Basic Set Theory

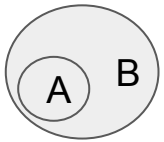
- Naively, sets are just well-determined collections[1]
  - Examples
    - Presidents of the United States
    - Letters of the alphabet
    - Failure modes of gas turbines
  - We will refer to such a collection as a "***class"***.
- The basic set relation is ***membership***.
  - Symbol: ∈
  - AKA "***is a***"
- Create a model containing the following declarations
  - PresidentOfUSA is a class.
  - GeorgeWashington is a member of this class.

Solution

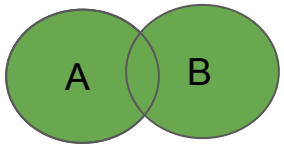
```
uri "http://sadl.org/BasicSetTheory1.sadl"alias bst1.
PresidentOfUSA is a class .
GeorgeWashington is a PresidentOfUSA.
```

Set Operations

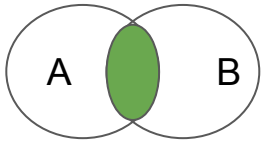
- Two sets A and B are "***the same***" if they have exactly the same members
- Subset: A is a subset of B if every member of A is also a member of B
  - Symbol: ⊆
  - Also referred to as sub-class or "***is a type of***"
  - Proper subset: A is a proper subset of B if it is a subset but is not the same



- Union: the union of sets A and B is the set containing all members of A and all members of B
  - Symbol: ∪
  - Union is referred to by "***or***", meaning "x ∈ (A ∪ B) if x ∈ A **or** x ∈ B"

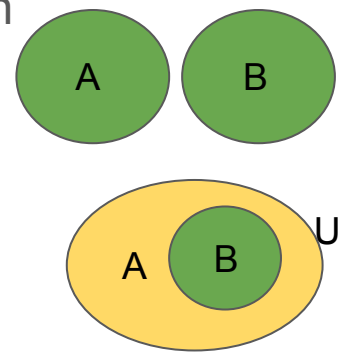


- Intersection: the intersection of sets A and B is the set containing the elements in both A and B
  - Symbol: ∩
  - Intersection is referred to by "***and***", meaning x ∈ (A ∩ B) if x ∈ A **and** x ∈ B



More about Sets

- Disjoint Sets: sets which cannot have members in common
  - "***A and B are disjoint.***"
  - "***{A, B} are disjoint.***"
- Compliment: within the universe U, A is everything not in B
  - "***{A, B} are types of U.***"
    "***A is not B.***"



- Extend/modify the previous model to contain the following:
  - MilitaryCommander is a class.
  - GeorgeWashington belongs to the intersection of PresidentsOfUSA and MilitaryCommander.
  - BillClinton and HarryTruman belong to the PresidentsOfUSA class.
  - Musician is the same as the union of the classes Singer and Instrumentalist.
  - WindInstrumentalist is a subclass of Instrumentalist.
  - Singers and WindInstrumentalist are disjoint.
  - Within the universe of Food, Vegetable is the complement of Meat.

Solution

```
uri "http://sadl.org/BasicSetTheory2.sadl"alias bst2.

PresidentOfUSA is a class .
MilitaryCommander is a class.

GeorgeWashington is a {PresidentOfUSA and MilitaryCommander}.
{BillClinton, HarryTruman } are instances of PresidentOfUSA.

Singer is a class.
Instrumentalist is a class.
Musician is the same as {Singer or Instrumentalist}.

WindInstrumentalist is a type of Instrumentalist.
Singer and WindInstrumentalist are disjoint.

Food is a class.
{Meat, Vegetable} are types of Food.
Vegetable is the same as not Meat.
```

Additional Information

- Two Sets of Interest
  - The set of all things (owl:Thing): every set is a subset of this set
  - The empty set, which has no elements: is a subset of every set
- Upper-level Ontologies
  - Lots of choices, see https://en.wikipedia.org/wiki/Upper_ontology.