TOR I - Discrete structures

– Problem Set 6 –

1. Show that $(A \cup C) \cap (B \setminus C) = (A \cap B) \setminus C$.
2. (last union property) Show that $A \subseteq C \wedge B \subseteq C \Rightarrow A \cup B \subseteq C$.
3. (one-but-last intersection property) Show that $A \subseteq B \Leftrightarrow A \cap B = A$.
4. (one-but-last set-difference property) Show that $(A \cap B) \setminus B = \emptyset$.
5. Determine the following sets:
   (i) $\{\emptyset, \{\emptyset\}\} \setminus \emptyset$
   (ii) $\{\emptyset, \{\emptyset\}\} \setminus \{\emptyset\}$
   (iii) $\{\emptyset, \{\emptyset\}\} \setminus \{\{\emptyset\}\}$
   (iv) $\{1, 2, 3, \{1\}, \{5\}\} \setminus \{2, \{3\}, 5\}$
6. Which of the following propositions are true for arbitrary sets A, B and C:
   (a) $A \in B \wedge B \in C \implies A \in C$.
   (b) $A \subseteq B \wedge B \in C \implies A \in C$.
   (c) $A \cap B \subseteq \overline{C} \wedge A \cup C \subseteq B \implies A \cap C = \emptyset$.
   (d) $A \neq B \wedge B \neq C \implies A \neq C$.
   (e) $A \subseteq \overline{(B \cup C)} \wedge B \subseteq \overline{(A \cup C)} \implies B = \emptyset$.
7. Let A, B and C be arbitrary sets. Show the following propositions:
   (a) $A \subseteq B \Leftrightarrow A \cap \overline{B} = \emptyset$.
   (b) $A \setminus B = \overline{B} \setminus \overline{A}$.