CS 260 Homework 2

Semanti Basu

July 14, 2017

Question 1

- g1(n) is $O(g3(n))$ for even n $\leq$ 0 and g1(n) is $\Omega(g3(n))$ for odd values of n
- g3(n) is $O(g1(n))$ for odd n $\geq$ 1 and g1(n) is $\Omega(g3(n))$ for even values of n
- g2(n) is $O(g3(n))$ for 0 $\leq$ n $\leq$ 100 and g2(n) is $\Omega(g3(n))$ for n>100
- g3(n) is $O(g2(n))$ for n > 100 and g3(n) is $\Omega(g2(n))$ for n<100
- g1(n) is $O(g2(n))$ for even and odd n > 100 and g1(n) is $\Omega(g2(n))$ for all n except even n < 100
- g2(n) is $O(g1(n))$ for all n except odd n>100 and g2(n) is $\Omega(g1(n))$ for all n except n < 100

Question 2

1. 17 is $O(1)$

$$c = 17 \tag{1}$$
$$17 <= 17 * 1 \tag{2}$$
$$c = 17 \tag{3}$$
$$no = 0 \tag{4}$$

c and no exist so true(proved)

2. $\frac{n(n-1}{2}$ is $O(n^2)$

$$c = \frac{1}{2} \tag{5}$$
$$\frac{n^2}{2} - \frac{n}{2} <= n^2/2 \tag{6}$$
$$-\frac{n}{2} <= 0 \tag{7}$$
$$n >= 0 \tag{8}$$
$$c = \frac{1}{2} \tag{9}$$
$$no = 0 \tag{10}$$

c and no exist so true(proved)

3. max(n^3 , $10n^2$) is $O(n^3)$

$$ifmax = n^3 \tag{11}$$
$$c = 1 \tag{12}$$
$$n^3 <= n^3 \tag{13}$$
$$n = 0 \tag{14}$$
$$no = 0, c = 1 \tag{15}$$
$$ifmax = 10n^2 \tag{16}$$
$$Supposec = 10 \tag{17}$$
$$10n^2 <= 10n^3 \tag{18}$$
$$10n^2(1-n) <= 0 \tag{19}$$
$$Thus, no = 0 \tag{20}$$
$$c = 10 \tag{21}$$
$$\tag{22}$$

c and no exist so true(proved)

4.

$$c = 1 \tag{23}$$
$$n^k + 1 = n * n^k = n^k + n^k + ..n^k \tag{24}$$
$$1^k + 2^k + ... + n^k < n^k + ...n^k \tag{25}$$
$$c = 1 \tag{26}$$
$$n0 = 0 \tag{27}$$
$$\sum_{k=1}^{n} i^k = O(n^k + 1) \tag{28}$$
$$\tag{29}$$

$$1^k + 2^k + ...\frac{n}{2}^k .... + n^k >= \frac{n}{2}^k + ... + n^k \tag{30}$$

$$\frac{n}{2}^k + ... + n^k >= \frac{n}{2}^k + ... + \frac{n}{2}^k \tag{31}$$

$$\frac{n}{2}^k + ... + \frac{n}{2}^k = \frac{n}{2}^(k + 1) \tag{32}$$

$$\sum_{k=1}^{n} i^k >= \frac{n^(k+1)}{2^(k+1)} \tag{33}$$

$$c = \frac{1}{2^k + 1} \tag{34}$$

$$n0 = 0 \tag{35}$$

$$\sum_{k=1}^{n} i^k = \Omega(n^k + 1) \tag{36}$$

$$\tag{37}$$

5. a polynomial cannot increase faster or slower than its highest power. So it is both $O(n^k)$ and $\Omega(n^k)$

Question 3

In increasing order:
$\frac{1}{3}^n$,17,log (log n),log n,$(\log n)^2$,$n^{\frac{1}{2}}$,$n^{\frac{1}{2}}(\log n)^2$,$\frac{n}{\log n}$,n,$\frac{3}{2}^n$

Question 4

1.

$$T(n) = T(\frac{n}{2}) + 1 \tag{38}$$

$$\tag{39}$$

2. Solving using Master theorem

$$T(n) = T(\frac{n}{2}) + 1 \tag{40}$$
$$a = 1 \tag{41}$$
$$b = 2 \tag{42}$$
$$f(n) = n^0 \tag{43}$$
$$c = \log_2 1 = 0 \tag{44}$$
$$f(n) = n^c \tag{45}$$
$$\log n^0 = 1 \tag{46}$$
$$T(n) = O(logn) \tag{47}$$
$$T(n) = \Omega(logn) \tag{48}$$
$$\tag{49}$$

Question 5

The loop continues to execute till end element position is reached. However, if x exists as the last element, then it will not be removed. This can be remedied by using for loop to execute through the entire length of list to make sure the last element is checked as well.

Also, the program will not work if the loop is empty. An if statement should be introduced and deletion should be proceeded with only when list is non-empty.

Question 6

$$END = \frac{n(n+1)}{2} + (n) \tag{50}$$
$$FIRST = 1 + \frac{n(n+1)}{2} \tag{51}$$
$$NEXT = \frac{n(n+1)}{2} + \frac{(n-1)n}{2} + n \tag{52}$$
$$NEXT = n(n+1) \tag{53}$$
$$\tag{54}$$