TOR I – Discrete structures

– Problem Set 7 –

1. Find $\mathcal{P}(A)$, where $A = \{a, b, c, d\}$.

2. Let $A = \{\{1, 2, 3\}, \{4, 5\}, \{6, 7, 8\}\}$.

   (i) Write down the elements of A.

   (ii) Is it true?
   (a) $1 \in A$ (b) $\{1, 2, 3\} \subseteq A$ (c) $\{6, 7, 8\} \in A$ (d) $\{\{4, 5\}\} \subseteq A$
   (e) $\emptyset \in A$ (f) $\emptyset \subseteq A$

3. Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

4. Let A, B in C be arbitrary subsets of the universal set $U = A \cup B \cup C$. Show the following propositions:

   (a) $A \setminus B \subseteq \overline{B}$.

   (b) $(A \setminus B) \cap B = \emptyset$.

   (c) $A \cap B \subseteq C \Leftrightarrow A \subseteq \overline{B} \cup C$.

   (d) $(A \setminus B) \cup B = A \Leftrightarrow B \subseteq A$.

   (e) If $B \subseteq A$, then$B \times B = (B \times A) \cap (A \times B)$.

   (f) Let A be a nonempty set. Which of the following sets

   $$\emptyset, \{\emptyset\}, A, \{A\}, \{A, \emptyset\}$$

   are elements and which are subsets of (i) $\mathcal{P}(A)$ and (ii) $\mathcal{P}(\mathcal{P}(A))$?

   (g) Is it true that $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$?