TOR I – Discrete Structures I

– Problem Set 2 –

1. Find the canonical disjunctive normal form (DNF) and the canonical conjuctive normal form (CNF) for the following propositions:

   (i) $\neg(A \wedge B) \Rightarrow (\neg B \Rightarrow A)$

   (ii) $\neg(A \vee B) \wedge (A \Rightarrow B)$

2. For the following compound proposition find a truth table, determine DNF, CNF and draw the corresponding circuit.

$$(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)).$$

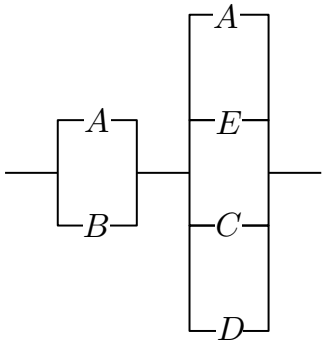
3. Find a compound proposition $\mathcal{I}$ such that

$$(A \Rightarrow (\mathcal{I} \Rightarrow \neg B)) \Rightarrow (A \wedge B) \vee \mathcal{I}$$

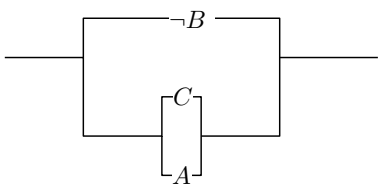
   is tautology.

4. For the following circuits find the corresponding compound propositions

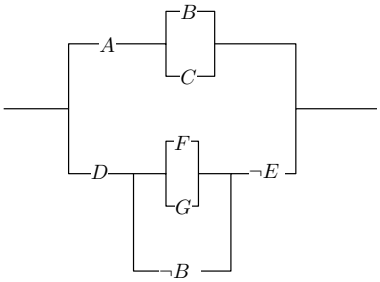
   (i)



   (ii)



(iii)



5. Simplify the following logical equivalence

$$(A \Rightarrow B) \vee (B \Rightarrow C).$$

6. Show that the following propositions are logical implications (a tautology where the main connective is implication).

   (i) $A \wedge (A \Rightarrow B) \Rightarrow B$

   (ii) $\neg B \wedge (A \Rightarrow B) \Rightarrow \neg A$

   (iii) $\neg A \wedge (A \vee B) \Rightarrow B$

   (iv) $(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$

   (v) $A \wedge (A \Leftrightarrow B) \Rightarrow B$

7. Are the following propositions logical implications?

   (i) $(A \Rightarrow B) \wedge (A \Rightarrow C) \wedge A \Rightarrow B \wedge C$

   (ii) $\neg(A \vee B) \wedge (A \vee C) \wedge (D \Rightarrow C) \Rightarrow D$

   (iii) $(A \Rightarrow B) \wedge (A \Rightarrow C) \wedge (D \wedge E \Rightarrow F) \wedge (C \Rightarrow E) \Rightarrow F$