TCS1 - 1st exam

UP FAMNIT

2.2.2022

Name: ____________________________________

Enrollment ID.: ________________________@student.upr.si

Exercise	Obtained points	Possible points
1		12
2		15
3		12
4		11
5		15
6		15
Total		80

Instructions:

1. You have **90 minutes** to complete the examination. As a courtesy to your classmates, we ask that you not leave during the last fifteen minutes.
2. When you finish please staple your pieces of papers that you would like to submit. The stapler will be provided. All pieces of the submitted paper must contain your name and student ID.
3. Please do not shuffle your solutions. The solution of each exercise should be uninterrupted on the same page, or at least on adjacent pages. Justify all your answers.
4. You may use one (1) double-sided A4 pages with notes that you have prepared. You may not use any other resources, including lecture notes, books, or other students.
5. Please sign the Honor Code statement below.

In recognition of and in the spirit of the University of Primorska Honor Code, I certify that I will neither give nor receive unpermitted aid on this examination.

Signature: ____________________________________

1. Let A, B and C be atomic propositions. We are given a proposition

$$\mathcal{I} = (\neg A \Leftrightarrow B) \Rightarrow \neg(C \wedge \neg A).$$

(a) Write the truth table of $\mathcal{I}$.

(b) Write the negation of $\mathcal{I}$ in disjunctive normal form.

(c) What can you say about the truth of A and C, conditioned on $B \sim 1$ and $\mathcal{I} \sim 0$?

(d) Draw the switching circuit equivalent to $\mathcal{I}$.

2. Are the following implications correct? If yes, prove it. Otherwise, find counterexample.

(a) $(A \Rightarrow C) \Rightarrow (A \vee C \Rightarrow B \vee C)$

(b) $(A \Rightarrow B \vee C) \Rightarrow (A \Rightarrow C)$

(c) $\neg A \wedge A \Rightarrow B$

3. For universe $S = \mathbb{R}^2$ define relation $\simeq$ as:

$$(a, b) \simeq (c, d) \iff 2a - b = 2c - d.$$

Prove that $\simeq$ is an equivalence relation on $\mathbb{R}^2$, or find a counterexample.

4. Let A, B and C be sets, and let $g \colon A \to B$ and $f \colon B \to C$ be functions. Prove:

(i) If f, g are surjective, then so is $f \circ g$.

(ii) If f, g are surjective and at the same time $f \circ g$ is not, then $f \circ g$ is injective.

5. Let $G = (V, E)$ be a graph with vertex-set $V = \{1, 2, 3, 4\}$ and edge-set

$$E = \{(1, 2), (3, 2), (4, 3), (1, 4), (2, 4)\}.$$

(a) Draw the graph.

Find

(b) maximal degree, i.e. $\Delta(G)$,

(c) minimal degree, i.e. $\delta(G)$,

(d) the size of biggest clique, i.e. $\omega(G)$,

(e) the size of biggest independent set, i.e. $\alpha(G)$, ter

(f) the minimal number of colours needed to color the graph, i.e. $\chi(G)$.

6. On $S = \{2, 4, 6, 10, 12, 20, 30, 60\}$ we work with the divisibility relation R:

$$xRy \Leftrightarrow x \text{ divides } y.$$

(i) Draw the Hasse diagram of R.

(ii) Find all R-minimal elements.

(iii) Find all R-maximal elements.

(iv) Find all non-empty subsets $U \subseteq S$, so that 6 is an R-lowerbound of U.

(v) Let $A = \{6, 10, 12\}$. If A admits an R-supremum, find it. If A admits an R-infimum, find it.

(vi) Is R a lattice?