CS 260 Homework 3

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Question 1

Bubble sort

- (1,7,3,2,0,5,0,8)
- (1,3,7,2,0,5,0,8)
- (1,3,2,7,0,5,0,8)
- (1,3,2,0,7,5,0,8)
- (1,3,2,0,5,7,0,8)
- (1,3,2,0,5,0,7,8)
- (1,3,2,0,5,0,7,8) first iteration
- (1,2,3,0,5,0,7,8)
- (1,2,0,3,5,0,7,8)
- (1,2,0,3,5,0,7,8)
- (1,2,0,3,0,5,7,8)
- (1,2,0,3,0,5,7,8)second iteration
- (1,0,2,3,0,5,7,8)
- (1,0,2,0,3,5,7,8)
- (1,0,2,0,3,5,7,8)third iteration
- (0,1,2,0,3,5,7,8)
- (0,1,0,2,3,5,7,8)
- (0,1,0,2,3,5,7,8)fourth iteration
- (0,0,1,2,3,5,7,8)

- (0,0,1,2,3,5,7,8) fifth iteration (sorted)

Selection sort

- (1,7,3,2,0,5,0,8)
- (1,7,3,2,0,5,0,8) first iteration
- (1,3,7,2,0,5,0,8) second iteration
- (1,3,2,7,0,5,0,8)
- (1,2,3,7,0,5,0,8) third
- (1,2,3,0,7,5,0,8)
- (1,2,0,3,7,5,0,8)
- (1,0,2,3,7,5,0,8)
- (0,1,2,3,7,5,0,8)fourth
- (0,1,2,3,5,7,0,8)fifth
- (0,1,2,3,5,0,7,8)
- (0,1,2,3,0,5,7,8)
- (0,1,2,0,3,5,7,8)
- (0,1,0,2,3,5,7,8)
- (0,0,1,2,3,5,7,8)(sorted)

Question 2

Initial array (22, 36, 6, 79, 26, 45, 75, 13, 31, 62, 27, 76, 33, 16, 62, 47) 47 is pivot

- (22, 36, 6, 26, 45, 13, 31, 27, 33, 16, 47, 76, 75, 62, 62, 79)
- (6, 13, 16, 26, 45, 36, 31, 27, 33, 22, 47, 76, 75, 62, 62, 79)
- (6, 13, 16, 26, 45, 36, 31, 27, 33, 22, 47, 76, 75, 62, 62, 79)
- (6, 13, 16, 22, 45, 36, 31, 27, 33, 26, 47, 76, 75, 62, 62, 79)
- (6, 13, 16, 22, 26, 36, 31, 27, 33, 45, 47, 76, 75, 62, 62, 79)
- (6, 13, 16, 22, 26, 36, 31, 27, 33, 45, 47, 76, 75, 62, 62, 79)
- (6, 13, 16, 22, 26, 31, 27, 33, 36, 45, 47, 76, 75, 62, 62, 79)
- (6, 13, 16, 22, 26, 27, 31, 33, 36, 45, 47, 76, 75, 62, 62, 79)

- (6, 13, 16, 22, 26, 27, 31, 33, 36, 45, 47, 76, 75, 62, 62, 79)
- (6, 13, 16, 22, 26, 27, 31, 33, 36, 45, 47, 62, 62, 76, 75, 79)
- (6, 13, 16, 22, 26, 27, 31, 33, 36, 45, 47, 62, 62, 75, 76, 79)

Question3

Part a

$$
\begin{align}
T(n) &= 4T(\frac{n}{3}) + n \tag{1}\\
&= 4[4T(\frac{n}{3}) + \frac{n}{3}] + n \tag{2}\\
&= 16T(\frac{n}{9}) + 4\frac{n}{3} + n \tag{3}\\
&= 64T(\frac{n}{27}) + 16\frac{n}{9} + 4\frac{n}{3} + n \tag{4}\\
&= 4^k T(\frac{n}{3^k}) + 3((\frac{4}{3})^k - 1)n \tag{5}\\
\frac{n}{3^k} &= 1 \tag{6}\\
k &= \log_3 n \tag{7}\\
Pluggingin : T(n) &= 4^{\log_3 n} T(1) + 3n((\frac{4}{3})^{\log_3 n - 1}) \tag{8}\\
&= O(n^{\log_3 4}) \tag{9}\\
& \tag{10}
\end{align}
$$

Part b

$$T(n) = 4T(\frac{n}{3}) + n^2 \tag{11}$$
$$= 4[4T(\frac{n}{3}) + \frac{n^2}{9}] + n^2 \tag{12}$$
$$= 16T(\frac{n}{9}) + 4\frac{n^2}{9} + n^2 \tag{13}$$
$$= 64T(\frac{n}{27}) + 16\frac{n^2}{81} + 4\frac{n^2}{9} + n^2 \tag{14}$$
$$= 4^k T(\frac{n}{3^k}) + \frac{9}{5}((1 - \frac{4}{9}^k))n^2 \tag{15}$$
$$\frac{n}{3^k} = 1 \tag{16}$$
$$k = \log_3 n \tag{17}$$
$$Pluggingin : T(n) = 4^{\log_3 n} T(1) + \frac{9}{5}((1 - \frac{4}{9}^{\log_3 n}))n^2 \tag{18}$$
$$= O(n^2) \tag{19}$$
$$\tag{20}$$

Part c

$$T(n) = 9T(\frac{n}{3}) + n^2 \tag{21}$$
$$= 9[9T(\frac{n}{9}) + \frac{n^2}{9}] + n^2 \tag{22}$$
$$= 81T(\frac{n}{9}) + 2n^2 \tag{23}$$
$$= 81[9T(\frac{n}{27}) + \frac{n^2}{81}] + 2n^2 \tag{24}$$
$$= 9^k T(\frac{n}{3^k}) + kn^2 \tag{25}$$
$$\frac{n}{3^k} = 1 \tag{26}$$
$$k = \log_3 n \tag{27}$$
$$Pluggingin : T(n) = 9^{\log_3 n} T(1) + \log_3 n n^2 \tag{28}$$
$$= n^2 + n^2 \log_3 n \tag{29}$$
$$= O(n^2 \log_3 n) \tag{30}$$
$$\tag{31}$$

Question4

Part a

$T(n) = T(n/2) + 1$ Using Master's theorem:

$$\begin{align} a &= 1 \tag{32} \\ b &= 2 \tag{33} \\ c &= \log_2 1 \tag{34} \\ &= 0 \tag{35} \\ f(n) &= n^0 \tag{36} \\ &= n^c \tag{37} \\ f(n) &= O(n^c \log_2 n^k) \tag{38} \\ k &= 0 \tag{39} \\ T(n) &= O(n^c \log_2 n^{k+1}) \tag{40} \\ &= O(\log_2 n) \tag{41} \\ T(n) &= \Omega(\log_2 n) \tag{42} \\ & \tag{43} \end{align}$$

Part b

$T(n) = 2T(n/2) + \log n$ Using Master's theorem:

$$\begin{align} a &= 2 \tag{44} \\ b &= 2 \tag{45} \\ c &= \log_2 2 \tag{46} \\ &= 1 \tag{47} \\ f(n) &= \log n \tag{48} \\ n^c = nn^c > f(n)&\text{n grows faster than log n} \tag{49} \\ &\text{By case 1 of Master theorem} \tag{50} \\ f(n) &= O(n^{c-\epsilon}) \tag{51} \\ T(n) &= O(n) \tag{52} \\ T(n) &= \Omega(n) \tag{53} \\ & \tag{54} \end{align}$$

Part c

$T(n) = 2T(n/2) + n$ Using Master's theorem:

$$\begin{aligned}
a &= 2 && (55)\\
b &= 2 && (56)\\
c &= \log_2 2 && (57)\\
&= 1 && (58)\\
f(n) &= n && (59)\\
&= n^c && (60)\\
&\text{By case 2 of Master's theorem} && (61)\\
f(n) &= O(n^c \log_2 n^k) && (62)\\
k &= 0 && (63)\\
T(n) &= O(n^c \log_2 n^{k+1}) && (64)\\
&= O(n \log_2 n) && (65)\\
T(n) &= \Omega(n \log_2 n) && (66)\\
& && (67)
\end{aligned}$$

Part d

$T(n) = 2T(n/2) + n^2$ Using Master's theorem:

$$\begin{aligned}
a &= 2 && (68)\\
b &= 2 && (69)\\
c &= \log_2 2 && (70)\\
&= 1 && (71)\\
f(n) &= n^2 && (72)\\
&\text{By case 3 of master theorem} && (73)\\
n^c < f(n)f(n) = \Omega(n^{c+\epsilon})T(n) &= O(f(n)) && (74)\\
&= O(n^2) && (75)\\
T(n) &= \Omega(f(n)) && (76)\\
&= \Omega(n^2) && (77)\\
& && (78)
\end{aligned}$$