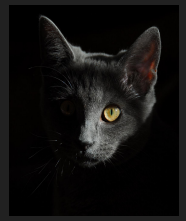
Given a sentence create an image

Supervised learning:

- Unique output (correct answer aka label) for each input.
- Input samples are independent.

Supervised learning

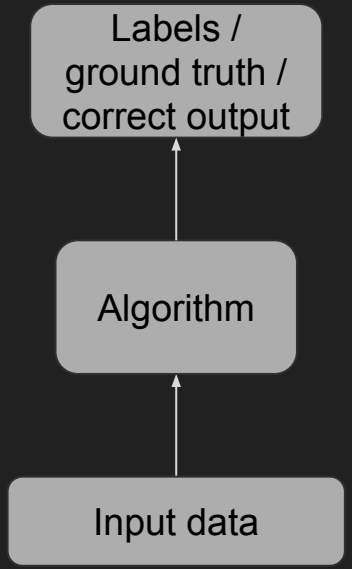
Goal: learn a mapping from data to labels.



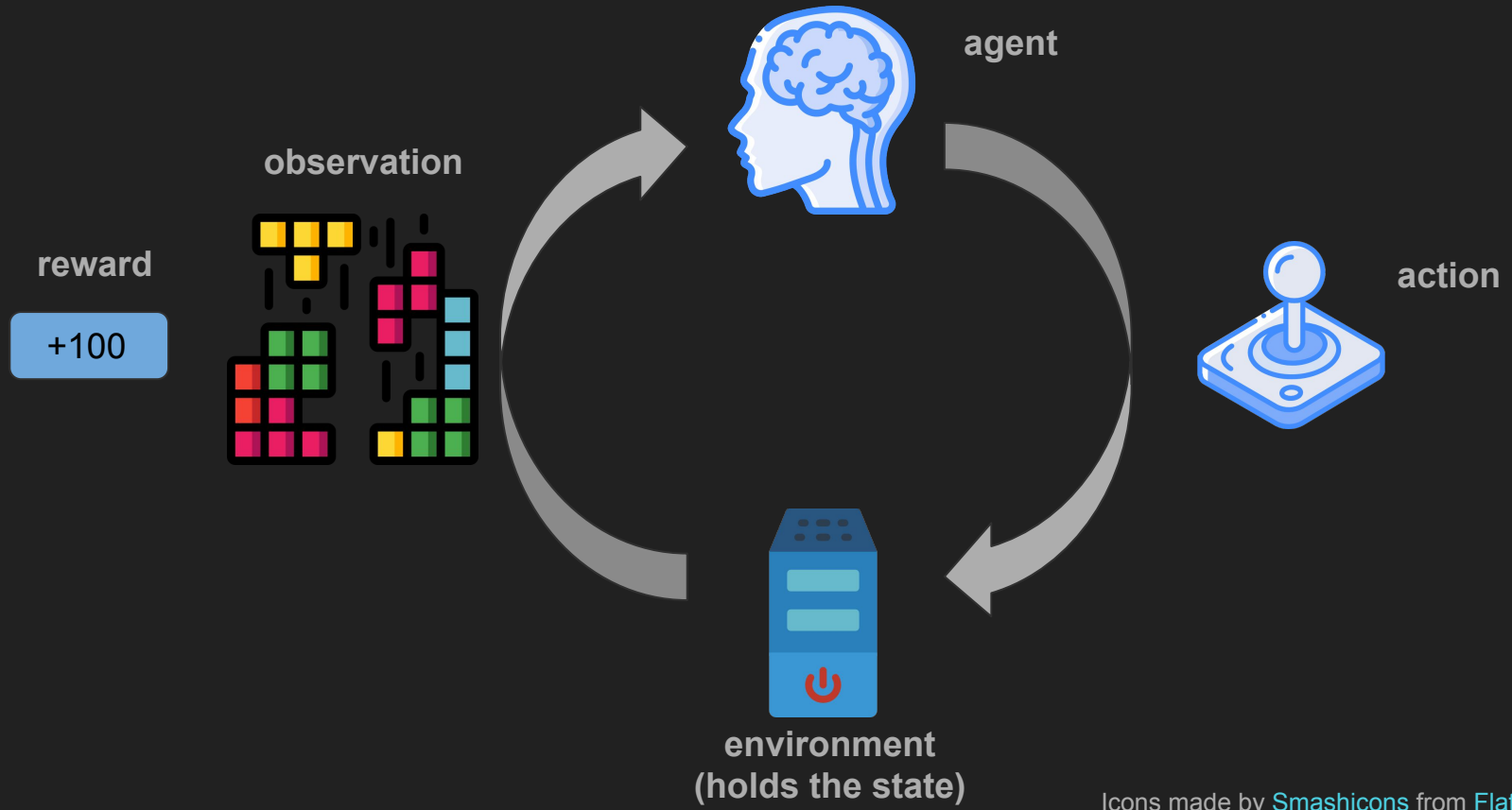
Dog or cat?

/ˌgʊd ˈmɔː.nɪŋ/

Speech to phonemes



Reinforcement learning



Icons made by Smashicons from Flaticon

Reinforcement learning success stories

nature
THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE

At last — a computer program that can beat a champion Go player PAGE 484

ALL SYSTEMS GO

© AP Photo/Lee Jin-man

CONSERVATION
SONGBIRDS À LA CARTE
Illegal harvest of millions of Mediterranean birds
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RESEARCH ETHICS
SAFEGUARD TRANSPARENCY
Don't let openness backfire on individuals
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WHEN GENES GOT 'SELFISH'
Dawkins's calling card forty years on
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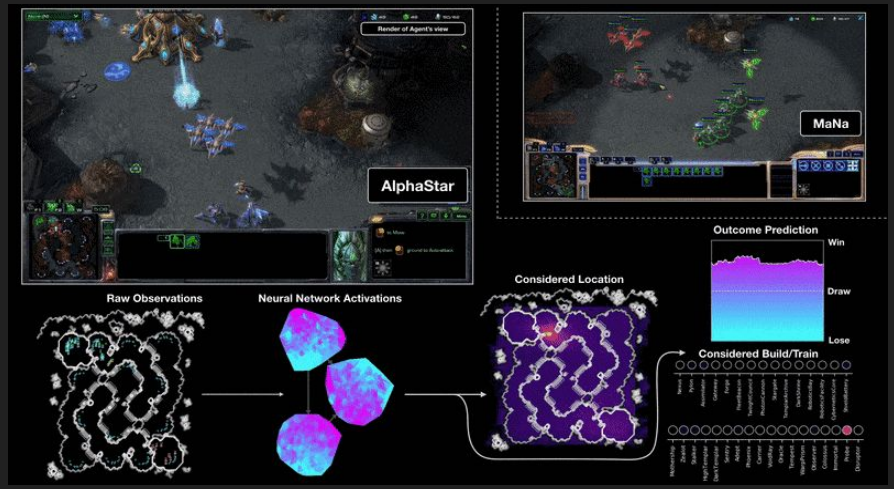
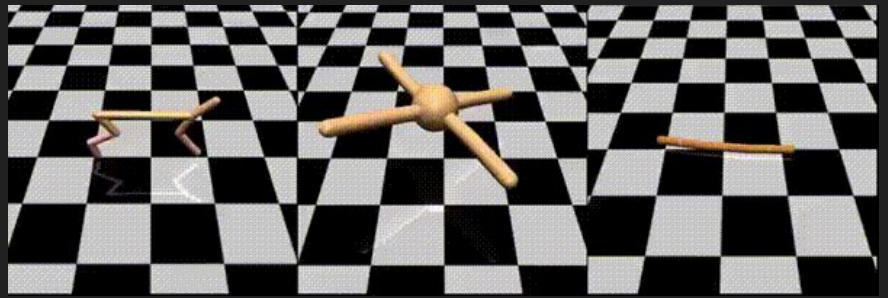
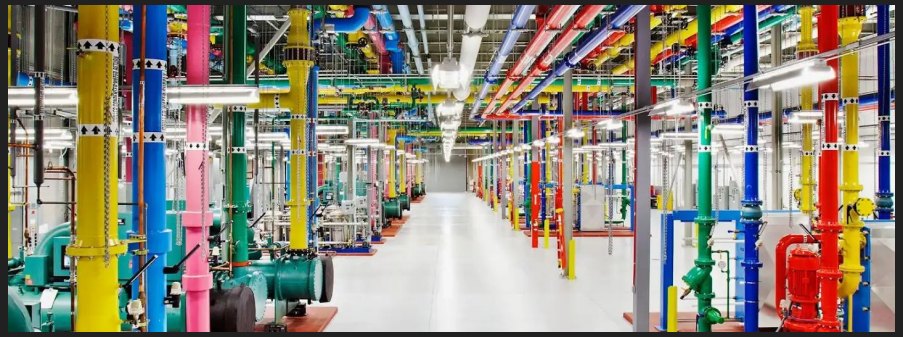


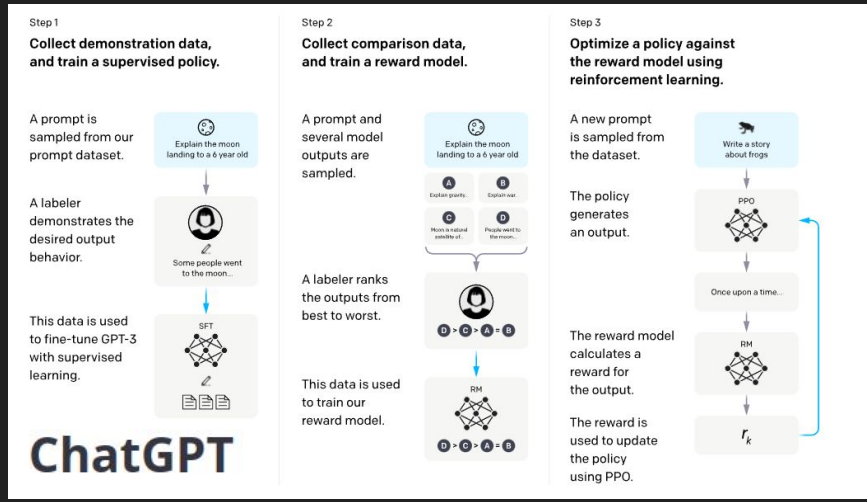


Images from Atari games, Starcraft, Dota 2, Mujoco



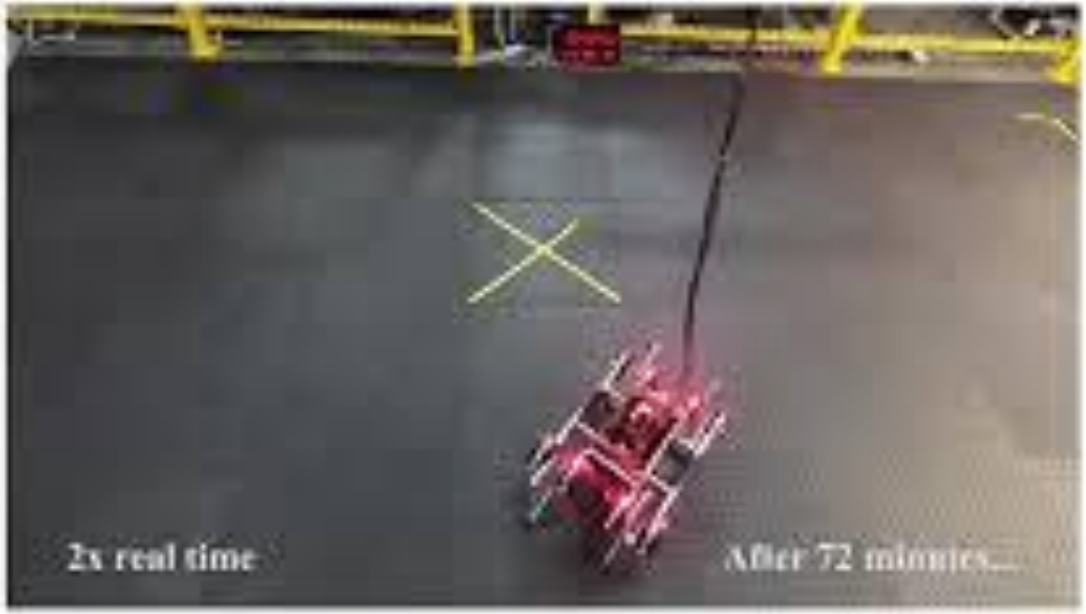
Datacenter cooling

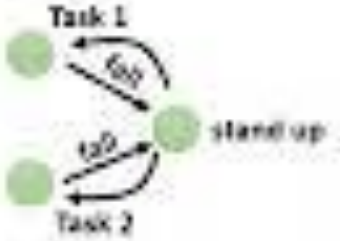
Robot farm



RLHF

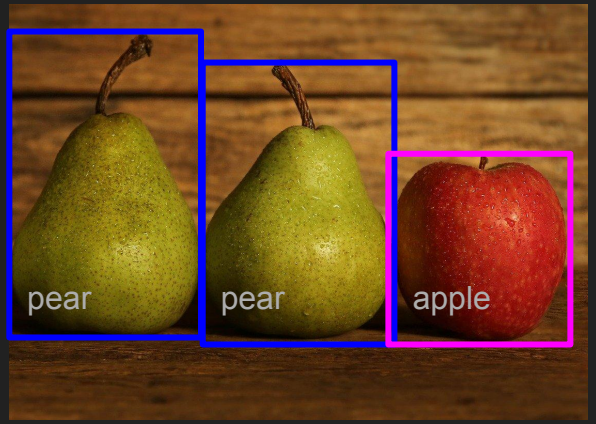
Making it more autonomous…



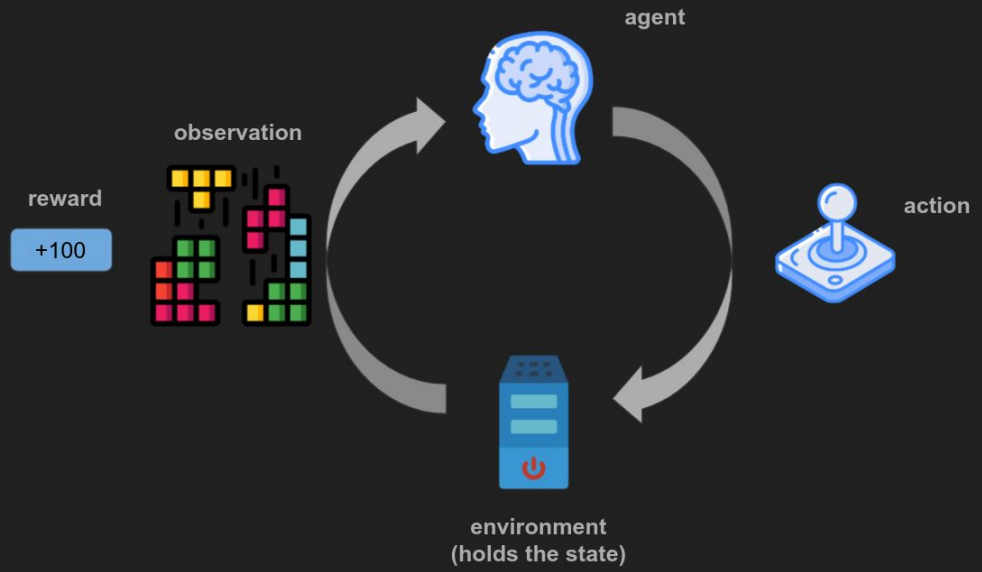


RL vs supervised learning

From single prediction to multi-step control



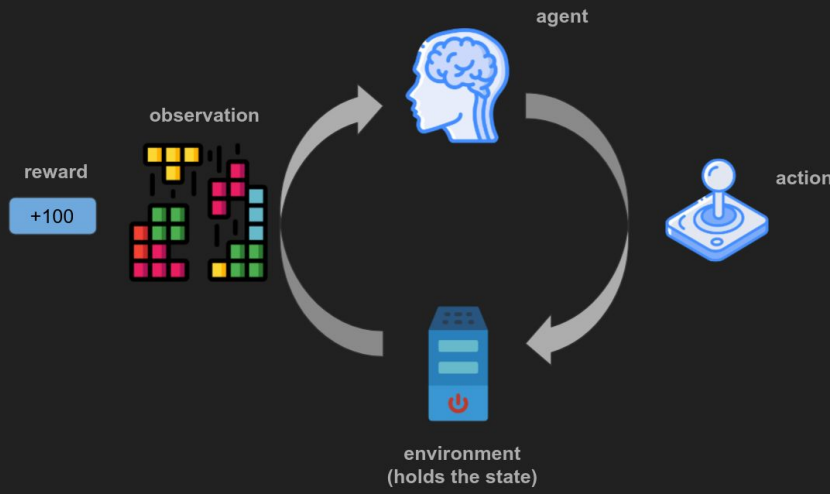
Predictions do not influence future inputs.

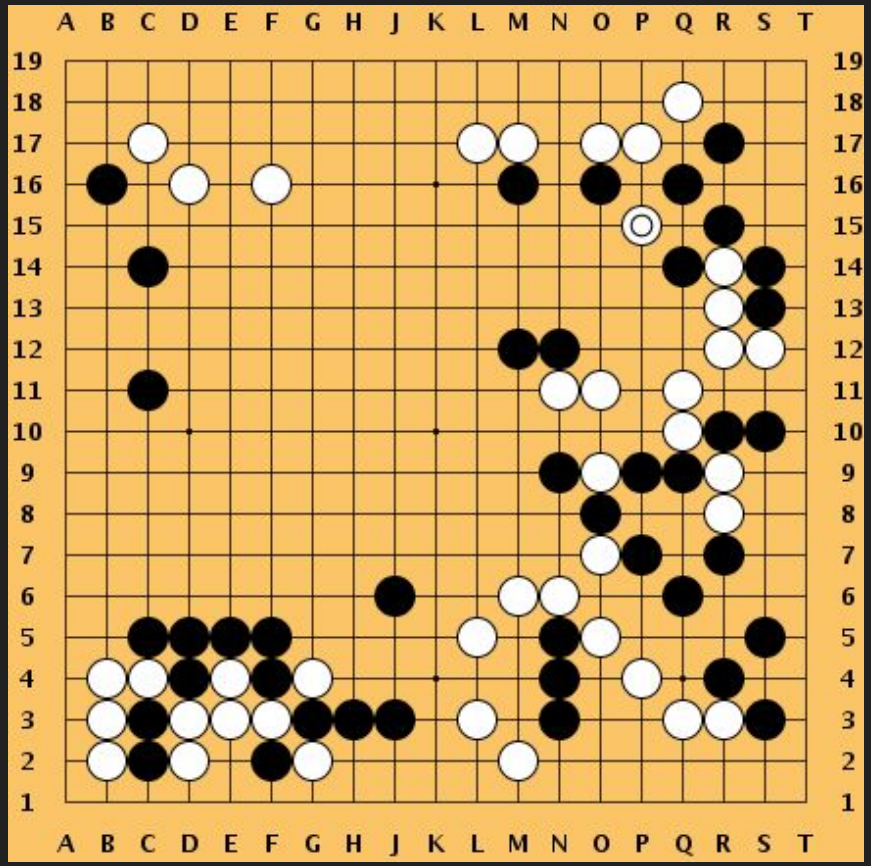


Predictions do influence future inputs.

What is the problem?

- Data is non-i.i.d.
  (current actions affect future observations)
- Sequence of actions
  (delayed feedback, credit assignment).
- Gradient of an arbitrary objective function
  (no supervision, only reward signal)
- Environment is stochastic.





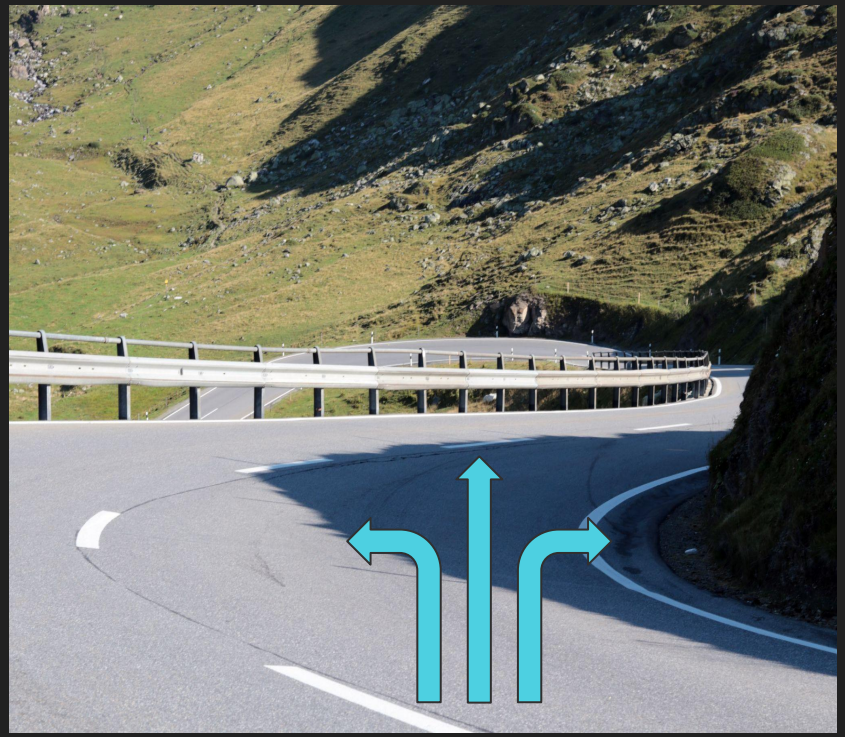
Agent: player.

State: positions of all stones.

Observation: positions of all stones.

Action: placing a stone.

Reward: +1 for winning, -1 for losing.



Agent: driver.

State: state of the world.

Observation: road view, dashboard.

Action: steering wheel, pedals.

Reward: +1 when arriving safely.

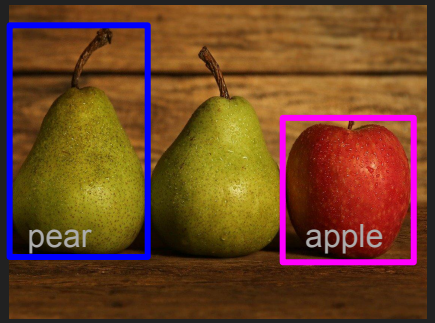
Image by S. Hermann & F. Richter from Pixabay

Supervised learning as a 1-step RL

agent = trained model

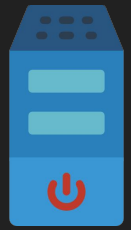
Observation = image

action = prediction



0.75

reward =
Mean average precision



Rest of the course

The next 3 lectures will touch on:

- Q-learning
- Policy gradients
- Alpha zero
- RL from Human Feedback
  - the key step from GPT-3 to ChatGPT

Deep Q-learning (DQN)

Approximate dynamic programming

Mnih et al. *Human-level control through deep reinforcement learning, Nature 518, 529–533 (February 2015, arxiv version Dec 2013)*

Gambler problem (Sutton & Barto, example 4.3)

- Gambler bets on coin flips. Probability of heads is p_h.
- In each round gambler decides on how much to bet (integral number of $) - x.
- Heads → he wins x$, tails → he loses x$.
- The game ends when the gambler has ≥ 100$ (he wins), or when the gambler loses all the money.

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Dynamic programming (DP):

$V_k(s)$ - chances of winning (reaching 100$) in at most k turns, having s$ currently

slido

Which recursive formula is correct?

ⓘ Start presenting to display the poll results on this slide.

Gambler problem (Sutton & Barto, example 4.3)

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Dynamic programming (DP):

$V_k(s)$ - chances of winning (reaching 100$) in at most k turns, having s$ currently

$V_k(s) = \max_{i=0,\dots,s} p_h V_{k-1}(s+i) + (1-p_h)V_{k-1}(s-i)$

$V_0(s) = 1$ if $s \geq 100$, 0 otherwise.

Value function

For a state s define:

- $V^*(s)$ - sum of rewards when starting from state s and acting optimally (how many points can I score **from now** till the end of the game)
- $V_k^*(s)$ - as above but for playing k turns

Current score **does not** count towards V!

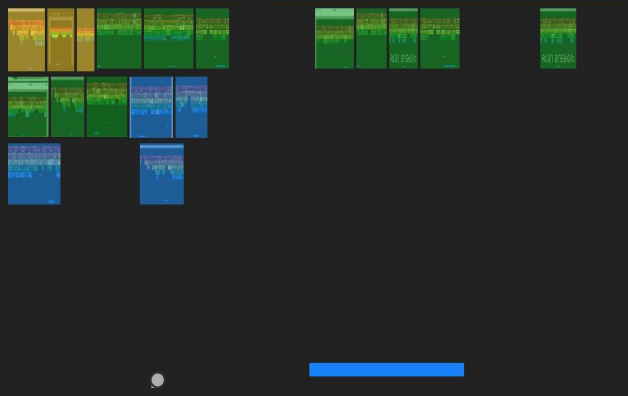
slido

Q: Which state has a higher V-value?

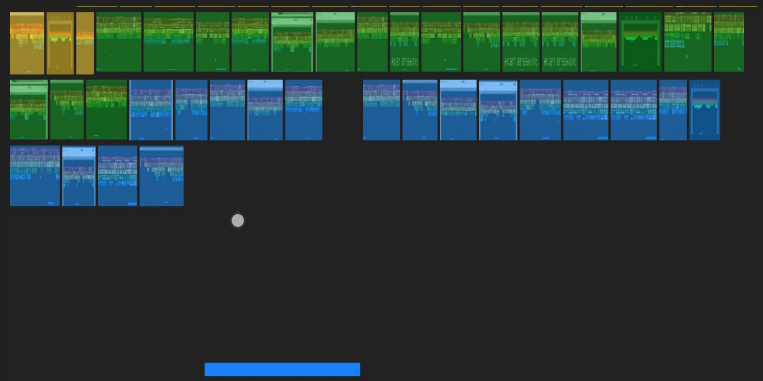
ⓘ Start presenting to display the poll results on this slide.

Value function

Score: 100



Score: 10



Q: Which state has a higher V-value?

Value function

$$V_k^*(s) = max_\pi \mathbb{E}[\sum_{t=0}^{k-1} R(s_t, a_t, s_{t+1}) | \pi, s_0 = s]$$

selecting policy

reward

Conditioning on the policy and initial state

Value iteration (Bellman update)

Start with $V_0(s) = 0$ for each state s.
For k = 1, …, H:
    For each state s:

$$V_k^*(s) = max_a \sum_{s'} P(s'|s,a) \cdot (R(s,a,s') + V_{k-1}^*(s'))$$

selecting action

Immediate reward

Look-up for the remaining k-1 steps

Discount factor

- Immediate reward of x is worth more than reward of x in the future
- Without discounting infinite total reward is possible
- $\gamma < 1$ (think 0.99)

$$V_k^*(s) = max_\pi \mathbb{E}[\sum_{t=0}^{k-1} \gamma^t R(s_t, a_t, s_{t+1}) | \pi, s_0 = s)]$$

$$V^*(s) = max_\pi \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, s_{t+1}) | \pi, s_0 = s)]$$

discounting

Value iteration (with discounts)

Start with $V_0(s) = 0$ for each state s.
For k = 1, …, H:
    For each state s:

$$V_k^*(s) = max_a \sum_{s'} P(s'|s,a) \cdot (R(s,a,s') + \gamma V_{k-1}^*(s'))$$

discounting

Theorem. *Value iteration converges. At convergence, we have found the optimal value function V* for the discounted infinite horizon problem, which satisfies the Bellman equations*

$$\forall S \in S: \quad V^*(s) = \max_a \sum_{s'} P(s'|s,a)\left[R(s,a,s') + \gamma V^*(s')\right]$$

$$\lim_{k\to\infty} V_k^*(s) = V^*(s)$$

source

Policy

Assume we have V*, but how to play the game?

$$\pi^*(s) = argmax_a \sum_{s'} P(s'|s,a) \cdot (R(s,a,s') + \gamma V^*(s'))$$

Requires knowing the game model
(transition probabilities, rewards, etc.)

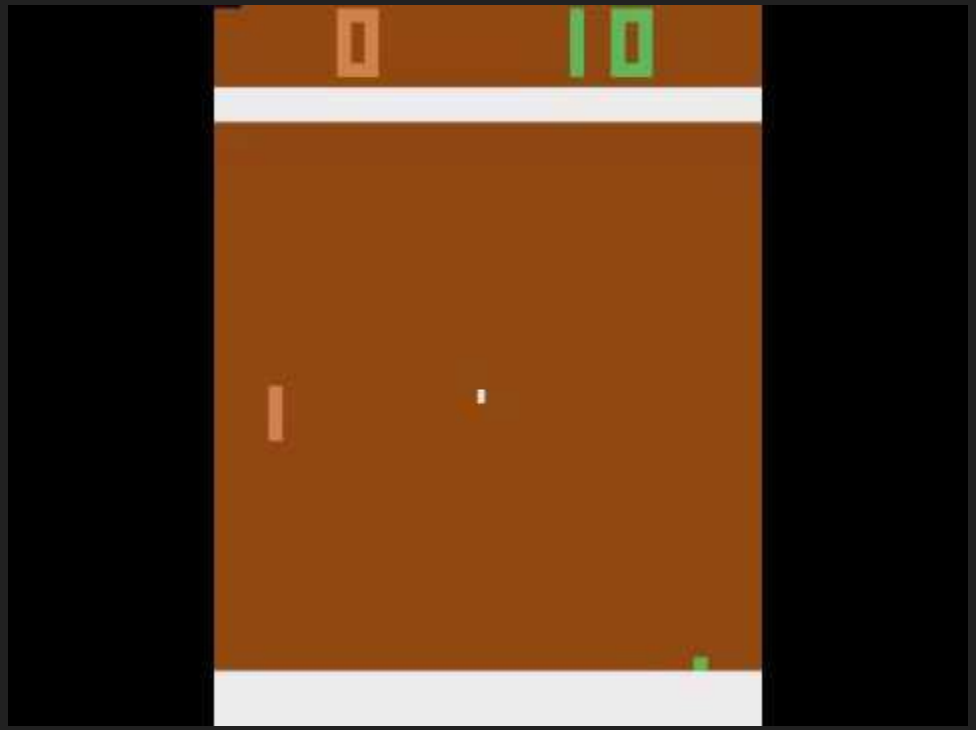
Q-learning

Q-values (action values)

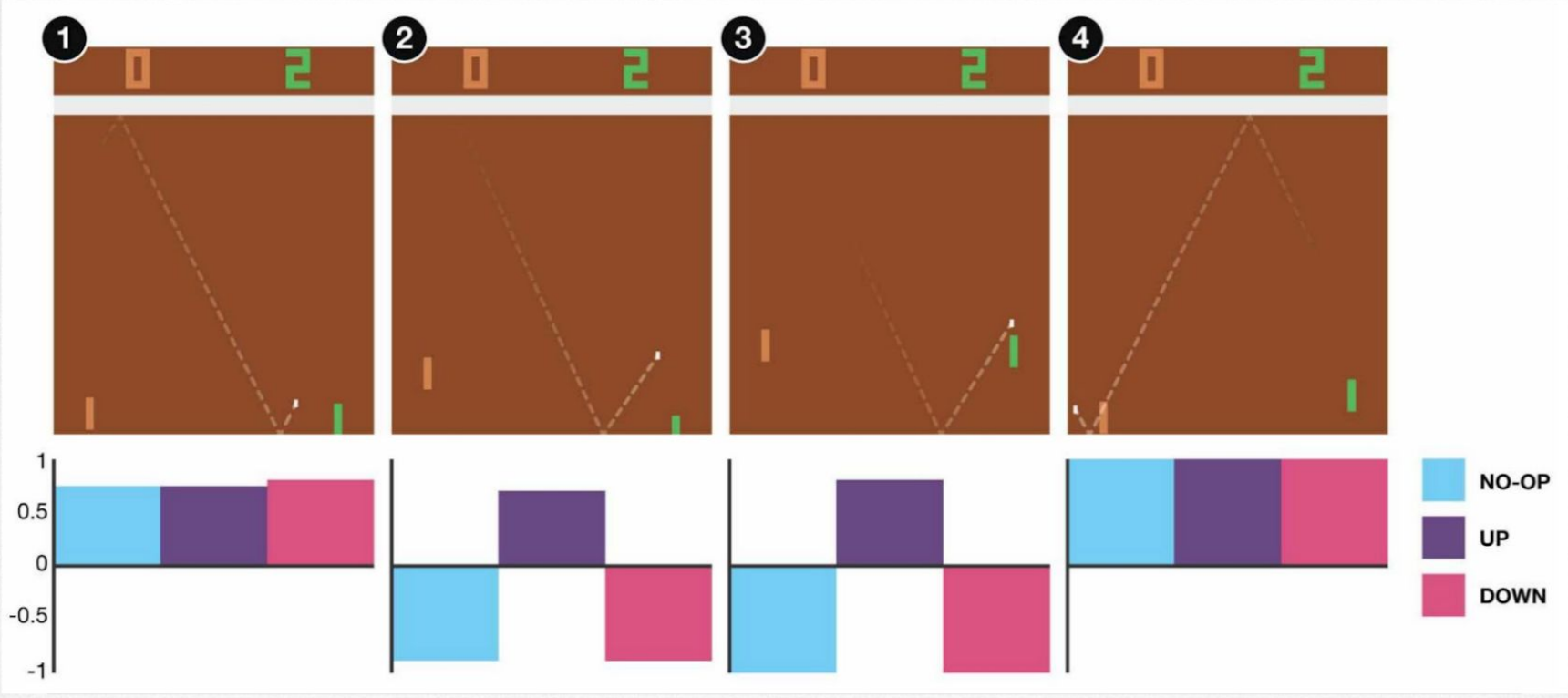
- Q*(s,a) - expected discounted reward, when starting in s, making action a and then playing optimally
- If we have Q-values, we can play the game!

$$\pi^*(s) = \arg\max_a Q^*(s, a)$$

Pong



Action Values on Pong



source

V*(s')

Bellman Equation:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$

Q-Value Iteration:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$$

source

Tabular Q-learning - TODO: k no longer means time horizon, just iter number

- Q-value iteration: $Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$
- Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$
- (Tabular) Q-Learning: replace expectation by samples
  - For an state-action pair (s,a), receive: $s' \sim P(s'|s,a)$
  - Consider your old estimate: $Q_k(s,a)$
  - Consider your new sample estimate:

    $$\text{target}(s') = R(s,a,s') + \gamma \max_{a'} Q_k(s',a')$$

  - Incorporate the new estimate into a running average:

    $Q_{k+1}(s,a) \leftarrow (1-\alpha) Q_k(s,a) + \alpha [\text{target}(s')]$

source

$$Q_{k+1}(s,a) \leftarrow Q_k(s,a) + \alpha \cdot \text{target}(s')$$

Q-learning converges, if:

- Each state is visited infinitely often
- Eventually the learning rate gets very small, but not too quickly

$$\sum_{t=0}^{\infty} \alpha_t(s,a) = \infty \qquad \sum_{t=0}^{\infty} \alpha_t^2(s,a) < \infty$$

In practice

- Use ε-greedy exploration strategy:
  - In a given state, with probability ε act randomly
  - With probability (1-ε) act greedily (according to the current estimates)
  - Restart after an episode finishes

- Anneal the learning rate

Basic Q-learning algorithm

Initialize $Q(s, a)$ for all s, a.

Get initial state s

Repeat until convergence

  Sample action a, get next state s'

  If s' is terminal:

    target $= R(s, a, s')$

    Sample new initial state s'

  else:

    target $= R(s, a, s') + \gamma \max_{a'} Q(s', a')$

  $Q(s, a) \leftarrow (1 - \alpha) Q(s, a) + \alpha \cdot$ target

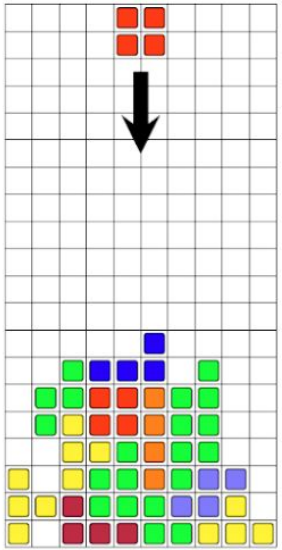
  $s \leftarrow s'$

Basic Q-learning visualization

Link to google drive: https://drive.google.com/file/d/1xFjUAYpsq1GDiI-1WxdPdBFrf0R8wAA2/view?usp=share_link

Approximate Q-learning

Too many states for tabulation



Tetris
10^60

Atari
10^308 (ram) 10^16992 (pixels)

Generalization across states

- Typically we can’t afford keeping a table of all states
- Therefore, the goal is to generalize:
  - Learn on a large enough sample of training states
  - Generalize to unseen states

Approximation by neural networks

Approximate Q-learning

New update rule that updates the current parameters Θ_k to new ones: Θ_{k+1}

$$\text{target} = R(s, a, s') + \gamma \max_{a'} Q_{\Theta_k}(s', a')$$

$$\Theta_{k+1} = \Theta_k - \alpha \nabla_\Theta [\frac{1}{2}(Q_\Theta(s, a) - \text{target}^2]|_{\Theta=\Theta_k}$$

Connection to tabular Q-learning

- Assume we have a parameter for every entry in the table.
- Analyze the gradient of MSE.

$$\theta \in \mathbb{R}^{|S|\times|A|}, \quad Q_\theta(s,a) = \theta_{sa}$$

$$\nabla_{\theta_{sa}} \left[\frac{1}{2}(Q_\theta(s,a) - \text{target})^2\right]$$

$$= \nabla_{\theta_{sa}} \left[\frac{1}{2}(\theta_{sa} - \text{target})^2\right]$$

$$= \theta_{sa} - \text{target}$$

Connection to tabular Q-learning

- Combine with the update rule.

$$\nabla_{\theta_{sa}} \left[\frac{1}{2}(Q_\theta(s,a) - \text{target})^2\right]$$
$$= \theta_{sa} - \text{target}$$

$$\theta_{sa} \leftarrow \theta_{sa} - \alpha(\theta_{sa} - \text{target})$$
$$= (1-\alpha)\cdot\theta_{sa} + \alpha \cdot \text{target}$$

- Compare to tabular Q-learning update.

$$Q(s,a) \leftarrow (1-\alpha)\cdot Q(s,a) + \alpha \cdot \text{target}$$

Approximate Q-learning

- Instead of a table, use a parameterized function $Q_\theta(s, a)$
- Can be a linear function in hand-designed features:

$$Q_\theta(s, a) = \Theta_0 f_0(s, a) + \Theta_1 f_1(s, a) + \cdots + \Theta_n f_n(s, a)$$

- Or a large neural network

DQN

DQN

- High-level idea - **make Q-learning look like supervised learning**.
- Two main ideas for stabilizing Q-learning.
- Apply Q-updates on batches of past experience instead of online:
  - **Experience replay** (Lin, 1993).
  - Previously used for better data efficiency.
  - Makes the data distribution more stationary.
- Use an older set of weights to compute the targets (**target network**):
  - Keeps the target function from changing too quickly.

$$L_i(\theta_i) = \mathbb{E}_{s,a,s',r\sim D}\left(\underbrace{r + \gamma \max_{a'} Q(s', a'; \theta_i^-)}_{\text{target}} - Q(s, a; \theta_i)\right)^2$$

"Human-Level Control Through Deep Reinforcement Learning", Mnih, Kavukcuoglu, Silver et al. (2015)

source

Target Network Intuition

- Changing the value of one action will change the value of other actions and similar states.
- The network can end up chasing its own tail because of bootstrapping.
- Somewhat surprising fact - bigger networks are less prone to this because they alias less.

$$L_i(\theta_i) = \mathbb{E}_{s,a,s',r\sim D}\left(\underbrace{r + \gamma \max_{a'} Q(s', a'; \theta_i^-)}_{\text{target}} - Q(s, a; \theta_i)\right)^2$$

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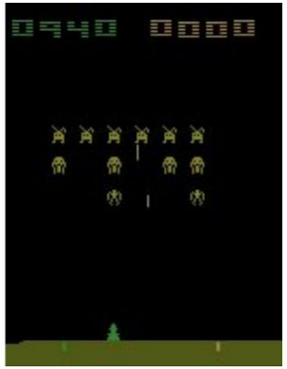


s'

DeepMind

source

DQN Training Algorithm

Algorithm 1: deep Q-learning with experience replay.

Initialize replay memory D to capacity N

Initialize action-value function Q with random weights θ

Initialize target action-value function $\hat{Q}$ with weights $\theta^{-} = \theta$

For episode = 1, M **do**

  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

  For $t = 1$,T **do**

    With probability ε select a random action a_t

    otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$

    Execute action a_t in emulator and observe reward r_t and image x_{t+1}

    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D

    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D

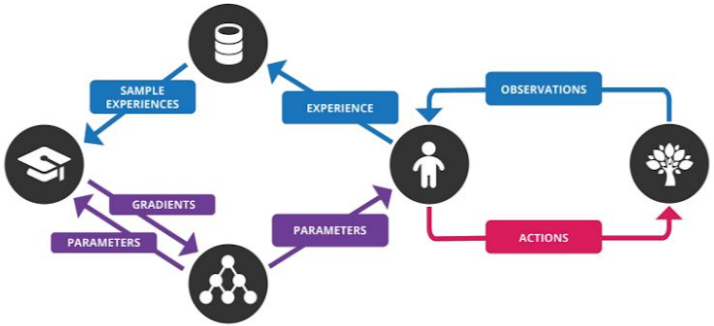
    Set $y_j = \begin{cases} r_j & \text{if episode terminates at step } j+1 \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^{-}) & \text{otherwise} \end{cases}$

    Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ

    Every C steps reset $\hat{Q} = Q$

  End For

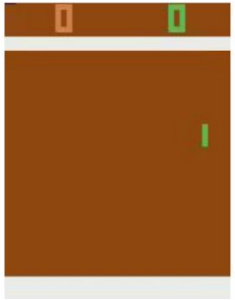
End For



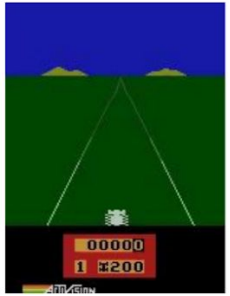
DeepMind

source

DQN on ATARI

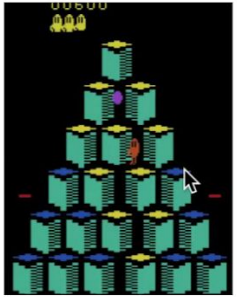


Pong



Enduro

Beamrider



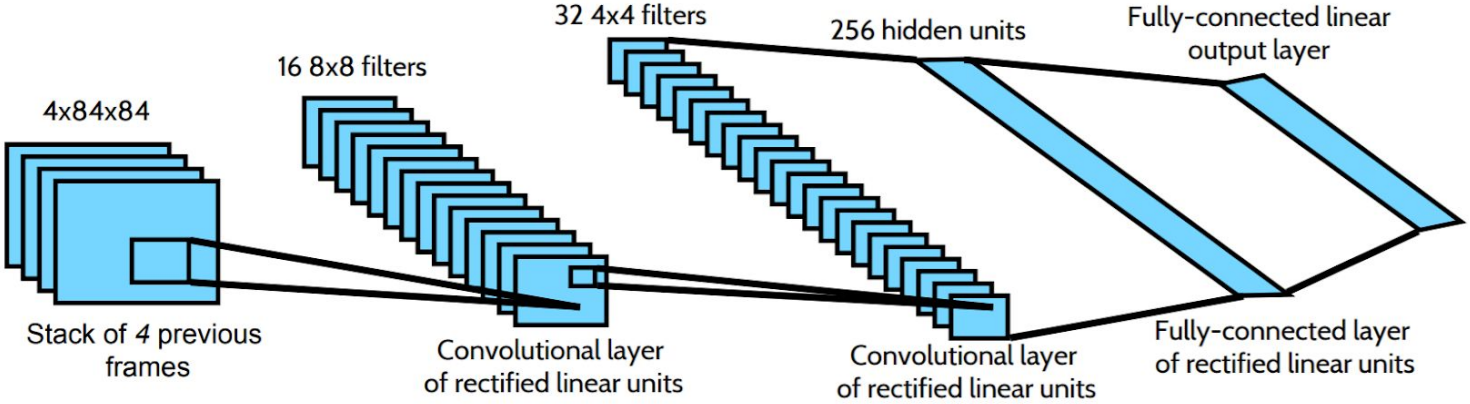
Q*bert

- 49 ATARI 2600 games.
- From pixels to actions.
- The change in score is the reward.
- Same algorithm.
- Same function approximator, w/ 3M free parameters.
- Same hyperparameters.
- Roughly human-level performance on 29 out of 49 games.

source

ATARI Network Architecture

- Convolutional neural network architecture:
  - History of frames as input.
  - One output per action - expected reward for that action $Q(s, a)$.
  - Final results used a slightly bigger network (3 convolutional + 1 fully-connected hidden layers).



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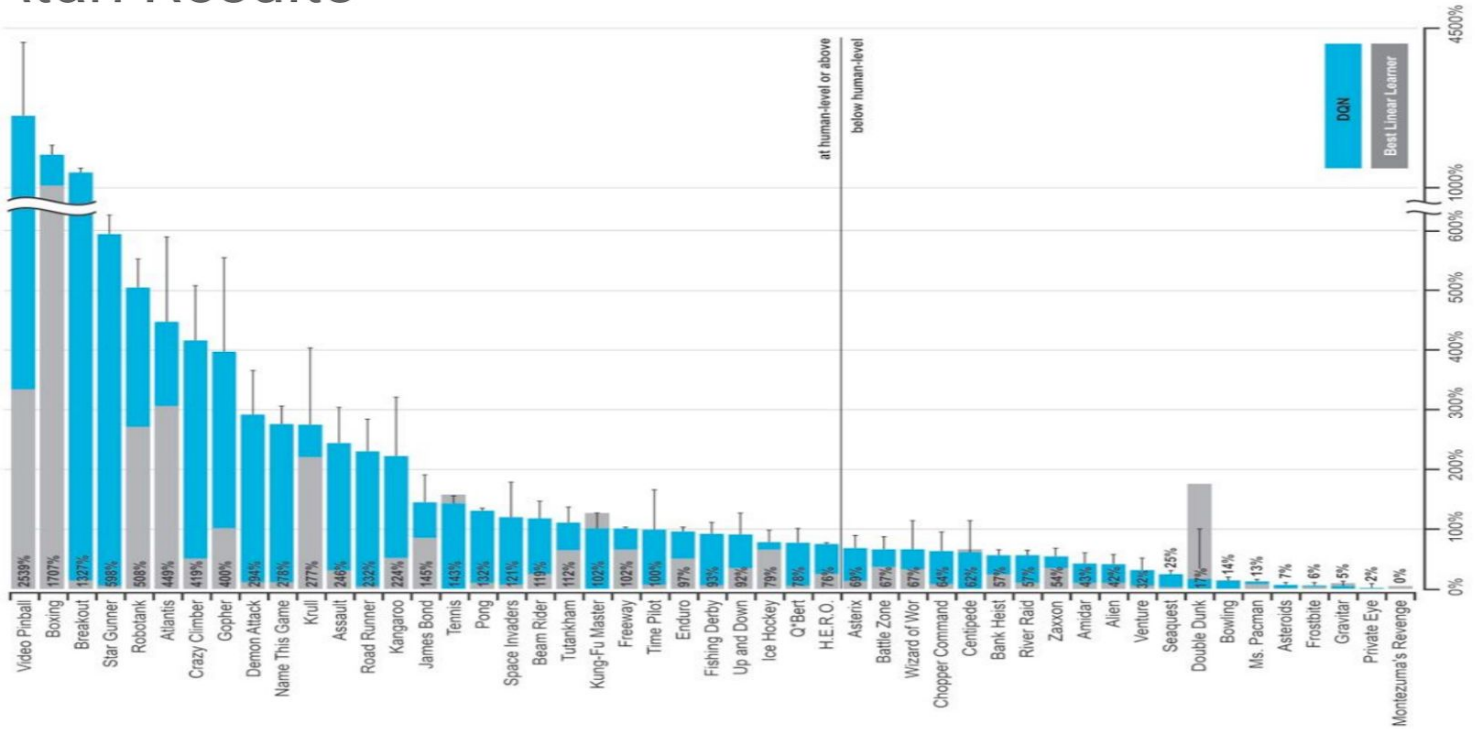
Atari Results



"Human-Level Control Through Deep Reinforcement Learning", Mnih, Kavukcuoglu, Silver et al. (2015)

source

Rest of the course

The next 3 lectures will touch on:

- Q-learning (done)
- Policy gradients
- Alpha zero
- RL from Human Feedback
  - the key step from GPT-3 to ChatGPT

Feedback is a gift

https://tinyurl.com/dnn-2025-12-17