TOR I – Discrete structures

– Problem Set 8 –

1. Let $S = \{1, 2, 3, 4, 5\}$. Is $R = \{(1, 2), (2, 3), (3, 5), (2, 4), (5, 1)\}$ a binary relation? Find the domain $\mathcal{D}R$ and the range $\mathcal{Z}R$ of R. Determine the inverse relation R^{-1} and $\mathcal{D}R^{-1}$ and $\mathcal{Z}R^{-1}$.

2. Let $R = \{(1, 1), (2, 1), (3, 3), (1, 5)\}$ and $T = \{(1, 4), (2, 1), (2, 2), (2, 5)\}$ be binary relations. Determine the compositions $R \circ T$ and $T \circ R$. Is it true that $R \circ T = T \circ R$?

3. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Define
$$R = \{(x, y) \mid x - y \text{ is divisible by } 3\} \quad \text{in} \quad T = \{(x, y) \mid x - y \geq 3\}.$$
Determine $R, T, R \circ R$.

4. Let $S = \mathbb{R}$. On S we define the relation R as follows
$$(\forall x)(\forall y)(xRy \Leftrightarrow y \geq x + 3).$$
Is R reflexive, symmetric, transitive or strict total?

5. Let $S = \{1, 2, 3, 4\}$. We have the following relations
   (i) $R_1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$,
   (ii) $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$,
   (iii) $R_3 = \{(1, 3), (2, 1)\}$,
   (iv) $R_4 = \emptyset$,
   (v) $R_5 = S \times S$.

   Which of the following properties hold for each relation: reflexive, symmetric, anti-symmetric, transitive?

6. Let R and S be symmetric relations. Show: $R \circ S$ symmetric $\Leftrightarrow R \circ S = S \circ R$.

7. Let $S = \{m \in \mathbb{N} \mid 1 \leq n \leq 10\}$ in $R = \{(m, n) \in S \times S \mid 3|m-n\}$. Is R an equivalence relation? If yes, determine the corresponding equivalence classes and the factor set.

8. Let $S = \mathbb{Z} \times \mathbb{Z}$ and define the relation R as follows
$$(a, b)R(c, d) \Leftrightarrow ad = bc.$$
Show that R is an equivalence relation and find the corresponding equivalence classes.

9. Let $S = \mathbb{R}^2$ and define the relation R as follows
$$(x_1, y_1)R(x_2, y_2) \Leftrightarrow x_1^2 + y_1^2 = x_2^2 + y_2^2.$$
Show that R is an equivalence relation and find the equivalenec class $R[(7, 1)]$.