Predictive log likelihood for Gaussian Process Classification with noisy step function likelihood

Daniel Korzekwa

01 February 2016

Predictive log likelihood

$$logp(y) = \sum_i logp(y_i) = \sum log\Phi(y_i \frac{\tilde{\mu}_i}{\sqrt{\tilde{\sigma}_i^2}}) \tag{1}$$

$$\tilde{\sigma}_i^2 = \sigma_i^2 + 1 \tag{2}$$

$$\tilde{\mu}_i = \mu_i \tag{3}$$

$$\partial logp(y) = \sum_i \frac{1}{\Phi(y_i \frac{\tilde{\mu}_i}{\sqrt{\tilde{\sigma}_i^2}})} \phi(y_i \frac{\tilde{\mu}_i}{\sqrt{\tilde{\sigma}_i^2}}) y_i (\frac{\partial \tilde{\mu}_i}{\sqrt{\tilde{\sigma}_i^2}} - \frac{\tilde{\mu}_i \partial \tilde{\sigma}_i^2}{2\tilde{\sigma}_i^3}) \tag{4}$$

Cavity parameters

$$\sigma_i^2 = k_i^{-1} \tag{5}$$

$$\mu_i = \sigma_i^2 h_i \tag{6}$$

$$\partial \sigma_i^2 = -k_i^{-1} \partial k_i k_i^{-1} \tag{7}$$

$$\partial \mu_i = \partial \sigma_i^2 h_i + \sigma_i^2 \partial h_i \tag{8}$$

Canonical cavity parameters

$$k_i = [\hat{\sigma}^2]_{ii}^{-1} - k_{\delta i} \tag{9}$$

$$h_i = [\hat{\sigma}^2]_{ii}^{-1} [\hat{\mu}]_i - h_{\delta i} \tag{10}$$

$$\partial k_i = -[\hat{\sigma}^2]_{ii}^{-1} [\partial \hat{\sigma}^2]_{ii} [\hat{\sigma}^2]_{ii}^{-1} \tag{11}$$

$$\partial h_i = -[\hat{\sigma}^2]_{ii}^{-1} [\partial \hat{\sigma}^2]_{ii} [\hat{\sigma}^2]_{ii}^{-1} [\hat{\mu}]_i + [\hat{\sigma}^2]_{ii}^{-1} [\partial \hat{\mu}]_i \tag{12}$$

GP posterior parameters

$$\hat{\sigma}^2 = \hat{k}^{-1} \tag{13}$$

$$\hat{\mu} = \hat{\sigma}^2 \hat{h} \tag{14}$$

$$\partial \hat{\sigma}^2 = -\hat{k}^{-1} \partial \hat{k} \hat{k}^{-1} \tag{15}$$

$$\partial \hat{\mu} = \partial \hat{\sigma}^2 \hat{h} + \hat{\sigma}^2 \partial \hat{h} \tag{16}$$

Canonical GP posterior parameters

$$\hat{k} = k_p + \sum_i k_{\delta i} \tag{17}$$

$$\hat{h} = h_p + \sum_i h_{\delta i} \tag{18}$$

$$\partial \hat{k} = \partial k_p \tag{19}$$

$$\partial \hat{h} = \partial h_p \tag{20}$$

Canonical GP prior parameters

$$k_p = [\sigma_p^2]^{-1} \tag{21}$$

$$h_p = k_p \mu_p \tag{22}$$

$$\partial k_p = -[\sigma_p^2]^{-1} \partial \sigma_p^2 [\sigma_p^2]^{-1} \tag{23}$$

$$\partial h_p = \partial k_p \mu_p + k_p \partial \mu_p \tag{24}$$