DTQ with a weak trapezoidal scheme

The family of stochastic differential equations considered by Anderson-Mattingly follow:

$$dX(t) = b(X(t))dt + \sum_{k=1}^{M} \sigma_k(X(t))\nu_k dW_k(t)$$

where $b : \mathbb{R}^d \to \mathbb{R}^d$, $\sigma_k : \mathbb{R}^d \to \mathbb{R}^+$, $\nu_k \in \mathbb{R}^d$. For our 1D case, we can simplify this SDE

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t$$

The Weak θ-midpoint trapezoidal algorithm is given as

$$\begin{aligned}
y^* &= Y_{i-1} + b(Y_{i-1})\theta h + \sigma(Y_{i-1})Z_i^1\sqrt{\theta h} \\
Y_i &= y^* + (\alpha_1 b(y^*) - \alpha_2 b(Y_{i-1}))(1-\theta)h + \sqrt{[\alpha_1\sigma^2(y^*) - \alpha_2\sigma^2(Y_{i-1})]^+}Z_2^i\sqrt{(1-\theta)h}
\end{aligned}$$

where $[x]^+$ is defined to be $\max\{x, 0\}$ and $\theta \in (0,1)$ is fixed and used to define the parameters α_1, α_2

$$\alpha_1 = \frac{1}{2}\frac{1}{\theta(1-\theta)}, \quad \alpha_2 = \frac{1}{2}\frac{(1-\theta)^2 + \theta^2}{\theta(1-\theta)}$$

On the basis of probability we can write:

$$\begin{aligned}
p(Y_i) &= \int_{y_{i-1}} p(Y_i, Y_{i-1})dy_{i-1} \\
&= \int_{y_{i-1}} p(Y_i|Y_{i-1})p(Y_{i-1})dy_{i-1}
\end{aligned}$$

If we discretize using the Euler-Maruyama method, the transition density $p(Y_i|Y_{i-1})$ is a Gaussian with mean $Y_{i-1} + b(Y_{i-1})h$ and variance $\sigma^2(Y_{i-1})h$. The introduction of the 2 step process changes that computation, so the transition density is computed as,

$$\begin{aligned}
p(Y_i|Y_{i-1}) &= \int_{y^*} p(y_i, y^*|y_{i-1})dy^* \\
&= \int_{y^*} \underbrace{p(y_i|y^*, y_{i-1})}_{\text{matrix A}}\underbrace{p(y^*|y_{i-1})}_{\text{matrix B}}dy^*
\end{aligned}$$

The matrix B is formed with variable y^* and y_{i-1}. The second part is a Gaussian with mean $\mu = Y_{i-1} + b(Y_{i-1})\theta h$ and variance $\sigma^2 = \sigma^2(Y_{i-1})\theta h$. The matrix A is formed by varying